Multiphase Flows – WS 2022/23
Problem Session 4 – **Solution**: Surface Tension

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Problem 1: Solution

a) Force balance for bubble: $(p_{s,0} - p_{a,0})\pi\left(\frac{4d_0}{2}\right)^2 = 2 \cdot 3\sigma \cdot 4\pi d_0$

$$=> p_{s,0} - p_{a,0} = \frac{6\sigma}{d_0}$$

Force balance for droplet: $(p_{i,0} - p_{s,0})\frac{\pi d_0^2}{4} = \sigma \cdot \pi d_0$

$$=> p_{i,0} - p_{s,0} = \frac{4\sigma}{d_0}$$

$$=> \Delta p_0 = p_{i,0} - p_{a,0} = \frac{10\sigma}{d_0}$$

Problem 1: Solution

Mass of air in bubble (constant)

b) Ideal gas EOS for air in bubble: $p_{s,1} = \frac{m}{V}\frac{R}{M}T = \frac{m_0}{\frac{4}{3}\pi\left(\frac{D_1}{2}\right)^3 - \frac{4}{3}\pi\left(\frac{d_0}{2}\right)^3}\frac{R}{M}T$ (1)

Force balance for bubble: $(p_{s,1} - 2p_{a,0})\pi\left(\frac{D_1}{2}\right)^2 = 2 \cdot 3\sigma \cdot \pi D_1$

=> $p_{s,1} = \frac{24\sigma}{D_1} + 2p_{a,0}$ (2)

Mass of air in bubble: $m_0 = \frac{p_{s,0}\cdot\frac{4}{3}\pi\left(\left(\frac{D_0}{2}\right)^3 - \left(\frac{d_0}{2}\right)^3\right)M}{RT}$ (3)

(2) & (3) in (1): $\frac{24\sigma}{D_1} + 2p_{a,0} = \frac{\left(p_{a,0}+\frac{6\sigma}{d_0}\right)\left(\left(\frac{D_0}{2}\right)^3 - \left(\frac{d_0}{2}\right)^3\right)}{\left(\frac{D_1}{2}\right)^3 - \left(\frac{d_0}{2}\right)^3}$ → to be solved for D_1

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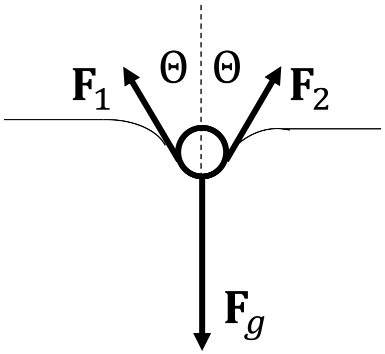
Problem 2: Solution

F_1 and F_2: surface tensions forces
$\rightarrow F_1 = F_2 = \sigma L$



Force balance in y-direction:

$$-mg + 2\sigma L \cdot \cos(\Theta) = 0$$

$$=> \quad m = \frac{2\sigma L \cdot \cos(\Theta)}{g}$$

Maximum for $\Theta = 0°$: $m_{\max} = 0.476\text{ g}$

Idealized assumptions:

- Force balance can be different at needle ends
- $\Theta = 0°$ will never be reached (infinite depth)

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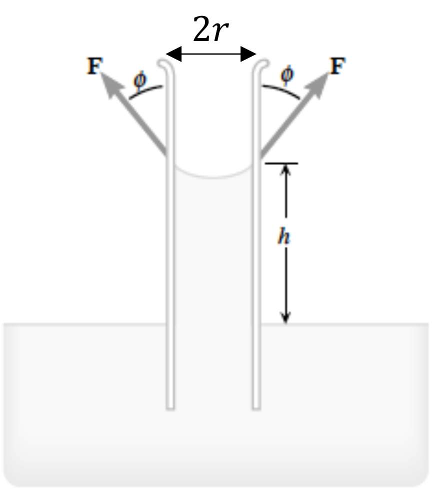
Problem 3: Capillary Action

$F = \sigma \cdot 2\pi r$

Force balance in y-direction:

$$\sigma \cdot 2\pi r \cdot \cos(\phi) = \rho g \pi r^2 h$$

$$=> h = \frac{2\sigma\cos(\phi)}{\rho g r}$$



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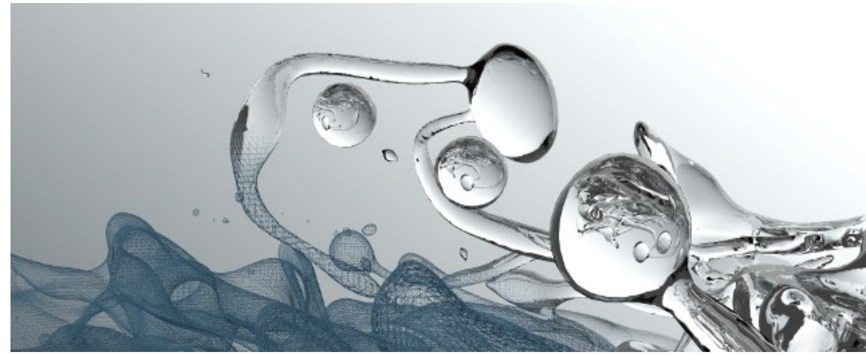
Problem 4: Solution

Thermodynamic equilibrium: $T_i = T_a,\ g_i = g_a$

But: $p_i \neq p \quad => \quad \Delta p = p_i - p = \frac{2\sigma}{r} \quad => \quad r = \frac{2\sigma(T)}{p_i - p} = \frac{2\sigma(T)}{p_{sat}(T) - p}$

Phase equilibrium: $g = g_i \quad => \quad g_\mathrm{v}(p, T) = g_\mathrm{l}\left(p + \frac{2\sigma(T)}{r}, T\right)$ => solve for r (in MATLAB)

Edit file getpressures.m and implement surface tension and pressure models



Thank you for your attention

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