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How to Implement Super-Twisting Controller based on Sliding Mode Observer?

Asif Chalanga[1] Shyam Kamal[2] L.Fridman[3] B.Bandyopadhyay[4] and J.A.Moreno[5]

Abstract— **Implementation of the Super-Twisting Control (STC) requires the first time derivative of the sliding surface must be Lipschitz in time. It is shown that if we will use STC based on the absolutely continuous estimation of the surface, controller can not be implemented. In this paper two methodologies are proposed to avoid the above problem. For simplicity we are considering here only perturbed double integrator which can be generalized for an arbitrary order. Numerical simulations are also presented to show the effectiveness of the proposed method.**

Index Terms— **Super-Twisting Control (STC), Super-Twisting observer (STO), higher order sliding mode observer.**

I. Introduction

Control, observation and identification of systems under uncertainties/pertubations is a major research topic in modern control theory. The sliding mode control [1], [2] methodology is one such robust control technique which has its roots in the relay control. One of the most intriguing aspects of sliding mode is the discontinuous nature of the control action whose primary function is to switch between two distinctively different structures about some predefined manifold such that a new type of system motion called sliding mode exists in a manifold. This peculiar system characteristic is claimed to result in a superb system performance which includes insensitivity to parameter variations and complete rejection of certain class of disturbances. Furthermore, the system possess new properties which are not present in original system.

The main disadvantage of sliding mode control is chattering [1], [2], [3]. To avoid chattering effect Super-Twisting algorithm [4] is proposed. Super-twisting control, is a continuous controller ensuring all the main properties of first order sliding mode control for the system with Lipschitz matched bounded uncertainties/disturbances.

The sliding mode approach has been exceptionally successful in the design of state feedback controllers. However, in most of the physical systems, the states are seldom available. Therefore, the state feedback sliding mode control cannot be directly used for implementations. In this situation, one has to resort to dynamic output feedback control methods [6] which are more practical than the sliding mode control based on state feedback.

The sliding mode observers are widely used due to the finite-time convergence, insensitivity with respect to uncertainties and also the estimation of the uncertainty [10]. A new generation of observers, based on the cascaded interconnection of the Super-Twisting Algorithm (STA) have been recently developed [11], [6]. The STA is a well-known second order sliding mode algorithm introduced in [5] and it has been widely used for control, observation [11] and robust exact differentiation. Finite time convergence and robustness for the STA has been proved by geometrical methods [4] and by means of Lyapunov based approach [7], [8]. The sliding mode observation based on cascaded STA has such attractive features as insensitivity (more than robustness) with respect to unknown inputs; and also offers the possibility of reconstructing the unknown input terms via the equivalent output injection. Also, its implementation does not need the separation principle to be proved.

There are several applications of the Super-Twisting Controller (STC), existing in the literature which is based on the (Super-Twisting Observer) STO [9] for the alleviate the chattering phenomenon. However, the control synthesis of these applications are based on the approximation, which has not sound analytic mathematical background and also poor precision.

The present paper is written in the the tutorial fashion and pointing out the major drawbacks of the existing STC design based on the STO, which is frequently reported in the literature for getting absolutely continuous control signal and also suggested the possible remedy.

Main Contribution

It is obvious that implementation of the Super-Twisting Control (STC) requires the first time derivative of the chosen sliding manifold must be Lipschitz in time. It is shown that if we will use STC based on the absolutely continuous estimation of the surface, controller can not be implemented. In this paper two methodologies are proposed to avoid the above problem. For simplicity we are considering here only perturbed double integrator, which can be generalized for the arbitrary order same as [9].

Structure of the Paper

The paper is organized as follows. Section II discusses the standard sliding mode STC based on STO for perturbed

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double integrator. Section III details the STC based on Super-Twisting Output Feedback (STOF). Higher order sliding mode observer based absolutely continuous control of perturbed double integrator is discussed in Section IV . Section V contains numerical results followed by the concluding Section.

II. Standard Sliding Mode STC based on STO for Perturbed Double Integrator

Consider the dynamical system of the following form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u + \rho_1 \\ y &= x_1 \end{aligned} \tag{1}$$

where y is the output variable, ρ_1 is a non vanishing Lipschitz disturbance and $|\dot{\rho}_1| < \rho_0$. Our aim is to reconstruct the states of the system and then design super-twisting controller based on the estimated information. Although this is already reported in the literature, however in the next subsection we are going to show that existing methodology is not stand on the sound mathematical background.

