# МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ НАЦІОНАЛЬНОМУ УНІВЕРСИТЕТІ "ЛЬВІВСЬКА ПОЛІТЕХНІКА"

Кафедра систем штучного інтелекту

# Розрахункова робота

3 дисципліни «Дискретна математика»

#### Виконала:

Студентка групи КН-115

Рокицька Анастасія

Викладач:

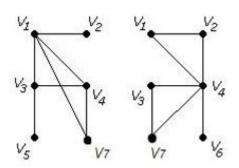
Мельникова Н.І.

Львів – 2019р.

#### **BAPIAHT 20**

#### Завдання №1

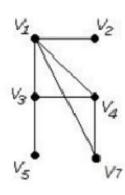
Виконати наступні операції над графами: 1) знайти доповнення до першого графу, 2) об'єднання графів, 3) кільцеву сумму G1 та G2 (G1+G2), 4) розмножити вершину у другому графі, 5) виділити підграф A - що скадається з 3-х вершин в G1 6) добуток графів.

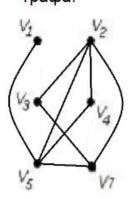


1)

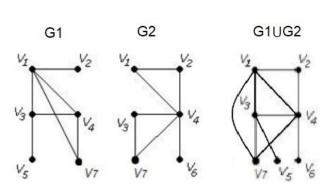
Граф:

Доповнення до графа:

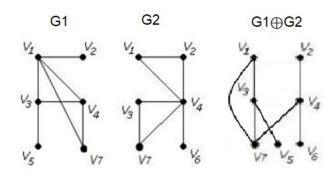




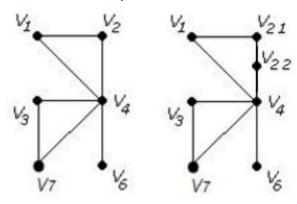
2)



3)



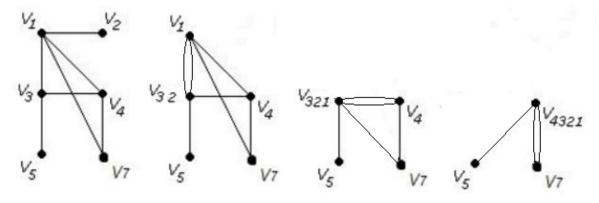
# 4) Розщеплення вершини V2



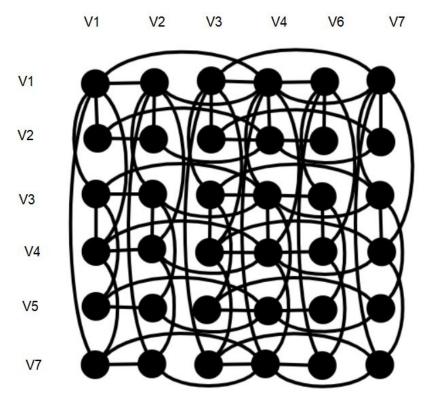
5)

G1: V1={V1,V2,V3,V4,V5,V7}

A: V2={V1,V2,V3} G1\A={V4321,V5,V7}

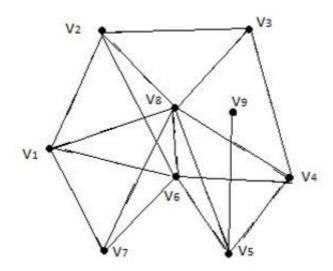


# 6) G1×G2



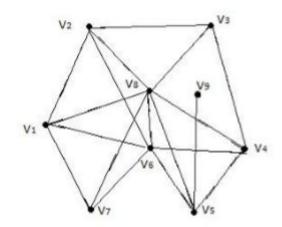
# Завдання №2

Скласти таблицю суміжності для орграфа.



