Chapter:5

Measures of Central Tendency

INTRODUCTION

A single expression representing the whole group and conveying a fairly adequate idea about the whole group is known as the *average*. Averages are generally the central part of the distribution and, therefore, they are also called the *measures of central tendency*. There are five types of central tendencies or averages which are commonly used. These are:

1. Arithmetic mean or average or mean (AM)
2. Median
3. Mode
4. Geometric mean (GM)
5. Harmonic mean (HM)

CHARACTERISTICS OF A GOOD AVERAGE

According to Professor G.U. Yule, a good average must have the following characteristics:

1. It should be rigidly defined so that different persons may not interpret it differently.
2. It should be easy to understand and easy to calculate.
3. It should be based on all the observations of the data.
4. It should be easily subject to further mathematical calculations.
5. It should be least affected by the fluctuations of the sampling.
6. It should not be unduly affected by the extreme values.
7. It should be easy to interpret.
8. It should have sampling stability. It means that if the average is computed for similar groups, the result should also be similar.

ARITHMETIC MEAN

Central value or average, obtained arithmetically, is known as Arithmetic Mean. It is the most common average used in our day-to-day life. Depending upon whether all the items in the data are to be considered of equal or unequal importance we get three sub-types of arithmetic mean. Accordingly arithmetic mean is of the following types.

Simple arithmetic mean: It is most commonly used of all the averages. It is the value which we get by dividing the sum of the items of the same series by the total number of observations.

Individual series: If the values of n items are $x_1, x_2, x_3, \ldots, x_n$ be the values of variable x, then simple arithmetic mean, denoted by $\bar{x}$, is obtained by dividing the sum of the values of all the items by the total number of observations. Symbolically

$$\bar{x} = \frac{x_1 + x_2 + - + - + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\text{Sum of observations}}{\text{Number of values}}$$

$\sum_{i=1}^{n} x_i$ = Sum of individual observations of the variables.

n = Number of observations.

Direct Method

EXAMPLE 5.1: Find the arithmetic mean of the marks obtained by 10 students of 10th class in Mathematics. The marks obtained are 25, 30, 21, 55, 47, 10, 15, 17, 45, 35.

Solution: Let the marks be denoted by x.
Given that values of x are: 25, 30, 21, 55, 47, 10, 15, 17, 45, 55.

$$\sum_{i=1}^{n} x_i = 25 + 30 + 21 + 55 + 47 + 10 + 15 + 17 + 45 + 35 = 300$$

$$n = 10$$

$$\text{Arithmetic mean } (\overline{x}) = \frac{300}{10} = 30$$

EXAMPLE 5.2: The following table gives the monthly income of 10 families in a city.
Income: Rs. 300, 700, 400, 1000, 600, 400, 300, 800, 500, 900.
Calculate the arithmetic mean.

Solution: Let the income be denoted by x.
Given that values of x are: Rs. 300, 700, 400, 1000, 600, 400, 300, 800, 500, 900.

$$\sum_{i=1}^{n} x_i = 300 + 700 + 400 + 1000 + 600 + 400 + 300 + 800 + 500 + 900 = 5900$$

$$n = 10$$

$$\text{Arithmetic mean } (\overline{x}) = \frac{5900}{10} = 590$$

Thus, the average monthly income is Rs. 590.

Short-cut Method

The arithmetic mean can also be calculated by a short-cut method to minimise the calculation.

Formula

$$\bar{x} = A + \frac{\sum d}{n},$$

where

$\bar{x}$ = Arithmetic mean

A = Assumed mean

$\sum d$ = Sum of the deviations

n = Number of values

$d = (x - A)$

x = Variables

EXAMPLE 5.3: Calculate the arithmetic mean from the following data.

Roll No.	1	2	3	4	5	6	7	8	9	10
Marks	33	35	44	34	41	45	39	46	36	37

Solution: Let the marks be denoted by x.
Assumed mean is the selected value of the marks (any value of the variable x).
Assumed mean is denoted by $A = 45$.

Roll No.	*Marks (x)*	$(x - 45) = d$
1	33	$33 - 45 = -12$
2	35	$35 - 45 = -10$
3	44	$44 - 45 = -1$
4	34	$34 - 45 = -11$
5	41	$41 - 45 = -4$
6	45	$45 - 45 = 0$
7	39	$39 - 45 = -6$
8	46	$46 - 45 = 1$
9	36	$36 - 45 = -9$
10	37	$37 - 45 = -8$
Total		$\Sigma d = -60$

$$\bar{x} = 45 + \left[\frac{-60}{10}\right] = 45 - 6 = 39$$

$$\bar{x} = 39$$

Discrete series: Let $x_1, x_2, x_3, \ldots, x_n$ be the variables and $f_1, f_2, f_3, \ldots, f_n$ be their corresponding frequencies, then their mean $\bar{x}$ is given by

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + f_3x_3 + \ldots + f_nx_n}{f_1 + f_2 + f_3 + \ldots + f_n} = \frac{\sum_{i=1}^{n} f_ix_i}{\sum_{i=1}^{n} f_i} = \frac{\sum_{i=1}^{n} f_ix_i}{N}$$

where $N = f_1 + f_2 + f_3 + \ldots + f_n$

Direct Method

EXAMPLE 5.4: Find the arithmetic mean from the following frequency table.

Marks	52	58	60	65	68	70	75
No. of students	7	5	4	6	3	3	2

Solution: Let the marks be denoted by x and the number of students by frequencies, i.e., f, so that we have the following table.

x	f	$f \cdot x$
52	7	$52 \times 7 = 364$
58	5	$58 \times 5 = 290$
60	4	$60 \times 4 = 240$
65	6	$65 \times 6 = 390$
68	3	$68 \times 3 = 204$
70	3	$70 \times 3 = 210$
75	2	$75 \times 2 = 150$
	30	1848

Here total frequencies $= N = \sum f = 30$ and $\sum fx = 1848$

$$\text{Arithmetic mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{1848}{30} = 61.6$$

EXAMPLE 5.5: Find the arithmetic mean from the following frequency table.

