



ECE 621

Signaling & Synchronization

Fall 2020

Topic 5

Equalization

Sameh A. Ibrahim
Ain Shams University
ICL

**(Courtesy of K. Yang – UCLA, S. Palermo – TAMU,
and E. Alon – UCB)**

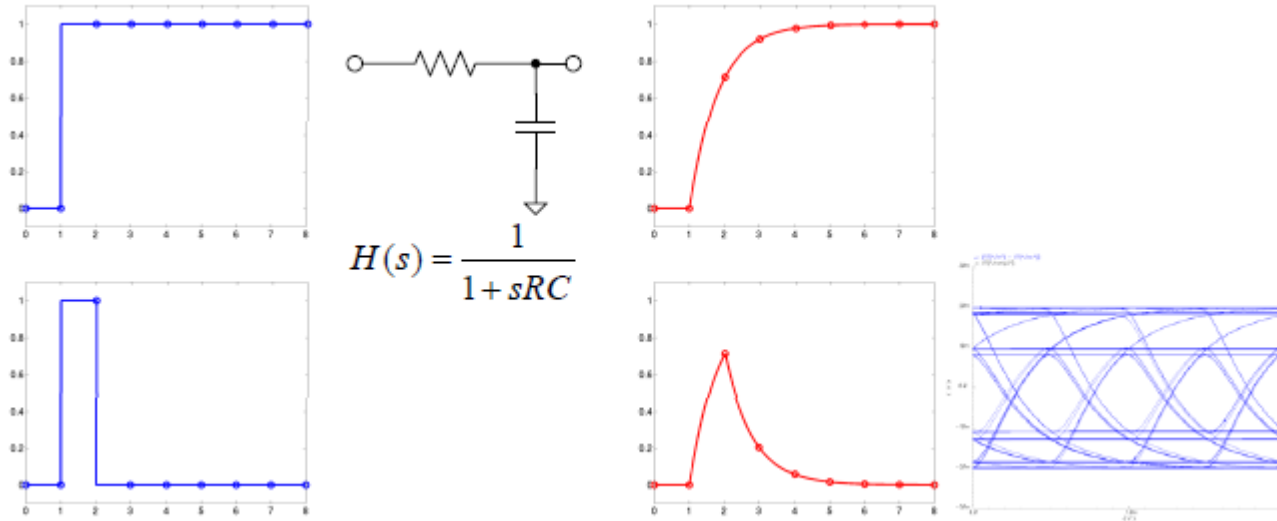
Outline

- **Equalization Theory and Circuits**
 - Overview
 - Classifications
- **Equalizer Implementations**
 - TX FIR
 - RX FIR
 - RX CTLE
 - RX DFE



Basic Concept (1)

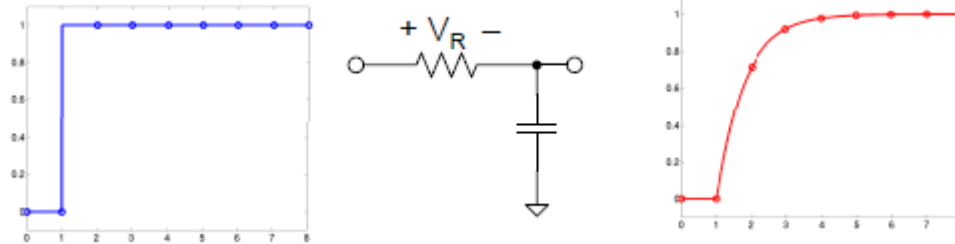
- Channel has limited bandwidth.
- A Simple model of channel: Single-pole lowpass filter



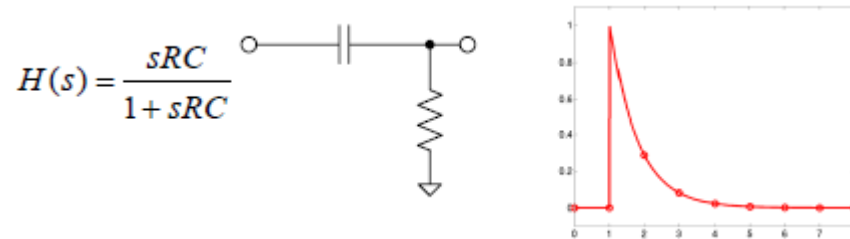
- Bandwidth limitation causes intersymbol interference (ISI)
- Can we transmit signals that exceeds the channel bandwidth?

Basic Concept (2)

- Where is the energy loss?



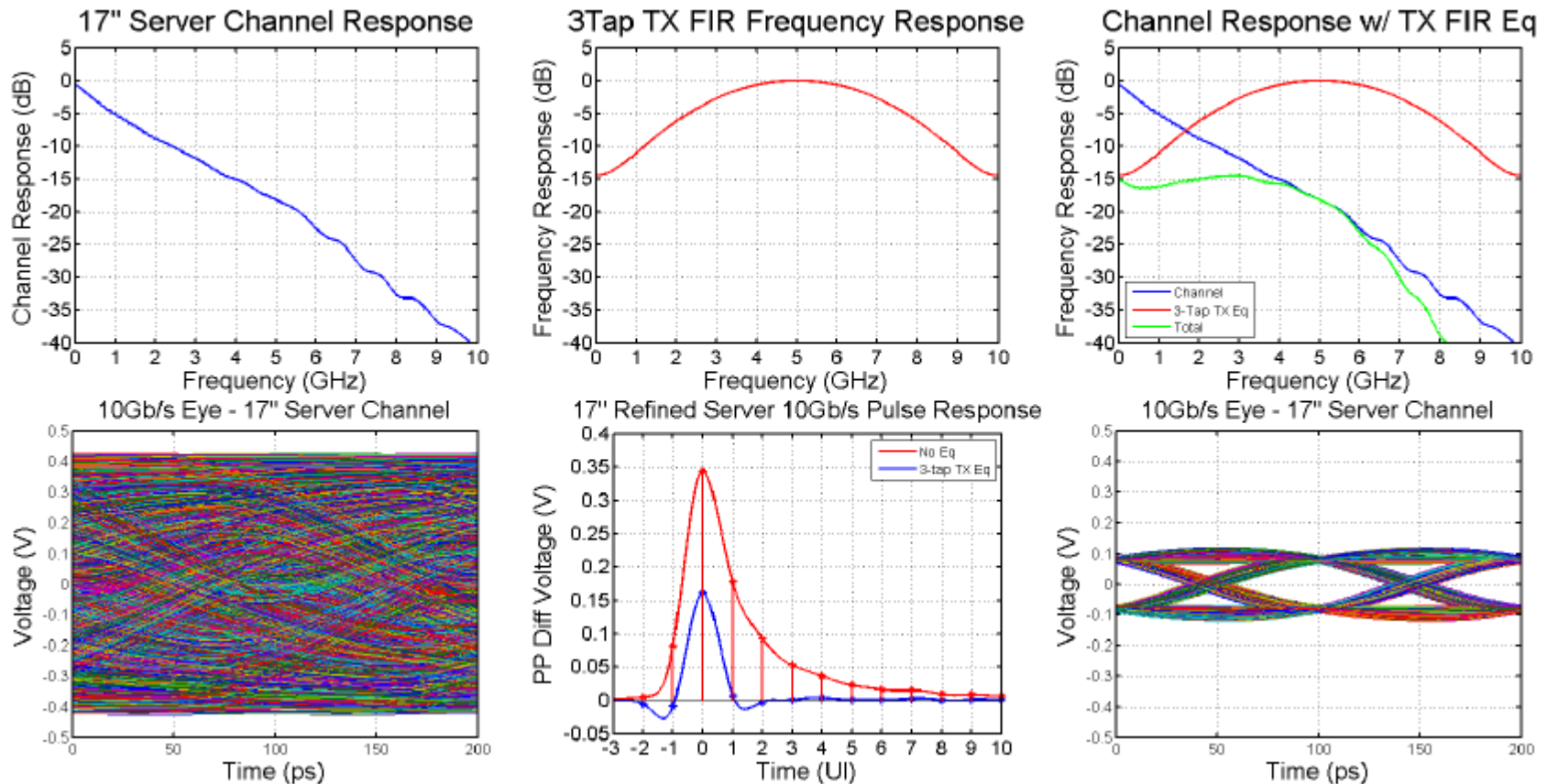
- Energy loss in R .
- If we add V_R back to output, we get the perfect signal.



- This circuit can help us to compensate for the lowpass filtering.
- What is this circuit? Highpass filter

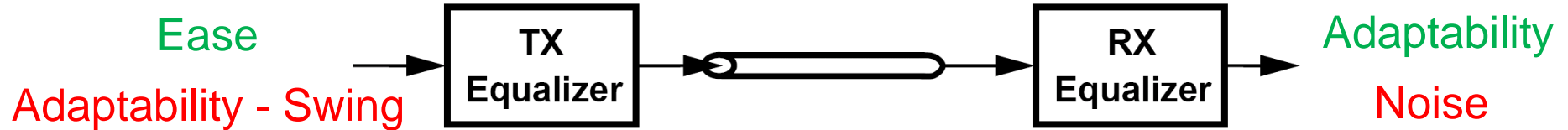
Channel Equalization

- Equalization goal is to flatten the frequency response out to the Nyquist Frequency and remove time-domain ISI.



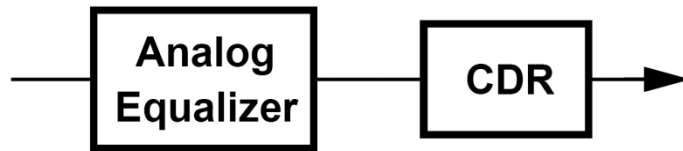
Equalizer Topologies

- TX vs. RX Equalization

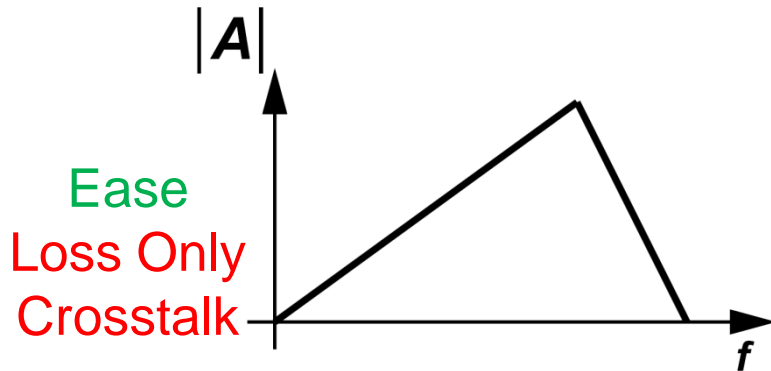


- Digital vs. Analog

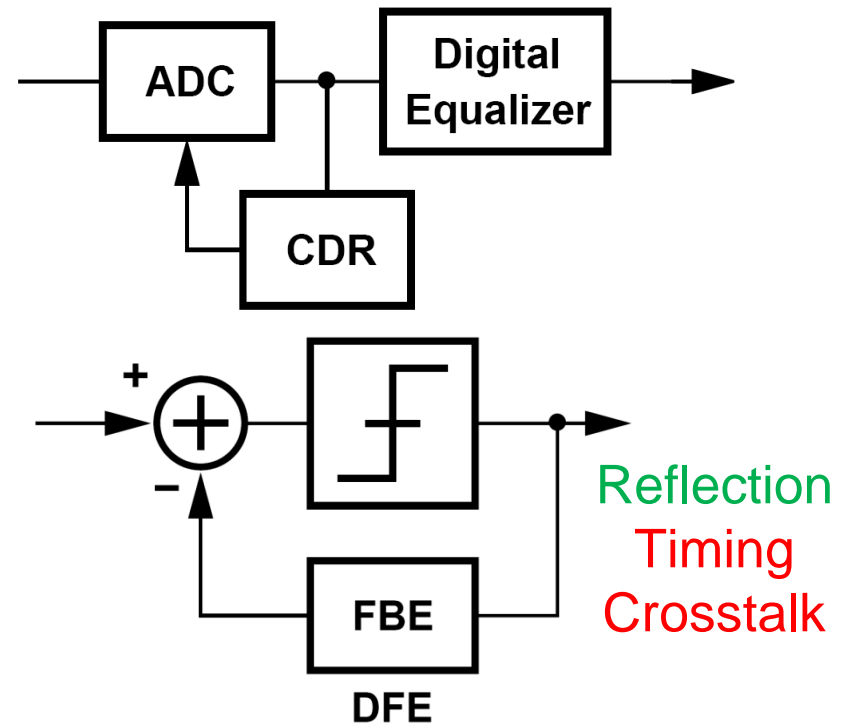
Add/Sub V/I using GmC, RC or L



- Linear vs. Non-Linear



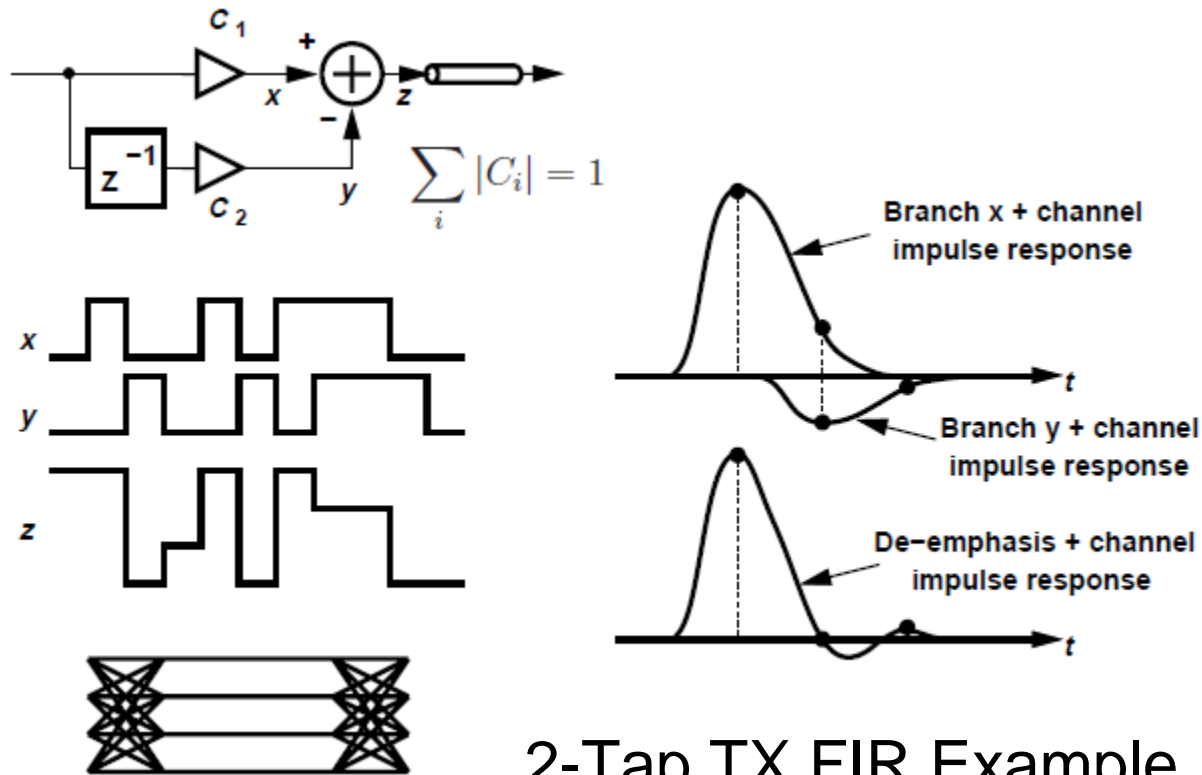
- Continuous vs. Discrete-Time



HPF, RC or L vs. FIR, IIR or DFE

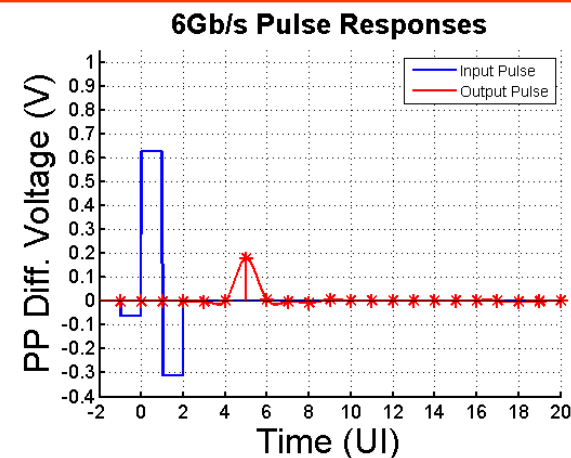
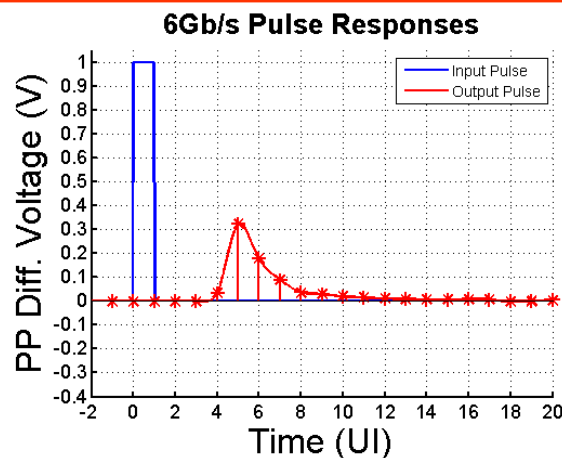
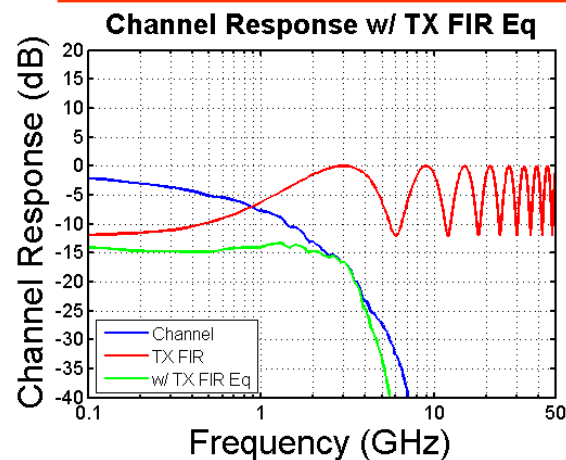
TX FIR Filter

- TX FIR filter pre-distorts transmitted pulse in order to invert channel distortion at the cost of attenuated transmit signal (de-emphasis).