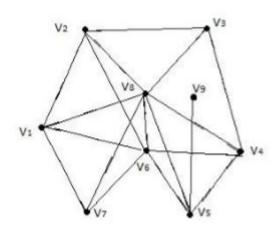
	V1	V2	V3	V4	V5	V6	V7	V8	V9
V1	0	1	0	0	0	1	1	1	0
V2	1	0	1	0	0	1	0	1	0
V3	0	1	0	1	0	0	0	1	0
V4	0	0	1	0	1	0	0	1	0
V5	0	0	0	1	0	1	0	1	1
V6	1	1	0	0	1	0	1	1	0
V7	1	0	0	0	0	1	0	1	0
V8	1	1	1	1	1	1	1	0	0
V9	0	0	0	0	1	0	0	0	0

Для графа з другого завдання знайти діаметр.



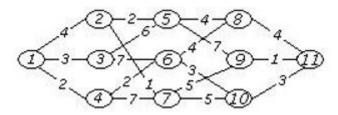
V1->V6->V5->V9 Діаметр графа дорівнює 3.

Завдання №4 Для графа з другого завдання виконати обхід дерева вшир



Вершина	BFS	Черга
V1	1	1
V2	2	12
V8	3	128
V6	4	1286
V7	5	12867
-	-	2867
V3	6	28673
-	-	8673
V4	7	86734
V5	8	867345
-	-	67345
-	-	7345
-	-	345
-	62	45
2	12	5
V9	9	59
-	-	9
<u> </u>		2

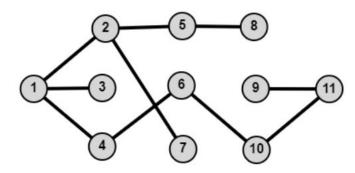
Знайти двома методами (Краскала і Прима) мінімальне остове дерево графа.



#### Прима:

V={11, 9, 10, 6, 8, 5, 2, 7, 1, 4, 3}

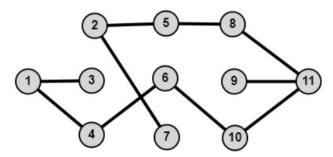
 $\mathsf{E} \! = \! \{ (11,\, 9),\, (11,\, 10),\, (10,\, 6),\, (6,\, 4),\, (4,\, 1),\, (1,\, 3),\, (1,\, 2),\, (2,\, 7),\, (2,\, 5),\, (5,\, 8) \}$ 



#### Краскала:

V={11, 9, 2, 7, 4, 6, 1, 5, 10, 3, 8}

 $E=\{(11, 9), (2, 7), (4, 6), (1, 4), (2, 5), (10, 11), (10, 6), (1, 3), (11, 8), (5, 8)\}$ 



Розв'язати задачу комівояжера для повного 8-ми вершинного графа методом «іди у найближчий», матриця вагів якого має вигляд:

	1	2	3	4	5	6	7	8
1	90	4	6	5	1	2	3	5
2	4	90	5	1	5	1	5	1
3	6	5	00	5	6	1	5	7
	5	1	5	00	6	4	5	5
4 5 6	1	1	6	∞ 6	00	3	2	
6	2	1	1	4	3	90	2	2
7	3	5	5	5	2	2	00	2
8	5	1	7	5	2	2	2	00

	1	2	4	5	6	7	8		1				
1	∞	4	5	1	2	3	5		6	2	4	5	7
2	4	∞	1	5	1	5	1	163	∞	4	5	1	3
4	5	1	∞	6	4	5	5	2	4	∞	1	5	5
5	1	5	6	00	3	2	2	4	5	1	∞	6	5
63	2	1	4	3	∞	2	2	5	1	5	6	00	2
7	3	5	5	2	2	00	2	7	3	5	5	2	00
8	5	1	5	2	2	2	00	8	5	1	5	2	2
500				7 5		824		 1		8			

