

A topos-theoretic proof of Shelah's eventual categoricity conjecture.

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Abstract

Assuming the Generalized Continuum Hypothesis (GCH), we give a topos-theoretic proof of Shelah's eventual categoricity conjecture for abstract elementary classes (AEC) with amalgamation. The proof is based on infinitary generalizations of Deligne and Joyal's completeness theorems for certain infinitary intuitionistic logics, by means of which it is possible to attack the question of categoricity in a topos-theoretic way. Using recent results on the Scott adjunction developed by Henry and Di Liberti, we prove under GCH that an AEC with amalgamation which is categorical in a pair of cardinals is also categorical in all cardinals in between. Under some extra natural assumptions on the AEC, we also deduce a new downward categoricity transfer. We also explain how these methods can be adapted to shed some light on the categoricity spectrum of more general accessible categories.