Marks	30	40	50	60	70	80	90
No. of students	15	20	10	15	20	15	5

Solution: Let the marks be denoted by x and the number of students by frequencies f.

x	f	fx
30	15	$30 \times 15 = 450$
40	20	$40 \times 20 = 800$
50	10	$50 \times 10 = 500$
60	15	$60 \times 15 = 900$
70	20	$70 \times 20 = 1400$
80	15	$80 \times 15 = 1500$
90	5	$90 \times 5 = 450$
	100	5700

Arithmetic mean $(\overline{x}) = \dfrac{\sum f_i x_i}{\sum f_i}$

Here $\quad \Sigma f_i x_i = 5700$ and $\Sigma f_i = N = 100$

Arithmetic mean $(\overline{x}) = \dfrac{5700}{100} = 57$

Thus, arithmetic mean of x is 57.

Short-Cut Method

According to this method, the formula is $\bar{x} = A + \dfrac{\sum fd}{N}$

where

$\bar{x}$ = Arithmetic mean
A = Assumed mean
f = Frequency
d = Deviation = $(x - A)$
x = Variables
N = Total frequency

$\sum fd$ = Multiply these deviations (d) by their respective frequency (f) and obtain the total, i.e., $\sum fd$.

EXAMPLE 5.6: Calculate the average from the following data.

Wages (Rs.)	10	15	20	25	30	35	40
No. of persons	5	2	3	10	3	2	5

Solution: Let the wages be denoted by x and the number of persons by f.

X	$d = (x - A) = (x - 20)$	f	$f \times d$
10	$10 - 20 = 10$	5	$5 \times - 10 = -50$
15	$15 - 20 = 5$	2	$2 \times - 5 = -10$
20	$20 - 20 = 0$	3	$3 \times 0 = 0$
25	$25 - 20 = 5$	10	$10 \times 5 = 50$
30	$30 - 20 = 10$	3	$10 \times 3 = 30$
35	$35 - 20 = 15$	2	$15 \times 2 = 30$
40	$40 - 20 = 20$	5	$20 \times 5 = 100$
		30	150

Here $N = \Sigma f = 30, \Sigma fd = 150, A = 20$

$$\overline{x} = A + \frac{\sum fd}{N} = 20 + \frac{150}{30} = 20 + 5 = 25$$

Hence, $\overline{x} = 25$

Step Deviation Method

For this method, the formula is $\bar{x} = A + \dfrac{\sum fd}{N} \times C$

where

$\bar{x}$ = Arithmetic mean

A = Assumed mean

f = Frequency

d = Deviation = $\dfrac{x - A}{C}$

C = Size of the class interval

x = Variables

N = Total frequency

$\sum fd$ = Multiply these deviations (d) by their respective frequency (f) and obtain the total, i.e., $\sum fd$.

EXAMPLE 5.7: Calculate the average from the following data.

Wages (Rs.)	10	15	20	25	30	35	40
No. of persons	5	2	3	10	3	2	5

Solution: Let the wages be denoted by x and the number of persons by f.

X	$d = \frac{x-25}{5}$	f	fd
10	$\frac{10-25}{5} = \frac{-15}{5} = -3$	5	$5 \times -3 = -15$
15	$\frac{15-25}{5} = \frac{-10}{5} = -2$	2	$2 \times -2 = -4$
20	$\frac{20-25}{5} = \frac{-5}{5} = -1$	3	$3 \times -1 = -3$
25	$\frac{25-25}{5} = \frac{0}{5} = 0$	10	$10 \times 0 = 0$

30	$\frac{30-25}{5} = \frac{5}{5} = 1$	3	$3 \times 1 = 3$
35	$\frac{35-25}{5} = \frac{10}{5} = 2$	2	$2 \times 2 = 4$
40	$\frac{40-25}{5} = \frac{15}{5} = 3$	5	$5 \times 3 = 15$
		30	0

Here $A = 25, \sum fd = 0, N = 30, C = 5$

$$\bar{x} = A + \frac{\sum fd}{N} \times C = 25 + \frac{0}{30} \times 5 = 25 + 0 = 25$$

Hence, $\bar{x} = 25$

Continuous series: Let $x_1, x_2, x_3, \ldots, x_n$ be the mid values of the class intervals and $f_1, f_2, f_3, \ldots, f_n$ be their corresponding frequencies, then their mean $\bar{x}$ is given by

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + f_3x_3 + \ldots + f_nx_n}{f_1 + f_2 + f_3 + \ldots + f_n} = \frac{\sum_{i=1}^{n} f_ix_i}{\sum_{i=1}^{n} f_i} = \frac{\sum_{i=1}^{n} f_ix_i}{N}$$

where $N = f_1 + f_2 + f_3 + \ldots + f_n$

$$\text{Midpoint} = \frac{\text{Lower limit of class interval + Upper limit of class interval}}{2}$$

EXAMPLE 5.8: Calculate the arithmetic mean for the following data.

Class intervals	0–10	10–20	20–30	30–40	40–50	50–60
Frequency	4	8	14	10	9	5

Solution: Let the class interval be denoted by CI and frequency by fx is denoted by mid values of the class intervals.

CI	x	f	$f \times x$
0–10	$\frac{0+10}{2} = \frac{10}{2} = 5$	4	$4 \times 5 = 20$
10–20	$\frac{10+20}{2} = \frac{30}{2} = 15$	8	$8 \times 15 = 120$
20–30	$\frac{20+30}{2} = \frac{50}{2} = 25$	14	$14 \times 25 = 350$
30–40	$\frac{30+40}{2} = \frac{70}{2} = 35$	10	$10 \times 35 = 350$
40–50	$\frac{40+50}{2} = \frac{90}{2} = 45$	9	$9 \times 45 = 405$
50–60	$\frac{50+60}{2} = \frac{110}{2} = 55$	5	$5 \times 55 = 275$
		50	$= 1520$

Here $$N = 50, \sum fx = 1520$$

$$\bar{x} = \frac{\sum fx}{N} = \frac{1520}{50} = 30.4$$

$$\bar{x} = 30.4$$

Step Deviation Method

For this method, the formula is $\bar{x} = A + \frac{\sum fd}{N} \times C$

where $\bar{x}$ = Arithmetic mean

A = Assumed mean (select any value of the mid values of the class intervals)

f = Frequency

d = Deviation = $\frac{x - A}{C}$

C = Size of the class interval

x = Mid values of the class intervals

N = Total frequency

$\sum fd$ = Multiply these deviations (d) by their respective frequency (f) and obtain the total, i.e., $\sum fd$.

