CMOS INVERTER

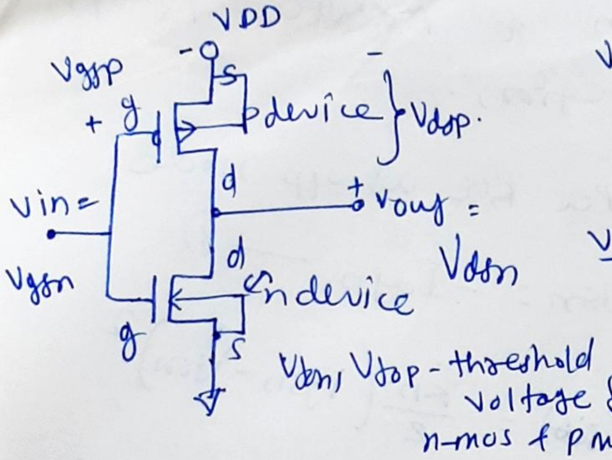


fig ① cmos inverter

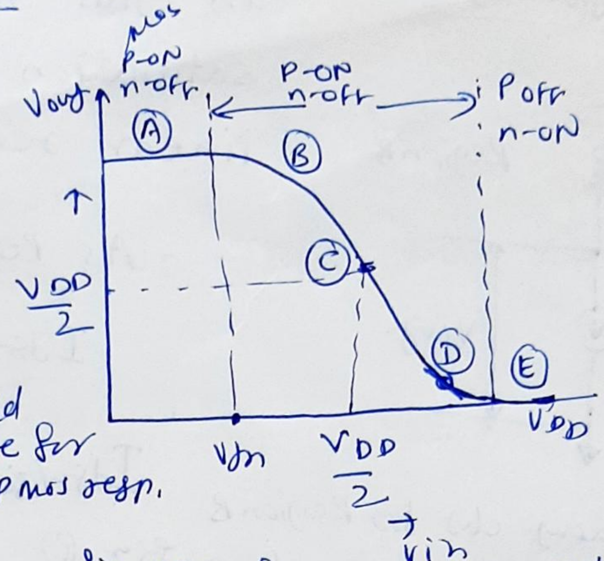


fig VTC for cmos inverter (D.C transfer characteristic & operating region)

Vin = 0 , Vout = Vdsp = logic 1 = VDD

Vin = 1 Vout = Vdsn = logic 0 = 0V

Region A.

Region A is defined as $0 \le Vin \le Vton$ n-device is cut off $\therefore Idsn = 0$ & pdevice is in linear region. Since $Idsn = -Idsp$ $\therefore Idsp = 0$

from fig ①

$\therefore Vs = VDD$

$\therefore Vd = Vout$

$$Vdsp = Vout - VDD \quad — ①$$

$Idsp = 0 \quad \therefore Vdsp = 0$ put in equn ①

$\therefore$ equn ① $\Rightarrow$ $\boxed{Vout = VDD}$ — ②

Region B

In this region n-device is in saturation & p device is in linear region.

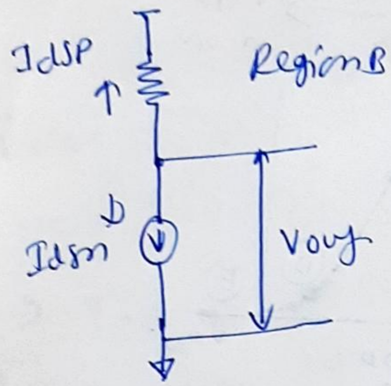


fig: Equivalent ckt for Region B

∴ As per KCL at o/p node

$$I_{dsn} = -I_{dsp} \quad \text{--- (3)}$$

$$I_{dsn(sat)} = \frac{k_n}{2}(V_{gsn} - V_{tn})^2$$

from fig (1)

$$\therefore V_{gsn} = V_{gn} - V_{sn} = V_{in} - 0 = V_{in}$$

$$\therefore I_{dsn} = \frac{k_n}{2}(V_{in} - V_{tn})^2 \quad \text{--- (4)}$$

&

$$I_{dsp} = \frac{-k_p}{2}\left[2(V_{gsp} - V_{tp}) \cdot V_{dsp} - V_{dsp}^2\right] \quad \text{--- 5}$$

$$V_{gsp} = V_{gp} - V_{sp} = V_{in} - V_{DD} \quad \& \quad \text{--- (6)}$$

$$V_{dsp} = V_{dp} - V_{sp} = V_{out} - V_{DD} \quad \text{--- (7)}$$

put eqn (6) & (7) in eqn (5)

$$\therefore I_{dsp} = \frac{-k_p}{2}\left[2(V_{in} - V_{DD} - V_{tp})(V_{out} - V_{DD}) - (V_{out} - V_{DD})^2\right] \quad \text{--- (8)}$$

