Understanding Confusion Matrices

Confusion Matrix

- Accuracy is only a single number.
- Sometimes we would like to understand *for which cases* we make the errors. This is where the confusion matrix helps.

For all test examples

Predicted Label

True Label		Safe Loan	Unsafe
	Safe Loan		
	Unsafe		

Confusion Matrix – Binary Classification

Default Truth	Model Prediction
0	0
1	1
0	1
0	1
0	0
1	1
0	0
0	0
1	1
1	0

Predicted class Actual class	Default	No Default	Total
Default	3	1	4
No Default	2	4	6
Total	5	5	10

Typically we call one class **positive** and the other **negative.**

Defaut : **-**
No Default : **+**

A Key Analytical Framework: Expected Value

- The expected value computation provides a framework that is useful in organizing thinking about data-analytic problems
- It decomposes data-analytic thinking into:
  - the structure of the problem,
  - the elements of the analysis that can be extracted from the data, and
  - the elements of the analysis that need to be acquired from other sources
- The general form of an expected value calculation:

$$EV = p(o_1) \times v(o_1) + p(o_2) \times v(o_2) + p(o_3) \times v(o_3) + ..$$

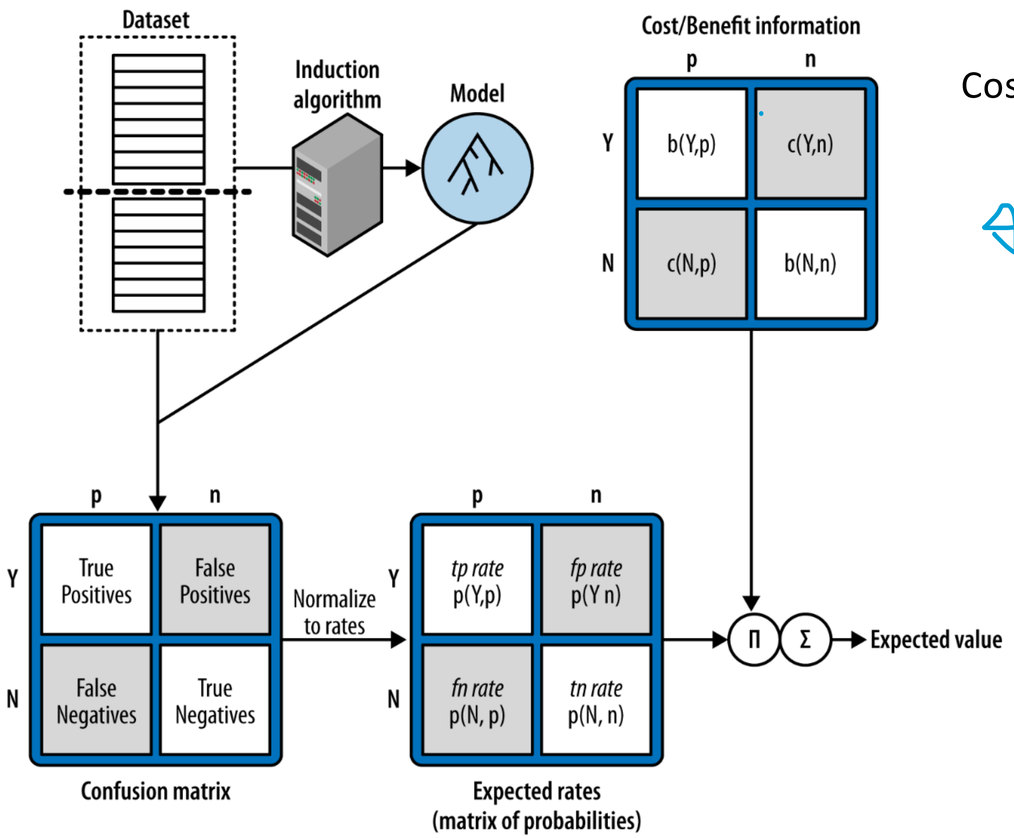
Expected Value Framework in Use Phase

Online marketing:

- Expected benefit of targeting $= p_R(x) \times v_R + [1 - p_R(x)] \times v_{NR}$
- Product Price: $200
- Product Cost: $100
- Targeting Cost: $1

$$p_R(\boldsymbol{x}) \times \$99 - [1 - p_R(\boldsymbol{x})] \times \$1 > 0$$
$$p_R(\boldsymbol{x}) > 0.01$$