EXAMPLE 5.9: Calculate the average marks by the step deviation method.

Marks	0–10	10–20	20–30	30–40	40–50	50–60
No. of students	25	40	50	35	30	20

Solution: Let the marks be denoted by class intervals, i.e., CI and the number of students by frequencies fx is mid values of the class intervals.
Here $A = 35$, $C = 10$.

CI	x	$d = \frac{x - A}{C}$	f	$f \times d$
0–10	$\frac{0+10}{2} = \frac{10}{2} = 5$	$\frac{5-35}{10} = \frac{-30}{10} = -3$	25	$25 \times -3 = -75$
10–20	$\frac{10+20}{2} = \frac{30}{2} = 15$	$\frac{15-35}{10} = \frac{-20}{10} = -2$	40	$40 \times -2 = -80$

20–30	$\frac{20+30}{2}=\frac{50}{2}=25$	$\frac{25-35}{10}=\frac{-10}{10}=-1$	50	$50\times-1=-50$
30–40	$\frac{30+40}{2}=\frac{70}{2}=35$	$\frac{35-35}{10}=\frac{0}{10}=0$	35	$35\times0=0$
40–50	$\frac{40+50}{2}=\frac{90}{2}=45$	$\frac{45-35}{10}=\frac{10}{10}=1$	30	$30\times1=30$
50–60	$\frac{50+60}{2}=\frac{110}{2}=55$	$\frac{55-35}{10}=\frac{20}{10}=2$	20	$20\times2=40$
			200	$=-135$

Here $\sum f = 200$ and $\sum fd = -135$

$$\bar{x} = A + \frac{\sum fd}{N} \times C = 35 + \frac{(-135)}{200} \times 10 = 35 - 6.75 = 28.25$$

Hence, $\bar{x} = 28.25$

EXAMPLE 5.10: Calculate the average income from the following data.

Wages (in Rs.)	10–19	20–29	30–39	40–49	50–59	60–69	70–79
No. of persons	2	3	5	10	5	3	2

Solution: Let the wages be denoted by class intervals 'CI' and no. of persons by frequencies 'f', and mid values by 'x' of the class intervals, A is assumed mean.

CI	x	$d = \frac{x-A}{C}$	f	$f \times d$
10–19	$\frac{10+19}{2} = \frac{29}{2} = 14.5$	$\frac{14.5-44.5}{10} = \frac{-30}{10} = -3$	2	$2 \times -3 = -6$
20–29	$\frac{20+29}{2} = \frac{49}{2} = 24.5$	$\frac{24.5-44.5}{10} = \frac{-20}{10} = -2$	3	$3 \times -2 = -6$
30–39	$\frac{30+39}{2} = \frac{69}{2} = 34.5$	$\frac{34.5-44.5}{10} = \frac{-10}{10} = -1$	5	$5 \times -1 = -5$
40–49	$\frac{40+49}{2} = \frac{89}{2} = 44.5$	$\frac{44.5-44.5}{10} = \frac{0}{10} = 0$	10	$10 \times 0 = 0$
50–59	$\frac{50+59}{2} = \frac{109}{2} = 54.5$	$\frac{54.5-44.5}{10} = \frac{10}{10} = 1$	5	$5 \times 1 = 5$

60–69	$\frac{60+69}{2}=\frac{129}{2}=64.5$	$\frac{64.5-44.5}{10}=\frac{20}{10}=2$	3	$3\times 2=6$
70–79	$\frac{70+79}{2}=\frac{149}{2}=74.5$	$\frac{74.5-44.5}{10}=\frac{30}{10}=3$	2	$2\times 3=6$
			30	0

Here $\quad A = 44.5,\ C = 10,\ \Sigma f = 30,\ \Sigma fd = 0$

$$\bar{x} = A + \frac{\sum fd}{N} \times C = 44.5 + \frac{(0)}{30} \times 10 = 44.5 + 0 = 44.5$$

Thus, average income is Rs. 44.5.

Weighted Arithmetic Mean

If $w_1, w_2, w_3, \ldots, w_n$ are the weights assigned to the values $x_1, x_2, x_3, \ldots, x_n$ respectively, then the weighted arithmetic mean is defined as:

$$\text{Weighted arithmetic mean} = \frac{w_1x_1 + w_2x_2 + w_3x_3 + \ldots w_nx_n}{w_1 + w_2 + w_3 + \ldots w_n} = \frac{\sum w_ix_i}{\sum w_i}$$

EXAMPLE 5.11: A student gets the following percentage in B.A. in 2005: English–70, Hindi–65, Mathematics–75, Physics–70 and Chemistry–66. Find out the weighted arithmetic mean if weights are 2, 2, 4, 3 and 4 respectively.

Solution: Let the marks be denoted by x and weights by w.

Subject	x	w	$w \times x$
English	70	2	$2 \times 70 = 140$
Hindi	65	2	$2 \times 65 = 130$
Mathematics	75	4	$4 \times 75 = 300$
Physics	70	3	$3 \times 70 = 210$
Chemistry	66	4	$4 \times 66 = 264$
		15	$= 1044$

Here $\qquad \sum w = 15$ and $\sum wx = 1044$

$$\text{Weighted arithmetic mean } (\bar{X}_w) = \frac{\sum wx}{\sum w} = \frac{1044}{15} = 69.6$$

EXAMPLE 5.12: The following table gives the number of students in different classes in a secondary school and their fees. Find the average fee per students.

Class	*No. of students*	*Fees*
5th	60	40
6th	75	50
7th	90	60
8th	85	65
9th	65	70

Solution: Let the number of students be denoted by x and fees by w.

Class	x	w	$w \times x$
5th	60	40	$40 \times 60 = 2400$
6th	75	50	$50 \times 75 = 3750$
7th	90	60	$60 \times 90 = 5400$
8th	85	65	$65 \times 85 = 5525$
9th	65	70	$70 \times 65 = 4550$
		285	$= 21,625$

Here $\sum w = 285$ and $\sum wx = 21,625$

$$\text{Weighted arithmetic mean } (\bar{X}_w) = \frac{\sum wx}{\sum w} = \frac{21625}{285} = 75.88$$

Combined Mean

If we are given the mean of two series, and their size, then the combined mean for the resultant series can be obtained by the formula:

$$\bar{X} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

where $\bar{X}$ = Combined mean of the two series
$\bar{x}_1$ = Mean of the first series
$\bar{x}_2$ = Mean of the second series
n_1 = Size of the first series
n_2 = Size of the second series

EXAMPLE 5.13: Average monthly production of a factory for the first 8 months is 2,584 units and for the remaining four months it is 2,416 units. Calculate the average monthly production for the year.