	2	4	5 1 6 3	7	8
2	∞	1	5	5	1
4	1	∞	6	5	5
5361	5	6	∞	2	2
7	5	5	2	∞	2
8	1	5	2	2	00

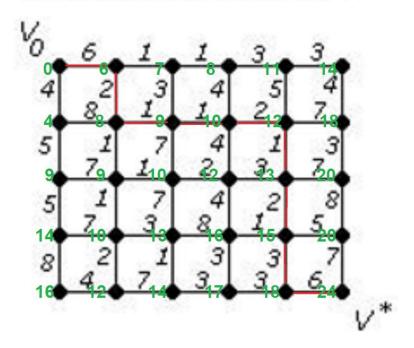
	2	4	7 5 1 6 3	8
2	∞	1	5	1
4	1	∞	5	5
75361	5	5	∞	2
8	1	5	2	00

	2	4	8 7 5 1 6 3
2	∞	1	1
4	1	∞	5
875361	1	5	00

	2 8 7 5 1 6 3	4
2875361	00	1
4	1	00

3->6->1->5->7->8->2->4->3 Найкоротший шлях=1+2+1+2+2+1+1+5=15

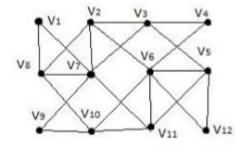
За допомогою алгоритму Дейкстри знайти найкоротший шлях у графі між парою вершин  $V_0$  і  $V^*$  .



Найкоротший шлях у графі між вершинами дорівнює 24.

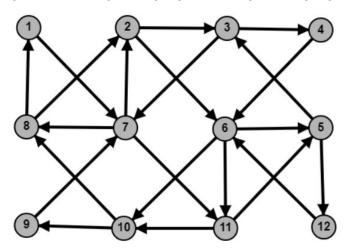
## Завдання №8

Знайти ейлеровий цикл в ейлеровому графі двома методами: а) Флері; б) елементарних циклів.



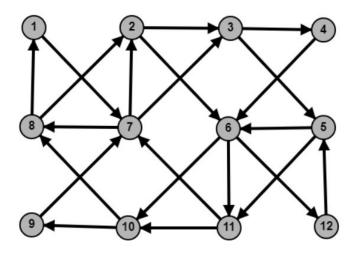
# а) Флері:

8->1->7->2->3->4->6->5->1->2->6->11->5->3->7->11->10->9->7->8->2->6->10->8



# б)елементарних циклів:

8->1->7->2->6->10->8->2->3->4->6->12->5->11->10->9->7->3->5->6->11->7->8



Спростити формули (привести їх до скороченої ДНФ).

$$(x \vee \overline{z})(\overline{y} \vee z)$$

$$(x \land \neg y) \lor (x \land z) \lor (\neg z \land \neg y) \lor (\neg z \land z) = \\ = (x \land \neg y) \lor (x \land z) \lor (\neg z \land \neg y) \lor F = \\ = (x \land \neg y) \lor (x \land z) \lor (\neg z \land \neg y) = \\ = ((x \land \neg y) \land (z \land \neg z)) \lor ((x \land z) \land (y \land \neg y)) \lor ((\neg z \land \neg y) \land (x \land \neg x)) = \\ = (x \land \neg y \land z) \lor (x \land \neg y \land \neg z) \lor (x \land y \land z) \lor (x \land \neg y \land \neg y \land \neg z) = \\ = (x \land \neg y \land z) \lor (x \land \neg y \land \neg z) \lor (x \land y \land z) \lor (\neg x \land \neg y \land \neg z) = \\ = (x \land z) \lor (\neg z \land \neg y)$$

