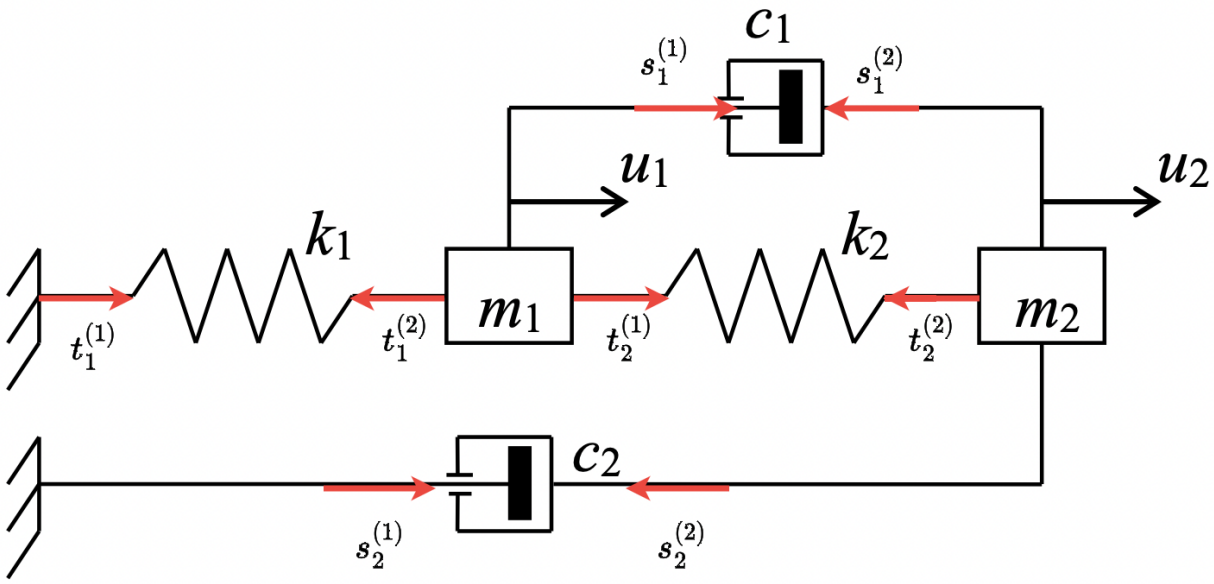
Задание

Составить уравнения движения для систем.

Система 1



Составим уравнения:

$$\begin{cases} m_1\ddot{u}_1 + t_1^{(2)} + t_2^{(1)} + s_1^{(1)} = 0 \\ m_2\ddot{u}_2 + t_2^{(2)} + s_2^{(2)} + s_1^{(2)} = 0 \end{cases} \tag{1}$$

$$\begin{cases} t_1^{(2)} = k_1 u_1 \\ t_2^{(1)} = k_2(u_1 - u_2) \\ t_2^{(2)} = k_2(u_2 - u_1) \\ s_1^{(1)} = c_1(\dot{u}_1 - \dot{u}_2) \\ s_2^{(2)} = c_2\dot{u}_2 \\ s_1^{(2)} = c_1(\dot{u}_2 - \dot{u}_1) \end{cases} \tag{2}$$

Подставляем (2) в (1):