Solution:

Here $\bar{x}_1$ = Mean of the first production = 2,584
$\bar{x}_2$ = Mean of the second production = 2,416
n_1 = Size of the first = 8
n_2 = Size of the second = 4

$$\bar{X} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{8 \times 2584 + 4 \times 2416}{8+4} = 2528$$

EXAMPLE 5.14: A firm of readymade garments makes both men's and women's shirts. Its profit average is 6% of the sales. Its profits in men's shirts average 8% of sales and women's shirts comprise 60% of output. What are the average profits per sale rupee in women's shirts?

Solution: Here $\overline{X} = 6$, $\overline{x}_1 = 8$, $\overline{x}_2 = ?$, $n_1 = 40$, $n_2 = 60$.
Assuming that the total output is 100, we are required to find out $\overline{x}_2$. We know that

$$\overline{X} = \frac{n_1\overline{x}_1 + n_2\overline{x}_2}{n_1 + n_2} = \frac{40 \times 8 + 60 \times n_2}{40 + 60}$$

$$6 = \frac{320 + 60\overline{x}_2}{100}$$

$$\overline{x}_2 = \frac{600 - 320}{60} = \frac{280}{60} = 4.66$$

Thus, the average profit in women's shirt is 4.66 per cent of sales, or Rs. 0.0466 per sale rupee.

Merits

1. It can be easily calculated.
2. Its calculation is based on all the observations.
3. It is easy to understand.
4. It is rightly defined by the mathematical formula.
5. It is least affected by sampling fluctuations.

Demerits

1. It cannot be calculated if all the values are not known.
2. The extreme values have greater effect on mean.
3. It cannot be determined for the qualitative data such as beauty, love, honesty.

Uses

1. Estimates are always obtained by mean.
2. A common man uses mean for calculating average marks obtained by a student.
3. It is frequently used in practical statistics.

MEDIAN

It is the middlemost point or the central value of the variable in a set of observations arranged either in ascending order or in descending order of their magnitudes.

According to Prof. Ghosh and Chowdhury " Median is the value of that item in a series which divides the series into two equal parts, one part consisting of all values less and the other all values greater than it.

Individual Series

To find the value of median in this case, the terms are arranged in ascending or descending order first; then the middle term taken is median.

When the number of terms is odd

The terms are arranged in ascending order or descending order and then $\left(\frac{n+1}{2}\right)^{th}$ term value is taken of the order as median.

EXAMPLE 5.15: Find median from the following data.

17	19	21	13	16	18	24	22	20

Solution:

Given values are 17, 19, 21, 13, 16, 18, 24, 22, 20.

First arrange the terms in ascending order or descending order

13	16	17	18	19	20	21	22	24

Total number of values $n = 9$ (odd number)

$$\text{Median} = \left(\frac{n+1}{2}\right) = \left(\frac{9+1}{2}\right) = \frac{10}{2} = \text{5th value of the order is 19}$$

$\therefore$ Median = 19

When the number of terms is even

The terms are arranged in ascending order or descending order and then mean of the two midd terms is taken as median.

i.e. $$\text{Median} = \frac{\left(\frac{n}{2}\right) + \left(\frac{n}{2}+1\right)}{2} \text{ value of the order}$$

where n is the number of items.

EXAMPLE 5.16: Find the median of the following data.

87	71	32	28	69	85	53	90	60	56

Solution:

Given values are 87, 71, 32, 28, 69, 85, 53, 90, 60, 56.

First arrange the terms in ascending order or descending order.

28	32	53	56	60	69	71	85	87	90

Total number of values n =10 (even number)

$$\left(\frac{n}{2}\right) = \left(\frac{10}{2}\right) = \text{5th value of the order is 60}$$

$$\left(\frac{n}{2}+1\right) = \left(\frac{10}{2}+1\right) = \text{(5 + 1)th value of the order is 69}$$

i.e. $$\text{Median} = \frac{\left(\frac{n}{2}\right) + \left(\frac{n}{2}+1\right)}{2} \text{ value of the order}$$

$$\text{Median} = \left(\frac{60+69}{2}\right) = \frac{129}{2} = 64.5$$

$$\text{Median} = 64.5$$

Discrete series: The discrete series involves frequencies, in order to find out the median in such a case, it is necessary to divide the total frequency into two equal parts.

1. Arrange the data either in ascending order or in descending order.
2. Calculate cumulative frequencies.
3. Find out the value of the middle item by applying the formula.

   $$\text{Median} = \text{size of } \left(\frac{N+1}{2}\right)\text{th item}$$

4. Find out the total in the cumulative frequency column which is either equal to $\left(\frac{N+1}{2}\right)$th or next higher than that.
5. Locate the value of the variable corresponding to the cumulative frequency. This value of the variable is the value of the median.

EXAMPLE 5.17: Determine the median from the following data.

Size	5	10	15	20	25	30	35
Frequency	2	3	4	6	10	5	2

Solution:

Let the size be denoted by x, frequency by f and cumulative frequency by cf.

x	f	cf
5	2	$0 + 2 = 2$
10	3	$2 + 3 = 5$
15	4	$5 + 4 = 9$
20	6	$9 + 6 = 15$
25	10	$15 + 10 = 25$
30	5	$25 + 5 = 30$
35	2	$30 + 2 = 32$
	32	

N = Total frequency = 32

$$\text{Median} = \text{Size of } \left(\frac{N+1}{2}\right)\text{th item} = \text{Size of } \left(\frac{32+1}{2}\right) = \left(\frac{33}{2}\right) = 16.5$$

The above table shows that all items from 16 to 25 have their values of the variable x is 25. Since 25th item lies in this interval, therefore, its value is 25.