Using Expected Value to Frame Classifier Evaluation



Cost-Benefit Matrix

Conditional Probability

A rule of basic probability is:

$$p(x, y) = p(y) \times p(x \mid y)$$

$$\begin{aligned}\text{Expected profit} \\ &= p(Y, p) \times b(Y, p) + p(N, p) \times b(N, p) \\ &+ p(N, n) \times b(N, n) + p(Y, n) \times b(Y, n)\end{aligned}$$

$$\begin{aligned}\text{Expected profit} \\ &= p(Y|p) \times p(p) \times b(Y, p) + p(N|p) \times p(p) \times b(N, p) \\ &+ p(N|n) \times p(n) \times b(N, n) + p(Y|n) \times p(n) \times b(Y, n)\end{aligned}$$

$$\begin{aligned}\text{Expected profit} \\ &= p(p) \times [p(Y|p) \times b(Y, p) + p(N|p) \times b(N, p)] \\ &+ p(n) \times [p(N|n) \times b(N, n) + p(Y|n) \times b(Y, n)]\end{aligned}$$

Online marketing:

- Expected benefit of targeting $= p_R(x) \times v_R + [1 - p_R(x)] \times v_{NR}$
- Product Price: $200
- Product Cost: $100
- Targeting Cost: $1

Total $T = 110$

Positive $P = 61$ Negative $N = 49$

Positive	Negative
$p(p) = 0.55$	$p(n) = 0.45$
$p(Y\|p) = 56/61 = 0.92$	$p(Y\|n) = 7/49 = 0.14$
$p(N\|p) = 5/61 = 0.08$	$p(N\|n) = 42/49 = 0.86$

$$p_R(\boldsymbol{x}) \times \$99 - [1 - p_R(\boldsymbol{x})] \times \$1 > 0$$
$$p_R(\boldsymbol{x}) > 0.01$$

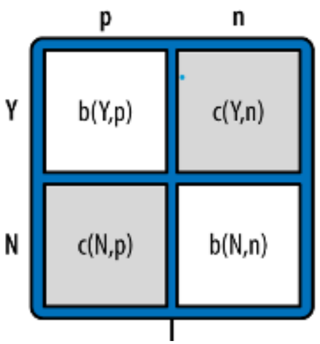
$$\text{ected profit} = p(\boldsymbol{p}) \times [p(\boldsymbol{Y}|\boldsymbol{p}) \times b(\boldsymbol{Y},\boldsymbol{p}) + p(\boldsymbol{N}|\boldsymbol{p}) \times b(\boldsymbol{N},\boldsymbol{p})] + p(\boldsymbol{n}) \times [p(\boldsymbol{N}|\boldsymbol{n}) \times b(\boldsymbol{N},\boldsymbol{n}) + p(\boldsymbol{Y}|\boldsymbol{n}) \times b(\boldsymbol{Y},\boldsymbol{n})]$$

$$= 0.55 \times [0.92 \times b(\boldsymbol{Y},\boldsymbol{p}) + 0.08 \times b(\boldsymbol{N},\boldsymbol{p})] + 0.45 \times [0.86 \times b(\boldsymbol{N},\boldsymbol{n}) + 0.14 \times b(\boldsymbol{Y},\boldsymbol{n})]$$

$$= 0.55 \times [0.92 \times 99 + 0.08 \times 0] + 0.45 \times [0.86 \times 0 + 0.14 \times (-1)]$$

$$= 50.1 - 0.063 \approx \$\mathbf{50.04}$$

A cost-benefit matrix for the marketing example



$$\begin{array}{cc} & \textbf{Actual} \\ \textbf{Predicted} & \begin{array}{cc} & \begin{array}{cc} \mathbf{p} & \mathbf{n} \end{array} \\ \begin{array}{c} \mathbf{Y} \\ \mathbf{N} \end{array} & \begin{pmatrix} 99 & -1 \\ 0 & 0 \end{pmatrix} \end{array} \end{array}$$

b(Y,p) = 200 – 100 – 1 = 99
c(Y, n) = 0 – 1 = -1
c(N,p) and b(N,n) are equal to zero because we didn’t produce nor targeted these people. So they couldn’t buy neither.