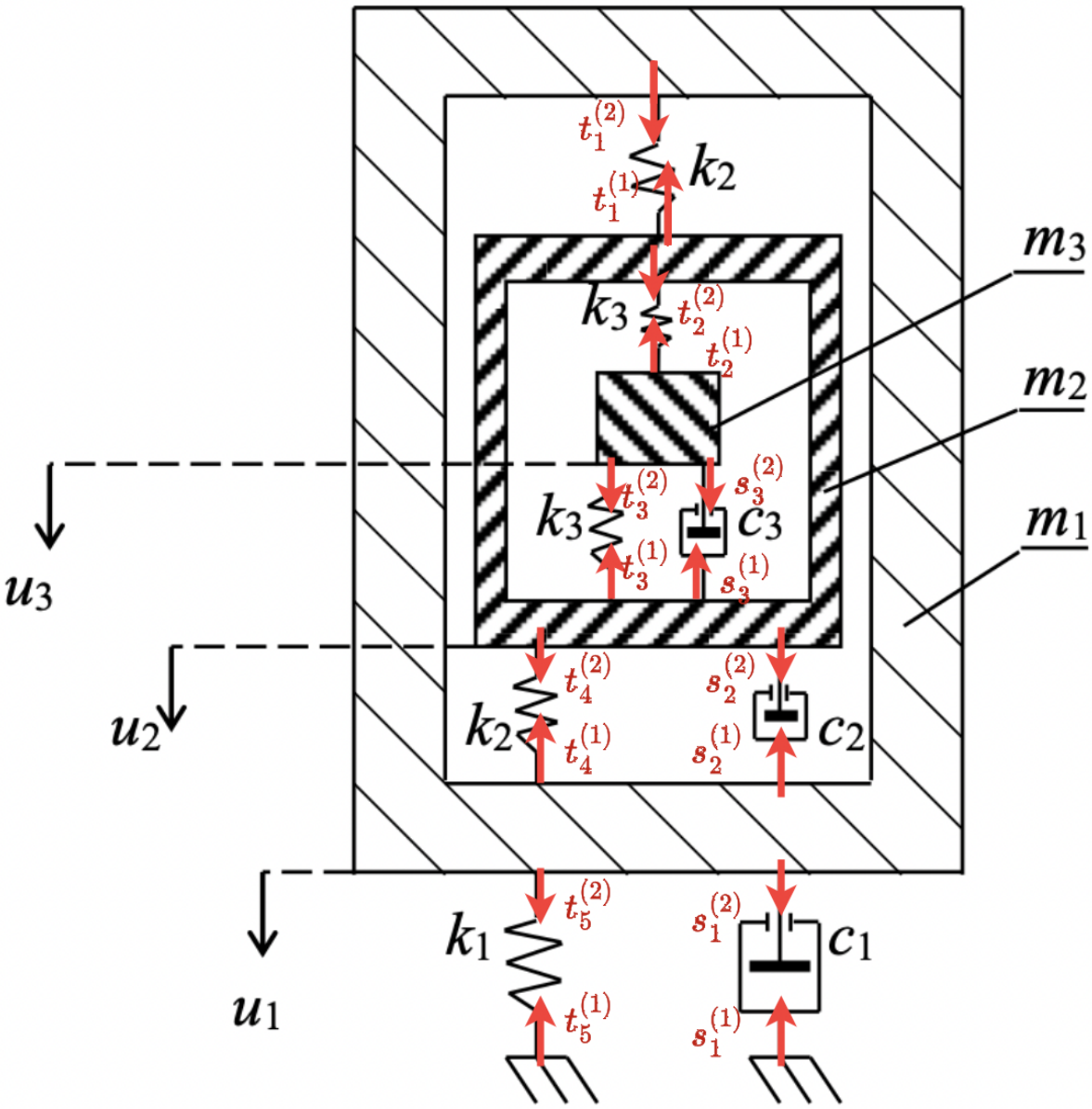
$$\begin{cases} m_1\ddot{u}_1 + k_1u_1 + k_2(u_1 - u_2) + c_1(\dot{u}_1 - \dot{u}_2) = 0 \\ m_2\ddot{u}_2 + k_2(u_2 - u_1) + c_2\dot{u}_2 + c_1(\dot{u}_2 - \dot{u}_1) = 0 \end{cases}$$

В матричном виде:

$$M\ddot{q} + C\dot{q} + Kq = 0$$

$$M = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}, \quad C = \begin{pmatrix} c_1 & -c_1 \\ -c_1 & c_1 + c_2 \end{pmatrix}, \quad K = \begin{pmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{pmatrix}$$

Система 2



Составим уравнения:

$$\begin{cases} m_1\ddot{u}_1 + t_1^{(2)} + t_4^{(1)} + t_5^{(2)} + s_1^{(2)} + s_2^{(1)} = 0 \\ m_2\ddot{u}_2 + t_1^{(1)} + t_2^{(2)} + t_4^{(2)} + t_3^{(1)} + s_2^{(2)} + s_3^{(1)} = 0 \\ m_3\ddot{u}_3 + t_2^{(1)} + t_3^{(2)} + s_3^{(2)} = 0 \end{cases} \tag{3}$$

$$\begin{cases} t_5^{(2)} = k_1 u_1 \\ s_1^{(2)} = c_1 \dot{u}_1 \\ t_4^{(1)} = k_2(u_1 - u_2) \\ s_2^{(1)} = c_2(\dot{u}_1 - \dot{u}_2) \\ t_4^{(2)} = k_2(u_2 - u_1) \\ s_2^{(2)} = c_2(\dot{u}_2 - \dot{u}_1) \\ t_3^{(1)} = k_3(u_2 - u_3) \\ s_3^{(1)} = c_3(\dot{u}_2 - \dot{u}_3) \\ t_3^{(2)} = k_3(u_3 - u_2) \\ s_3^{(2)} = c_3(\dot{u}_3 - \dot{u}_2) \\ t_2^{(1)} = k_3(u_3 - u_2) \\ t_2^{(2)} = k_3(u_2 - u_3) \\ t_1^{(1)} = k_2(u_2 - u_1) \\ t_1^{(2)} = k_2(u_1 - u_2) \end{cases} \tag{4}$$

Подставляем (4) в (3):

$$\begin{cases} m_1\ddot{u}_1 + k_2(u_1 - u_2) + k_2(u_1 - u_2) + k_1 u_1 + c_1\dot{u}_1 + c_2(\dot{u}_1 - \dot{u}_2) = 0 \\ m_2\ddot{u}_2 + k_2(u_2 - u_1) + k_3(u_2 - u_3) + k_2(u_2 - u_1) + k_3(u_2 - u_3) + c_2(\dot{u}_2 - \dot{u}_1) + c_3(\dot{u}_2 - \dot{u}_3) = 0 \\ m_3\ddot{u}_3 + k_3(u_3 - u_2) + k_3(u_3 - u_2) + c_3(\dot{u}_3 - \dot{u}_2) = 0 \end{cases}$$

В матричном виде:

$$M\ddot{q} + C\dot{q} + Kq = 0$$

$$M = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \quad C = \begin{pmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{pmatrix}, \quad K = \begin{pmatrix} k_1 + 2k_2 & -2k_2 & 0 \\ -2k_2 & 2k_2 + 2k_3 & -2k_3 \\ 0 & -2k_3 & 2k_3 \end{pmatrix}$$