# Програмна реалізація:

# 4)Вшир:

```
#include <iostream>
#include <queue>
using namespace std;
int main()
{
 queue<int> Queue;
 int mas[9][9]{ {0, 1, 0, 0, 0, 1, 1, 1, 0},
           { 1,0,1,0,0,1,0,1,0 },
           \{0,1,0,1,0,0,0,1,0\},\
           \{0,0,1,0,1,1,0,1,0\},\
           \{0,0,0,1,0,1,0,1,1\},\
           { 1,1,0,1,1,0,1,1,0 },
           { 1,0,0,0,0,1,0,1,0 },
           { 1,1,1,1,1,1,0,0 },
           {0,0,0,0,1,0,0,0,0};
 int nodes[9];
 for (int i = 0; i < 9; i++)
    nodes[i] = 0;
 Queue.push(0);
 while (!Queue.empty())
 {
    int node = Queue.front();
    Queue.pop();
    nodes[node] = 2;
    for (int j = 0; j < 9; j++)
       if (mas[node][j] == 1 \&\& nodes[j] == 0)
      {
         Queue.push(j);
         nodes[j] = 1;
      }
    cout << node + 1 << endl;
 }
 cin.get();
 return 0;
}
```

```
1
2
6
7
8
3
4
5
```

# 5) Прима:

```
#include <iostream>
using namespace std;
void main()
{
        setlocale(LC_ALL, "ukr");
        int a, b, i, j, min, ne = 1, mincost = 0;
        int visited[15] = \{0\};
        int path[15] = \{ 0 \};
        int num = 0;
        int n = 11;
        \{0,0,4,3,2,0,0,0,0,0,0,0,0,0\}
                                   \{0,4,0,0,0,2,0,1,0,0,0,0,0\},
                                \{0,3,0,0,0,6,7,0,0,0,0,0,0\},
                                \{0,2,0,0,0,0,2,7,0,0,0,0\},
                                \{0,0,2,6,0,0,0,0,4,7,0,0\},\
                       \{0,0,0,7,2,0,0,0,4,0,3,0\},\
                       \{0,0,1,0,7,0,0,0,0,5,5,0\},\
                    \{0,0,0,0,0,4,4,0,0,0,0,4\},
                    \{0,0,0,0,0,7,0,5,0,0,0,1\},
                       \{0,0,0,0,0,0,3,5,0,0,0,3\},\
                       \{0,0,0,0,0,0,0,0,4,1,3,0\}
                     };
        int arr2[12][12];
        for (i = 1; i < 12; i++)
                 for (j = 0; j < 12; j++) {
                          arr2[i][j] = arr[i][j];
                          if (arr2[i][j] == 0)
                                   arr2[i][j] = 999;
                 }
        visited[11] = 1;
        cout << "\n\tПошук мінімального остового дерева за методом Прима: ";
        while (ne < n)
        {
                 min = 999;
                 for (i = 1; i \le n; i++) {
                          for (j = 1; j \le n; j++) {
                                   if (arr2[i][j] < min) {
                                            if (visited[i] != 0)
                                            {
                                                     min = arr2[i][j];
                                                     a = i;
                                                     b = j;
                                            }
                                   }
                          }
```

```
}
              if (visited[a] == 0 || visited[b] == 0)
                     path[num] = b;
                     num++;
                     ne++;
                     mincost += min;
                     visited[b] = 1;
                     cout << "\n\t( " << a << ", " << b << ")";
              }
              arr2[a][b] = arr2[b][a] = 999;
       cout << "\n\n";
       cout << "\tV={";
       cout << 11 << ", ";
       for (int i = 0; i < n - 1; i++)
              cout << path[i];
              if (i < n - 2) cout << ", ";
       }
       cout << "}";
       cout << "\n\tВага остового дерева = " << mincost;
       cout << endl << endl;
 Пошук мінімального остового дерева за методом Прима:
 (11,
 (11, 10)
 (10, 6)
   6,
         4)
         1)
 (4,
 (1,
        3)
 (1,
         2)
 (2,
         7)
 (2,
         5)
 V=\{11, 9, 10, 6, 4, 1, 3, 2, 7, 5, 8\}
 Вага остового дерева = 25
б) Краскала:
#include <iostream>
using namespace std;
struct Rib
{
       int v1, v2, weight;
```

