Extreme value theory and wildfire modeling

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1. Fitting a generalized Pareto distribution to burned area data
2. Causality

August complex wildfire

- List of California wildfires from 2003 to 2024 [1]

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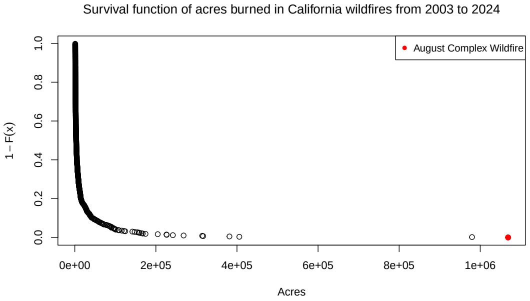
- The August Complex (August-November 2020, Northern California) burned more than 1 million acres



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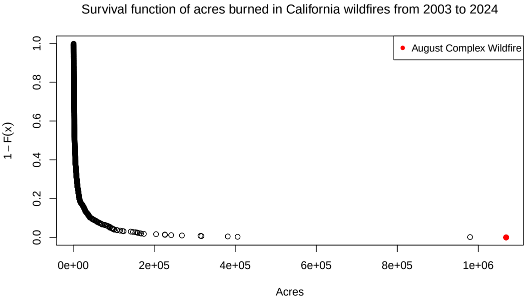

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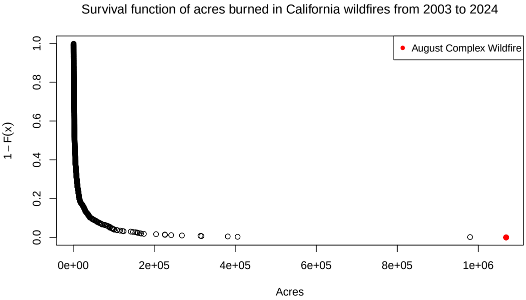

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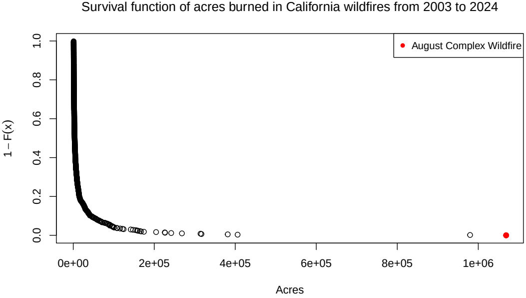

- Question : How unusual is the August Complex wildfire?
- One can say that such an event can happen once every 20 years...
- however this estimation is based on small sample of observations

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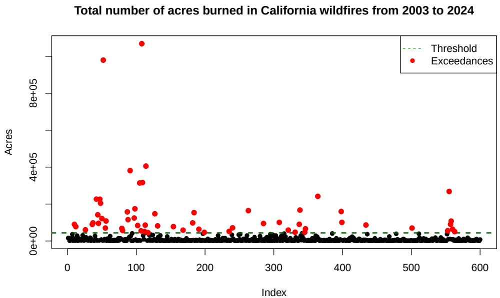
Peaks over thresholds

- Let $X_1, \ldots, X_n$ denote the number of acres burned on California wildfires from 2003 to 2024
- We use the following model

$$\lim_{u \to \infty} \mathsf{P}\left(X - u \leqslant x \mid X > u\right) = G(x) \tag{1}$$

→ Condition $X > u$: "peaks over thresholds"

→ $Y = X - u$ is the exceedance loss above the high threshold u



Generalized Pareto distribution

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Theorem (Gnedenko (1943); Fisher and Tippett (1928))

The limit distribution G in (1) is necessarily a generalized Pareto distribution (gpd):

$$G_{\gamma,\,\alpha}(x) = \begin{cases} 1 - (1 + \gamma x/\alpha)^{-1/\gamma}, & \gamma \neq 0, \\ 1 - e^{-x/\alpha}, & \gamma \neq 0, \end{cases}$$

where $\alpha > 0$, and $1 + \gamma x/\alpha > 0$ when $\gamma \neq 0$.

