Multiphase Flows – WS 2022/23
Problem Session 10: Process Engineering Applications

Aranya Dan, M. Tech.

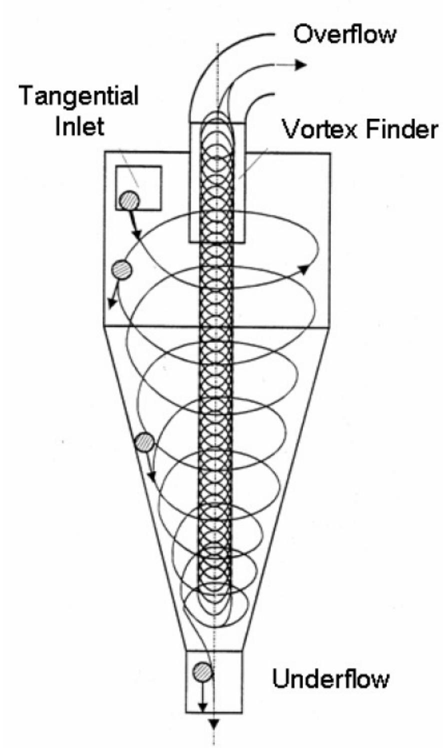
Institute for Combustion Technology
RWTH Aachen University

Problem Session 10: Process Engineering Applications

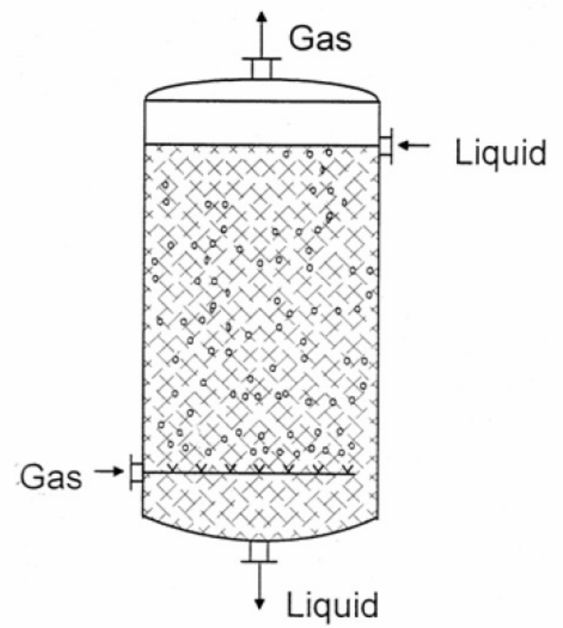
Process Reactors

- Physical processes
  - Bubble column
  - Fluidized bed
  - Fixed beds
  - Bio-reactor

- Thermal and chemical reactors
  - Bubble column
  - Fluidized bed
  - Bio-reactor



Hydrocyclone



Bubble column

Problem Session 10: Process Engineering Applications

Problem 1: Sedimentation of Single Particles (1)

We investigate the sedimentation of single particles at different conditions.

a) *Compute the steady-state sedimentation velocity of a single particle with diameter* $d_P = 0.02\ \mathrm{mm}$ *in water at* $20\ °\mathrm{C}$.

b) *Compute the steady-state sedimentation velocity of a single particle with diameter* $d_P = 0.2\ \mathrm{mm}$ *in air at* $1000\ °\mathrm{C}$.

c) *Compute the time after which an initially quiescent particle with diameter* $d_P = 0.02\ \mathrm{mm}$ *has reached* $99\ \%$ *of its steady-state sedimentation velocity in water at* $20\ °\mathrm{C}$. *The acceleration factor for the accelerated liquid is* $\alpha = 0.5$.

Particle density: $\rho_P = 5200\ \frac{\mathrm{kg}}{\mathrm{m}^3}$

Properties of water $(20\ °\mathrm{C})$: $\rho_{W,20} = 998.2\ \frac{\mathrm{kg}}{\mathrm{m}^3}$, $\eta_{W,20} = 1.004 \cdot 10^{-3}\ \frac{\mathrm{kg}}{\mathrm{m \cdot s}}$

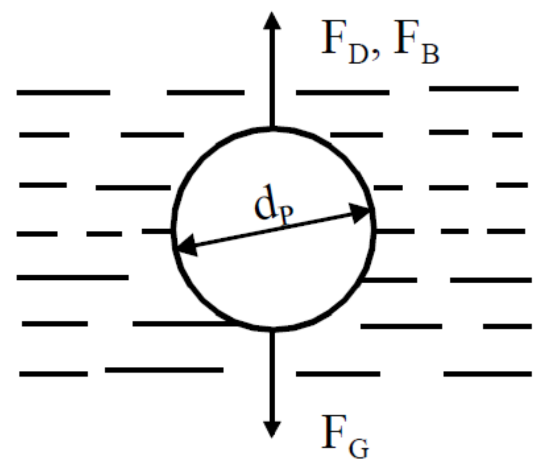
Properties of air $(1000\ °\mathrm{C})$: $\rho_{A,1000} = 0.2733\ \frac{\mathrm{kg}}{\mathrm{m}^3}$, $\eta_{A,1000} = 47.93 \cdot 10^{-6}\ \frac{\mathrm{kg}}{\mathrm{m \cdot s}}$