}Graph[100];

```
struct sort_rib {
        int v1;
        int v2;
        int weight;
}sort;
void Fill_Struct(int number_of_ribs) {
        for (int i = 0; i < number_of_ribs; i++) {
                 cout << "Firts point: ";
                 cin >> Graph[i].v1;
                 cout << "Second point: ";</pre>
                 cin >> Graph[i].v2;
                 int sort;
                 if (Graph[i].v1 > Graph[i].v2) {
                         sort = Graph[i].v1;
                         Graph[i].v1 = Graph[i].v2;
                         Graph[i].v2 = sort;
                 cout << "The rib [" << Graph[i].v1 << ";" << Graph[i].v2 << "] = ";
                 cin >> Graph[i].weight;
                 cout << endl;
        }
void Sort_Sructure(int number_of_ribs) {
        for (int s = 1; s < number_of_ribs; s++) {
                 for (int i = 0; i < number_of_ribs - s; i++) {
                         if (Graph[i].weight > Graph[i + 1].weight) {
                                  sort.v1 = Graph[i].v1;
                                  sort.v2 = Graph[i].v2;
                                  sort.weight = Graph[i].weight;
                                  Graph[i].v1 = Graph[i + 1].v1;
                                  Graph[i].v2 = Graph[i + 1].v2;
                                  Graph[i].weight = Graph[i + 1].weight;
                                  Graph[i + 1].v1 = sort.v1;
                                  Graph[i + 1].v2 = sort.v2;
                                  Graph[i + 1].weight = sort.weight;
                         }
                 }
        }
void Show_Struct(int number_of_ribs) {
        for (int i = 0; i < number_of_ribs; i++) {
                 cout << "The rib [" << Graph[i].v1 << ";" << Graph[i].v2 << "] = " << Graph[i].weight <<
endl;
        }
void Algo_Kraskala(int number_of_ribs, int amount_of_points)
{
        int weighttree = 0;
```

```
int* parent = new int[amount_of_points];
        int v1, v2, weight;
        int to_change, changed;
        for (int i = 0; i < amount_of_points; i++)
        {
                parent[i] = i;
        for (int i = 0; i < number_of_ribs; i++)
        {
                v1 = Graph[i].v1;
                v2 = Graph[i].v2;
                weight = Graph[i].weight;
                if (parent[v2] != parent[v1])
                         cout << "The rib [" << Graph[i].v1 << ";" << Graph[i].v2 << "] = " <<
Graph[i].weight << endl;
                         weighttree += weight;
                         to_change = parent[v1];
                         changed = parent[v2];
                         for (int j = 0; j < amount_of_points; j++)
                         {
                                 if (parent[j] == changed)
                                 {
                                          parent[j] = to_change;
                                 }
                         }
                }
        }
        delete[] parent;
        cout << "The weight of the tree: " << weighttree;</pre>
}
int main() {
        cout << "Enter an amount of points" << endl;
        int q;
        cin >> q;
        int amount_of_points = q + 1;
        cout << "Enter a number of ribs" << endl;
        int number_of_ribs;
        cin >> number_of_ribs;
        Fill_Struct(number_of_ribs);
        Sort_Sructure(number_of_ribs);
        cout << "After sorting" << endl;</pre>
        Show_Struct(number_of_ribs);
        cout << "Tree" << endl;
        Algo_Kraskala(number_of_ribs, amount_of_points);
}
```

```
Enter an amount of
                    Firts point: 6
11
                    Second point: 8
Enter a number of
                    The rib [6;8] = 4
18
Firts point: 1
                    Firts point: 6
Second point: 2
                    Second point: 10
The rib [1;2] = 4
                    The rib [6;10] = 3
Firts point: 1
                    Firts point: 7
Second point: 3
                    Second point: 9
The rib [1;3] = 3
                    The rib [7;9] = 5
Firts point: 1
                    Firts point: 7
Second point: 4
                    Second point: 10
The rib [1;4] = 2
                    The rib [7;10] = 5
Firts point: 2
                    Firts point: 8
Second point: 5
                    Second point: 11
The rib [2;5] = 2
                    The rib [8;11] = 4
Firts point: 2
                    Firts point: 9
Second point: 7
                    Second point: 11
The rib [2;7] = 1
                    The rib [9;11] = 1
Firts point: 3
                    Firts point: 10
Second point: 5
The rib [3;5] = 6
                    Second point: 11
                    The rib [10;11] = 3
                    After sorting
Firts point: 3
Second point: 6
                    The rib [2;7]
The rib [3;6] = 7
                    The rib [9;11] = 1
                    The rib [1;4] = 2
                    The rib [2;5]
Firts point: 4
                                  = 2
Second point: 6
                    The rib [4;6]
                    The rib [1;3]
The rib [4;6] = 2
                                  = 3
                                          Tree
                    The rib [6;10] = 3
                                          The rib [2;7] = 1
                    The rib [10;11] = 3
Firts point: 4
                                          The rib [9;11] = 1
Second point: 7
                    The rib
                            [1;2] = 4
                                          The rib [1;4] = 2
                    The rib [5;8]
The rib [4;7] = 7
                                          The rib [2;5] = 2
                    The rib [6;8]
                                          The rib [4;6] = 2
Firts point: 5
                    The rib [8;11] = 4
                                          The rib [1;3] = 3
Second point: 8
The rib [5;8] = 4
                    The rib
                            [7;9]
                    The rib [7;10] = 5
                                          The rib [6;10] = 3
                                          The rib [10;11] = 3
                    The rib [3;5] = 6
                                          The rib [1;2] = 4
                    The rib [3;6]
Firts point: 5
                    The rib [4;7]
Second point: 9
                                          The rib [5;8] = 4
The rib [5;9] = 7
                   The rib [5;9]
                                          The weight of the tree: 25
```

