Seminar 6 - test 911 -

ex 1 : $\begin{cases} x' = 2y \\ y' = -5x \end{cases}$

$\frac{dy}{dx} = -\frac{5x}{2y}$ ... $2y^2 + 5x^2 = c$, $c \in \mathbb{R}$

$H(x,y) := 5x^2 + 2y^2$ $\forall (x,y) \in \mathbb{R}^2$

$\frac{\partial H}{\partial x} \cdot f_1 + \frac{\partial H}{\partial y} \cdot f_2 = 0$ in $\mathbb{R}^2$

TRUE

ex 3:

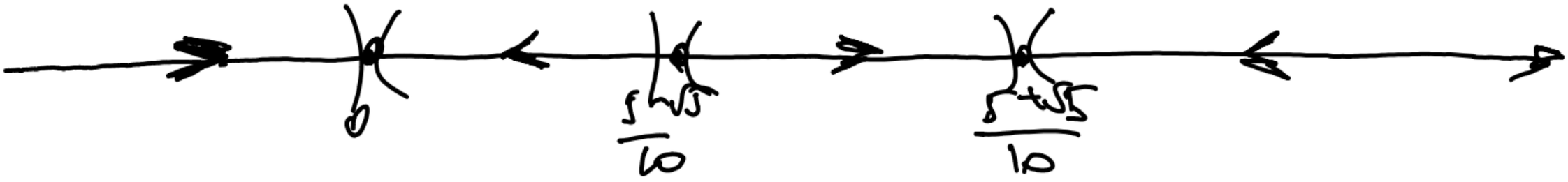
$$x' = -\frac{1}{5}x + x^2 - x^3$$

- ph. portrait
- $\varphi(t, 0) = ?$
- $\varphi(t, \frac{1}{4})$, $\varphi(t, \frac{1}{2})$, $\varphi(t, 1)$ - propr.

eq. point: $-\frac{1}{5}x + x^2 - x^3 = 0.$

$$x_1 = 0 \quad , \quad x_{2,3} = \frac{5 \pm \sqrt{5}}{10}$$

x		0		$\frac{5-\sqrt{5}}{10}$		$\frac{5+\sqrt{5}}{10}$	
$f(x)$	$+$	0	$-$	0	$+$	0	$-$



$\varphi(t,0) = 0 \quad \Leftarrow 0\text{-eq. p.}$

$\varphi(t,\frac{1}{4})$ = s. decr., lim ; $\frac{1}{4} \in (0, \frac{5-\sqrt{5}}{10})$

$\varphi(t,\frac{1}{2})$ = s. decr. lim $\frac{1}{2} \in (\frac{5-\sqrt{5}}{10}, \frac{5+\sqrt{5}}{10})$

$\varphi(t,1)$ = s. decr., lim $1 \in (\frac{5+\sqrt{5}}{10}, \infty)$

ex3 :

$$\begin{cases} x' = x - 2xy \\ y' = x - y \end{cases}$$

- eq. p.
- type, st.

(a) $\begin{cases} x - 2xy = 0 \\ x - y = 0 \end{cases} \Rightarrow \cdots$ $\begin{matrix} (x_1, y_1) = (0,0) \\ (x_2, y_2) = \left(\frac{1}{2}, \frac{1}{2}\right) \end{matrix}$ } eq p.

(b) $Jf(x,y) = \begin{pmatrix} 1-2y & -2x \\ 1 & -1 \end{pmatrix}$

→ $\eta_1^* = (0,0) \Rightarrow A_1 = Jf(\eta_1^*) = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}$

$X' = Jf(\eta_1^*) \cdot X$, $\lambda_1 = 1$, $\lambda_2 = -1$

↳ saddle

$\Rightarrow \eta_1^* = \underline{\underline{\text{unstable}}}$

→ $\eta_2^* = (\frac{1}{2}, \frac{1}{2}) \Rightarrow A_2 = Jf(\eta_2^*) = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}$

$X' = Jf(\eta_2^*) \cdot X$, $\lambda_{12} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

↳ focus

$\Rightarrow \eta_2^*$ = attractor.