A. Problem of Designing Super-Twisting Controller Using Super-Twisting Observer for the System (1)

The super-twisting observer dynamics to estimate the states is

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + z_1 \\ \dot{\hat{x}}_2 &= u + z_2 \end{aligned} \tag{2}$$

where $z_1 = k_1|e_1|^{\frac{1}{2}}\text{sign}(e_1)$ and $z_2 = k_2\text{sign}(e_1)$ are correction terms. Let us define the error $e_1 = x_1 - \hat{x}_1$ and $e_2 = x_2 - \hat{x}_2$ and the error dynamics is

$$\begin{aligned} \dot{e}_1 &= e_2 - z_1 \\ \dot{e}_2 &= -z_2 + \rho_1 \end{aligned}$$

So e_1 and e_2 will converge to zero in finite time $t > T_0$, by selecting the appropriate gains k_1 and k_2 [8]. For this, one can say that $x_1 = \hat{x}_1$ and $x_2 = \hat{x}_2$ after finite time $t > T_0$.

System represented by (1) is relative degree two system with respect to output variable $y = x_1$, therefore direct super-twisting control $u = -k_1|x_1|^{1/2}\text{sign}(x_1) - \int_0^t k_2\text{sign}(x_1)d\tau$ or $u = -k_1|\hat{x}_2|^{1/2}\text{sign}(\hat{x}_2) - \int_0^t k_2\text{sign}(\hat{x}_2)d\tau$ is not applicable. Therefore, we have to define sliding manifold of the form

$$s = c_1x_1 + \hat{x}_2. \tag{3}$$

Remark *1:* It has to note that one can not define sliding surface $s = c_1x_1 + x_2$, because we do not have exact information of x_2. However, using super-twisting observer [5], [11] (2), we are able to extract information of x_2 as $\hat{x}_2$. Therefore, taking sliding manifold as (3) is much reasonable. Also, one can not take sliding surface as $s = c_1\hat{x}_1 + \hat{x}_2$, because we have exact information of x_1, so exact sliding manifold for applying control is given by (3).

Taking the time derivative of (3) (for designing the super-twisting control), one can write

$$\dot{s} = c_1\dot{x}_1 + \dot{\hat{x}}_2. \tag{4}$$

After finite time $t > T_0$, when observer start extracting the exact information of the states, then one can substitute $\dot{x}_1 = \hat{x}_2$. Also using (2) and (4), one can further write

$$\dot{s} = c_1\hat{x}_2 + u + k_2\text{sign}(e_1). \tag{5}$$

Therefore, the system (1) in the co-ordinate of x_1 and s by using (3) and (4) could be written as

$$\begin{aligned} \dot{x}_1 &= s - c_1x_1 \\ \dot{s} &= c_1\hat{x}_2 + u + k_2\text{sign}(e_1). \end{aligned} \tag{6}$$

Now suppose that we will follow the standard way of super-twisting control design (which is existing in the literature) as

$$u = -c_1\hat{x}_2 - \lambda_1|s|^{\frac{1}{2}}\text{sign}(s) - \int_0^t \lambda_2\text{sign}(s)d\tau. \tag{7}$$

where λ_1 and λ_2 are the designed parameters for the control. The closed loop system after applying the control (7) to (6)

$$\begin{aligned} \dot{x}_1 &= s - c_1x_1 \\ \dot{s} &= -\lambda_1|s|^{\frac{1}{2}}\text{sign}(s) - \int_0^t \lambda_2\text{sign}(s)d\tau + k_2\text{sign}(e_1) \end{aligned} \tag{8}$$

or

$$\begin{aligned} \dot{x}_1 &= s - c_1x_1 \\ \dot{s} &= -\lambda_1|s|^{\frac{1}{2}}\text{sign}(s) + L + k_2\text{sign}(e_1) \\ \dot{L} &= -\lambda_2\text{sign}(s) \end{aligned} \tag{9}$$

where L is some fictitious state variable.