# 6)Комівояжера:

```
#include <iostream>
using namespace std;
const int inf=1E9,NMAX=16;
int n,i,j,k,m,temp,ans,d[NMAX][NMAX],t[1<<NMAX][NMAX];
bool get(int nmb,int x)
{ return (x&(1<<nmb))!=0; }
int main()
{
    cin >>n;
    for (i=0;i<n;++i)</pre>
```

```
for (j=0;j< n;++j) cin>>d[i][j];
  t[1][0]=0; m=1<<n;
  for (i=1;i< m;i+=2)
    for (j=(i==1)?1:0;j< n;++j)
       t[i][j]=inf;
       if (j>0 && get(j,i))
          temp=i^(1<<j);
          for (k=0;k< n;++k)
             if (get(k,i) \&\& d[k][j]>0) t[i][j]=min(t[i][j],t[temp][k]+d[k][j]);
       }
  for (j=1,ans=inf;j< n;++j)
    if (d[j][0]>0) ans=min(ans,t[m-1][j]+d[j][0]);
  if (ans==inf) cout<<-1; else cout<<ans;
}
```

## 7)Дейкстри:

```
\{0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,7,0,3,0,0,0,0,2,0,0,0,0\},
      \{0,0,0,0,0,0,0,0,0,0,0,0,0,0,7,0,0,0,0,3,0,8,0,0,0,0,1,0,0,0\}
      };
int shortest_dist[30];
bool visited[30] = \{0\};
int parent[30];
int getNearest() {
   int minValue = 999, minNode = 0;
   for (int i = 0; i < V; i++) {
      if (!visited[i] && shortest_dist[i] < minValue) {</pre>
         minValue = shortest dist[i];
         minNode = i;
      }
   }
   return minNode;
}
int main(void) {
   V = 30;
   for (int i = 0; i < V; i++) {
      for (int j = 0; j < V; j++) {
         if (incidental_matrix[i][j] == 0 && i != j) {
            incidental_matrix[i][j] = 999;
         }
      }
   starting_point = 0;
   for (int i = 0; i < V; i++) {
      parent[i] = i;
      shortest_dist[i] = INF;
   shortest_dist[starting_point] = 0;
```

```
for (int i = 0; i < V; i++) {
                 int nearest = getNearest();
                 visited[nearest] = true;
                 for (int adj = 0; adj < V; adj++) {
                         if (incidental_matrix[nearest][adj] != INF && shortest_dist[adj] >
shortest_dist[nearest] + incidental_matrix[nearest][adj]) {
                                  shortest_dist[adj] = shortest_dist[nearest] +
incidental_matrix[nearest][adj];
                                  parent[adj] = nearest;
                         }
                 }
        cout << "Cost : \t\t\tPath" << endl;
        cout << shortest_dist[V - 1] << "\t\t" << " ";
        cout << V << " ";
        temp = parent[V - 1];
        while (temp != starting_point) {
                 if (temp == 1) {
                         cout << " <- " << temp + 1 << " <- 1";
                 }
                 else {
                         cout << " <- " << temp + 1 << " ";
                 }
                 temp = parent[temp];