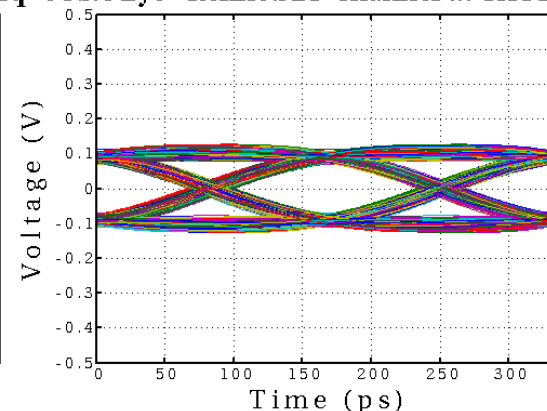
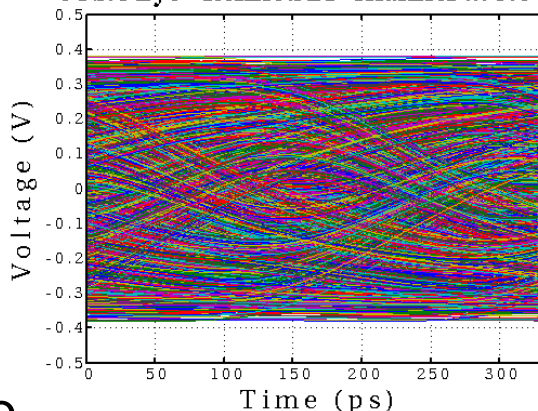


2-Tap TX FIR Example

6 Gb/s TX FIR Equalization Example

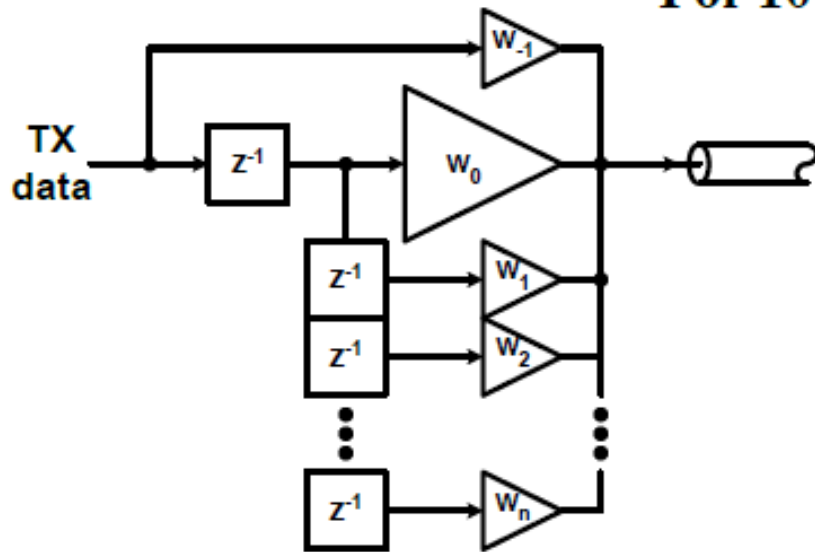


6Gb/s Eye - Refined BP Channel w/ No Eq **6Gb/s Eye - Refined BP Channel w/ TX FIR Eq**



Time-Domain Representation

For 10Gbps : $W(z) = -0.131 + 0.595z^{-1} - 0.274z^{-2}$



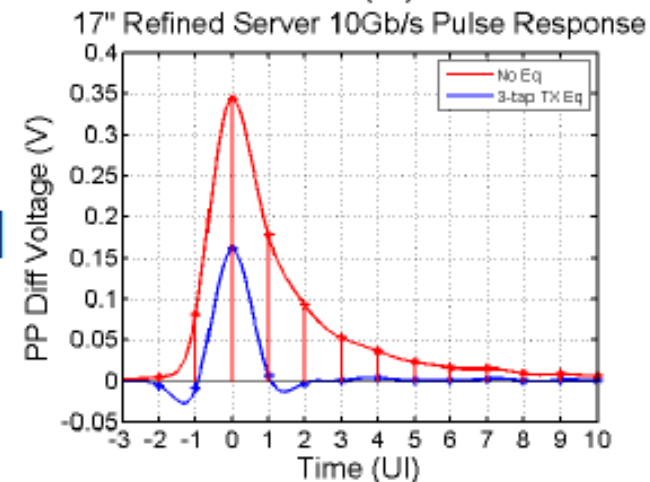
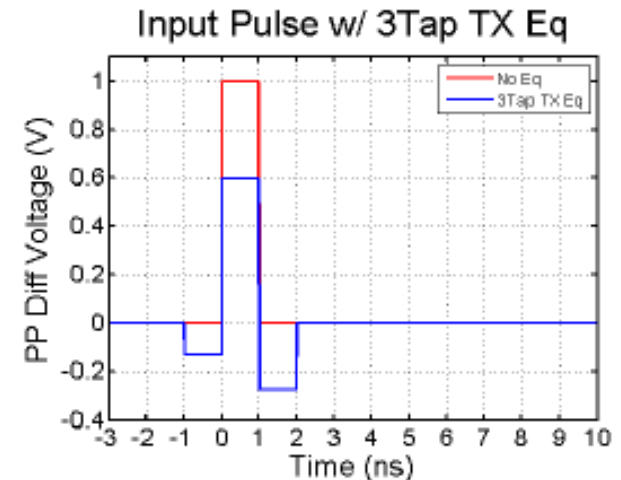
$$W = [-0.131 \quad 0.595 \quad -0.274]$$

Low Frequency Response (Sum Taps)

$$[\dots \quad 1 \quad 1 \quad 1 \quad \dots] * [-0.131 \quad 0.595 \quad -0.274] = [\dots \quad 0.190 \quad 0.190 \quad 0.190 \quad \dots]$$

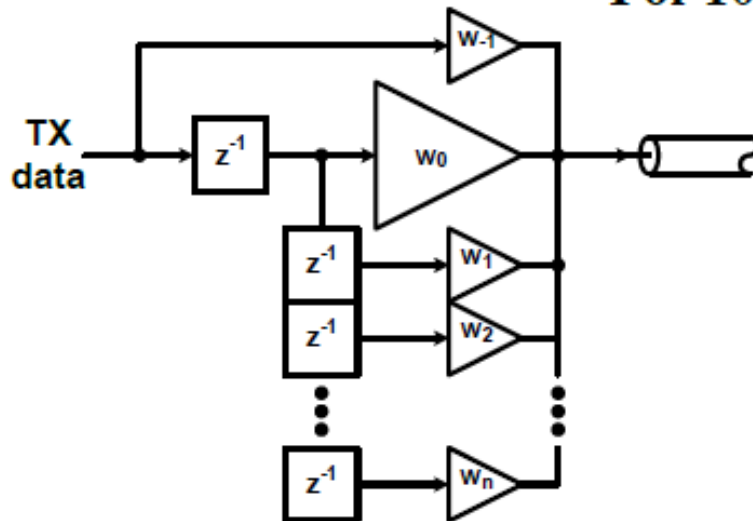
Nyquist Frequency Response (Sum Taps w/ Alternating Polarity)

$$[\dots \quad -1 \quad 1 \quad -1 \quad \dots] * [-0.131 \quad 0.595 \quad -0.274] = [\dots \quad 1 \quad -1 \quad 1 \quad \dots]$$



Frequency-Domain Representation

For 10Gbps : $W(z) = -0.131 + 0.595z^{-1} - 0.274z^{-2}$

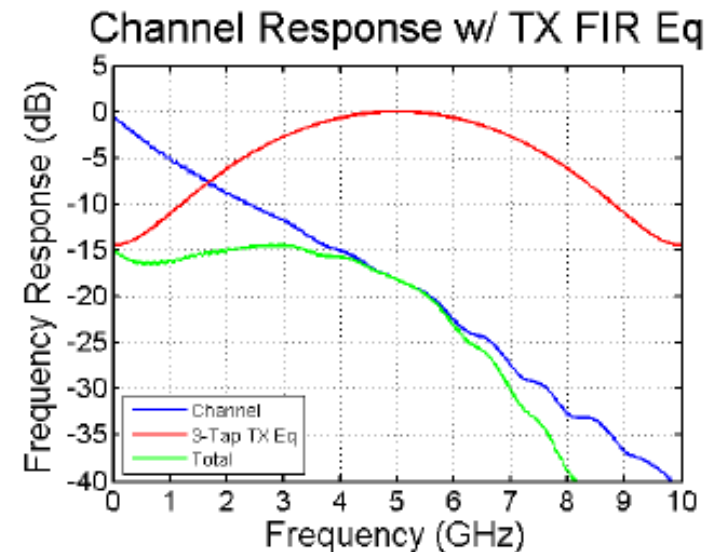


$$W(z) = -0.131 + 0.595z^{-1} - 0.274z^{-2}$$

w/ $z = e^{j2\pi fT_s} = \cos(2\pi fT_s) + j\sin(2\pi fT_s)$

Low Frequency Response ($f = 0$)

$$z = \cos(0) + j\sin(0) = 1 \Rightarrow W(f = 0) = 0.190 \Rightarrow -14.4\text{dB}$$



Nyquist Frequency Response $\left(f = \frac{1}{2T_s}\right)$

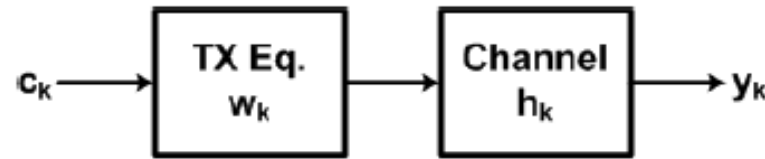
$$z = \cos(\pi) + j\sin(\pi) = -1 \Rightarrow W\left(f = \frac{1}{2T_s}\right) = -1 \Rightarrow 0\text{dB}$$

Note: $T_s = T_b = 100\text{ps}$

- **Equalizer has 14.4dB of frequency peaking**
 - Attenuates DC at -14.4dB and passes Nyquist frequency at 0dB

TX FIR Coefficient Selection (1)

- One approach to set the TX FIR coefficients is a Minimum Mean-Square Error (MMSE) Algorithm



channel output vector, y

Rows = $k+n+\ell-2$

where k = channel pulse model length

$$\begin{bmatrix} y(0) \\ y(1) \\ \dots \\ y(l+n+k-3) \end{bmatrix} = \begin{bmatrix} h(0) & 0 & 0 & \dots & 0 & 0 \\ h(1) & h(0) & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & h(k-1) & h(k-2) \\ 0 & 0 & 0 & \dots & 0 & h(k-1) \end{bmatrix} \begin{bmatrix} w(0) & 0 & 0 & \dots & 0 & 0 \\ w(1) & w(0) & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & w(n-1) & w(n-2) \\ 0 & 0 & 0 & \dots & 0 & w(n-1) \end{bmatrix} \begin{bmatrix} c(0) \\ c(1) \\ \dots \\ c(l-1) \end{bmatrix}$$

TX Eq "w" Matrix

Rows = $n+\ell-1$ where n = tap number

Columns = ℓ = input symbol number

Channel "h" Matrix

Rows = $k+n+\ell-2$

Columns = $n+\ell-1$

ℓ input symbols, c

TX FIR Coefficient Selection (2)

- **Total system**

$$\begin{bmatrix} y(0) \\ y(1) \\ \dots \\ y(l+n+k-3) \end{bmatrix} = \begin{bmatrix} h(0) & 0 & 0 & \dots & 0 & 0 \\ h(1) & h(0) & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & h(k-1) & h(k-2) \\ 0 & 0 & 0 & \dots & 0 & h(k-1) \end{bmatrix} \begin{bmatrix} w(0) & 0 & 0 & \dots & 0 & 0 \\ w(1) & w(0) & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & w(n-1) & w(n-2) \\ 0 & 0 & 0 & \dots & 0 & w(n-1) \end{bmatrix} \begin{bmatrix} c(0) \\ c(1) \\ \dots \\ c(l-1) \end{bmatrix}$$

- **Multiplying input symbols by TX Eq., $wc = w * c$**

$$\begin{bmatrix} y(0) \\ y(1) \\ \dots \\ y(l+n+k-3) \end{bmatrix} = \begin{bmatrix} h(0) & 0 & 0 & \dots & 0 & 0 \\ h(1) & h(0) & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & h(k-1) & h(k-2) \\ 0 & 0 & 0 & \dots & 0 & h(k-1) \end{bmatrix} \begin{bmatrix} wc(0) \\ wc(1) \\ \dots \\ wc(n+l-1) \end{bmatrix}$$

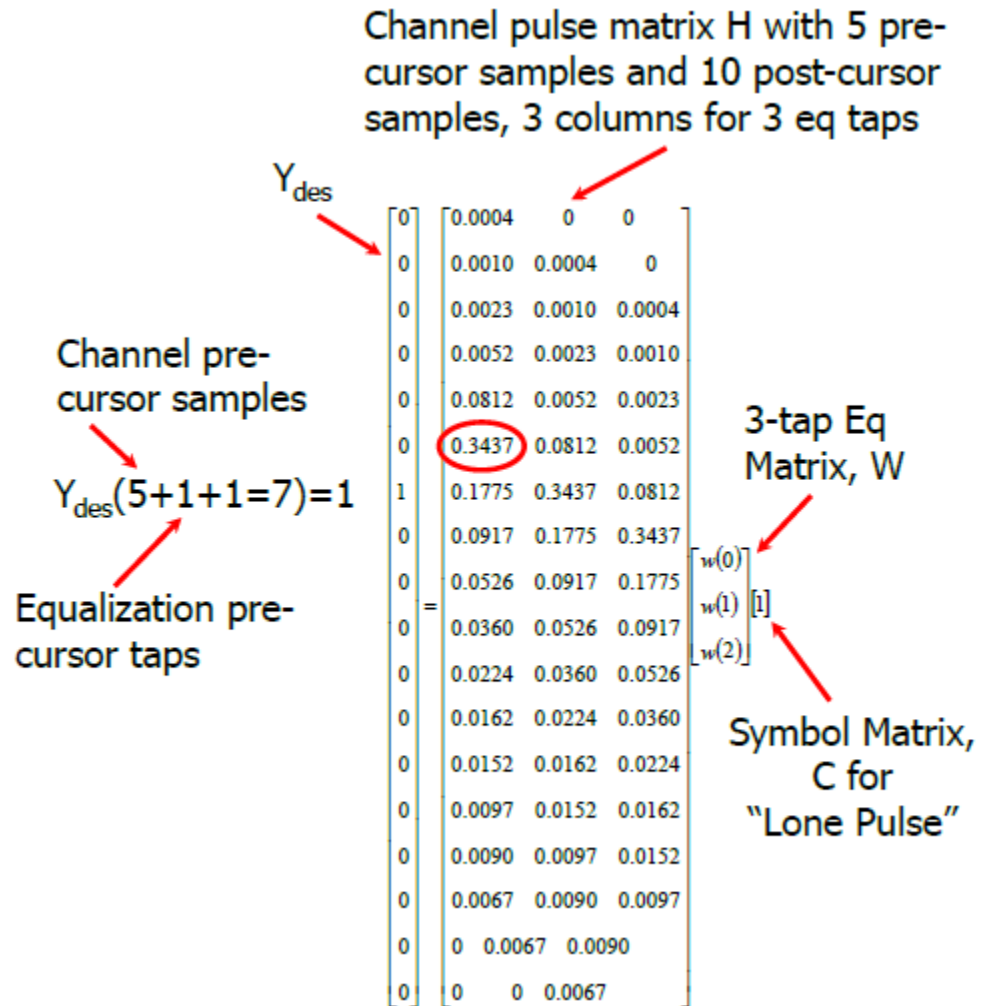
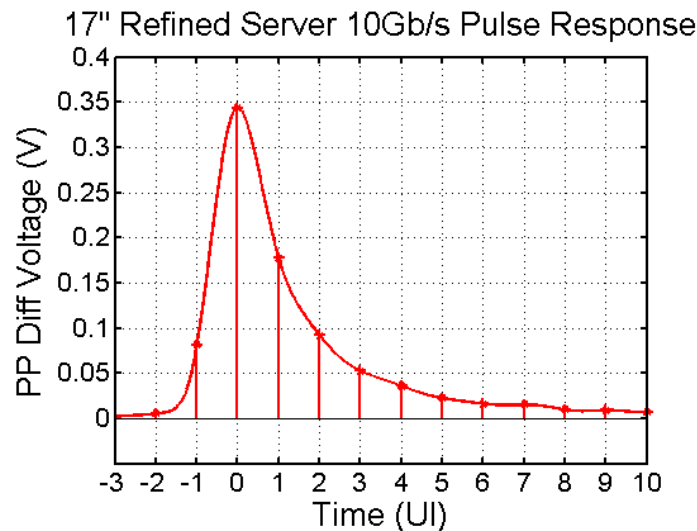
- **We desire the output vector, y , to be ISI free**

$$y_{des} = \begin{cases} y_{des}(n) = 1, n = \text{Channel pre-cursor sample \#} + \text{Eq precursor tap \#} + 1 \\ y_{des}(n) = 0, n \neq \text{Channel pre-cursor sample \#} + \text{Eq precursor tap \#} + 1 \end{cases}$$