According to eqn (1) equate eqn (4) & (8)

$$\therefore \frac{k_n}{2}(V_{in} - V_{tn})^2 = -\left\{\frac{-k_p}{2}\left[2(V_{in} - V_{DD} - V_{tp})(V_{out} - V_{DD}) - (V_{out} - V_{DD})^2\right]\right\}$$

$$\therefore \frac{k_n}{k_p}(V_{in} - V_{tn})^2 = 2(V_{in} - V_{DD} - V_{tp})(V_{out} - V_{DD}) - (V_{out} - V_{DD})^2 \quad \text{--- (9)}$$

$$\therefore (V_{out}-V_{DD})^2 - 2(V_{in}-V_{DD}-V_{TOP})(V_{out}-V_{DD}) + \frac{k_n}{k_p}(V_{in}-V_{Ton})^2 = 0 \quad \text{——— (10)}$$

In above quadratic eqn

$$\therefore a = 1, \quad b = -2(V_{in}-V_{DD}-V_{TOP}) \quad \& \quad c = \frac{k_n}{k_p}(V_{in}-V_{Ton})^2$$

$$\therefore V_{out}-V_{DD} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{2(V_{in}-V_{DD}-V_{TOP}) \pm \sqrt{4(V_{in}-V_{DD}-V_{TOP})^2 - 4(1)\left(\frac{k_n}{k_p}(V_{in}-V_{Ton})^2\right)}}{2}$$

$$\therefore V_{out}-V_{DD} = (V_{in}-V_{DD}-V_{TOP}) + \sqrt{(V_{in}-V_{DD}-V_{TOP})^2 - \frac{k_n}{k_p}(V_{in}-V_{Ton})^2}$$

$$\therefore V_{out} = \cancel{V_{DD}} + V_{in} - \cancel{V_{DD}} - V_{TOP} + \sqrt{\left[\underbrace{(V_{in}-V_{TOP})}_{a} - \underbrace{V_{DD}}_{b}\right]^2 - \frac{k_n}{k_p}(V_{in}-V_{Ton})^2}$$

$$= (V_{in}-V_{TOP}) + \sqrt{(V_{in}-V_{TOP})^2 - \underline{2(V_{in}-V_{TOP})V_{DD}} + \underline{V_{DD}^2} - \frac{k_n}{k_p}(V_{in}-V_{Ton})^2}$$

taking $-2V_{DD}$ common

$$V_{out} = V_{in}-V_{TOP} + \sqrt{(V_{in}-V_{TOP})^2 - 2\left(V_{in}-V_{TOP}-\frac{V_{DD}}{2}\right)V_{DD} - \frac{k_n}{k_p}(V_{in}-V_{Ton})^2}$$

$$\boxed{\therefore V_{out} = V_{in}-V_{TOP} + \sqrt{(V_{in}-V_{TOP})^2 - 2\left(V_{in}-V_{TOP}-\frac{V_{DD}}{2}\right)V_{DD} - \frac{k_n}{k_p}(V_{in}-V_{Ton})^2} \quad \text{——— (11)}}$$

Region C

In this region both n-device & p-devices are in saturation.

$$\therefore \quad I_{dsp} = -\frac{k_p}{2}(V_{gsp} - V_{tp})^2 \quad \text{--- (1)}$$

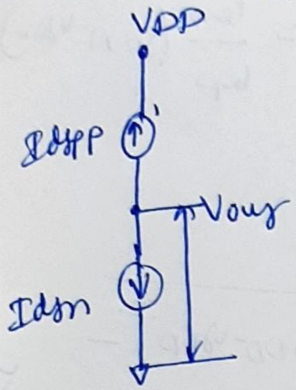


fig: Equivalent ckt for CMOS inverter in region C

Put $V_{gsp} = V_{gp} - V_{sp} = V_{in} - V_{DD}$ in eqn (1)

$$\therefore \quad I_{dsp} = -\frac{k_p}{2}(V_{in} - V_{DD} - V_{tp})^2 \quad \text{--- (2)}$$

$$I_{dsn} = \frac{k_n}{2}(V_{gsn} - V_{tn})^2 \quad \text{--- (3)}$$

Put $V_{gsn} = V_{gn} - V_{sn} = V_{in} - 0 = V_{in}$ in eqn (3) $\rightarrow$ eqn (4)

$$I_{dsn} = \frac{k_n}{2}(V_{in} - V_{tn})^2 \quad \text{--- (4)}$$

~~equate~~. By applying KCL at o/p node

$$I_{dsn} = -I_{dsp}$$

$$\therefore \frac{k_n}{2}(V_{gsn} - V_{tn})^2 = \frac{k_p}{2}(V_{in} - V_{DD} - V_{tp})^2$$

$$\frac{k_n}{k_p}(V_{in} - V_{tn})^2 = (V_{in} - V_{DD} - V_{tp})^2 \quad \text{--- (5)}$$

taking square root of eqn (5) & neglecting +ve roots

$$\therefore -\sqrt{\frac{k_n}{k_p}}(V_{in} - V_{tn}) = V_{in} - V_{DD} - V_{tp}$$

