Multiphase Flows – WS 2022/23
Problem Session 11: Statistical Modeling

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Agenda

- Problem Session 9: Linear momentum coupling
- Problem Session 10 (today): Statistical modeling
- Problem Session 11: Applications

Problem Session 11: Statistical Modeling

Volume-Averaged Equations

- Averages of quantity B typically expressed in terms of volume average $\bar{B}$ or phase average $\langle B \rangle$

➢ For continuous phase density: $\bar{\rho}_c = \alpha_c \langle \rho_c \rangle$

➢ Volume-averaged gradient operations for quantity B (for detailed derivation: see [1]):

$$\overline{\frac{\partial B}{\partial t}} = \frac{\partial \bar{B}}{\partial t} + \frac{1}{V} \int_{S_d} B(v_i n_i + \dot{r}) dS$$

$$\overline{\frac{\partial B}{\partial x_i}} = \frac{\partial \bar{B}}{\partial x_i} - \frac{1}{V} \int_{S_d} B n_i dS$$

with V: control volume, S_d: surface of dispersed phase in control volume, n_i: normal outward vector of particle surface, r: particle radius

[1] Crowe et al., Multiphase Flows with Droplets and Particles, *CRC Press*, 2012.

Problem 1: Volume-Averaged Continuity Equation

The general continuity equation for the carrier phase of a two-phase mixture is given as

$$\frac{\partial \rho_c}{\partial t} + \frac{\partial \rho_c u_i}{\partial x_i} = 0.$$

Derive the volume averaged continuity equation of the carrier phase!

Problem 2: Rosin-Rammler Distribution

The Rosin-Rammler distribution is frequently used to describe droplet size distributions during fuel injection. In terms of the cumulative mass distribution, it can be expressed as

$$F_m(D) = 1 - \exp\left(-\left(\frac{D}{\delta}\right)^n\right)$$

a) *Derive the corresponding PDF.*

b) *For the parameters* $\delta = 144\ \mu m$ *and* $n = 2$*, compute the mean diameter and variance by numerical integration in MATLAB. Plot the resulting PDF and CDF.*

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Problem 3: Monte-Carlo Sampling (in MATLAB)

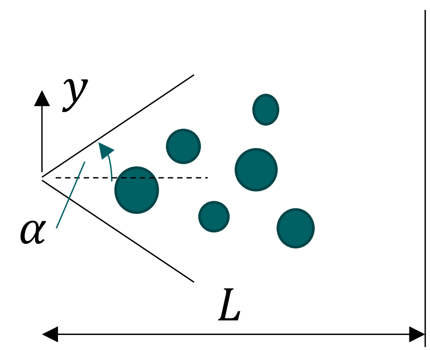
Solid particles are injected into a large chamber. Their size is assumed to be log-normally distributed inside their minimum and maximum bounds of $0.2\ mm$ *and* $10\ mm$*, respectively, with parameters* $D_m = 2mm$ *and variance* $\nu = 0.2\ mm^2$*. The particles are ejected by the nozzle at a constant absolute velocity of* $u_0 = 100\ m/s$*, while their injection angle* α *is uniformly distributed between* $-\pi/8$ *and* $+\pi/8$*. The carrier phase has a constant velocity in y-direction of* $v_c = 10\ m/s$*.*

Simulate the droplet motion by Monte-Carlo sampling with the given distributions of particle diameter D *and injection angle* α*. Visualize the particle distribution in terms of vertical position* y_L*, residence time in the chamber* t_L*, and x-velocity* u_L *at* $x = L = 1m$ *as well as their joint distributions. Compute the corresponding mean values and standard deviations.*

Use MATLAB's built-in functions rand() and lognrnd() for sampling.

Other parameters:

$$\rho_\mathrm{d} = 1000\,\frac{kg}{m^3},\ \mu_\mathrm{c} = 1.846\cdot 10^{-5}\,\frac{kg}{m\cdot s},\ \rho_\mathrm{c} = 1.2\,\frac{kg}{m^3}$$



Thank you for your attention

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