Lone-Pulse Equalization Example

- With lone-pulse equalization, $l=1$ input symbols, i.e. $c=[1]$



TX FIR Coefficient Selection (3)

- We can calculate the error w.r.t. a desired output

$$E = Y - Y_{des} = HW_C - Y_{des} = HW - Y_{des} \text{ with pulse input}$$

- Computing the error matrix norm²

$$\|E\|^2 = W^T H^T H W - 2Y_{des}^T H W + Y_{des}^T Y_{des}$$

- Differentiating this w.r.t. tap matrix taps to find taps which yield minimum error norm²

$$\frac{d}{dW} \|E\|^2 = 2W^T H^T H - 2Y_{des}^T H = 0$$

$$W^T H^T H = Y_{des}^T H$$

- Solving for optimum TX Eq taps, W

$$W_{ls} = (H^T H)^{-1} H^T Y_{des}$$

- This will yield a W matrix to produce a value of “1” at the output cursor, i.e. an FIR filter with gain

- Need to normalize by the total abs(tap) sum for TX FIR realization

$$W_{lsnorm}(n) = \frac{W_{ls}(n)}{\sum_{i=1}^n |W_{ls}(n)|}$$

Equalization

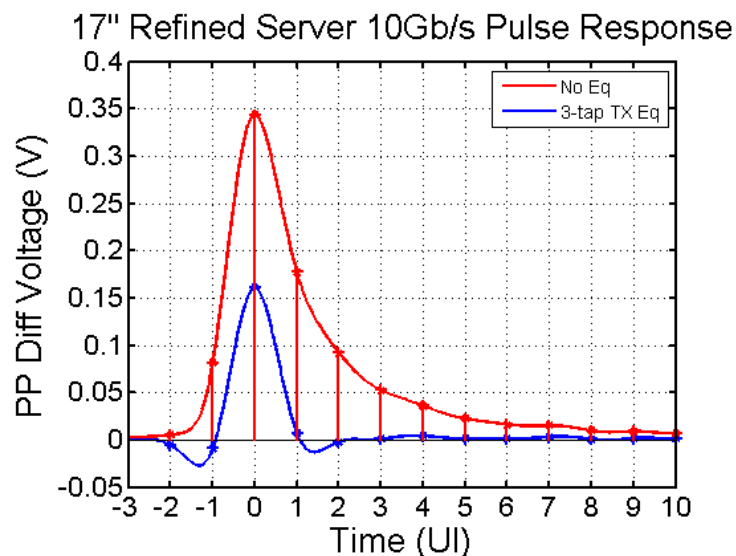


TX FIR Tap Resolution

- Using the above MMSE algorithm for the Refined Server Channel at 10Gb/s

$$W_L = \begin{bmatrix} -0.8180 \\ 3.7245 \\ -1.7184 \end{bmatrix} \xrightarrow{\text{normalizing by 6.2609}} W_{L\text{norm}} = \begin{bmatrix} -0.1307 \\ 0.5949 \\ -0.2745 \end{bmatrix}$$

$$W(z) = -0.131 + 0.595z^{-1} - 0.274z^{-2}$$
$$\begin{bmatrix} 1_{pre} & 1_{main} & 1_{post} \end{bmatrix}$$
$$\begin{bmatrix} -0.131 & 0.595 & -0.274 \end{bmatrix}$$

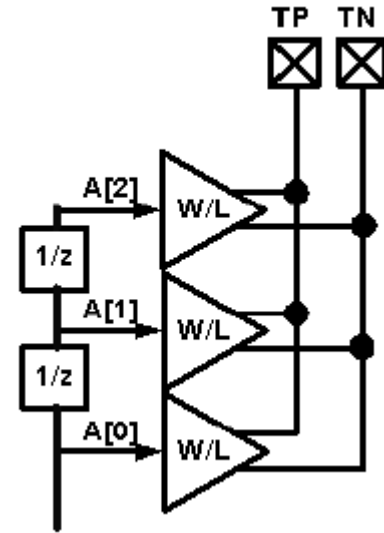


- Generally, TX DAC resolution is limited to between 4 to 6 bits.
- Mapping these equalization coefficients with this resolution may impact performance.

TX FIR Circuit Architectures

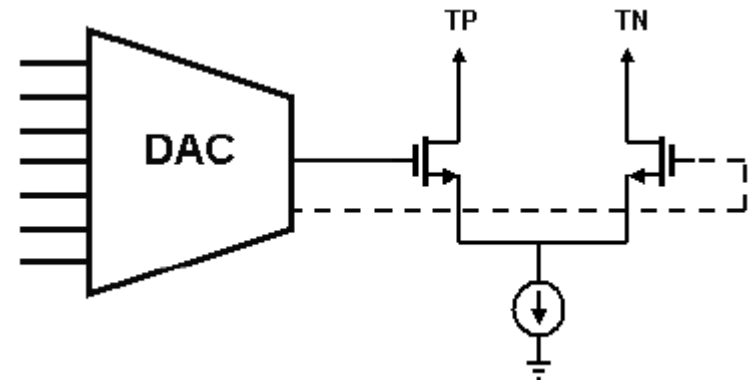
- **Direct FIR**

- Parallel output drivers for output taps
- Each parallel driver must be sized to handle its potential maximum current
- Lower power & complexity
- Higher output capacitance



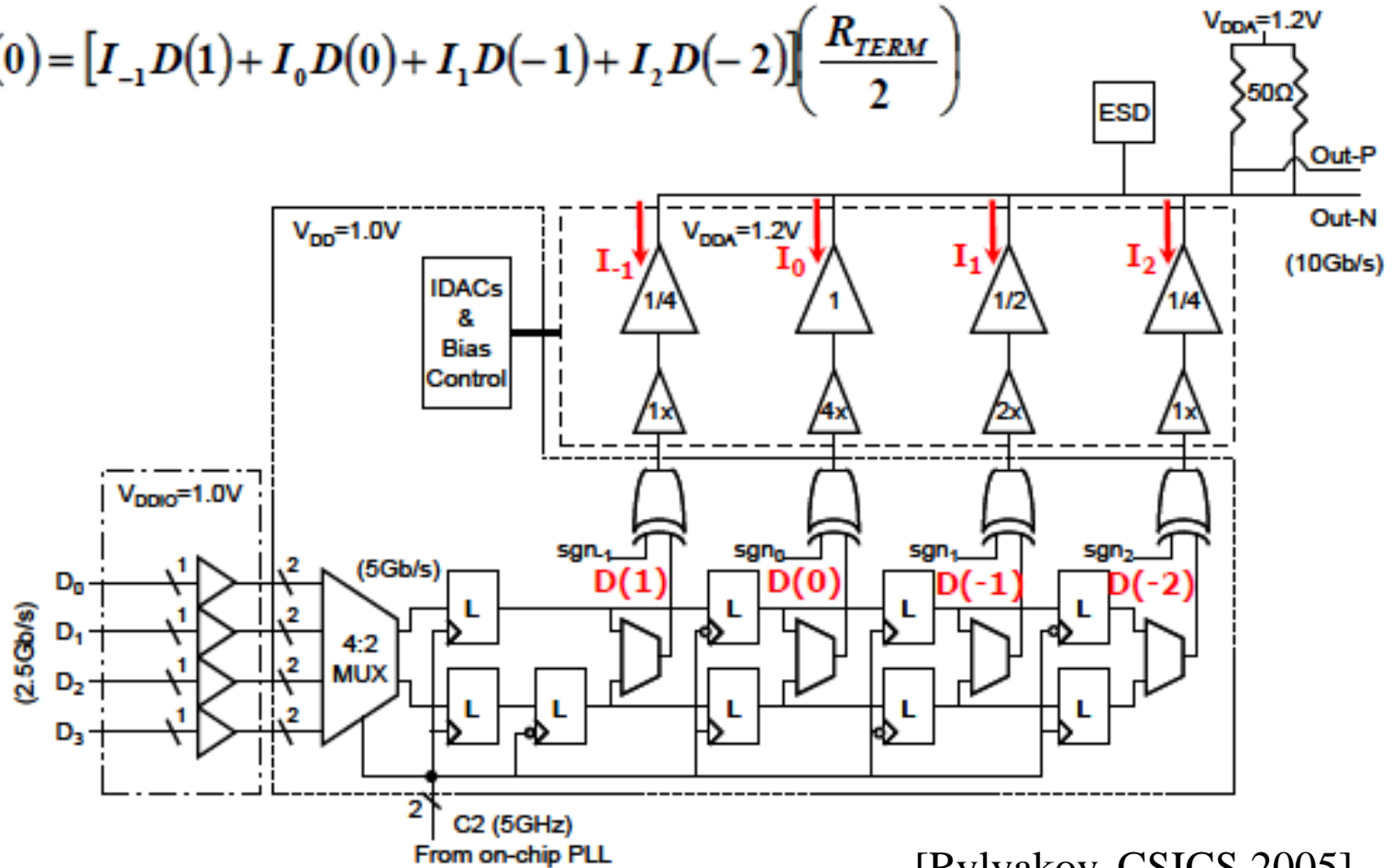
- **Segmented DAC**

- Minimum sized output transistors to handle peak output current
- Lowest output capacitance
- Most power & complexity
- Need mapping table (RAM)
- Very flexible in equalization



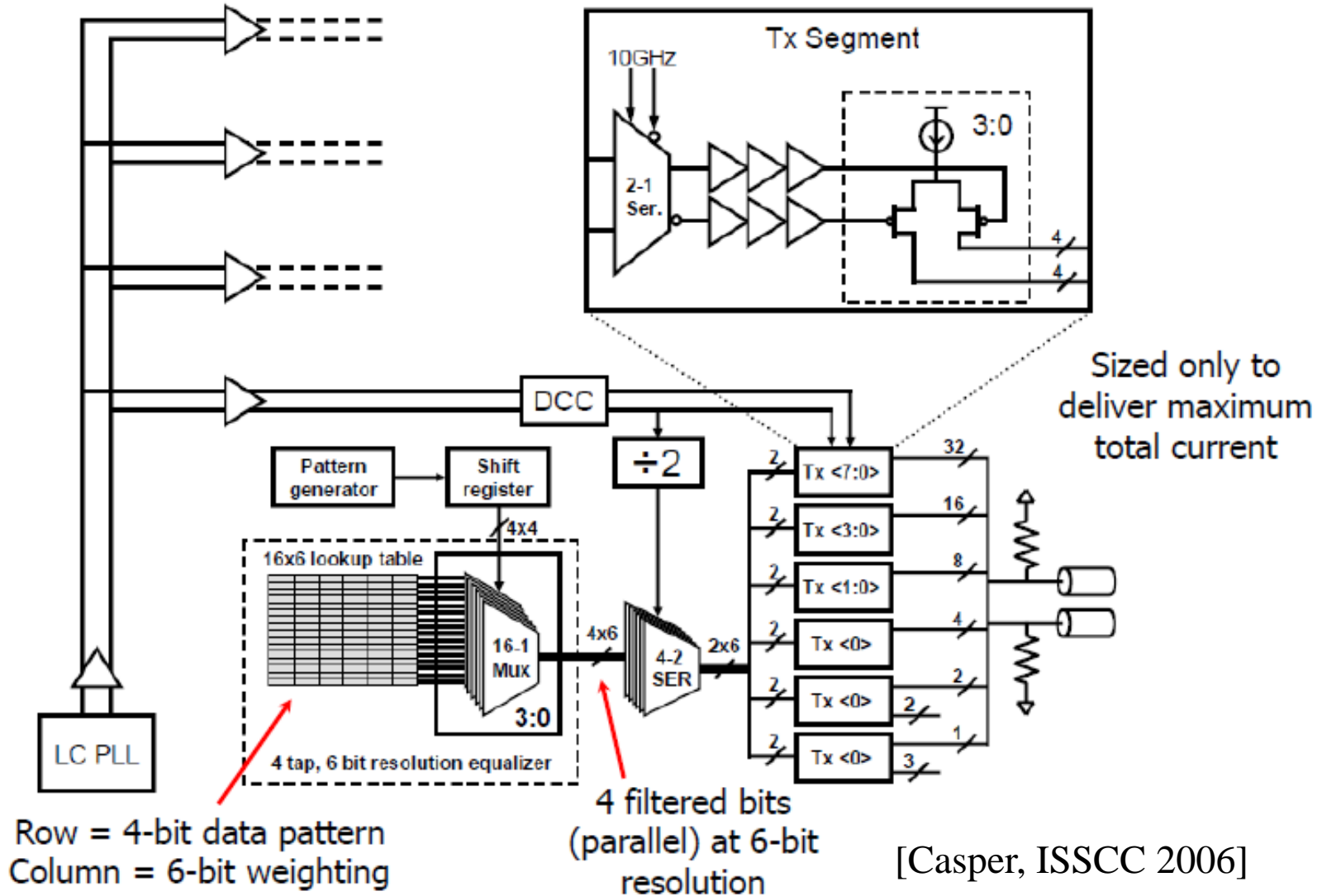
Direct FIR Example

$$V_{out}(0) = [I_{-1}D(1) + I_0D(0) + I_1D(-1) + I_2D(-2)] \left(\frac{R_{TERM}}{2} \right)$$

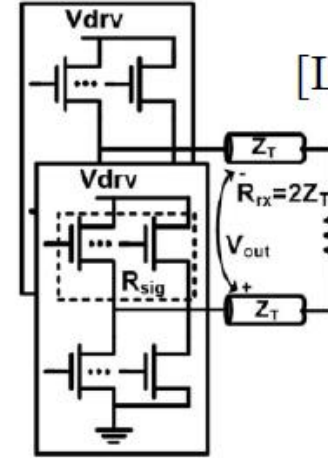
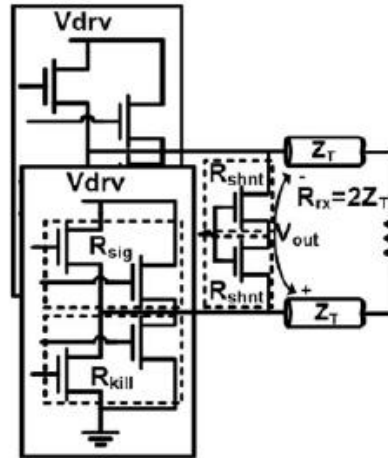
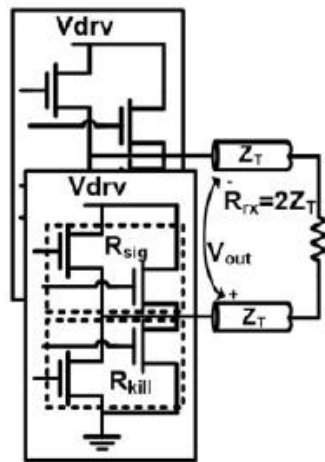


[Rylyakov, CSICS 2005]

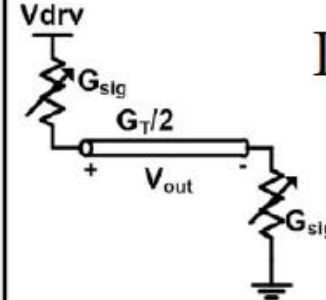
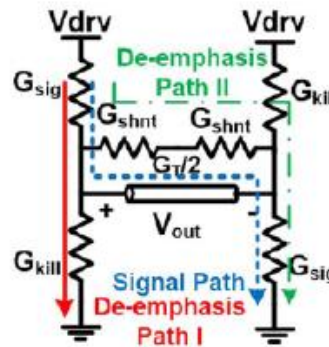
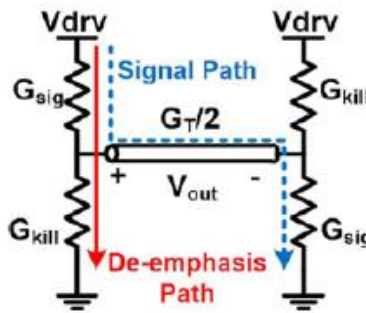
Segmented DAC Example



Pre-Emphasis in VM Drivers



[Lu, JSSC Aug 2013]



Decoders needed

- Reduce output (de-emphasis) by a path to ground in combination with the main signal path.
- Amplitude changes without impedance.
- Power loss is high.
- Shunt impedance in parallel with channel
- G_{kill} can be removed completely.
- Termination is not affected.
- Power is reduced with swing.
- Power is minimum when amplitude reduces.
- But termination is corrupted.