$\therefore \left(1+\sqrt{\frac{k_n}{k_p}}\right) \cdot V_{in} = V_{DD} + V_{tp} + \sqrt{\frac{k_n}{k_p}} \cdot V_{tn}$

$$\therefore \quad V_{in} = \frac{V_{DD} + V_{tp} + V_{tn}\sqrt{\frac{k_n}{k_p}}}{1 + \sqrt{k_n/k_p}} \quad \text{--- (6)}$$

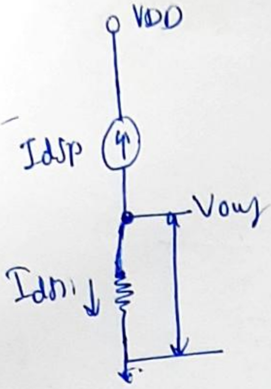
at $k_n = k_p$ & $V_{tn} = -V_{tp}$ eqn (6) $\rightarrow$ (7)

$\boxed{V_{in} = V_{DD}/2}$ & $\boxed{V_{out} = \frac{V_{DD}}{2}}$

Region D

In region D, $\frac{V_{DD}}{2} < V_{in} \le V_{DD} + V_{tp}$

the p-device is in saturation & n-device is in Linear region.



Equivalent diagram of cmos inverter in Regione

$$\therefore I_{dsp\,(sat)} = \frac{-k_p}{2}\left(V_{gsp} - V_{tp}\right)^2 \quad (12)$$

Put $V_{gsp} = V_{gp} - V_{sp} = V_{in} - V_{DD}$ in equ^n (12)

$$\therefore I_{dsp}\,(sat) = \frac{-k_p}{2}\left(V_{in} - V_{DD} - V_{tp}\right)^2 \quad (13)$$

$$I_{dsn} = \frac{k_n}{2}\left[2(V_{gsn} - V_{tn}) \cdot V_{dsn} - V_{dsn}^2\right] \quad (14)$$

Put $V_{gsn} = V_{in}$ in equ^n (14) & $V_{dsn} = V_{out}$ in equ^n (14)

$$\therefore I_{dsn} = \frac{k_n}{2}\left[2(V_{in} - V_{tn}) \cdot V_{out} - V_{out}^2\right] \quad (15)$$

As per KCL of o/p node

$$I_{dsn} = -I_{dsp}$$

equate equ^n (13) & (15)

$$\therefore +\frac{k_p}{2}\left(V_{in} - V_{DD} - V_{tp}\right)^2 = \frac{k_n}{2}\left[2(V_{in} - V_{tn})V_{out} - V_{out}^2\right] \quad (16)$$

$$\frac{k_p}{k_n}\left(V_{in} - V_{DD} - V_{tp}\right)^2 = 2(V_{in} - V_{tn})V_{out} - V_{out}^2 \quad (18)$$

$$V_{out}^2 - 2(V_{in} - V_{tn})V_{out} + \frac{k_p}{k_n}\left(V_{in} - V_{DD} - V_{tp}\right)^2 = 0 \quad (18)$$

Eqn (18) is quadratic equation having

$a = 1$, $b = -2(v_{in} - v_{ton})$ & $c = \frac{k_p}{k_n}(v_{in} - V_{DD} - V_{top})^2$.

$$\therefore V_{out} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2(v_{in} - v_{ton}) \pm \sqrt{4(v_{in} - v_{ton})^2 - 4\frac{k_p}{k_n}(v_{in} - V_{DD} - V_{top})^2}}{2}$$

$$\boxed{V_{out} = (V_{in} - v_{ton}) - \sqrt{(v_{in} - v_{ton})^2 - \frac{k_p}{k_n}(v_{in} - V_{DD} - V_{top})^2}} \quad (19)$$

Region E

This region is defined by $v_{in} \geq V_{DD} - V_{top}$

in this region p-device is cutoff $\therefore I_{dsp} = 0$ & the n-device is in Linear mode.

Here, $V_{gsp} = v_{in} - V_{DD}$ which is more +ve than V_{tp}.

$$\boxed{\therefore V_{out} = 0}$$