B. Discussion of above Mathematical Transformation

It is clear from the above mathematical transformation that, when one can use super twisting observer (STO) to estimate the state of second order uncertain system (1) and follow the standard way of the STC design as (7) by selecting sliding manifold as (3). Then second order sliding motion is never start in the (9), because $\dot{s}$ contains the non-differentiable term $k_2\text{sign}(e_1)$, which exclude the possibility of lower two subsystem of (9) to act as the super-twisting and finally start the second order sliding motion (so that $s = \dot{s} = 0$ in finite time).

In the next subsection we are going to propose the possible methodology of the control design such that non-differentiable term $k_2\text{sign}(e_1)$ is cancel out and then lower two subsystem of (9) act as the super-twisting and finally second order sliding is achieved.

C. Existence Condition of Sliding Mode using Super-Twisting Algorithm

The main aim here, is to design u, such that sliding motion occurs in finite time. For this purpose control is selected according to the following Proposition

***Proposition** 1:* The control input u which is defined as

$$u = -c_1\hat{x}_2 - k_2\text{sign}(e_1) - \lambda_1|s|^{\frac{1}{2}}\text{sign}(s) - \int_0^t \lambda_2\text{sign}(s)d\tau \tag{10}$$

where, $\lambda_1 > 0$ and $\lambda_2 > 0$ are selecting according to [5], [8], leads to the establishment second order sliding in finite time, which further implies asymptotic stability of x_1 and x_2.

Proof: The closed loop system after substituting (10) into (6)

$$\begin{aligned} \dot{x}_1 &= s - c_1x_1 \\ \dot{s} &= -\lambda_1|s|^{\frac{1}{2}}\text{sign}(s) - \int_0^t \lambda_2\text{sign}(s)d\tau \end{aligned} \tag{11}$$

or

$$\begin{aligned} \dot{x}_1 &= s - c_1x_1 \\ \dot{s} &= -\lambda_1|s|^{\frac{1}{2}}\text{sign}(s) + \nu \\ \dot{\nu} &= -\lambda_2\text{sign}(s) \end{aligned} \tag{12}$$

Last two equation of (12) has same structure as super-twisting. Therefore, one can easily observe that after finite time $t > T_0$, $s = 0$, which further implies, that the closed loop system is given as

$$\begin{aligned} \dot{x}_1 &= -c_1x_1 \\ \hat{x}_2 &= -c_1x_1 \end{aligned} \tag{13}$$

Therefore, both states x_1 and $\hat{x}_2$ are asymptotic stability by choosing $c_1 > 0$. Also, when observer estimating the exact state $\hat{x}_2 = x_2$ after finite time, then x_2 also going to zero simultaneously as $\hat{x}_2$. ■

It is clear from the mathematical derivation of control (10) that, when one can use super twisting observer (STO) to estimate the state of second order uncertain system (1) and then design super twisting based controller (STC) by selecting sliding manifold as (3), the control is still discontinuous, because it contains the discontinuous term $k_2\text{sign}(e_1)$. Therefore, absolutely continuous control design based on STO-STC is not possible. Before, going to propose the solution of above mentioned problem, it is necessary to discuss the existing methodology which is not mathematically sound but still a lot of literature has been based on it.

III. STC based on Super-Twisting Output Feedback (STOF)

In the existing strategy it is reported that first consider the following sliding sliding surface

$$s = c_1x_1 + x_2 \tag{14}$$

assuming that all states information are available. After that for realizing the control expression based on super-twisting , take the first time derivative of sliding surface s using (14)

$$\dot{s} = c_1\dot{x}_1 + \dot{x}_2 \tag{15}$$

Now substitute $\dot{x}_1$ and $\dot{x}_2$ from (1) into (15), one can write

$$\dot{s} = c_1x_2 + u + \rho_1 \tag{16}$$

Now design control as

$$u = -c_1x_2 - \lambda_1|s|^{\frac{1}{2}}\text{sign}(s) - \int_0^t \lambda_2\text{sign}(s)d\tau \tag{17}$$

assuming that every states information are available. After substituting the control (17) into (16), one can write

$$\dot{s} = -\lambda_1|s|^{\frac{1}{2}}\text{sign}(s) - \int_0^t \lambda_2\text{sign}(s)d\tau + \rho_1, \tag{18}$$

or

$$\begin{aligned} \dot{s} &= -\lambda_1|s|^{\frac{1}{2}}\text{sign}(s) + \nu \\ \dot{\nu} &= -\lambda_2\text{sign}(s) + \dot{\rho}_1. \end{aligned} \tag{19}$$