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- We obtain the log-likelihood function

$$l((\gamma, \alpha); Y_1, \ldots, Y_{N_u}) = \begin{cases} -N_u \ln(\alpha) - (1 + 1/\gamma) \sum_{i=1}^{N_u} \ln\left(1 + \gamma Y_i/\alpha\right), & \gamma \neq 0, \\ \alpha^{-1} \left(N_u(1-u) - \sum_{i=1}^{N_u} Y_i \right), & \gamma \neq 0, \end{cases} \tag{2}$$

which we maximize subject to $\alpha > 0$, and $1 + \gamma Y_i/\alpha$ for all i, when $\gamma \neq 0$

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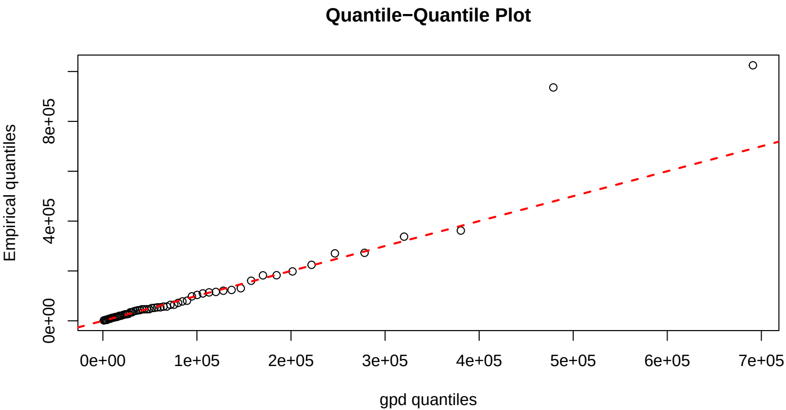
- We consider the QQ-plot

$$\left(Q_{\hat{\gamma},\hat{\alpha}}(\frac{i}{N_u+1}), Y_{i,N_u}\right), \qquad i = \ldots, N_u,$$

  where Y_{i,N_u} denote the i-th ranked observation
- See the plot on the next slide

QQ-plot of the fitting: Part 2



How unusual is the August Complex wildfire?

- Again, we assume that the exceedance losses $Y_1, \ldots, Y_{N_u} \sim G_{\hat{\gamma}, \hat{\alpha}}$
- Let $x > u$. In our example, x is the number of acres burned in the August Complex wildfire and recall u, the 0.9 quantile

$$\begin{aligned}
\mathsf{P}(X > x) &= \mathsf{P}(X > u)\,\mathsf{P}(X > x \mid X > u) \\
&= 0.9 \times \mathsf{P}(X - u > x - u \mid X > u) \\
&= 0.9 \times \mathsf{P}(Y > x - u \mid X > u) \\
&= 0.9 \times \left(1 + \hat{\gamma}\frac{x - u}{\hat{\alpha}}\right)^{-1/\hat{\gamma}} \approx 0.006648479
\end{aligned}$$

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1 Fitting a generalized Pareto distribution to burned area data

2 Causality

Possible causes

We aim to explore the causal relationships between the *number of wildfires*, *acres burned*, and factors such as heatwaves, windspeed, drought, humidity, hurrican force ...

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- Recall that we are studying the August Complex wildfire (August-November 2020)
- SHASTA DAM station, located in California, USA, is in the area where this wildfire occurred
- We use data provided by the National Oceanic and Atmospheric Administration (NOAA) [2]

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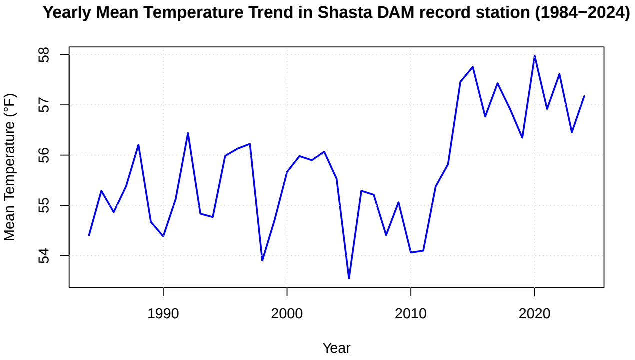
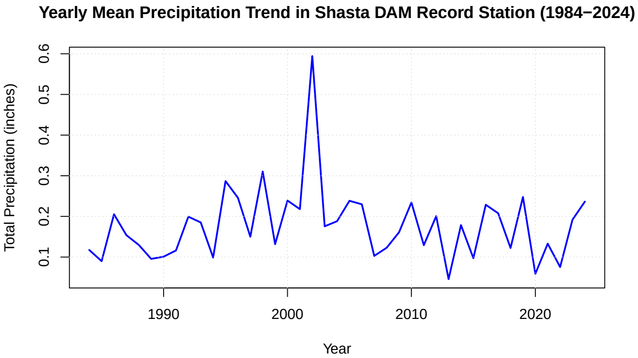
Possible causes: heatwaves



The year 2020 recorded the highest mean temperature within the period from 1984 to 2024.

Possible causes: droughts



The year 2020 experienced the second-lowest precipitation within the period from 1984 to 2024.

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