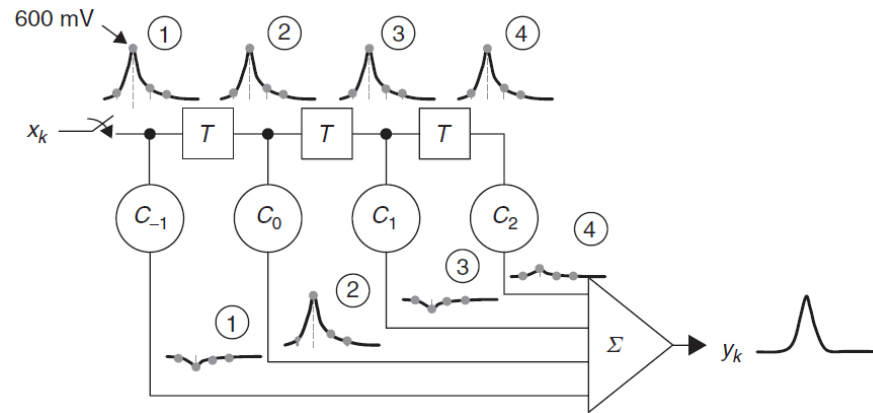
Equalization

Outline

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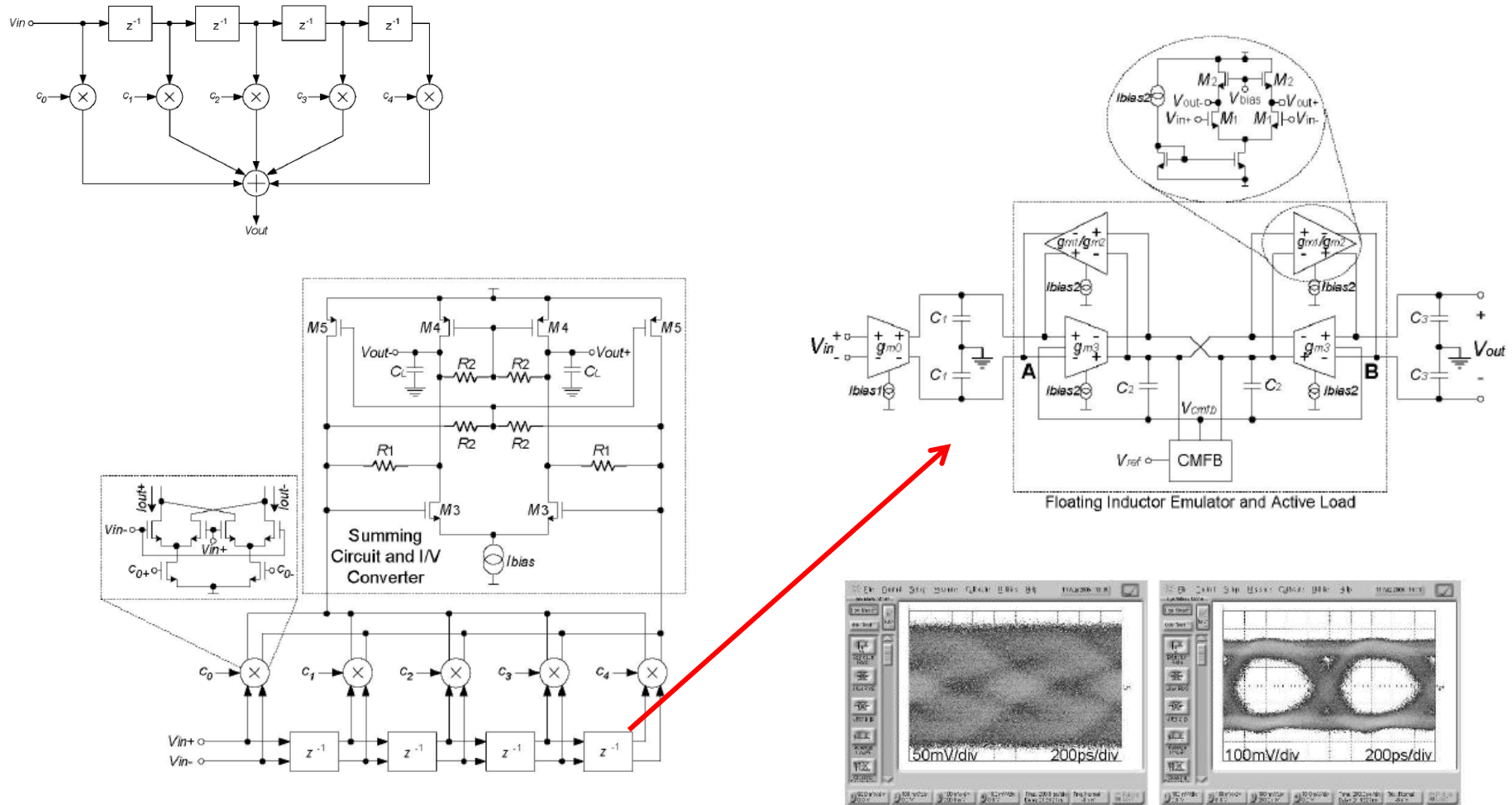
RX FIR Equalization



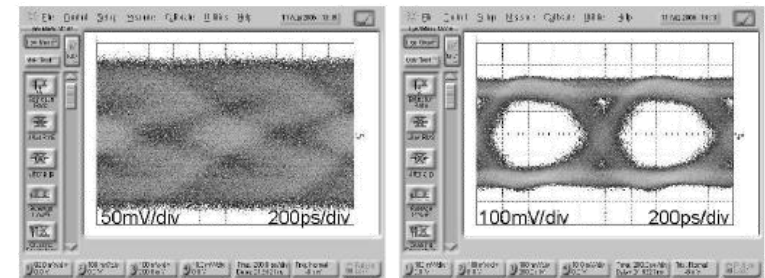
- **Delay analog input signal and multiply by equalization coefficients.**
- **Pros**
 - With sufficient dynamic range, can amplify high frequency content (rather than attenuate low frequencies).
 - Can cancel ISI in pre-cursor and beyond filter span.
 - Filter tap coefficients can be adaptively tuned without any back-channel.
- **Cons**
 - Amplifies noise/crosstalk.
 - Implementation of analog delays
 - Tap precision

Analog RX FIR Equalization Example

- 5-tap equalizer with tap spacing of $T_b/2$



[Hernandez-Garduno, ISSCC 2007]



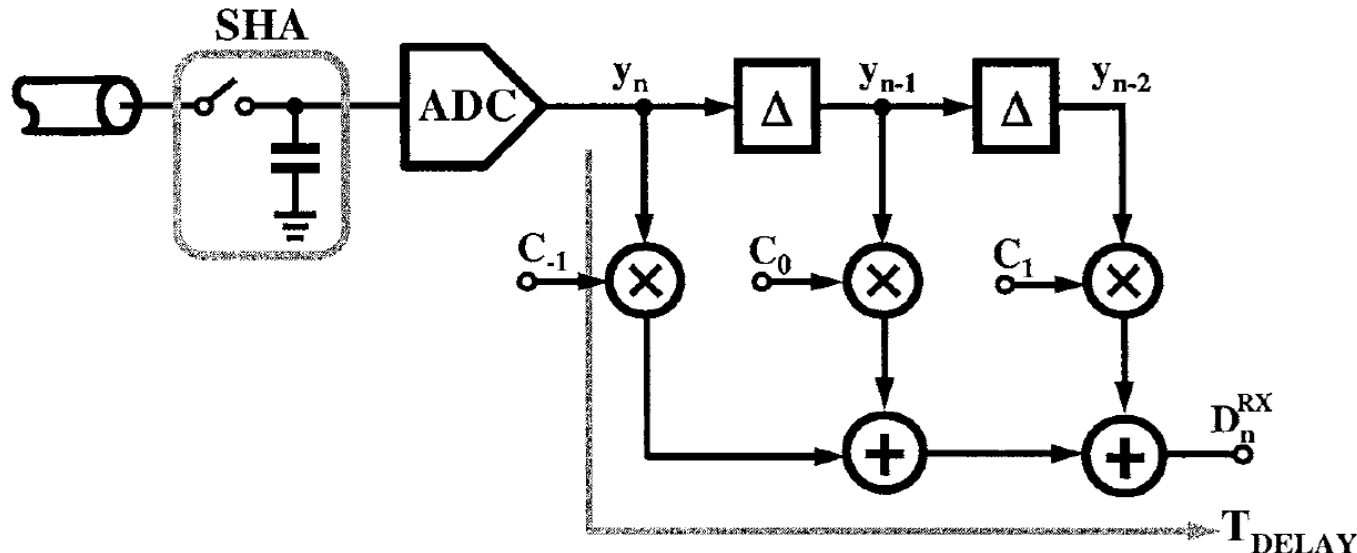
Before Equalizer: 23meters

After Equalizer: 23meters

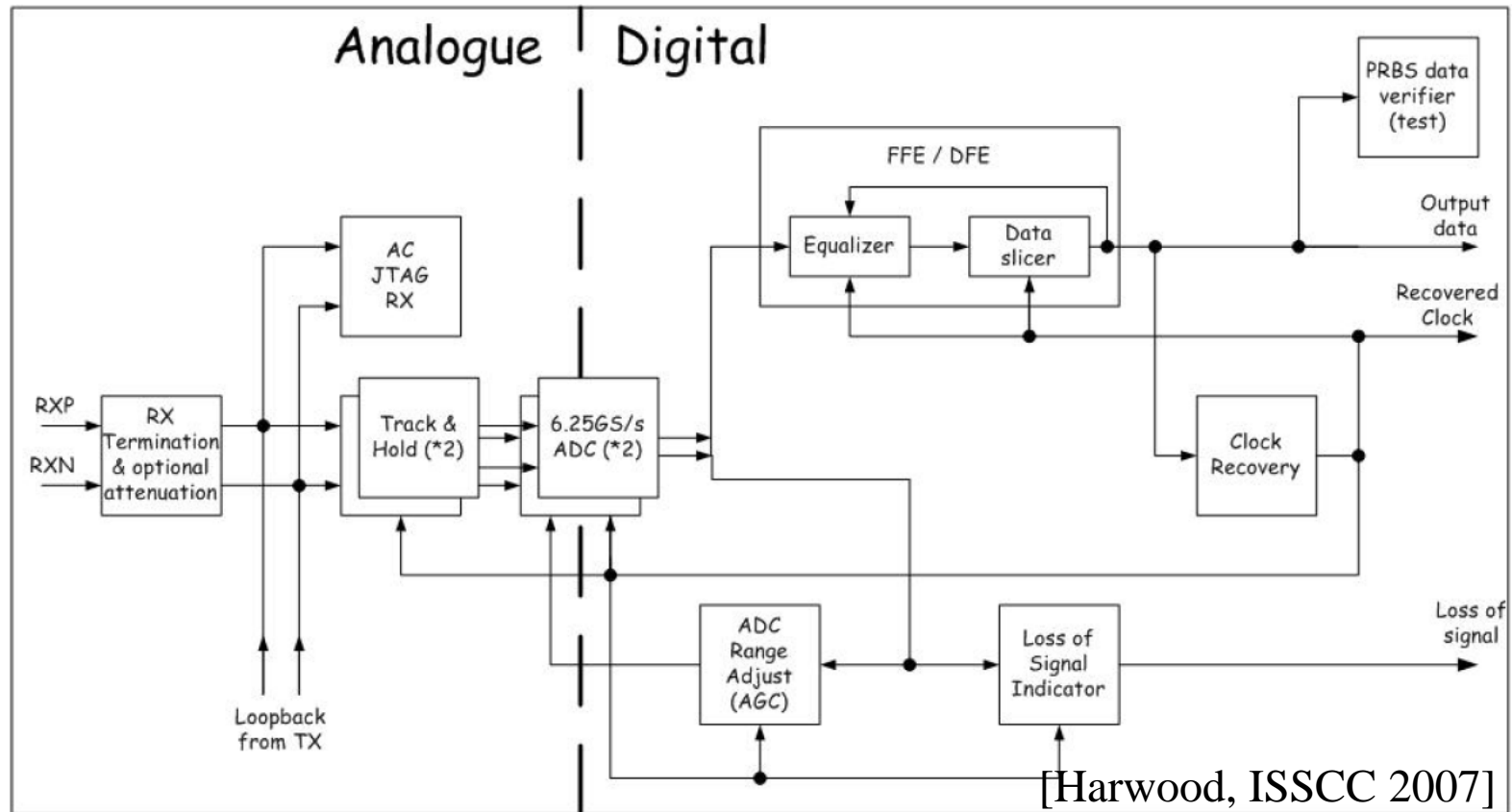
Equalization

Digital RX FIR Equalization

- Digitize the input signal with high-speed low/medium resolution ADC and perform equalization in digital domain
 - Digital delays, multipliers, adders
 - Limited to ADC resolution
- Power can be high due to very fast ADC



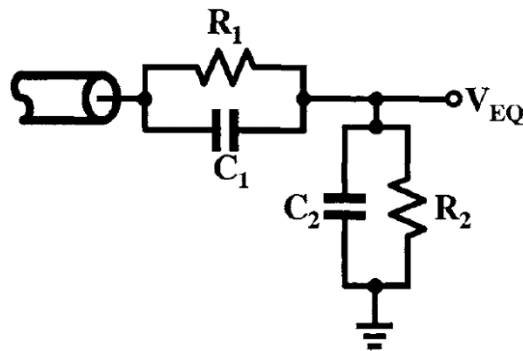
Digital RX FIR Equalization Example



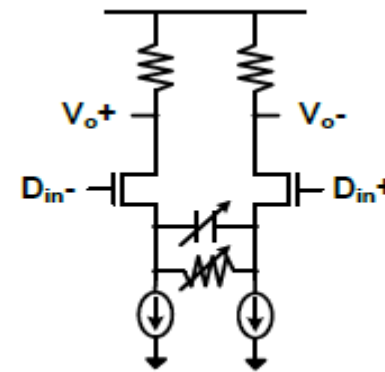
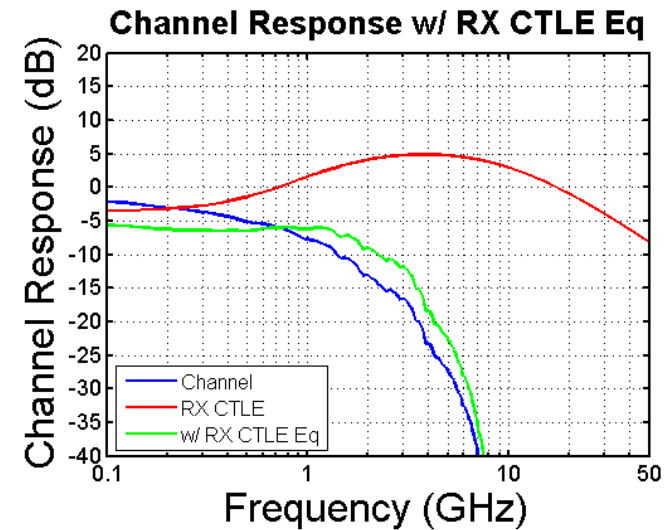
- 12.5GS/s 4.5-bit Flash ADC in 65nm CMOS
- 2-tap FFE & 5-tap DFE
- XCVR power (inc. TX) = 330mW, Analog = 245mW, Digital = 85mW

Continuous-Time Linear Equalizer (CTLE)

- Passive R-C (or L) can implement high-pass transfer function to compensate for channel loss.
- Cancel both precursor and long-tail ISI.
- Can be purely passive or combined with an amplifier to provide gain.



