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$$\int B^T DB \, dV$$

$$B = \begin{bmatrix} \dfrac{\partial [N]}{\partial x} \\ \\ \dfrac{\partial [N]}{\partial y} \end{bmatrix} \qquad \text{где} \quad \begin{matrix} N_1 = L_1(2L_1 - 1) & N_2 = 4L_1L_2 \\ N_3 = L_2(2L_2 - 1) & N_4 = 4L_2L_3 \\ N_5 = L_3(2L_3 - 1) & N_6 = 4L_3L_1 \end{matrix}$$

$$\begin{cases} x = x(L_1, L_2, L_3) \\ y = y(L_1, L_2, L_3) \end{cases}$$

$\beta = \overline{1,6}:$

$$\begin{cases} \frac{\partial N_\beta}{\partial L_1} = \frac{\partial N_\beta}{\partial x} \cdot \frac{\partial x}{\partial L_1} + \frac{\partial N_\beta}{\partial y} \cdot \frac{\partial y}{\partial L_1} \\ \frac{\partial N_\beta}{\partial L_2} = \frac{\partial N_\beta}{\partial x} \cdot \frac{\partial x}{\partial L_2} + \frac{\partial N_\beta}{\partial y} \cdot \frac{\partial y}{\partial L_2} \end{cases} \quad \Leftrightarrow \quad \begin{bmatrix} \frac{\partial N_\beta}{\partial L_1} \\ \frac{\partial N_\beta}{\partial L_2} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial x}{\partial L_1} & \frac{\partial y}{\partial L_1} \\ \frac{\partial x}{\partial L_2} & \frac{\partial y}{\partial L_2} \end{bmatrix}}_{J} \cdot \begin{bmatrix} \frac{\partial N_\beta}{\partial x} \\ \frac{\partial N_\beta}{\partial y} \end{bmatrix}$$

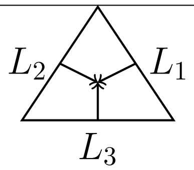
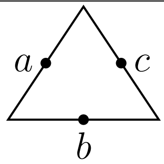
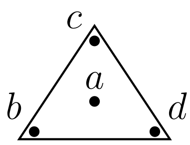
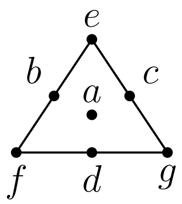
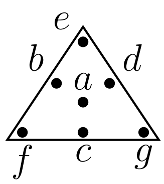
$$\begin{bmatrix} \frac{\partial N_\beta}{\partial x} \\ \frac{\partial N_\beta}{\partial y} \end{bmatrix} = J^{-1} \cdot \begin{bmatrix} \frac{\partial N_\beta}{\partial L_1} \\ \frac{\partial N_\beta}{\partial L_2} \end{bmatrix}$$

$$\frac{\partial N_\beta}{\partial L_1} = \frac{\partial N_\beta}{\partial L_1} \cdot \underbrace{\frac{\partial L_1}{\partial L_1}}_{1} + \frac{\partial N_\beta}{\partial L_2} \cdot \underbrace{\frac{\partial L_2}{\partial L_1}}_{0} + \frac{\partial N_\beta}{\partial L_3} \cdot \underbrace{\frac{\partial L_3}{\partial L_1}}_{-1} = \frac{\partial N_\beta}{\partial L_1} - \frac{\partial N_\beta}{\partial L_3}$$

<u>Пример:</u>

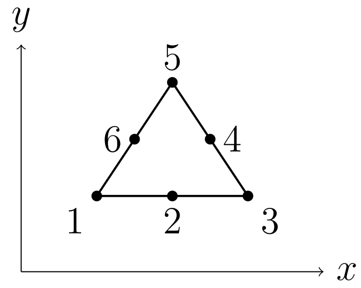
$$N_4 = 4L_2L_3 = 4L_2(1 - L_1 - L_2) \Rightarrow \frac{\partial N_4}{\partial L_1} = -4L_2$$

$$Z = \int\limits_0^1 \int\limits_0^{1-L_2} f(L_1, L_2, L_3)|J| \; dL_1 dL_2 = \sum_{i=1}^{n} W_i g(L_1, L_2, L_3), \text{ где } g = f \cdot |J|$$