Hence, median = 25 marks

Example:5.18 Calculate the median for the following data

No. of Students	6	4	16	7	8	2
Marks	20	9	25	50	40	80

Sol: Let the number of students be denoted by f, marks by x and cumulative frequency by cf. Arrange the marks in ascending order and prepare the table as follows.

x	f	cf
9	4	0+4 =4
20	6	4+6=10
25	16	10+16=26
40	8	26+8=34
50	7	34+7=41
80	2	41+2 =43
	43	

Total frequency =N=43.

Median = Size of $\left(\frac{N+1}{2}\right)$ th item = Size of $\left(\frac{43+1}{2}\right)$ = 22

The above table shows that all items from 11 to 26 have their values of the variable x is 25. Hence , median =25 marks.

Continuous Series

In continuous series, median cannot be located in a strait forward method. In this case , the median lies in a class interval , i.e., between the lower and upper limit of a class interval. For exact value, we have to interpolate the median with the help of a formula

$$Median = l + \frac{\frac{N}{2} - m}{f} \times C$$

where $N = \sum f$ = Total frequency

$l =$ Lower limit of the class in which the median lies.

$m =$ Cumulative frequency of the class preceding the median class

f = Frequency of the class in which the median lies

$C =$ Width of the class interval of the class in which the median lies.

Example:5.19 calculate the median from the following data.

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	7	18	34	50	35	20	6

Solution: Let the class interval be denoted by CI and the frequency by f.

CI	f	cf
0–10	7	$0 + 7 = 7$
10–20	18	$7 + 18 = 25$
20–30	34	$25 + 34 = 59$
30–40	50	$59 + 50 = 109$
40–50	35	$109 + 35 = 144$
50–60	20	$144 + 20 = 164$
60–70	6	$164 + 6 = 170$
	170	

Here $N = 170, \ \dfrac{N}{2} = \dfrac{170}{2} = 85, \ m = 59, f = 50, \ l = \dfrac{30+30}{2} = \dfrac{60}{2} = 30, \ C = 10$

$$\text{Median} = l + \frac{\frac{N}{2} - m}{f} \times C = 30 + \frac{85 - 59}{50} \times 10 = 30 + \frac{26}{50} \times 10 = 30 + 5.2$$

$$\text{Median} = 35.2$$

EXAMPLE 5.20: The following table gives the marks obtained by students in Chemistry. Find the median.

Marks	10–19	20–29	30–39	40–49	50–59	60–69	70–79
No. of students	7	16	24	27	22	15	9

Solution: Let the marks be denoted by CI and number of students by frequency *f*.

CI	*Class boundary*	*f*	*cf*
10–19	9.5–19.5	7	0 + 7 = 7
20–29	19.5–29.5	16	7 + 16 = 23
30–39	29.5–39.5	24	23 + 24 = 47
40–49	39.5–49.5	27	47 + 27 = 74
50–59	49.5–59.5	22	74 + 22 = 96
60–69	59.5–69.5	15	96 + 15 = 111
70–79	69.5–79.5	9	111 + 9 = 120
		120	

Here $N = 120, \dfrac{N}{2} = \dfrac{120}{2} = 60, m = 47, f = 27, l = \dfrac{40+39}{2} = \dfrac{79}{2} = 39.5, C = 10$

$$\text{Median } = l + \frac{\frac{N}{2} - m}{f} \times C = 39.5 + \frac{60-47}{27} \times 10 = 39.5 + \frac{13}{27} \times 10 = 39.5 + 4.8148$$

$$\text{Median } = 44.3148$$

Merits

1. It is easily understood
2. 2 It is not affected by extreme values
3. It can be located graphically
4. It is the best measure for qualitative data such as beauty, intelligence etc.
5. It can be easily located even if the class intervals in the series are unequal.

Demerits

1. It is not subject to algebraic treatments.
2. It cannot represent the irregular distribution series.
3. It does not have sampling stability.
4. It is a positional average and is based on the middle item.

Uses

1. It is useful in those cases where numerical measurements are not possible .
2. It is also useful in those cases where mathematical calculations cannot be made in order to obtain the mean.
3. It is generally used in studying phenomenon like skill, honesty , intelligence etc.

MODE

Mode is the value in a series which occurs most frequently, i.e., has the maximum frequency. In other words , mode represents that value which is most frequent or typical or predominant. According to Croxton and Cowden, " The mode of a distribution is the value at the point around which the item tends to be most heavily concentrated. It may be regarded as the most typical of a series of values."

Individual Series

If each term of the series occurs only once, then there is no mode, otherwise the value that occurs the maximum number of times is known as mode.

Example: 5.21 Find the mode from the following data.

2	4	5	7	7	7	9	6	8

In this data 7 is repeated 3 times. So the mode is 7. This called unimodal.

Example:5.22 Find the mode from the following data.

3	8	11	34	9	8	8	3	4	6	3

In this data 3 is repeated 3 times and 8 is also repeated 3 times. So the modes are 3 and 8. This is bimodal.

Example: 5.23 Find the mode from the following data.

13	11	24	15	17	11	26	24	11	34	17	17	24	12	20	11	17

Solution: In this data 11 is repeated 4 times, 17 is repeated 4 times and 24 is repeated 3 times. So modes are 11, 17 and 24. This is called multimodal.

Discrete Series

Inspection method: Here the mode is known by Inspection method only. Here that variable is the mode where the frequency is the highest.

But this method is applicable if

(1) There is a gradual rise or fall in the sequence of frequencies.
(2) The highest frequency and the next highest frequency are not too close.
(3) Maximum frequency is not repeated.

EXAMPLE 5.24: Find the mode for the following data.

x	4	7	11	16	25
f	3	9	14	21	13

Solution: In the above given series, highest frequency is 21 and variable corresponding to this frequency is 16. Thus, the mode is 16.

But, however sometimes it becomes impossible to locate the mode by inspection as concentration of frequencies is not in the unique manner or fashion as desired for this method.

For such a distribution we have to prepare (1) Grouping Table and (2) Analysis Table. A grouping table has the following six columns:

I. Grouping table

It has six steps as given below:

1. Frequencies are taken and written in the 1st column as f(I).
2. Frequencies are added in two(s) and written in the 2nd column as f(II).
3. Except the first item and the remaining frequencies are added in two(s) and then written in the 3rd column as f(III).
4. Frequencies are added in 3's and written in 4th column as f(IV).
5. Leaving the first frequency, the remaining frequencies are added in 3's, and written in the 5th column as f(V).
6. Leaving the first two frequencies the remaining frequencies are added in 3's and written in the 6th column as f(VI).