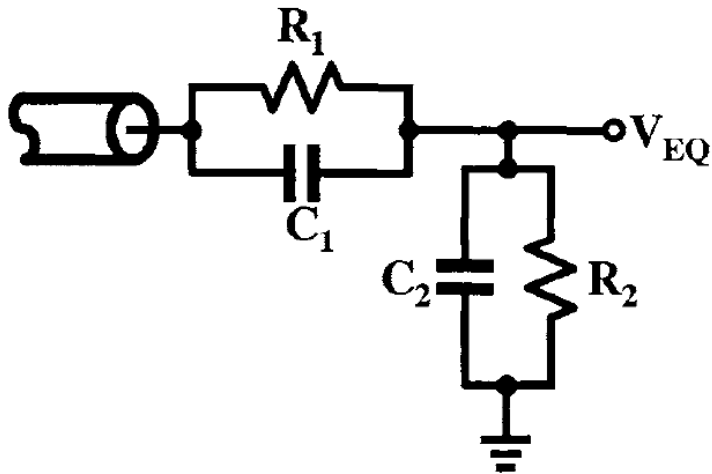
Passive CTLE



Active CTLE

Passive CTLE

- Passive structures offer excellent linearity, but no gain at Nyquist frequency.



$$H(s) = \frac{R_2}{R_1 + R_2} \frac{1 + R_1 C_1 s}{1 + \frac{R_1 R_2}{R_1 + R_2} (C_1 + C_2) s}$$

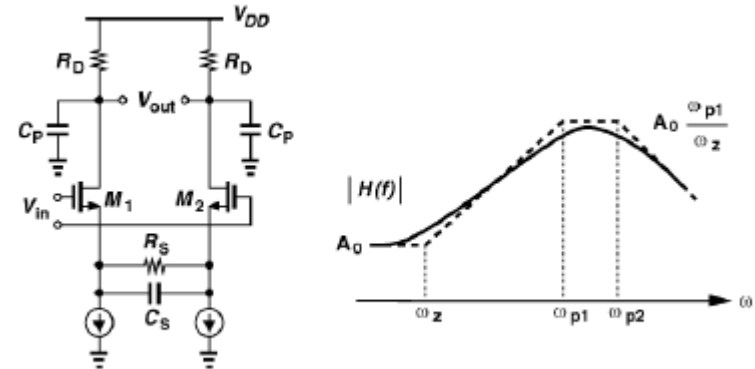
$$\omega_z = \frac{1}{R_1 C_1}, \quad \omega_p = \frac{1}{\frac{R_1 R_2}{R_1 + R_2} (C_1 + C_2)}$$

$$\text{DC gain} = \frac{R_2}{R_1 + R_2}, \quad \text{HF gain} = \frac{C_1}{C_1 + C_2}$$

$$\text{Peaking} = \frac{\text{HF gain}}{\text{DC gain}} = \frac{\omega_p}{\omega_z} = \frac{R_1 + R_2}{R_2} \frac{C_1}{C_1 + C_2}$$

Active CTLE

- Input amplifier with RC degeneration can provide frequency peaking with gain at Nyquist frequency.
- Potentially limited by gain-bandwidth of amplifier.
- Amplifier must be designed for input linear range.
- Often TX eq. provides some low frequency attenuation.
- Sensitive to PVT variations and can be hard to tune.
- Generally limited to 1st-order compensation.



$$H(s) = \frac{g_m}{C_p} \frac{s + \frac{1}{R_S C_S}}{\left(s + \frac{1 + g_m R_S / 2}{R_S C_S} \right) \left(s + \frac{1}{R_D C_p} \right)}$$

$$\omega_z = \frac{1}{R_S C_S}, \quad \omega_{p1} = \frac{1 + g_m R_S / 2}{R_S C_S}, \quad \omega_{p2} = \frac{1}{R_D C_p}$$

$$\text{DC gain} = \frac{g_m R_D}{1 + g_m R_S / 2}, \quad \text{Ideal peak gain} = g_m R_D$$

$$\text{Ideal Peaking} = \frac{\text{Ideal peak gain}}{\text{DC gain}} = \frac{\omega_{p1}}{\omega_z} = 1 + g_m R_S / 2$$

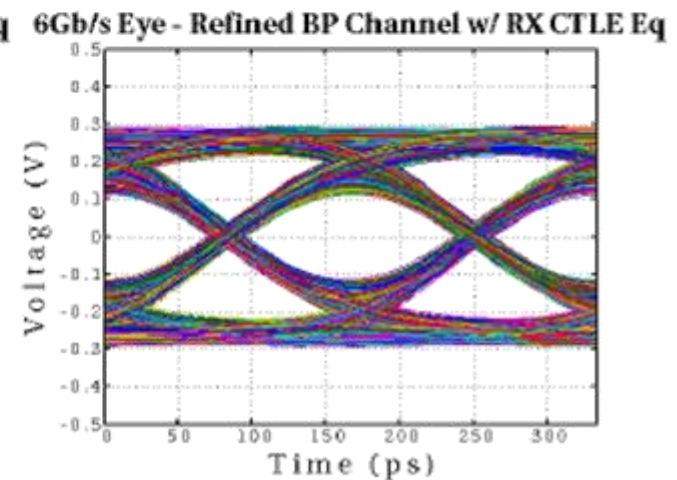
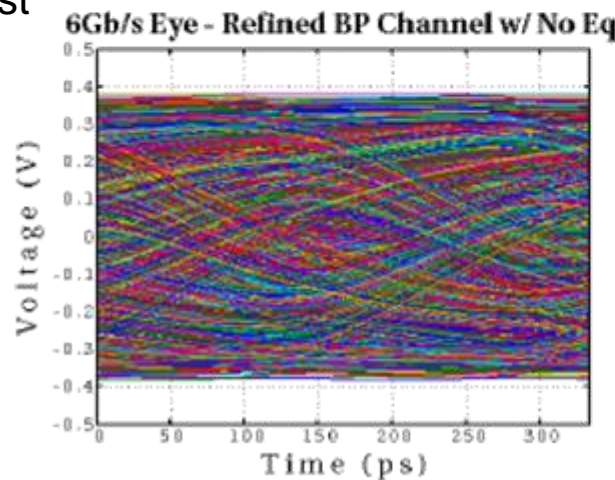
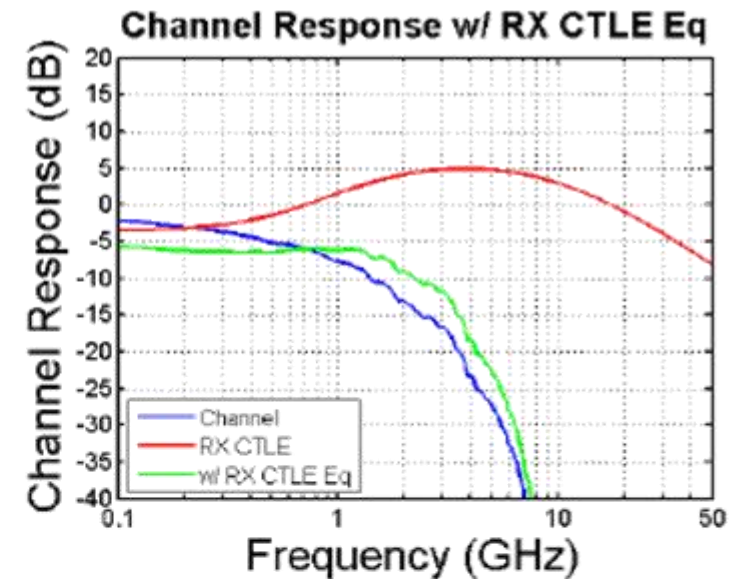
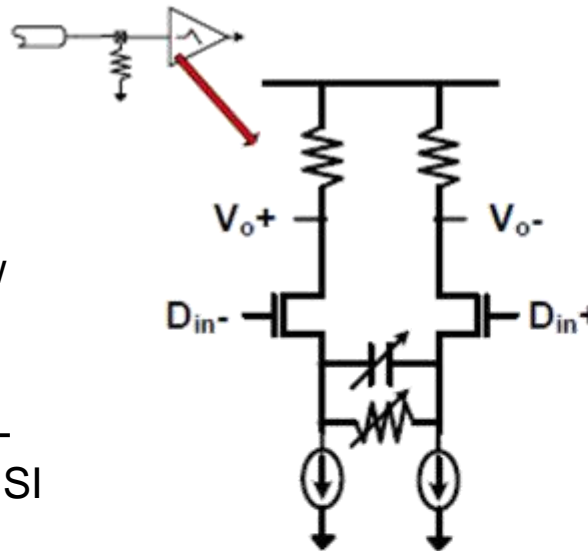
Active CTLE Example

- Pros

- Provides gain and equalization with low power and area overhead
- Can cancel both pre-cursor and long-tail ISI

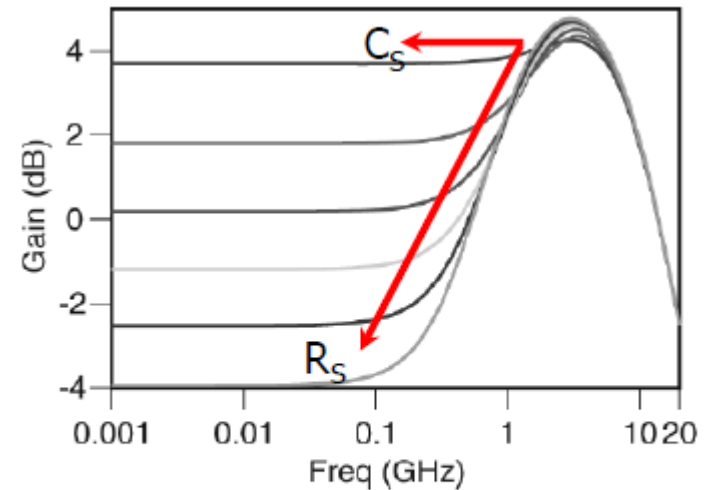
- Cons

- Generally limited to 1st order compensation
- Amplifies noise/crosstalk
- PVT sensitivity
- Can be hard to tune



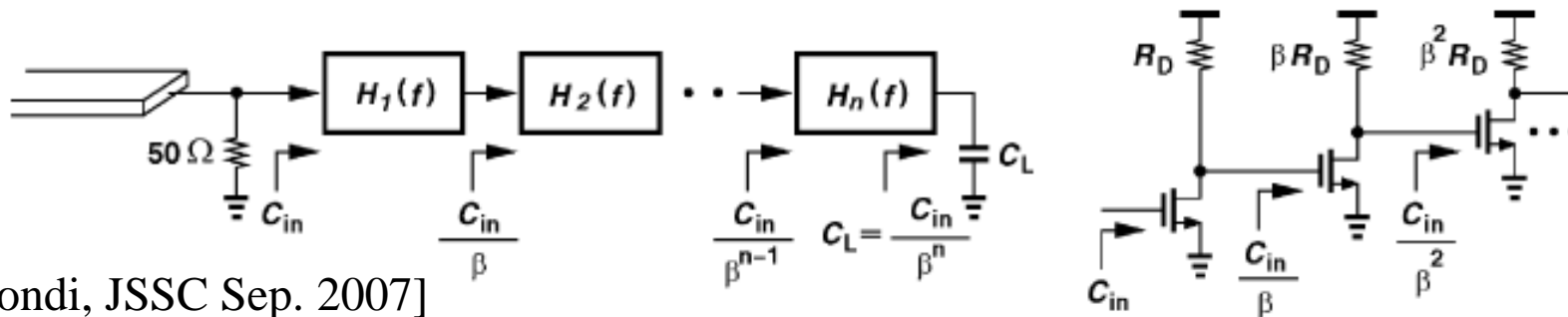
Active CTLE Tuning

- Tune degeneration resistor and capacitor to adjust zero frequency and 1st pole which sets peaking and DC gain.
- Increasing C_S moves zero and 1st pole to a lower frequency w/o impacting (ideal) peaking.
- Increasing R_S moves zero to lower frequency and increases peaking (lowers DC gain)
 - Minimal impact on 1st pole



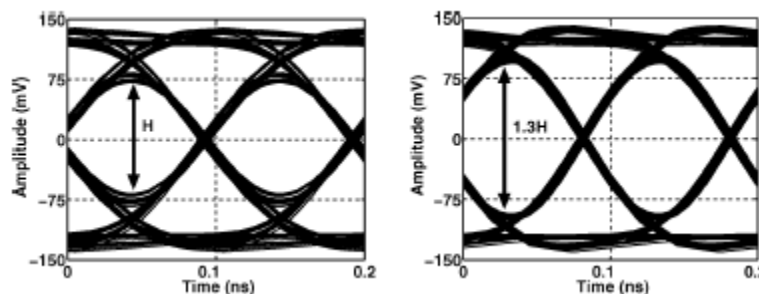
$$\omega_z = \frac{1}{R_S C_S}, \quad \omega_{p1} = \frac{1 + g_m R_S / 2}{R_S C_S}$$

Active CTLE Cascading



[Gondi, JSSC Sep. 2007]

- Multiple stages are required for higher peaking with reasonable DC gain.
- Some gain stages can be used.
- Reverse scaling provides bandwidth improvement in applications where the input impedance need not be very high.

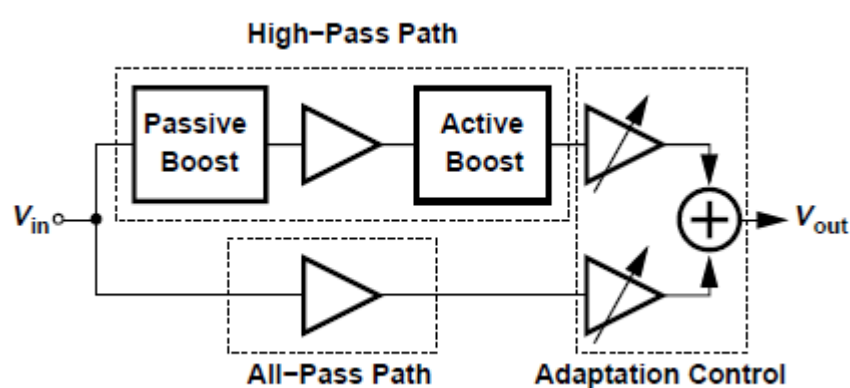


Not Scaled

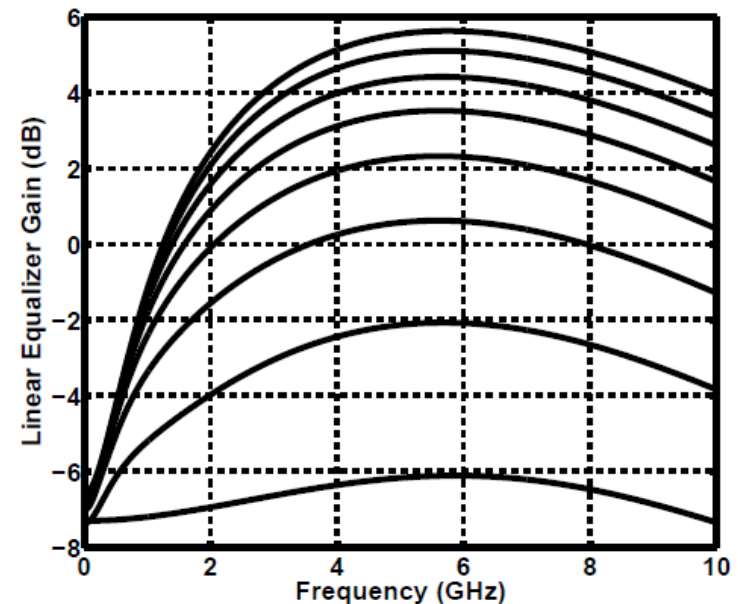
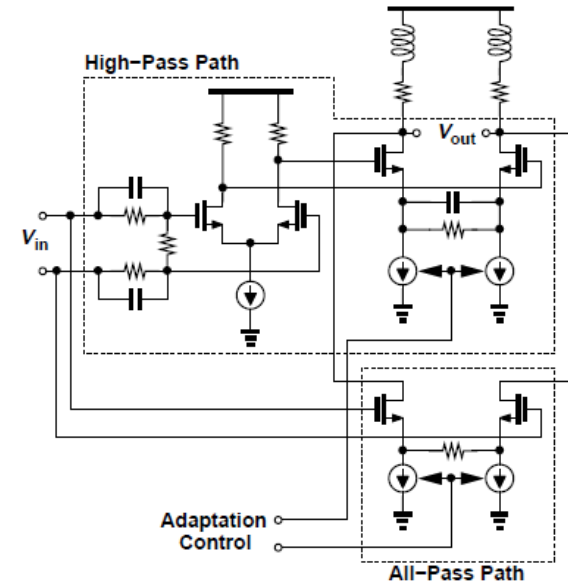
Scaled

Equalization

Dual-Path CTLE



- Adding a high-pass filter to an all-pass filter results in peaking.
- Phase matching between the two paths is required, especially for cascaded stages.
- Tuning is done by varying the relative gains.



