Problem 5

y_t - dependant variable $\quad \bar{y} = \frac{1}{n}\sum_{t=1}^{n} y_t = 17.21$

y_t^* - new variable $\quad y_t^* = y_t + 10 \Rightarrow \bar{y}_t^* = \frac{1}{n}\sum_{t=1}^{n}(y_t + 10) = \bar{y} + 10$

LRM: $\begin{cases} \hat{y}_t^* = c_0 + c_1 x_t \\ \hat{y}_t = b_0 + b_1 x_t \end{cases}$

Lets find the formula for tho coefficients b_0, b_1, c_0, c_1:

$Q = \sum_{i=1}^{n} e_i = \sum_{i=1}^{n}(y_i - a_0 - a_1 x_i)^2 \rightarrow \min \Rightarrow \begin{cases} \frac{\partial Q}{\partial a_0} = 0 = -2\sum_{i=1}^{n}(y_i - a_0 - a_1 x_i) \\ \frac{\partial Q}{\partial a_1} = 0 = -2\sum_{i=1}^{n}(y_i - a_0 - a_1 x_i)x_i \end{cases} \Rightarrow$

$\Rightarrow :/n \Rightarrow \begin{cases} \frac{1}{n}\sum_{i=1}^{n} y_i = a_0 + a_1 \frac{1}{n}\sum_{i=1}^{n} x_i \\ \frac{1}{n}\sum_{i=1}^{n} x_i y_i = \frac{1}{n} a_0 + \frac{1}{n} a_1 \sum_{i=1}^{n} x_i \end{cases} \Rightarrow \begin{cases} \bar{y} = a_0 + a_1 \bar{x} \\ \frac{1}{n}\sum_{i=1}^{n} x_i y_i = a_0 \bar{x} + \frac{1}{n} a_1 \sum_{i=1}^{n} x_i^2 \end{cases}$

$\Rightarrow \begin{cases} \boxed{a_0 = \bar{y} - a_1 \bar{x}} \\ a_1 = \dfrac{\frac{1}{n}\sum_{i=1}^{n} x_i y_i - \bar{x}\bar{y}}{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}} = \boxed{\dfrac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2} = a_1} \end{cases}$

a). Thus:

$$b_1 = \frac{\frac{1}{n}\sum_{t=1}^{n}(y_t - \bar{y})(x_t - \bar{x})}{\frac{1}{n}\sum_{t=1}^{n}(x_t - \bar{x})^2} \qquad c_1 = \frac{\frac{1}{n}\sum_{t=1}^{n}(\overbrace{y_t + 10}^{y_t^*} - \overbrace{\bar{y} - 10}^{\bar{y}_t^*})(x_t - \bar{x})}{\frac{1}{n}\sum_{t=1}^{n}(x_t - \bar{x})^2}$$

$$\boxed{c_1 = b_1}$$

$$b_0 = \bar{y} - b_1 \bar{x} \qquad c_0 = \bar{y}_t^* - c_1 \cdot \bar{x} = 10 + \bar{y} - \underset{b_1}{b_1}\bar{x}$$

$$\boxed{c_0 = b_0 + 10}$$

$$\hat{y}_t^* = c_0 + c_1 x_t = b_0 + 10 + b_1 x_t = \boxed{\hat{y}_t + 10 = \hat{y}_t^*}$$

Regressand $y_t \Rightarrow R^2 = \frac{ESS}{TSS} = \frac{\sum_{t=1}^{n}(\hat{y}_t - \bar{y})^2}{\sum_{t=1}^{n}(y_t - \bar{y})^2}$

Regressand $y_t \Rightarrow R_*^2 = \frac{ESS_*}{TSS_*} = \frac{\sum_{t=1}^{n}(\hat{y}_t^* - \bar{y}^*)^2}{\sum_{t=1}^{n}(y_t^* - \bar{y}^*)^2} = \frac{\sum_{t=1}^{n}(\hat{y}_t + 10 - \bar{y} - 10)^2}{\sum_{t=1}^{n}(y_t + 10 - \bar{y} - 10)^2} = \frac{ESS}{TSS}$

Answer: $\boxed{R^2 = R_*^2}$ the residual sum of squares would not change!

1) $\boxed{R^2 = R^2_*}$

And the same if we use any constant : $\boxed{y_t^* = y_t + const}$
even if const is less then 0.

$\left(\begin{array}{l} y_t^* = y_t - 10 \\ const = -10 < 0 \end{array} \right)$

$\overline{y_t^*} = \bar{y} + const$

Analogous as in (0):

$\begin{cases} \hat{y}_t^* = c_0 + c_1 x_t \\ \hat{y}_t = b_0 + b_1 x_t \end{cases}$; $\begin{cases} c_1 = b_1 \\ c_0 = b_0 + const \end{cases}$; $\{ \hat{y}_t^* = \hat{y}_t + const$

$$\underline{R^{*2}} = \frac{ESS^*}{TSS^*} = \frac{\sum_{t=1}^{n} (\hat{y}_t^* - \bar{y}^*)^2}{\sum_{t=1}^{n} (y_t^* - \bar{y}^*)^2} = \frac{\sum_{t=1}^{n} (\hat{y}_t + \cancel{const} - \bar{y} - \cancel{const})^2}{\sum_{t=1}^{n} (y_t + \cancel{const} - \bar{y} - \cancel{const})^2} = \underline{R^2}$$

Proved!