   In each case, take maximum total and put it in a circle or a box to distinguish it from others.

II. Analysis table

It has the following steps:

1. Note down the height total in each column.
2. Note the variable in each column corresponding to that total.
3. Check if the total is of individual term or more (2 or 3) terms.
4. If the total consists of 2 or more frequencies, all such variables have to be marked by '√' or X.

5. Count '√' or X marks in each column.
6. Variable with maximum '√' or X marks denote mode.

The procedure of preparing a grouping table and analysis table shall be clear from the following example.

EXAMPLE 5.25: From the following data of the ages of different persons, determine the modal age.

Age	15	20	25	30	35	40	45	50	55
No. of persons	2	3	4	10	11	12	3	2	1

Solution: In this case it is not possible to state by inspection whether the mode is 30, 35 or 40. We need to determine it through grouping and analysis tables.

Grouping Table

Frequency

Age (x)	*Frequency f(I)*	*Of two f(II)*	*Of two leaving the first fr. f(III)*	*Of three f(IV)*	*Of three leaving the first fr. f(V)*	*Of three leaving the first two fr. f(VI)*
15	2					
20	3	5		9		
25	4		7			
30	10	14		33	17	25
35	11		21			
40	12	23		6		
45	3		15		26	
50	2	5				17
55	1		3			

Analysis Table
Age in Years

Variable	*I*	*II*	*III*	*IV*	*V*	*VI*	*Total*
15							–
20							–
25						x	1
30			x	x		x	3
35		x	x	x	x	x	5
40	x	x		x	x		4
45					x		1
50							–
55							–

Since the value 35 has occurred the maximum number of times, i.e., 5, the modal age is 35. It should be noted that by inspection, one is likely to say that the modal value is 40, since it has the maximum frequency. But it is not correct as revealed by the analysis table.

Continuous Series

The modal class can be determined either by inspection or with the help of a grouping table. After finding the modal class, we calculate the mode by the following formula.

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times C$$

where l = Lower limit of the modal class
f = Maximum frequency
f_1 = Maximum frequency of the class preceding value
f_2 = Maximum frequency of the class succeeding value
C = Class interval size

EXAMPLE 5.26: From the following table, calculate the mode.

Class intervals	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Frequency	9	11	14	20	15	10	8

Solution:

Class intervals	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Frequency	9	11	14	20	15	10	8

l = Lower limit of the modal class $= \dfrac{30+30}{2} = \dfrac{60}{2} = 30$

f = Maximum frequency = 20

f_1 = Maximum frequency of the class preceding value = 14

f_2 = Maximum frequency of the class succeeding value = 15

C = Class interval size = 10

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times C = 30 + \frac{20-14}{2\times 20 - 14 - 15} \times 10 = 30 + \frac{60}{11} \times 10 = 35.4545$$

Mode = 35.4545

EXAMPLE 5.27: Find the mode from the following data.

Class intervals	0–4	5–9	10–14	15–19	20–24	25–29	30–34
Frequency	3	11	21	32	25	17	9

Solution: As the difference is 1 between the upper limit of an interval and the lower limit of the next interval. Therefore, we have to deduct 0.5 from the lower and add 0.5 to the upper limit of each interval.

Class intervals	–0.5–4.5	4.5–9.5	9.5–14.5	14.5–19.5	19.5– 24.5	24.5–29.5	29.5–34.5
Frequency	3	11	21	32	25	17	9

$$l = \text{Lower limit of the modal class} = \frac{14.5+14.5}{2} = \frac{29}{2} = 14.5$$

$$f = \text{Maximum frequency} = 32$$

f_1 = Maximum frequency of the class preceding value = 21
f_2 = Maximum frequency of the class succeeding value = 25
C = Class interval size = 5

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times C = 14.5 + \frac{32 - 21}{2 \times 32 - 21 - 25} \times 5 = 14.5 + \frac{11}{18} \times 5 = 14.5 + 3.0556 = 17.5556$$

Mode = 17.5556

Asymmetrical distribution

A distribution in which mean, median and mode do not coincide is called asymmetrical distribution. If the distribution is moderately asymmetrical, then the mean, median and mode are connected by the formula

$$\text{Mode} = 3\ \text{Median} - 2\ \text{Mean}$$

EXAMPLE 5.28: If the value of the mode and mean is 60 and 66 respectively. Find the value of the median.

Solution: We know that

$$\text{Here mode} = 60 \text{ and mean} = 66$$

$$\text{Mode} = 3\ \text{Median} - 2\ \text{Mean}$$

$$\text{Median} = \frac{1}{3}(\text{Mode} + \text{Mean}) = \frac{1}{3}(60 + 2 \times 66) = 64$$

Merits

1. It is easy to understand
2. It is simple to calculate
3. It is not affected by extreme values.
4. It is a positional average and can be located easily by inspection.
5. It can be determined by graphic method.

Demerits

1. It is an average , which is ill-defined and indeterminate.
2. It is not further used in algebraic calculation.
3. In the case of bimodal class, the calculation is difficult as it involves grouping and analysis table.
4. It is not based on all observations.

Uses

1. It is used for the study of most popular fashion.
2. It is extensively used by businessman and commercial managements.

GEOMETRIC MEAN

So far we have studied three measures of central tendency- mean, median and mode. There are two other means which are occasionally used in economics and business. These are Geometric Mean and Harmonic Mean. These are also called Ratio-Averages because these are more suitable when the data comprise rates, percentages of ratios instead of actual quantities. Geometric mean is the n-th root of the product of N items of a series.