Now select $\lambda_1 > 0$ and $\lambda_2 > 0$ according to [8], which leads to second order sliding in finite time provided ρ_1 is Lipschitz and $|\dot{\rho}_1| < \rho_0$. When $s = 0$, then $x_1 = x_2 = 0$ asymptotically same as discussed above by selecting $c_1 > 0$. In last, it discussed that control (17) is not implementable because we do not have information of x_2, so replace x_2 by $\hat{x}_2$. It is argued that after finite time $x_1 = \hat{x}_1$ and $x_2 = \hat{x}_2$, therefore control signal applied to original system (1) is

$$u = -c_1\hat{x}_2 - \lambda_1|\hat{s}|^{\frac{1}{2}}\text{sign}(\hat{s}) - \int_0^t \lambda_2\text{sign}(\hat{s})d\tau \tag{20}$$

where $\hat{s} = c_1\hat{x}_1 + \hat{x}_2$, without considering the dynamics of $\dot{\hat{x}}_2$ for which control derivation is explicitly dependent and it contains the discontinuous term $k_2\text{sign}(e_1)$. One can easily see that average value of this discontinuous term is equal to negative of the disturbance. So whatever control we are applying for the real system is only approximate not the exact. However, the exact controller is always discontinuous which already discussed and mathematically proved in the above section.

Now in the next section we are going to proposed the new strategy, which gives the correct way to implement continuous STC, when only output information of the perturbed double integrator (1) is available.

IV. Higher Order Sliding Mode Observer based Continuous Control of Perturbed Double Integrator (1)

The higher order sliding mode observer (HOSMO) dynamics to estimate the states for the perturbed double integrator (1) is given as

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + z_1 \\ \dot{\hat{x}}_2 &= \hat{x}_3 + u + z_2 \\ \dot{\hat{x}}_3 &= z_3 \end{aligned} \tag{21}$$

where $z_1 = k_1|e_1|^{\frac{2}{3}}\text{sign}(e_1)$, $z_2 = k_2|e_1|^{\frac{1}{3}}\text{sign}(e_1)$ and $z_3 = k_3\text{sign}(e_1)$ are correction terms. Let us define the error $e_1 = x_1 - \hat{x}_1$ and $e_2 = x_2 - \hat{x}_2$ and the error dynamics is

$$\begin{aligned} \dot{e}_1 &= e_2 - z_1 \\ \dot{e}_2 &= -\hat{x}_3 - z_2 + \rho_1 \\ \dot{\hat{x}}_3 &= z_3 \end{aligned} \tag{22}$$

Now define the new variable $e_3 = -\hat{x}_3 + \rho_1$, if ρ_1 is Lipschitz and $|\dot{\rho}_1| < \rho_0$, then one can further write (22) as

$$\begin{aligned} \dot{e}_1 &= e_2 - z_1 \\ \dot{e}_2 &= e_3 - z_2 \\ \dot{e}_3 &= -z_3 + \dot{\rho}_1 \end{aligned} \tag{23}$$

So e_1, e_2 and e_3 will converge to zero in finite time $t > T_0$, by selecting the appropriate gains k_1, k_2 and k_3 [6]. After the convergence of error, one can find that $x_1 = \hat{x}_1$, $x_2 = \hat{x}_2$ and $\hat{x}_3 = \rho_1$ after finite time $t > T_0$. Now taking the time derivative of (3) (for designing the super twisting control), one can write

$$\dot{s} = c_1\dot{x}_1 + \dot{\hat{x}}_2. \tag{24}$$

After finite time $t > T_0$, when observer start extracting the exact information of the sates, then one can substitute $\dot{x}_1 = \hat{x}_2$. Also using (22) and (24), one can further write

$$\begin{aligned} \dot{s} &= c_1\hat{x}_2 + \hat{x}_3 + u + z_2 \\ &= c_1\hat{x}_2 + u + k_2|e_1|^{\frac{1}{3}}\text{sign}(e_1) + \int_0^t k_3\text{sign}(e_1)d\tau \end{aligned} \tag{25}$$

Therefore, the system (1) in the co-ordinate of x_1 and s by using (3) and (25) could be written as

$$\begin{aligned} \dot{x}_1 &= s - c_1 x_1 \\ \dot{s} &= c_1\hat{x}_2 + u + k_2|e_1|^{\frac{1}{3}}\text{sign}(e_1) + \int_0^t k_3\text{sign}(e_1)d\tau \end{aligned} \tag{26}$$