        cout << endl;
        return 0;
}
```

```
Cost : Path
24 30 <- 29 <- 23 <- 17 <- 11 <- 10 <- 9 <- 8 <- 2 <- 1
```

#### 8)а) Флері:

```
#include<iostream>
#include<vector>
#define NODE 132
using namespace std;
int graph[NODE][NODE] = {
                  \{0,0,0,0,0,0,1,1,0,0,0,0,0\},\
                  \{0,0,1,0,0,1,1,1,0,0,0,0,0\},\
                  \{0,1,0,1,1,0,1,0,0,0,0,0,0,0\},
                  \{0,0,1,0,0,1,0,0,0,0,0,0,0,0\}
                  \{0,0,1,0,0,1,0,0,0,0,1,1\},\
                  \{0,1,0,1,1,0,0,0,0,1,1,1\},
                  \{1,1,1,0,0,0,0,1,1,0,1,0\},\
                  \{1,1,0,0,0,0,1,0,0,1,0,0\},\
                  \{0,0,0,0,0,0,1,0,0,1,0,0\},\
                  \{0,0,0,0,0,1,0,1,1,0,1,0\},\
                  \{0,0,0,0,1,1,1,0,0,1,0,0\},\
```

```
\{0,0,0,0,1,1,0,0,0,0,0,0,0\}
};
int tempGraph[NODE][NODE];
bool findStartVert() {
        for (int i = 1; i < NODE; i++) {
                 int deg = 0;
                 for (int j = 0; j < NODE; j++) {
                          if (tempGraph[i][j])
                                   deg++;
                 }
                 if (deg % 2 != 0)
                          return false;
        return true;
}
bool isBridge(int u, int v) {
        int deg = 0;
        for (int i = 0; i < NODE; i++)
                 if (tempGraph[v][i])
                          deg++;
        if (deg > 1) {
                 return false;
        return true;
}
int edgeCount() {
        int count = 0;
        for (int i = 0; i < NODE; i++)
                 for (int j = i; j < NODE; j++)
                          if (tempGraph[i][j])
                                   count++;
        return count;
void fleuryAlgorithm(int start) {
        static int edge = edgeCount();
        for (int v = 0; v < NODE; v++) {
                 if (tempGraph[start][v]) {
                          if (edge <= 1 || !isBridge(start, v)) {
                                   cout << start + 1 << "--" << v + 1 << " ";
                                   tempGraph[start][v] = tempGraph[v][start] = 0;
                                   edge--;
                                   fleuryAlgorithm(v);
                          }
                 }
        }
}
int main() {
        for (int i = 0; i < NODE; i++)
                 for (int j = 0; j < NODE; j++)
                          tempGraph[i][j] = graph[i][j];
```

Euler Path Or Circuit: 8--1 1--7 7--2 2--3 3--4 4--6 6--2 2--8 8--7

7--3 3--5 5--6 6--10 10--9 9--7 7--11 11--5 5--12 12--6 6--11 11--10 10--8