Individual Series

If $x_1, x_2, x_3, \ldots, x_n$ are n values of a variable x, one of them, geometric mean G, is defined as

$$G = (x_1, x_2, x_3, \ldots, x_n)^{\frac{1}{n}} \tag{1}$$

The difficulty of calculating the nth root is overpowered with the help of logarithms. Now taking logarithms of both sides of (1), we get

$$G = (x_1, x_2, x_3, \ldots, x_n)^{\frac{1}{n}}$$

Taking logarithms on both sides

$$\log G = \log\,(x_1, x_2, x_3, \ldots, x_n)^{\frac{1}{n}} = \frac{1}{n}\log\,(x_1, x_2, \ldots, x_n)$$

$$\log G = \frac{\log x_1 + \log x_2 + \ldots + \log x_n}{n}$$

$$G = \text{Antilog of}\left(\frac{\sum \log x_i}{n}\right)$$

EXAMPLE 5.29: Compute the geometric mean for the following data: 10, 110, 120, 50, 52, 80, 37, 60.

Solution: Let us prepare the following table:

x *values*	*Value of* $\log x$
10	1.0000
110	2.0414
120	2.0792
50	1.6990
52	1.7160
80	1.9031
37	1.5682
60	1.7782
	13.7851

Here $$n = 8 \text{ and } \Sigma \log x = 13.7851$$

$$\log G = \frac{1}{n} \Sigma \log x = \frac{13.7851}{8} = 1.723$$

$$G = \text{Antilog of } 1.723 = 52.84$$

Discrete Series

In the case of discrete series, geometric mean of n values $x_1, x_2, \ldots, x_n$ of a variate x occurring with frequency $f_1, f_2, \ldots, f_n$ respectively is given by

$$G = \left[x_1^{f_1} \cdot x_2^{f_2} \cdot x_3^{f_3}, \ldots, x_n^{f_n}\right]^{\frac{1}{n}}$$

Taking log on both sides

$$\text{Log } G = \frac{f_1 \log x_1 + f_2 \log x_2 + \ldots + f_n \log x_n}{N}$$

$$G = \text{Antilog}\left[\frac{\sum_{i=1}^{n} f_i \log x_i}{N}\right]$$

Here $N = \sum f_i = \text{Total frequency}$

EXAMPLE 5.30: Calculate the geometric mean for the following data.

x	12	13	14	15	16	17
f	3	4	5	4	2	1

Solution: Let us prepare the following table in order to calculate the geometric mean for the given data.

x	*log x*	f	$f \times \log x$
12	1.0792	3	3.2376
13	1.1139	4	4.4556
14	1.1461	5	5.7305
15	1.1761	4	4.7044
16	1.2041	2	2.4082
17	1.2304	1	1.2304
		19	21.7667

Here $N = 19$ and $\frac{\sum f \log x}{N} = \frac{21.7667}{19} = 1.1456$

$G = \text{Antilog of } (1.1456) = 13.98$

$G = 13.98$

Continuous Series

In the case of continuous series, geometric mean of n values $x_1, x_2, \ldots, x_n$ of mid values of the class intervals and corresponding frequencies $f_1, f_2, \ldots, f_n$ respectively is given by

$$G = \left[x_1^{f_1} \cdot x_2^{f_2} \cdot x_3^{f_3}, \ldots, x_n^{f_n} \right]^{\frac{1}{n}}$$

Taking log on both sides

$$\text{Log } G = \frac{f_1 \log x_1 + f_2 \log x_2 + \ldots + f_n \log x_n}{N}$$

$$G = \text{Antilog} \left[\frac{\sum_{i=1}^{n} f_i \log x_i}{N} \right]$$

Here $N = \Sigma f_i =$ Total frequency

EXAMPLE 5.31: Find the geometric mean for the following data.

Marks	0–10	10–20	20–30	30–40	40–50
No. of students	4	8	10	7	6

Solution: Let the marks be denoted by class intervals (CI) and the number of student frequencies f.

CI	x	*Log x*	f	$f \times \log x$
0–10	$\frac{0+10}{2} = \frac{10}{2} = 5$	0.6990	4	$4 \times 0.6990 = 2.796$
10–20	$\frac{10+20}{2} = \frac{30}{2} = 15$	1.1761	8	$8 \times 1.1761 = 14.088$
20–30	$\frac{20+30}{2} = \frac{50}{2} = 25$	1.3979	10	$10 \times 1.3979 = 13.979$
30–40	$\frac{30+40}{2} = \frac{70}{2} = 35$	1.5441	7	$7 \times 1.5441 = 10.8087$
40–50	$\frac{40+50}{2} = \frac{90}{2} = 45$	1.6532	6	$6 \times 1.6532 = 9.9192$
			35	51.5909

Here $N = 35$ and $\frac{\sum f \log x}{N} = \frac{51.5909}{35} = 1.4740$

$G = \text{Antilog of } (1.4740) = 29.79$

$G = 29.79$

Merits

1. It is simple and lends itself to algebraic treatment.
2. It is useful in the construction of index number.
3. It is not much affected by the fluctuations of sampling.
4. It is based on all the observations.
5. It gives less weight to large items and large weight to small items.

Demerits

1. It cannot be easily understood.
2. It is relatively difficult to compute as it requires some special knowledge of logarithms.
3. It cannot be calculated when any value is zero or negative.
4. It cannot be obtained by inspection.

Uses

It is used in the cases where it is necessary to average ratios which express rate of change. It is also used for the construction of index numbers.

HARMONIC MEAN

Harmonic mean is based on arithmetic mean of reciprocals of the values of the variable. It may be the mean of the reciprocal of the values of the variable. It may be defined as the reciprocal of the values of the variable.

$$\text{Harmonic mean} = \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\ldots+\frac{1}{x_n}} = \frac{n}{\sum\left(\frac{1}{x_i}\right)}$$

where $x_1, x_2, x_3, \ldots, x_n$ different items of the variable and 'n' is the number of items.

Individual Series

EXAMPLE 5.32: Find the harmonic mean from the following data.

Values (x)	34	46	88	78	86	67	110	134

Solution: Let the values be denoted by x.