A. Existence Condition of Sliding Mode using Super-twisting Algorithm

The main aim here, is to design u, such that sliding motion occurs in finite time. For this purpose control is selected according to the following theorem

***Proposition** 2:* The control input u which is defined as

$$\begin{aligned} u = &-c_1\hat{x}_2 - k_2|e_1|^{\frac{1}{3}}\text{sign}(e_1) - \int_0^t k_3\text{sign}(e_1)d\tau \\ &- \lambda_1|s|^{\frac{1}{2}}\text{sign}(s) - \int_0^t \lambda_2\text{sign}(s)d\tau \end{aligned} \tag{27}$$

or

$$\begin{aligned} u = &-c_1\hat{x}_2 - \int_0^t k_3\text{sign}(e_1)d\tau \\ &- \lambda_1|s|^{\frac{1}{2}}\text{sign}(s) - \int_0^t \lambda_2\text{sign}(s)d\tau \end{aligned} \tag{28}$$

because observer is much faster than controller, which makes $e_1 = 0$ in finite time and if $\lambda_1 > 0$ and $\lambda_2 > 0$ are selecting according to [5], [8], leads to the establishment s equal to zero in finite time, it further implies asymptotic stability of x_1 and x_2.

Proof: Proof is the same as the Proposition 1. ■

B. Discussion of HOSMO based STC Design

It is clear from the STC control (27) expression based on HOSMO (21) is continuous. Also, when we design STC control based on HOSMO then one has to tune only observer gain according to the first derivative of disturbance, because it is necessary for the convergence of the error variables of the HOSMO. However, during controller design there is no explicit gain condition for the λ_2 with respect to disturbances.

One can also observe that STC (20) design based STOF (2), (which is propagating in the literature without any sound mathematical justification) requires two gains, one is STO observer gain k_2 based on the explicit maximum bound of the direct disturbance and another is λ_2, for the STC based on the maximum bound of the derivative of disturbance.

Therefore, one can conclude from the above observation that sound mathematical analysis reduces the two gains conditions with respect to disturbance by simply one gain condition. Also the precision of the sliding manifold is much improved by using the HOSMO based STC rather than STO based STC. Due to the increase of this precision of sliding variable precision of the states are also much effected. In other word if we talk about stabilization problem, then states are much closer to origin in the case of HOSMO based STC rather than STO based STC. We only talk about closeness of states variable with respect to equilibrium point, because only asymptotic stability is possible in the both of design methodology.

V. Numerical Simulation

For the simulation, initial conditions of perturbed double integrator (1), for the all three cases, STC-STO, STC-STOF and STO-HOSMO, is taken as $x_1 = 10$, $x_2 = 0$ and $\rho_1 = 2 + 3\sin(t)$. Other gains for the all three cases are selected as follows

- STC-STO
  - STC gains $k_1 = 3$ and $k_2 = 4$
  - STO gains $\lambda_1 = 3.5$ and $\lambda_2 = 6$
- STC-STOF
  - STC gains $k_1 = 2$ and $k_2 = 1$
  - STO gains $\lambda_1 = 3.5$ and $\lambda_2 = 6$
- STC-HOSMO
  - STC gains $k_1 = 2$ and $k_2 = 1$
  - STO gains $\lambda_1 = 6$, $\lambda_2 = 11$ and $\lambda_3 = 6$

Evolution of the output with respect to time and its precision (in the zoomed version of figure) after applying the STC based on STO, STOF and HOSMO are shown in the Fig.1, when the output has non-noisy measurement and Fig.6 in the case of noisy measurement of the amplitude 10^{-2}. Figs.2 and 7 show that the evolution of the sliding manifold and and its precision (in the zoomed version of figure) in the case of non-noisy and noisy measurement respectively. Error evolution along with precision, in the case of STO and HOSMO are shown in the Fig.3 and 8, with and without noisy measurement respectively. Figs.4 and 9 show that control evolution of STC based on STO and HOSMO, with and without noisy measurement. Similarly, control evolution with and without noisy measurement in the case of STC based on STOF is shown in Figs.5 and 10 respectively.