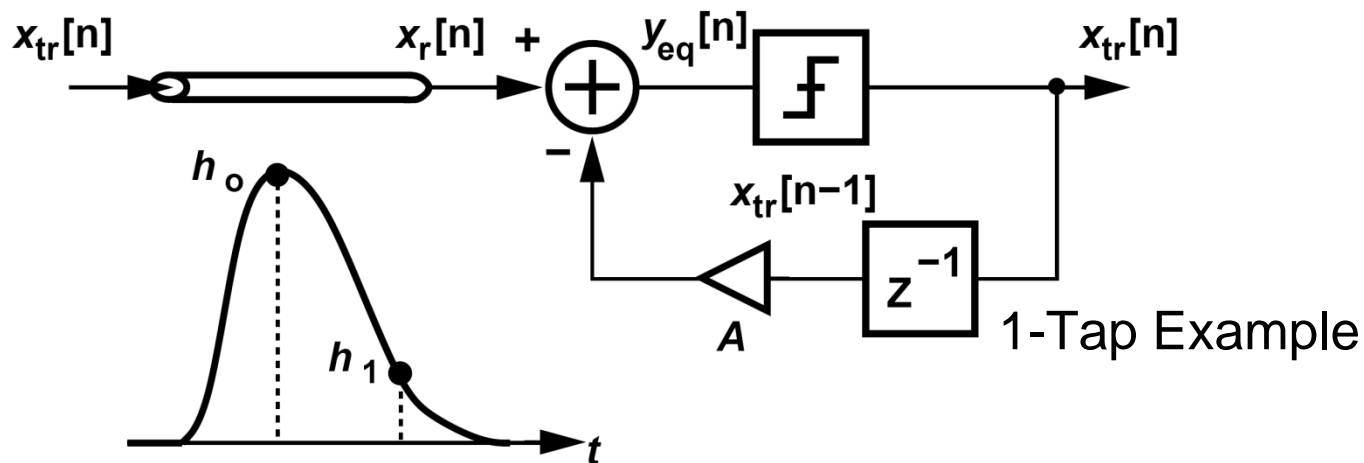
Equalization

Outline

- Equalization Theory and Circuits
 - Overview
 - Classifications
- Equalizer Implementations
 - TX FIR
 - RX FIR
 - RX CTLE
 - RX DFE



Decision Feedback Equalizer (DFE)



$$x_r[n] = h_0 x_{tr}[n] + h_1 x_{tr}[n-1]$$

$$y_{eq}[n] = x_r[n] - A x_{tr}[n-1]$$

$$= h_0 x_{tr}[n] + h_1 x_{tr}[n-1] - A x_{tr}[n-1]$$

- DFE is a non-linear equalizer.
- Slicer makes a symbol decision, i.e. quantizes input.
- ISI is then directly subtracted from the incoming signal via a feedback FIR filter.

RX DFE Pros & Cons

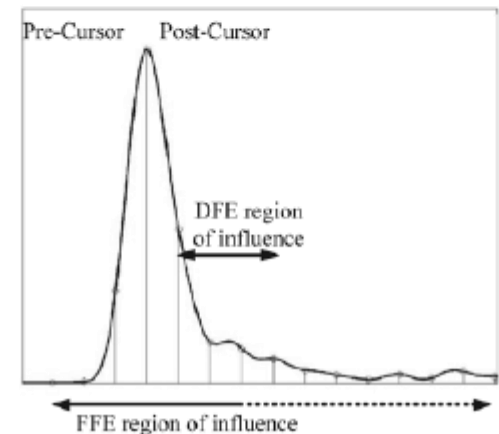
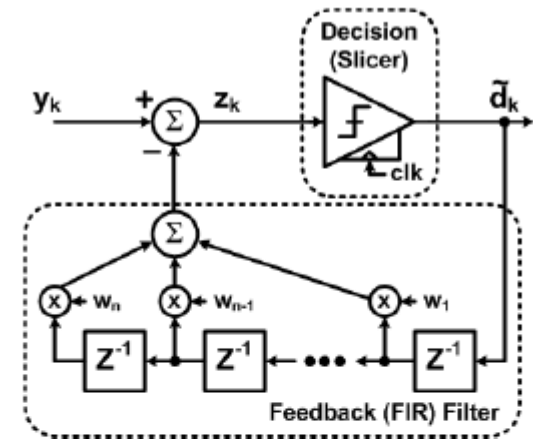
• Pros

- Can boost high frequency content without noise and crosstalk amplification.
- Filter tap coefficients can be adaptively tuned without any back-channel.
- Corrects both loss and reflections.

• Cons

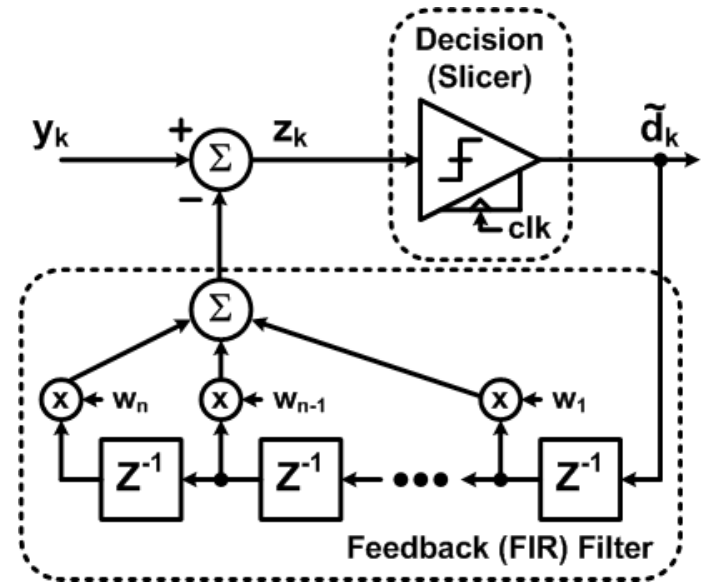
- Cannot cancel pre-cursor ISI.
- Chance for error propagation
 - Low in practical links (BER=10⁻¹²)
- Critical feedback timing path
- Timing of ISI subtraction complicates CDR phase detection.
- High power (especially, digital)

$$z_k = y_k - w_1 \tilde{d}_{k-1} \cdots - w_{n-1} \tilde{d}_{k-(n-1)} - w_n \tilde{d}_{k-n}$$



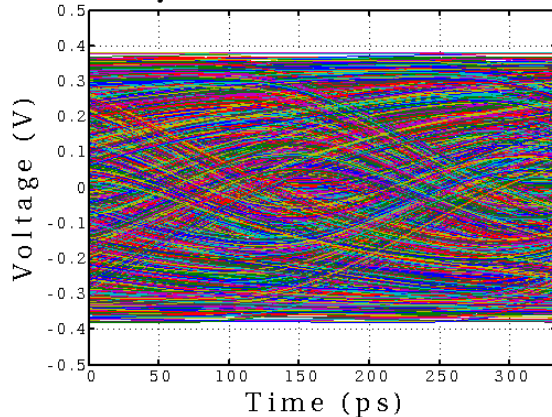
DFE Example

- If only DFE equalization, DFE tap coefficients should equal the unequalized channel pulse response values $[a_1 \ a_2 \ \dots \ a_n]$.
- With other equalization, DFE tap coefficients should equal the pre-DFE pulse response values.