X	*Reciprocals* $\left(\frac{1}{x}\right)$
34	$\left(\frac{1}{34}\right) = 0.0294$
46	$\left(\frac{1}{46}\right) = 0.0217$
88	$\left(\frac{1}{88}\right) = 0.0114$
78	$\left(\frac{1}{78}\right) = 0.0128$

86	$\left(\frac{1}{86}\right) = 0.0116$
67	$\left(\frac{1}{67}\right) = 0.0149$
110	$\left(\frac{1}{110}\right) = 0.0091$
134	$\left(\frac{1}{134}\right) = 0.0075$
	0.2525

Here $\quad n = 8 \text{ and } \Sigma\left(\frac{1}{x}\right) = \Sigma\left(\frac{1}{0.2525}\right)$

$$\text{Harmonic mean} = \frac{n}{\sum\left(\frac{1}{x}\right)} = \frac{8}{\frac{1}{0.2525}} = \frac{8}{3.9604} = 2.020$$

Harmonic mean = 2.020

Discrete Series

In the case of discrete series, harmonic mean of n values $x_1, x_2, \ldots, x_n$ of a variable x occurring with frequency $f_1, f_2, \ldots, f_n$ respectively is given by

$$\text{Harmonic mean} = \frac{f_1 + f_2 + f_3 + \ldots + f_n}{\dfrac{f_1}{x_1} + \dfrac{f_2}{x_2} + \dfrac{f_3}{x_3} + \ldots + \dfrac{f_n}{x_n}} = \frac{\sum f_i}{\sum f_i \times \dfrac{1}{x_i}} = \frac{N}{\sum \dfrac{f_i}{x_i}}$$

EXAMPLE 5.33: Compute the harmonic mean for the following data.

Marks	20	21	22	23	24	25
No. of students	4	2	7	1	3	1

Solution: Let the marks be denoted by x and the number of students by frequency f.

x	$\frac{1}{x}$	f	$f \times \frac{1}{x}$
20	$\frac{1}{20} = 0.0500$	4	$4 \times 0.05000 = 0.20000$
21	$\frac{1}{21} = 0.04762$	2	$2 \times 0.04762 = 0.09524$

22	$\frac{1}{22} = 0.04545$	7	$7 \times 0.04545 = 0.31815$
23	$\frac{1}{23} = 0.04348$	1	$1 \times 0.04348 = 0.04348$
24	$\frac{1}{24} = 0.04167$	3	$3 \times 0.04167 = 0.12501$
25	$\frac{1}{25} = 0.04000$	1	$1 \times 0.04000 = 0.04000$
		18	0.82188

Here $N = 18 \text{ and } \sum f \times \frac{1}{x} = 0.82188$

$$\text{Harmonic mean} = \frac{N}{\sum \frac{f_i}{x_i}} = \frac{18}{0.82188} = 21.9010$$

Harmonic mean $= 21.9010$

Continuous Series

In the case of continuous series, harmonic mean of n values $x_1, x_2, \ldots, x_n$ of mid values of the class intervals and corresponding frequencies $f_1, f_2, \ldots, f_n$ respectively is given by:

$$\text{Harmonic mean} = \frac{f_1 + f_2 + f_3 + \ldots + f_n}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \frac{f_3}{x_3} + \ldots + \frac{f_n}{x_n}} = \frac{\sum f_i}{\sum f_i \times \frac{1}{x_i}} = \frac{N}{\sum \frac{f_i}{x_i}}$$

EXAMPLE 5.34: Calculate the harmonic mean from the following data.

Marks	0–10	10–20	20–30	30–40	40–50
Frequencies	2	7	13	5	3

Solution: Let the marks be denoted by class intervals (CI) and frequencies by f.

CI	x	f	$\frac{f}{x}$
0–10	$\frac{0+10}{2} = \frac{10}{2} = 5$	2	$\frac{2}{5} = 0.400$
10–20	$\frac{10+20}{2} = \frac{30}{2} = 15$	7	$\frac{7}{15} = 0.467$
20–30	$\frac{20+30}{2} = \frac{50}{2} = 25$	13	$\frac{13}{25} = 0.520$

30–40	$\frac{30+40}{2} = \frac{70}{2} = 35$	5	$\frac{5}{35} = 0.143$
40–50	$\frac{40+50}{2} = \frac{90}{2} = 45$	3	$\frac{3}{45} = 0.067$
		30	1.597

Here $N = 30$ and $\sum \frac{f}{x} = 1.597$

$$\text{Harmonic mean} = \frac{N}{\sum \frac{f_i}{x_i}} = \frac{30}{1.597} = 18.79$$

Harmonic mean $= 18.79$.

Merits

1. It is based on all the observations of the series.
2. It is suitable for further algebraic treatment.
3. It is rigidly defined.

Demerits

1. It is difficult to compute.
2. Its value cannot be computed when there are both positive and negative items.
3. It is not popular.

Uses

1. It is useful in averages involving time rate and price.
2. It gives less weight to large item and more weight to small item.

Relation between AM, GM and HM is AM > GM > HM (OR) AM < GM < HM.

EXAMPLE 5.35: If the HM and GM of two numbers are in the ratio of 12:13, find the ratio of the number.

Solution: Let the two numbers be a and b.

According to the definition of HM = $\dfrac{2ab}{a+b}$

According to the definition of GM = $\sqrt{ab}$

Given that $$\frac{\text{HM}}{\text{GM}} = \frac{12}{13}$$

$$\Rightarrow \quad \frac{\dfrac{2ab}{a+b}}{\dfrac{\sqrt{ab}}{1}} = \frac{12}{13} \Rightarrow \frac{2ab}{a+b} \times \frac{1}{\sqrt{ab}} = \frac{12}{13} \Rightarrow \frac{\sqrt{ab}}{a+b} = \frac{12}{13} \times \frac{1}{2} = \frac{6}{13}$$

By cross- multiplication

$$13\sqrt{ab} = 6(a+b)$$

Divided by b

$$13\sqrt{\frac{a}{b}} = 6\left(\frac{a}{b}+1\right)$$

Put $\sqrt{\frac{a}{b}} = x \implies \frac{a}{b} = x^2$

$$13x = 6\left(x^2-1\right)$$

$$\implies x = \frac{3}{2} \; or \; x = \frac{2}{3} \implies \frac{a}{b} = \frac{9}{4} \; or \; \frac{a}{b} = \frac{4}{9}$$

Example:3.36 If AM of two numbers is 18 and GM is 16, find HM.

Sol. Let two numbers be a and b.

According to problem AM = 18 $\implies \frac{a+b}{2} = 18 \implies a+b = 36$.

GM = 16 = $\sqrt{ab} \implies ab = 256$,

HM = $\frac{2}{\frac{1}{a}+\frac{1}{b}} = \frac{2ab}{a+b} \implies \frac{2*256}{36} = 14.22$

END