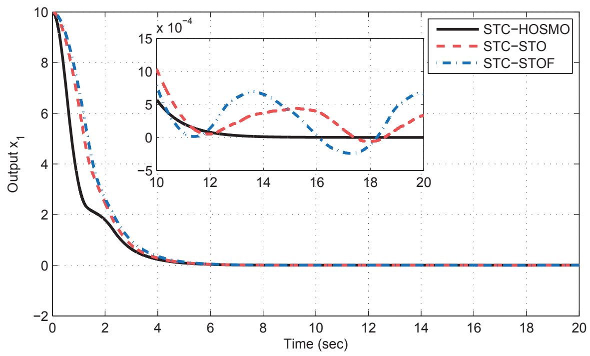

Fig. 1. Evolution of output w.r.t. time for the STC based on HOSMO, STOF and STO

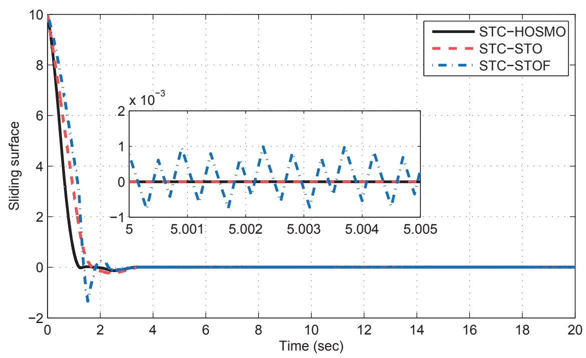

Fig. 2. Evolution of sliding manifold w.r.t. time for the STC based on HOSMO, STOF and STO

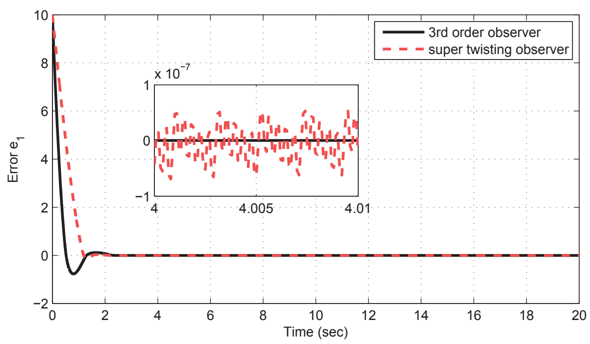

Fig. 3. Evolution of error w.r.t. time using STO and HOSMO

VI. Conclusion

It is shown in the paper that, if one wants to implement absolutely continuous STC signal for the perturbed double integrator, the derivative of the chosen manifold must be Lipschitz in the time. Therefore, we have the need of second