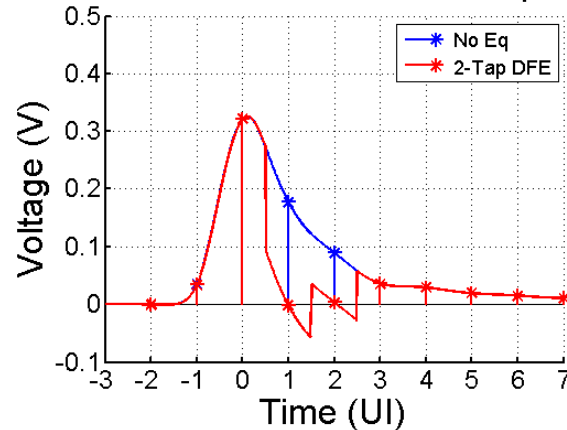


$$[w_1 \ w_2] = [a_1 \ a_2]$$

6Gb/s Eye - Refined BP Channel w/ No Eq

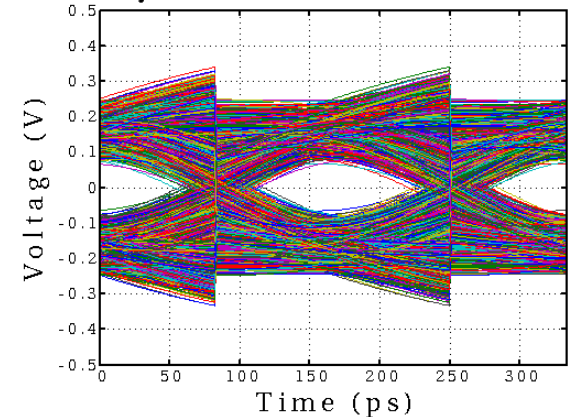


Refined BP Channel 6Gb/s Pulse Responses

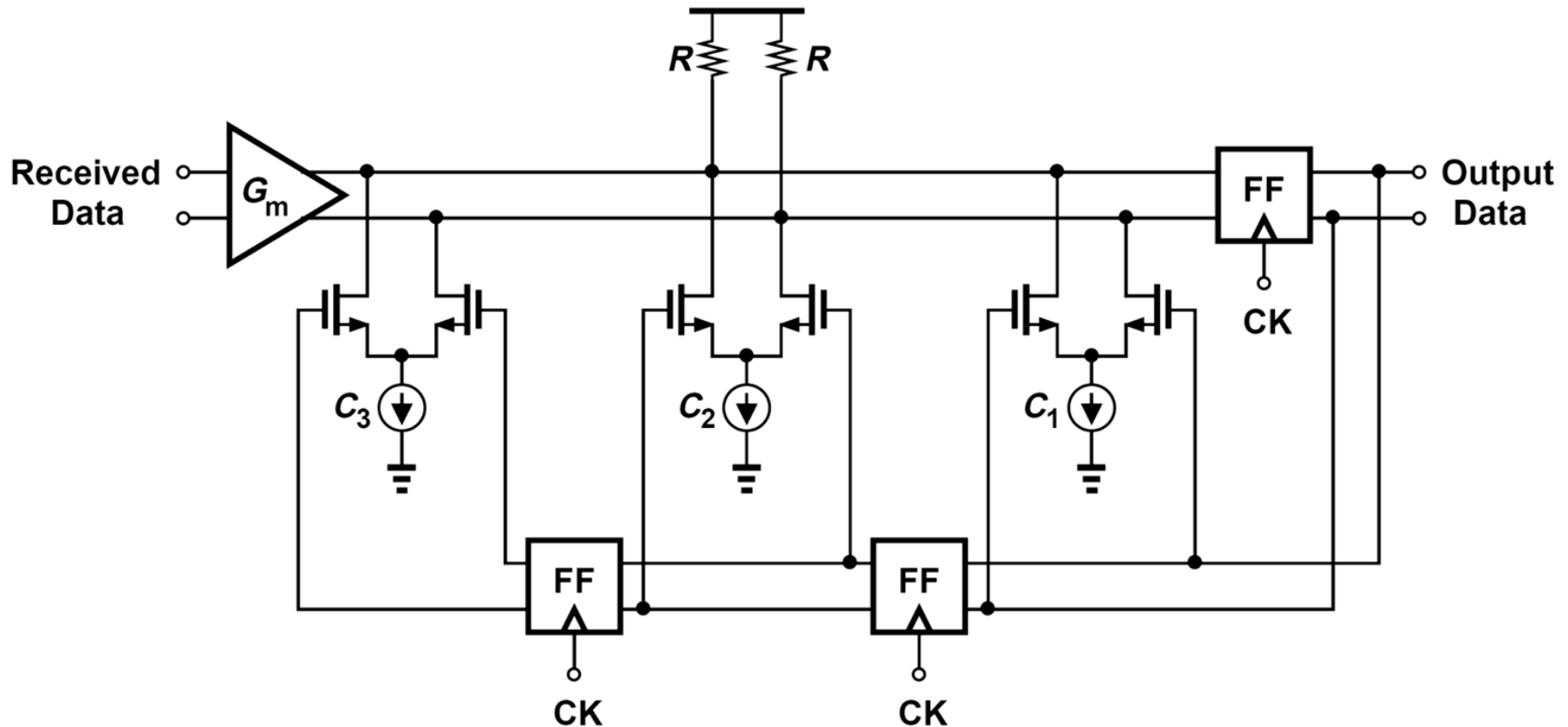


Equalization

6Gb/s Eye - Refined BP Channel w/ RX DFE Eq



Full-Rate DFE



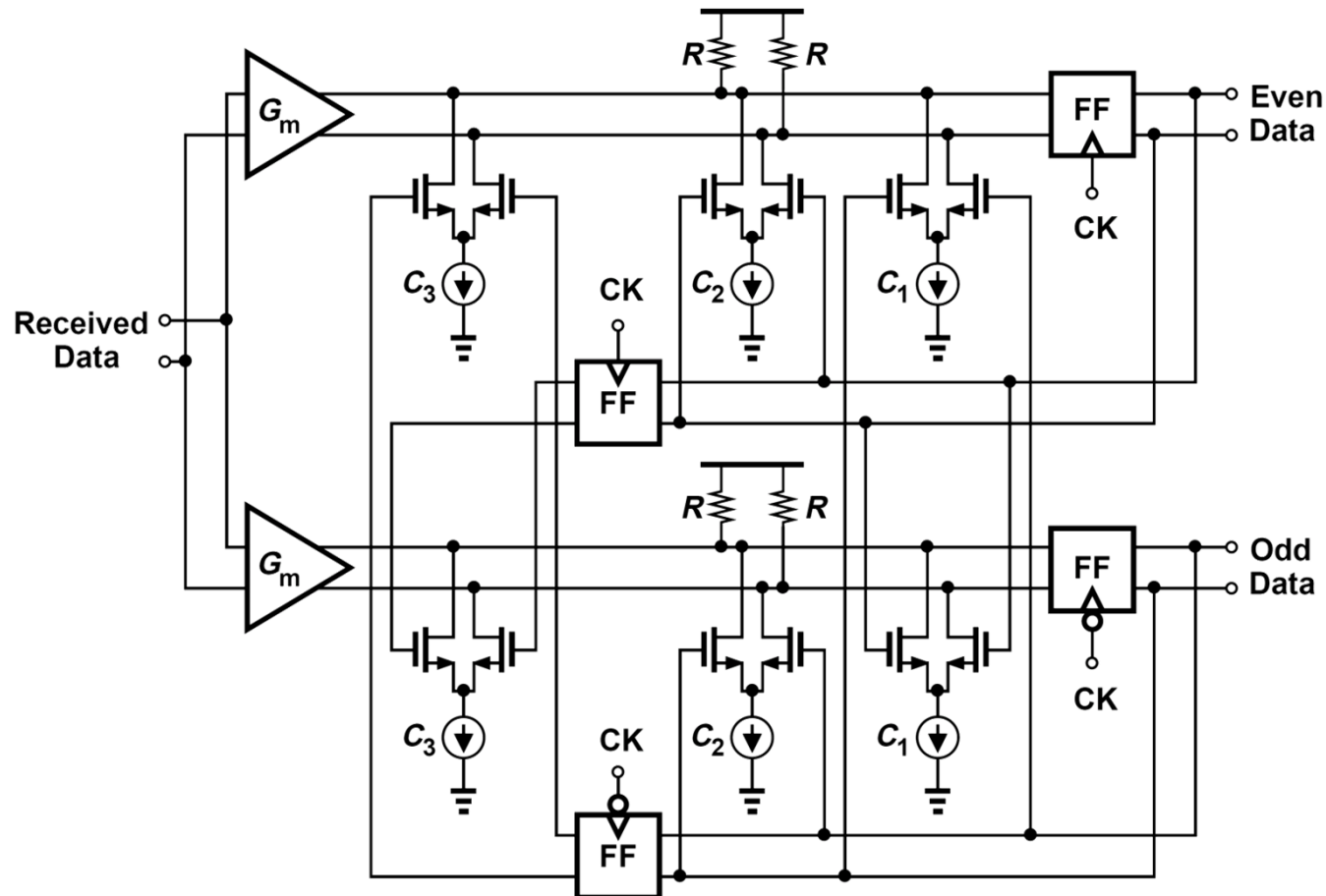
☺ Direct implementation

☹ Full-rate clock

☺ Lowest complexity

$$t_{cq} + t_{setup} + t_{FB} < UI$$

Half-Rate DFE



☺ Half-rate clock

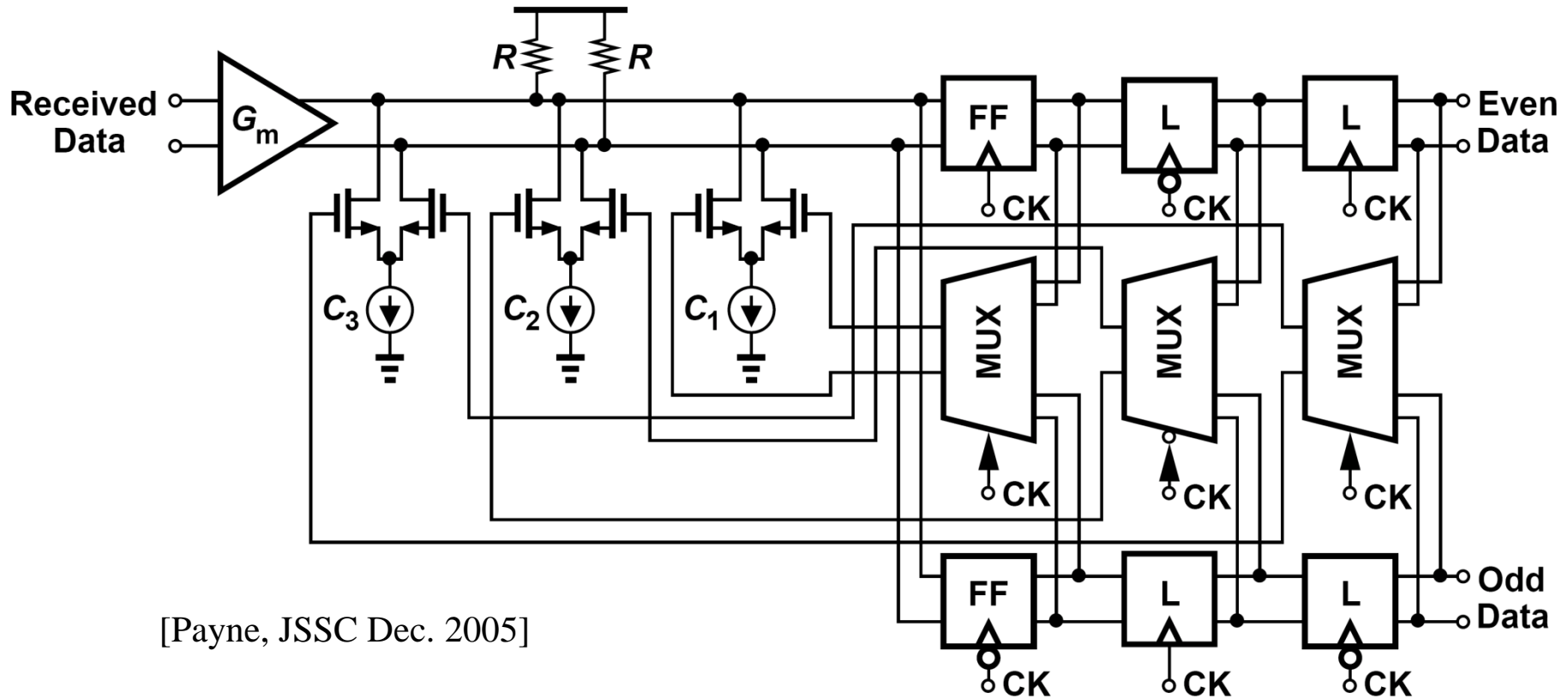
☹ Large area and power

☺ Relaxes FF design

$$t_{cq} + t_{setup} + t_{FB} < UI$$

Equalization

Multiplexed-Half-Rate DFE



☺ Half-rate clock

☺ Saves one summing node

☹ Larger delay

$$t_{cq,FF} + t_{setup} + t_{p,MUX} + t_{FB} < UI$$

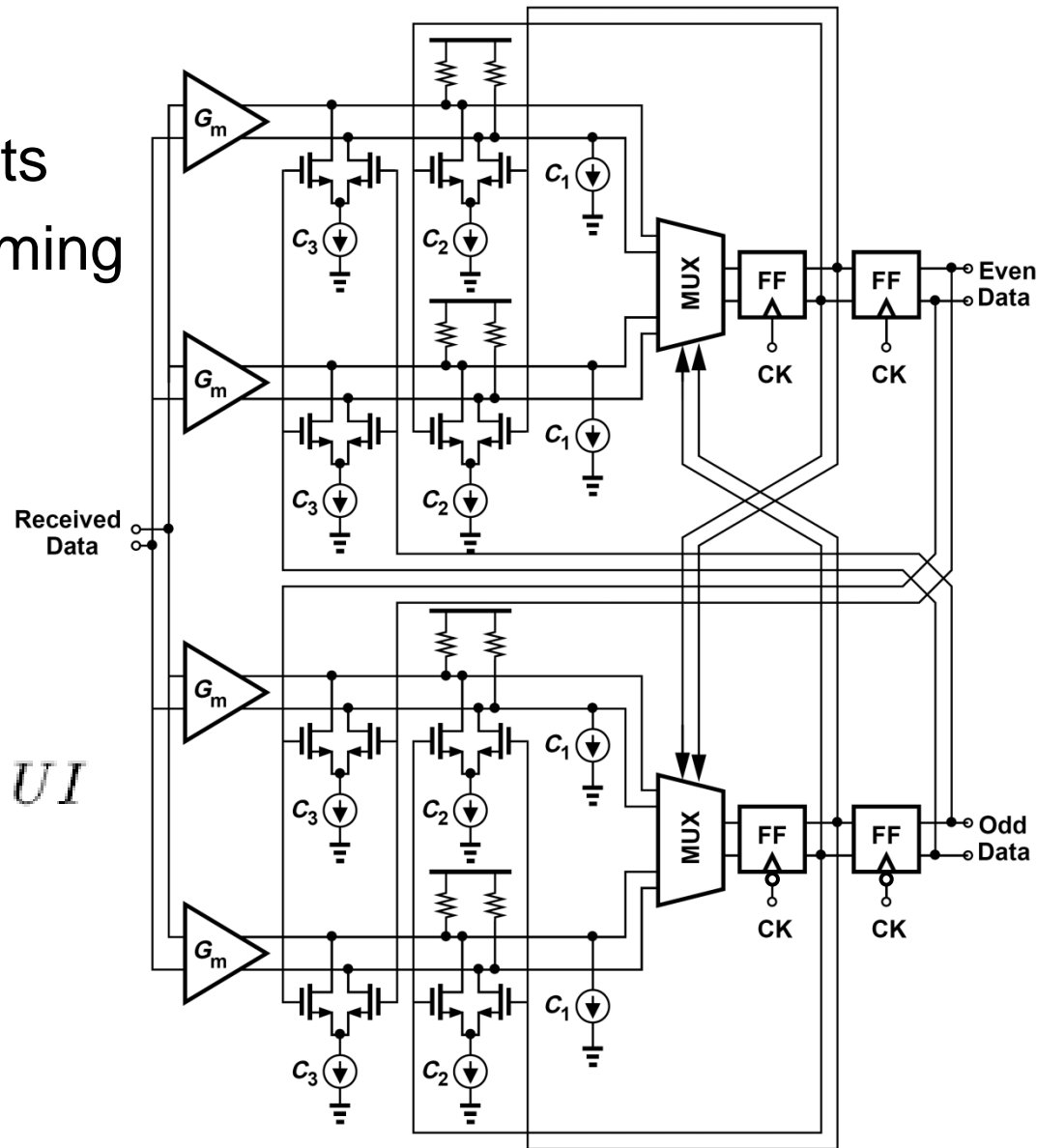
Half-Rate Loop-Unrolled DFE

- ☺ Half-rate clock
- ☺ Relaxes timing constraints
- ☺ Reduces loading at summing nodes

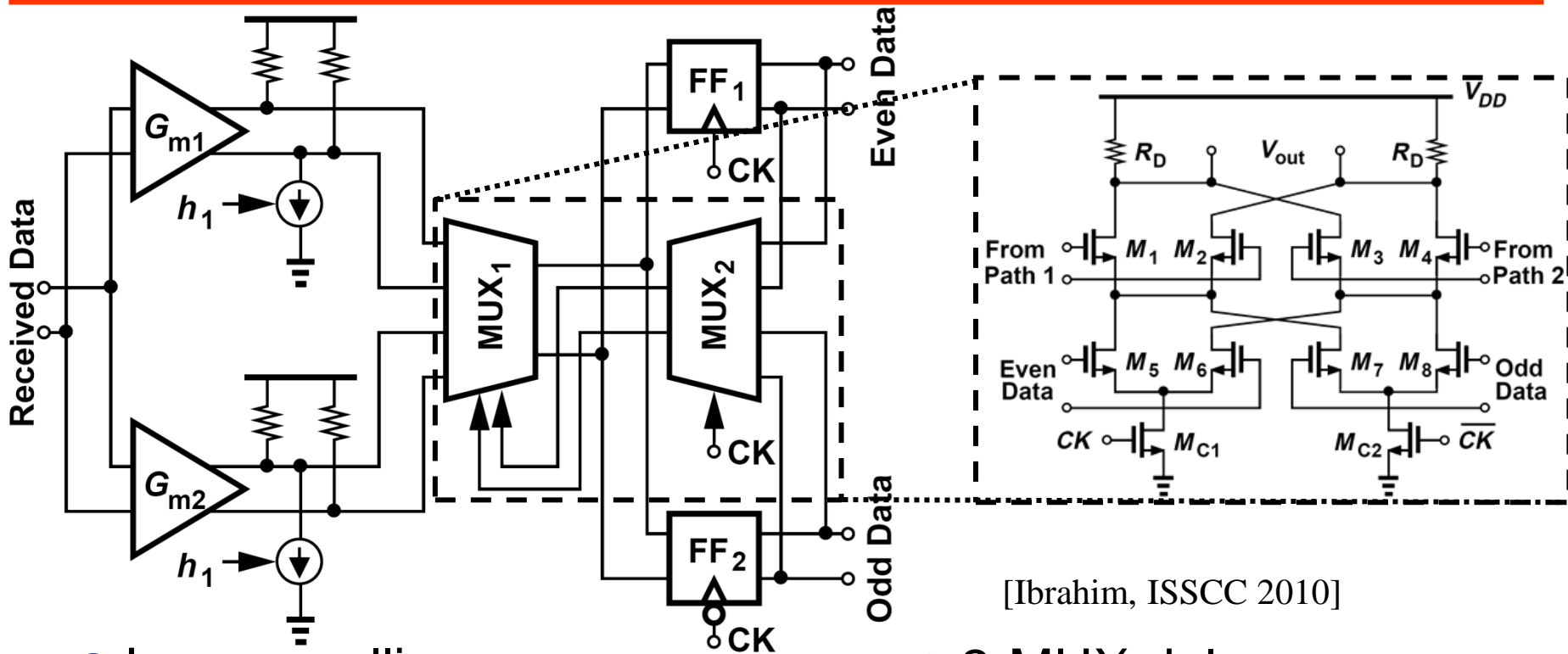
- ☹ High complexity
- ☹ Four summing nodes

$$t_{cq,FF} + t_{setup} + t_{sq,MUX} < UI$$

- Also called, lookahead, speculative or partial response DFE.



HR Loop-Unrolled Multiplexed DFE



[Ibrahim, ISSCC 2010]

☺ Loop unrolling

☺ Multiplexing

☺ Half-rate clock

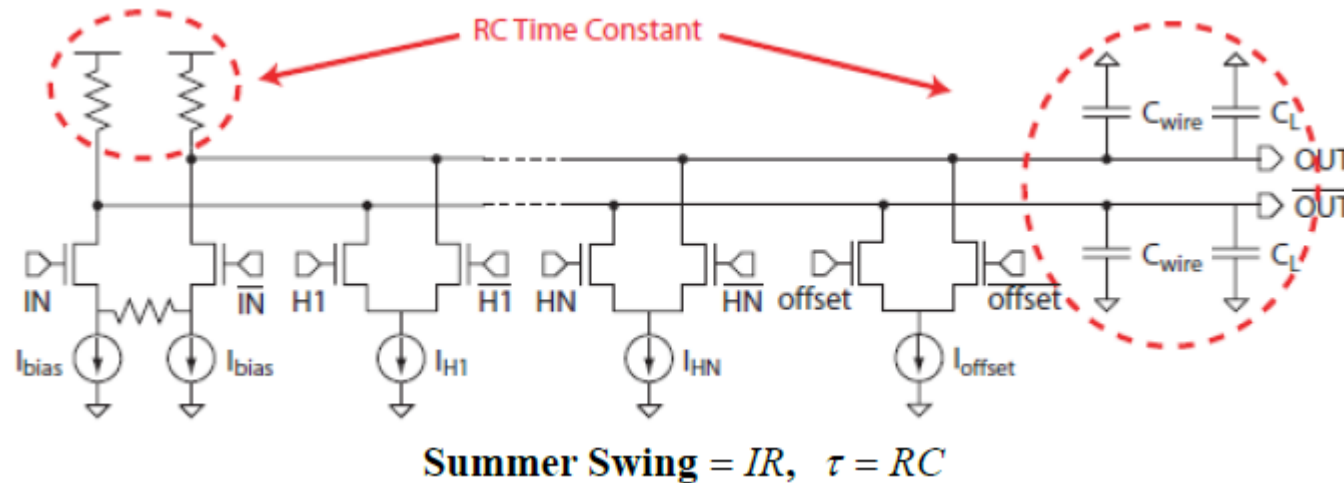
☺ Other taps can be readily added

☹ 2-MUX delay

$$t_{cq,FF} + t_{setup} + t_{sq,MUX_1} + t_{p,MUX_2} < UI$$

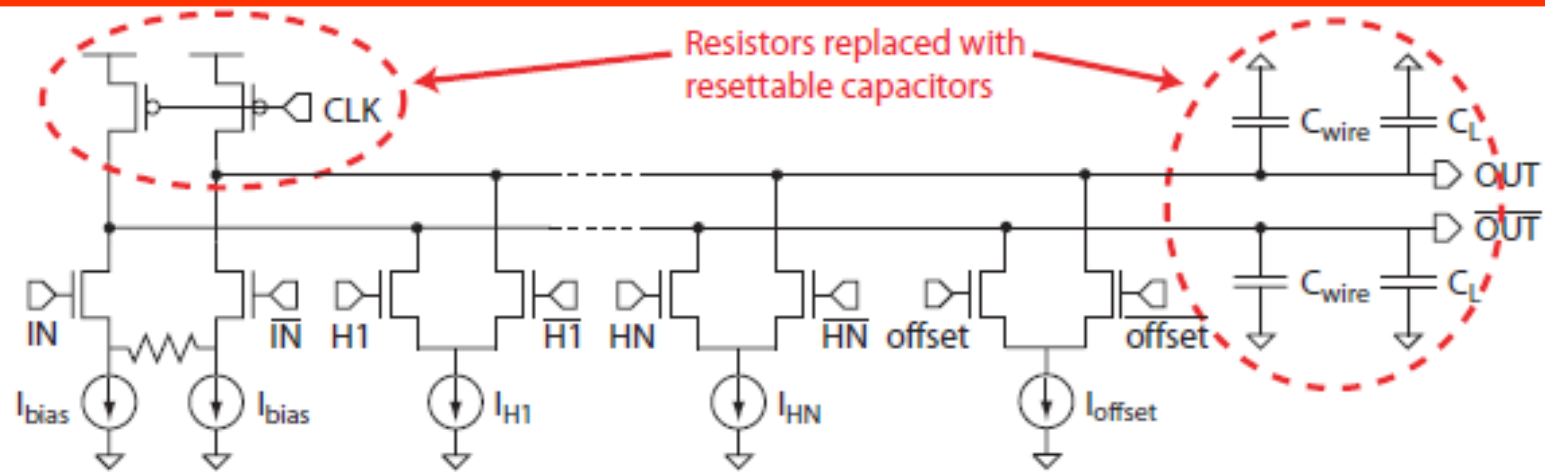
Equalization

DFE Resistive-Load Summer

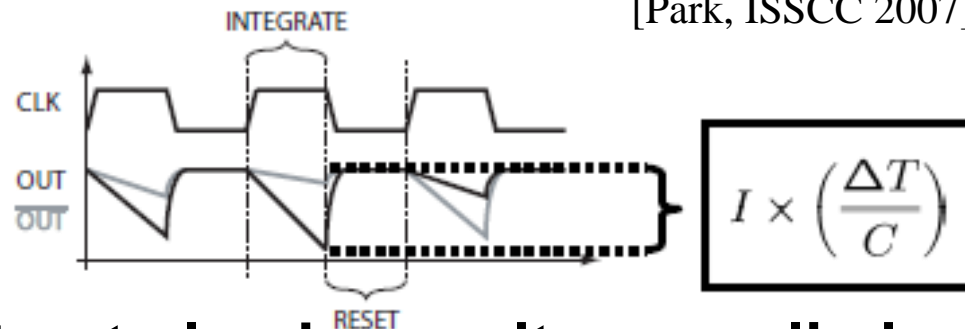


- Summer performance is critical for DFE operation.
- Summer must settle within a certain level of accuracy (>95%) for ISI cancellation.
- Trade-off between summer output swing and settling time.
- Can result in large bias currents for input and taps.