2) $\boxed{x_t^* = x_t + const = x_t + \alpha}$ (it is easier to write α then const)

$$\frac{1}{n}\sum_{t=1}^{n} x_t^* = \frac{1}{n}\sum_{t=1}^{n} x_t + \frac{1}{n}\sum_{t=1}^{n}\alpha = \bar{x} + \alpha$$

It is obvious that $y_t^* = y_t$; $\bar{y} = \bar{y}^*$

$\begin{cases} \hat{y}_t^* = c_0 + c_1 x_t^* \\ \hat{y}_t = b_0 + b_1 x_t \end{cases}$

$$c_1 = \frac{\frac{1}{n}\sum_{t=1}^{n} (y_t - \bar{y})(x_t^* - \bar{x}^*)}{\frac{1}{n}\sum_{t=1}^{n} (x_t^* - \bar{x}^*)^2} = \frac{\frac{1}{n}\sum_{t=1}^{n} (y_t - \bar{y})(x_t + \cancel{\alpha} - \bar{x} - \cancel{\alpha})}{\frac{1}{n}\sum_{t=1}^{n} (x_t + \cancel{\alpha} - \bar{x} - \cancel{\alpha})^2} = b_1$$

$$\Rightarrow \begin{cases} \boxed{c_1 = b_1} \\ c_0 = \bar{y} - c_1 \bar{x}^* = \bar{y} - b_1 \bar{x} - \alpha b_1 = \boxed{b_0 - \alpha b_1 = c_0} \end{cases}$$

$$\hat{y}_t^* = b_0 - \alpha b_1 + b_1 \bar{x}^* = b_0 - \cancel{\alpha b_1} + b_1 \bar{x} + \cancel{b_1 \alpha} = \hat{y}_t$$

Regress x_t^*

$$R^{*2} = \frac{\sum_{t=1}^{n} (\hat{y}_t^* - \bar{y})^2}{\sum_{t=1}^{n} (y_t - \bar{y})^2} = \frac{\sum_{t=1}^{n} (\hat{y}_t - \bar{y})^2}{\sum_{t=1}^{n} (y_t - \bar{y})^2} = R^2$$

Answer: if we do $x_t^* = x_t +$ const then $\boxed{R^2 = R^{*2}}$

APPLIED SCIENCES FACULTY.DS

3). Demeaned regression: $\begin{cases} y_t^* = y_t - \bar{y} = y_t + const_2 \\ x_t^* = x_t - \bar{x} = x_t + const_1 \end{cases}$

$const_2 = (-\bar{y})$ $const_1 = (-\bar{x})$

from(1) $\Downarrow$ $\Downarrow$ (from(2))

$R^{2*y} = R^2$ $R^{2*x} = R^2$

$\boxed{R^{2**} = R^2}$

$\begin{cases} \hat{y}_t^* = c_0 + c_1 x_t^* \\ \hat{y}_t = b_0 + b_1 x_t \end{cases}$; $\begin{cases} \bar{y}_t^* = 0 \\ \bar{x}_t^* = 0 \end{cases}$

$$c_1 = \frac{\frac{1}{n}\sum_{t=1}^{n}(y_t^* - \bar{y}^*)(x_t^* - \bar{x}^*)}{\frac{1}{n}\sum_{t=1}^{n}(x_t^* - \bar{x}^*)^2} = \frac{\frac{1}{n}\sum_{t=1}^{n}(y_t - \bar{y} - 0)(x_t - \bar{x} - 0)}{\frac{1}{n}\sum_{t=1}^{n}(x_t - \bar{x} - 0)^2}$$

$c_1 = b_1$

$c_0 = \bar{y}^* - c_1\bar{x}^* = 0$ $\Rightarrow$ $\hat{y}_t^* = b_1 x_t^* = \hat{y}_t - \underbrace{b_0 - b_1\bar{x}}_{} = \hat{y}_t - \bar{y}$

$b_1 x_t^* = b_1(x_t - \bar{x})$

$$R^{**2} = \frac{\sum_{t=1}^{n}(\hat{y}_t^* - \bar{y}^*)^2}{\sum_{t=1}^{n}(y_t^* - \bar{y}^*)^2} = \frac{\sum_{t=1}^{n}(\hat{y}_t - \bar{y} - 0)^2}{\sum_{t=1}^{n}(y_t - \bar{y} - 0)^2} = R^2$$

Proved! For a demeaned regression the residual sum of squares remains not changed.

$\boxed{R^{**2} = R^2}$