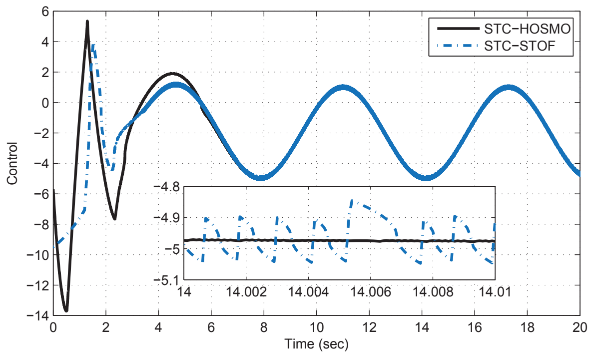

Fig. 4. Evolution of STC based on HOSMO and STOF w.r.t. time

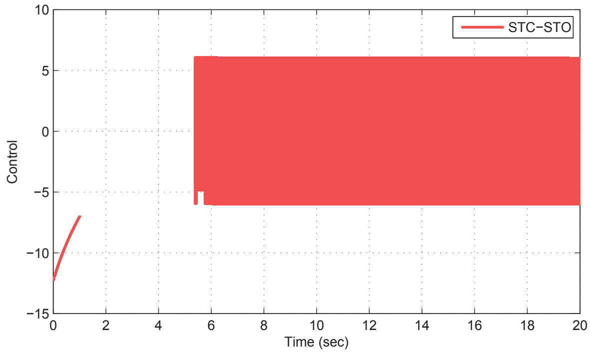

Fig. 5. Evolution of STC based on STO w.r.t. time

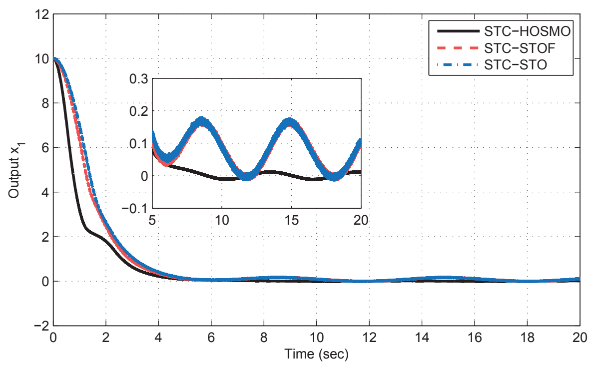

Fig. 6. Evolution of output w.r.t. time for the STC based on HOSMO, STOF and STO under noisy measurement

order observer/differentiators in this case. The same is also true for the higher order perturbed chain of integrators, when we want to synthesize absolutely continuous STC signal under the output information. Numerical simulations are also presented to support the effectiveness of the proposed methodology.

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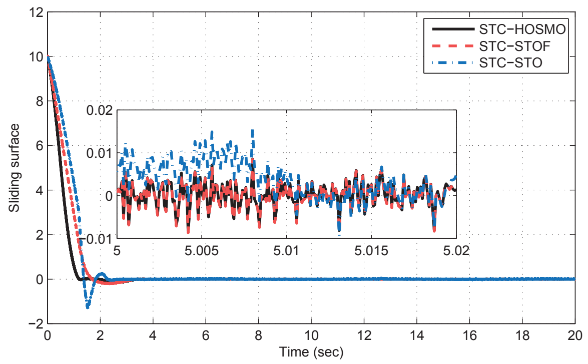

Fig. 7. Evolution of sliding manifold w.r.t. time for the STC based on HOSMO, STOF and STO under noisy measurement

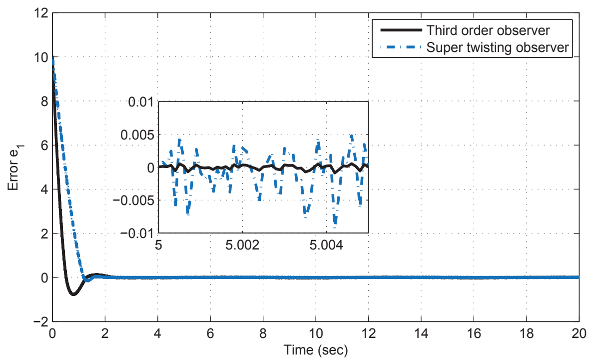

Fig. 8. Evolution of error w.r.t. time using STO and HOSMO under noisy measurement

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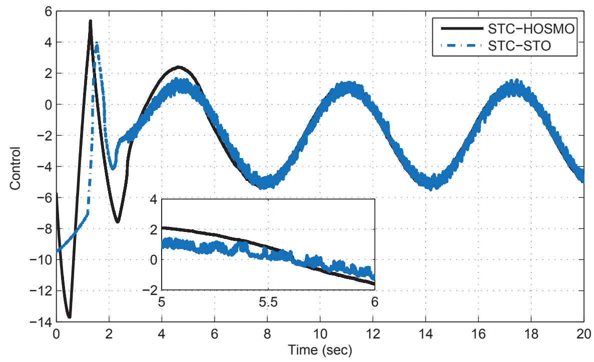

Fig. 9. Evolution of control STC based on HOSMO and STOF w.r.t. time under noisy measurement

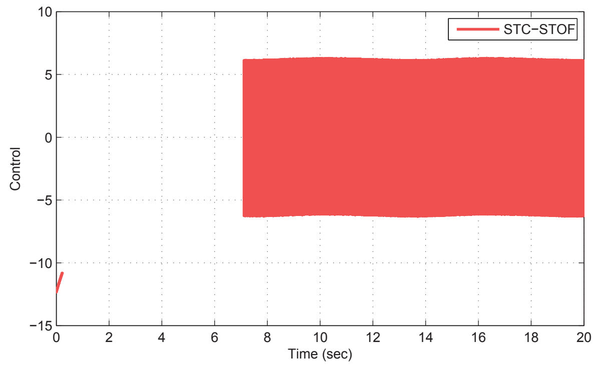

Fig. 10. Evolution of control STC based on STO w.r.t. time under noisy measurement

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