	Ошибка	$(\cdot)$	$L_1\ L_2\ L_3$	W_i
	$R = o(h^2)$	a	$\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}$	$\frac{1}{2}$
	$R = o(h^2)$	a	$\frac{1}{2}\ 0\ \frac{1}{2}$	$\frac{1}{6}$
		b	$\frac{1}{2}\ \frac{1}{2}\ 0$	$\frac{1}{6}$
		c	$0\ \frac{1}{2}\ \frac{1}{2}$	$\frac{1}{6}$
	$R = o(h^4)$	a	$\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}$	$\frac{27}{96}$
		b	$\frac{11}{15}\ \frac{2}{15}\ \frac{2}{15}$	$\frac{25}{96}$
		c	$\frac{2}{15}\ \frac{2}{15}\ \frac{11}{15}$	$\frac{25}{96}$
		d	$\frac{2}{15}\ \frac{11}{15}\ \frac{2}{15}$	$\frac{25}{96}$
	$R = o(h^4)$	a	$\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}$	$\frac{27}{120}$
		b	$\frac{1}{2}\ 0\ \frac{1}{2}$	$\frac{8}{120}$
		c	$0\ \frac{1}{2}\ \frac{1}{2}$	$\frac{8}{120}$
		d	$\frac{1}{2}\ \frac{1}{2}\ 0$	$\frac{8}{120}$
		e	$0\ 0\ 1$	$\frac{3}{120}$
		f	$1\ 0\ 0$	$\frac{3}{120}$
		g	$0\ 1\ 0$	$\frac{3}{120}$
		a	$\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}$	0.1125
	$R = o(h^6)$	b	$\beta\ \alpha\ \beta$	0.066197075
	$\alpha = 0.05961587$	c	$\beta\ \beta\ \alpha$	0.066197075
	$\beta = 0.47014206$	d	$\alpha\ \beta\ \beta$	0.066197075
	$\gamma = 0.10128651$	e	$\gamma\ \gamma\ \delta$	0.0629695
	$\Delta = 0.79742699$	f	$\delta\ \gamma\ \gamma$	0.0629695
		g	$\gamma\ \delta\ \gamma$	0.0629695

Пример.

$$\text{Вычислить: } \int\limits_S \frac{\partial N_4}{\partial x} \cdot \frac{\partial N_4}{\partial y}\; dxdy$$



Точка	Координаты точки	
1	1	1
3	3	2
5	2	3

$$\begin{cases} x = x_1 L_1 + x_3 L_2 + x_5 L_3 \\ y = y_1 L_1 + y_3 L_2 + y_5 L_3 \end{cases} \Rightarrow \begin{cases} x = L_1 + 3L_2 + 2L_3 \\ y = L_1 + 2L_3 + 3L_3 \end{cases}$$

$$N_4 = 4L_2 L_3$$

$$J = \begin{bmatrix} \frac{\partial x}{\partial L_1} & \frac{\partial y}{\partial L_1} \\ \frac{\partial x}{\partial L_2} & \frac{\partial y}{\partial L_2} \end{bmatrix} = \begin{bmatrix} 1-2 & 1-3 \\ 3-2 & 2-3 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ 1 & -1 \end{bmatrix} \Rightarrow |J| = 1 + 2 = 3$$

$$\begin{bmatrix} \frac{\partial N_4}{\partial x} \\ \frac{\partial N_4}{\partial y} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} -1 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} -4L_2 \\ 4L_3 - 4L_2 \end{bmatrix} \Rightarrow$$

$$\frac{\partial N_4}{\partial x} = \frac{1}{3}(8L_2 - 4L_3) \qquad \frac{\partial N_4}{\partial y} = \frac{-1}{3}(4L_2 + 4L_3)$$

$$\frac{\partial N_4}{\partial x} \cdot \frac{\partial N_4}{\partial y} = -\frac{1}{9}(8L_2 - 4L_3)(4L_2 + 4L_3) = \int\limits_S \frac{\partial N_4}{\partial x} \cdot \frac{\partial N_4}{\partial y}\; dxdy =$$

$$= \int\limits_0^1 \int\limits_0^{1-L_2} -\frac{1}{9} \cdot 3(8L_2 - 4L_3)(4L_2 + 4L_3)\, dL_1 dL_2 = -\frac{1}{3} \sum_{i=1}^{3} W_i \cdot g(L_1, L_2, L_3) = -\frac{12}{18}$$