DFE Integrating Summer



[Park, ISSCC 2007]

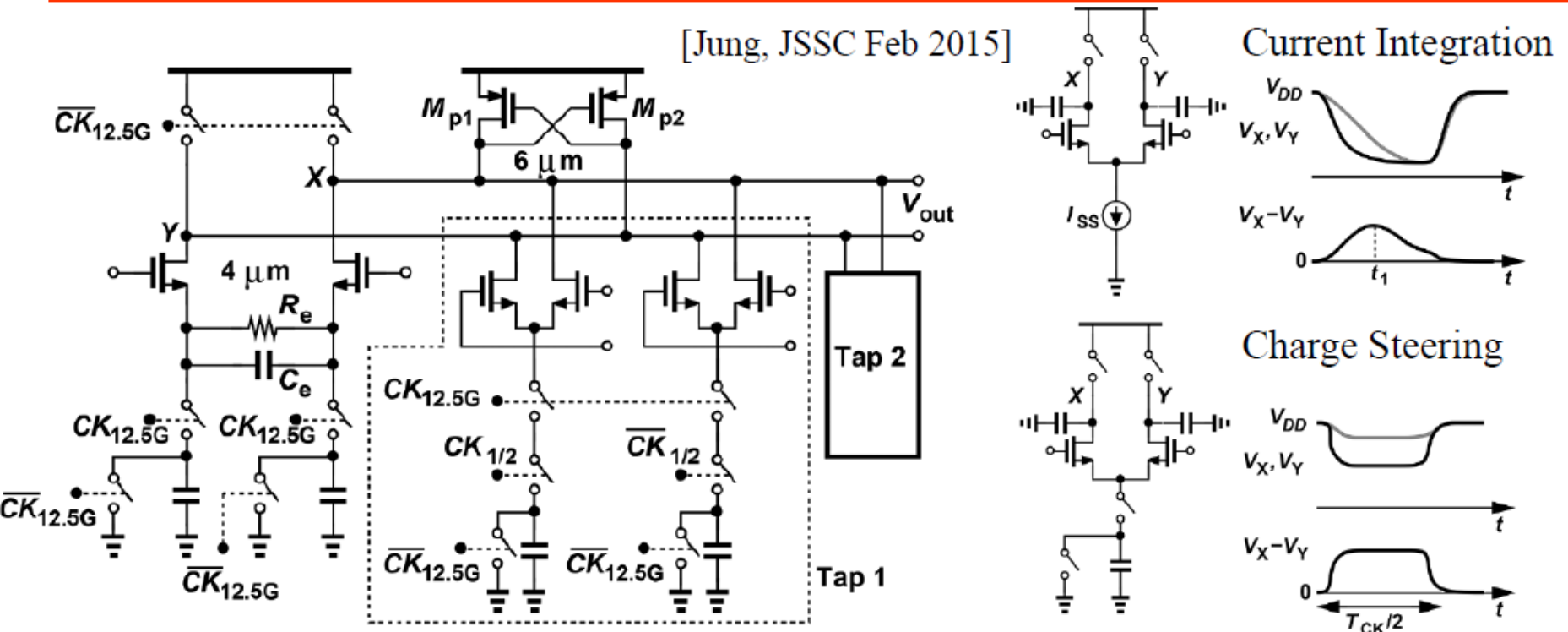


- Integrating current onto load capacitances eliminates RC settling time.
- Since $\Delta T / C > R$, bias current can be reduced for a given output swing.
 - Typically a 3x bias current reduction

Equalization

Charge-Steering Summer

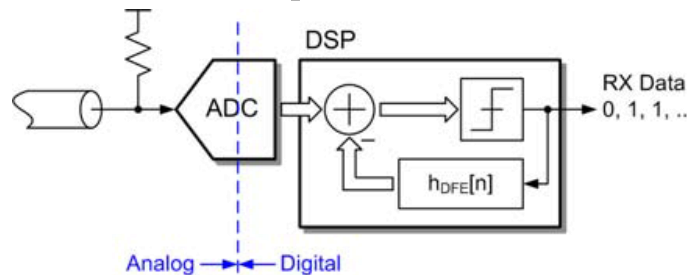
[Jung, JSSC Feb 2015]



- 25 Gb/s DFE with low power consumption
- Half-rate design so 12.5G clock is used.
- Multiplexed design so 6.25G clock is used to implement MUX.

Digital Implementation

- Similar to RX FIR, RX DFE can be implemented digitally after a high-speed ADC.



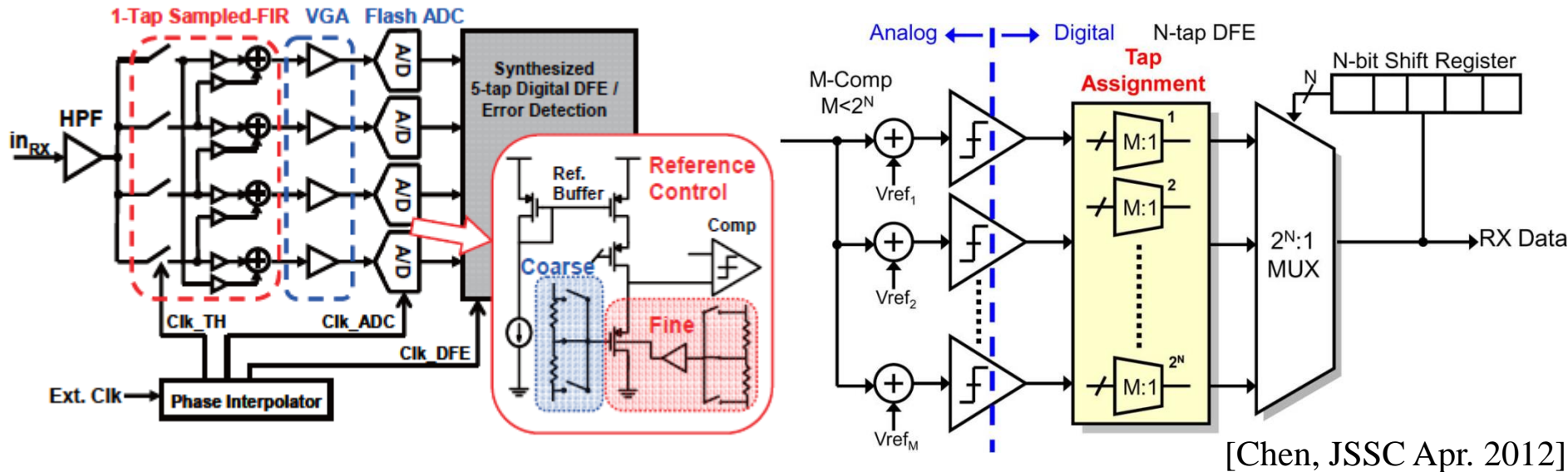
- **Pros**

- Ability to do both FIR and IIR and even coding all together (correct pre- & post-cursor)
- More programmable and flexible
- Better portability

- **Cons**

- Requires accurate D/A or A/D
- Very difficult to build at high-speeds
- High resolution at multi-GS/s
- Signal processing at multi-GHz

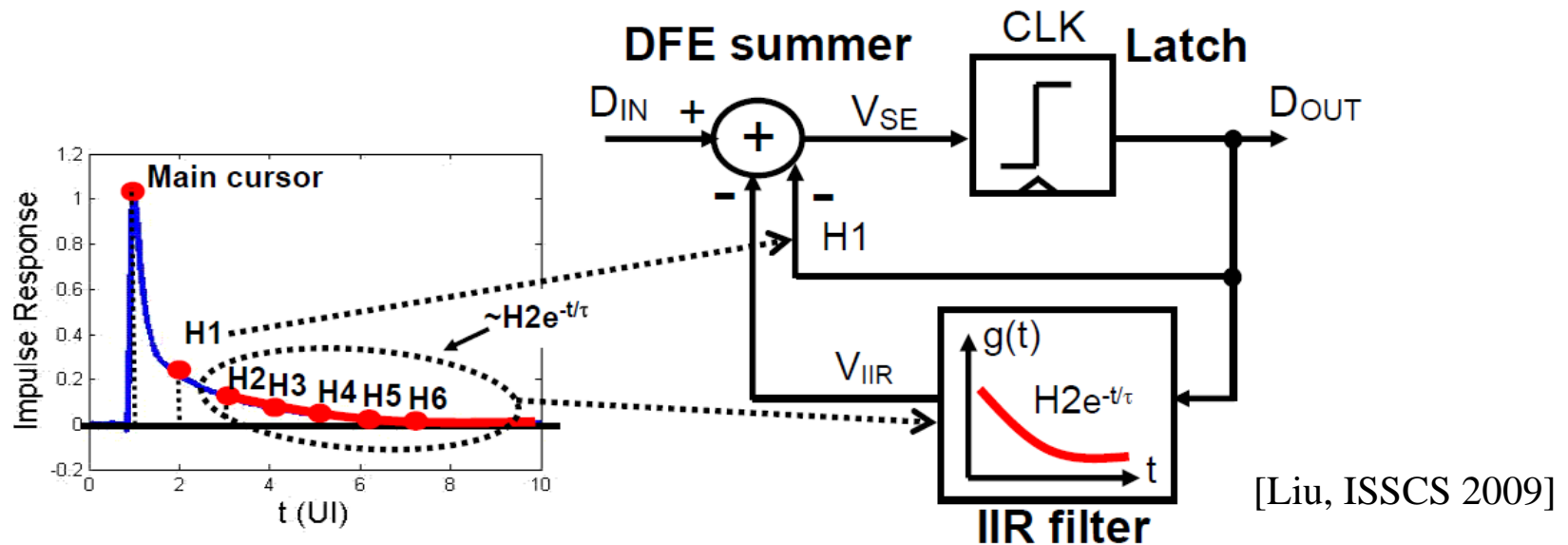
ADC-Based Equalizer Example



[Chen, JSSC Apr. 2012]

- HPF provides a low-power pre-filtering for post-cursor ISI.
- FIR cancels the pre-cursor ISI at lower power than digital.
- A VGA (up to 10dB) buffers the signal before the ADC and provides an alternative for ADC FSR adjustment.
- An ADC with non-uniform quantization levels is used to reduce number of comparators.
- A selection-based DFE is easy to implement in digital.

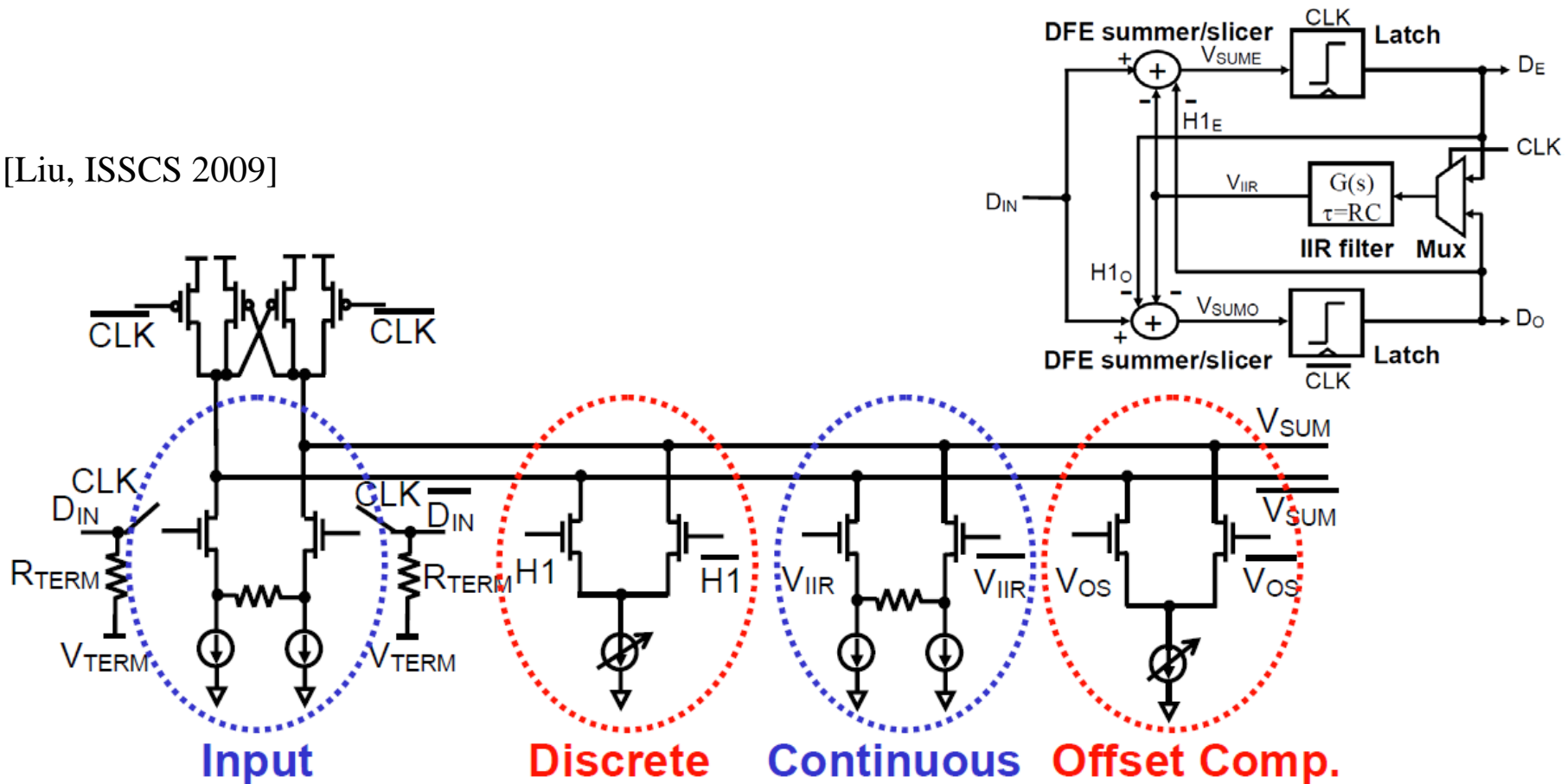
DFE with IIR Feedback



- Large 1st post-cursor $H1$ is canceled with normal FIR feedback tap.
- Smooth long tail ISI from 2nd post-cursor and beyond is canceled with low-pass IIR feedback filter.
- Note: channel needs to be smooth (not many reflections) in order for this approach to work well.

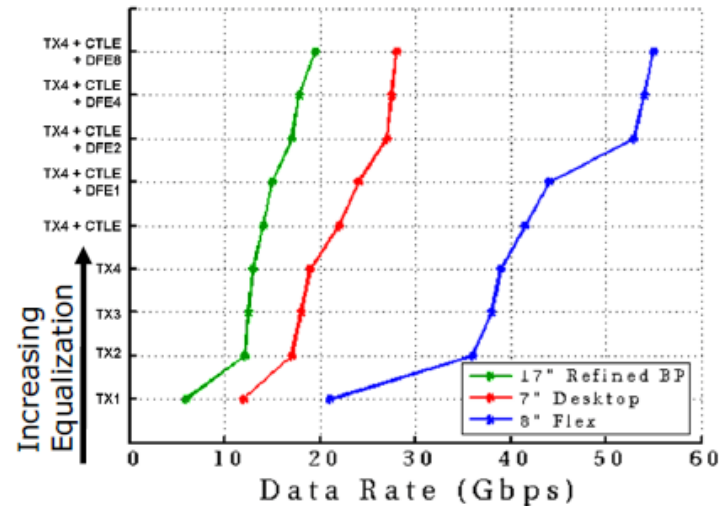
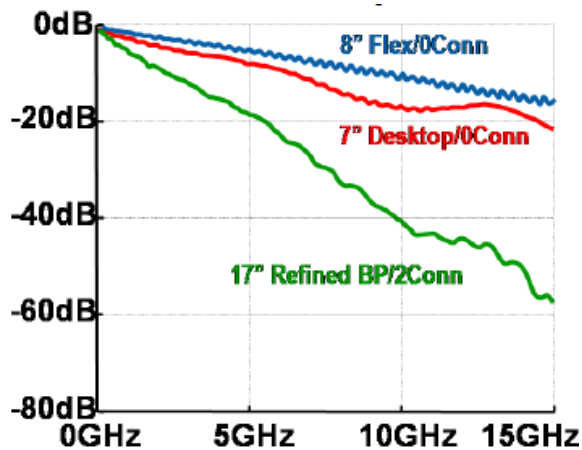
Merged Summer & Partial Slicer

[Liu, ISSCS 2009]



- Integrating summer with regeneration PMOS devices to realize partial slicer operation.

Equalization Effectiveness



Some observations

- Big initial performance boost with 2-tap TX eq.
- With only TX eq., not much difference between 2 to 4-tap.
- RX equalization, particularly DFE, allows for further performance improvement
 - Caution – hard to build fast DFEs due to critical timing path