Single particle motion in a resting fluid

Drag laws of spherical particles

- Force balance in vertical direction at a sphere settling in a steady state
  - Gravity force $F_G = V_{SP} \cdot \rho_P \cdot g$
  - Buoyancy force $F_B = V_{SP} \cdot \rho_L \cdot g$
  - Drag force $F_D = C_D \cdot \rho_L \cdot A_{SP} \cdot \frac{v^2}{2}$

$$F_G - F_B - F_D = 0$$



Single particle motion in a resting fluid

Drag laws of spherical particles

- Drag laws of spherical particles
  - $\mathrm{Re} \leq 0.1$
    - Creeping flows
    - Analytical solutions by solving the Navier-Stokes-Equation [Stokes and Oseen]

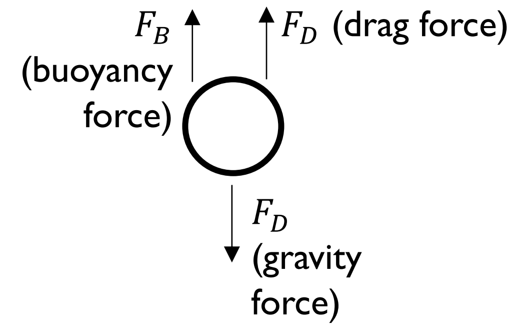
$$C_D = \frac{24}{\mathrm{Re}}$$

  - $0.1 < \mathrm{Re} < 2 \cdot 10^3$
    - Determined experimentally by owing a fixed sphere
    - The drag is increasingly dominated by inertial forces
    - The flow separates from the particle surface and eddies are generated downstream

$$C_D = \frac{24}{\mathrm{Re}} + \frac{4}{\sqrt{\mathrm{Re}}} + 0.4$$

Problem 1: Solution

Compute the steady-state sedimentation velocity of a single particle with diameter $d_P = 0.02\ \mathrm{mm}$ in water at $20\ °\mathrm{C}$.



a) Force balance at particle at steady-state:

$$F_G - F_B - F_D = 0$$

$$F_B = \frac{\pi}{6} d_P^3 \rho_W g$$

$$F_G = \frac{\pi}{6} d_P^3 \rho_P g$$

$$F_D = C_D \rho_W \frac{\pi}{4} d_P^2 \frac{v^2}{2}$$

Preliminary assumption:

Stokes flow $\rightarrow$ $C_D = \frac{24}{Re}$ (with $Re = \frac{d_P v \rho_W}{\eta_W}$)

$$=> \frac{\pi}{6} d_P^3 (\rho_P - \rho_W) g - \frac{24\eta_W}{d_P v \rho_W} \rho_W \frac{\pi}{4} d_P^2 \frac{v^2}{2} = 0$$

$$=> \quad v_\infty = 9.123 \cdot 10^{-4} \frac{\mathrm{m}}{\mathrm{s}}$$

Check assumption of Stokes flow: $Re = 0.018 < 0.1$
(assumption was correct)

Problem 1: Solution

Compute the steady-state sedimentation velocity of a single particle with diameter $d_P = 0.2\ \mathrm{mm}$ in air at $1000\ °\mathrm{C}$.

b) Assumption: $0.1 < Re < 2000$

=> Approximation of Kaskas: $C_D = \frac{24}{Re} + \frac{4}{\sqrt{Re}} + 0.4$ (1)

From force balance (as before): $C_D = \frac{3v^2\rho_L}{4d_P g(\rho_P - \rho_L)}$ (2)

(2)=(1): $\frac{3v^2\rho_L}{4d_P g(\rho_P - \rho_L)} = \frac{24\eta_L}{\rho_L d_P v} + 4\sqrt{\frac{\eta_L}{\rho_L d_P v}} + 0.4$

Iteration necessary => $v_\infty = 1.85\,\frac{\mathrm{m}}{\mathrm{s}}$

Problem 1: Solution

c) Time to reach 99% of steady-state velocity ($v_\infty = 9.123 \cdot 10^{-4}\,\frac{\mathrm{m}}{\mathrm{s}}$ already computed in (a))

Transient force balance (equation of motion): $F_G - F_B - F_T - F_D = 0$

$$=> \frac{\pi}{6} d_P^3 (\rho_P + \alpha \rho_w) \frac{dv}{dt} + \frac{24\eta_w}{d_P v \rho_w} \rho_w \frac{\pi}{4} d_P^2 \frac{v^2}{2} + \frac{\pi}{6} d_P^3 (\rho_W - \rho_P) g = 0 \quad (1)$$

(with $\alpha = V_{W,\mathrm{accel}}/V_P$)

From (a): $v_\infty = \frac{d_P^2 g(\rho_P - \rho_W)}{18\eta_W}$ => $\eta_W = \frac{d_P^2 g(\rho_P - \rho_W)}{18 v_\infty}$ (2)

Inserting (2) in (1): $\frac{v_\infty}{g(\rho_P - \rho_W)} (\rho_P + \alpha \rho_W) \frac{dv}{dt} = v_\infty - v$

Separation of variable & integration: $\int_0^{0.99 v_\infty} \frac{1}{v_\infty - v} dv = \int_0^{\tau} \frac{g(\rho_P - \rho_W)}{v_\infty(\rho_P + \alpha\rho_W)} dt$

$$=> -\ln(v_\infty - v)\,\Big|_0^{0.99 v_\infty} = \frac{g(\rho_P - \rho_W)}{v_\infty(\rho_P + \alpha\rho_W)} \tau$$

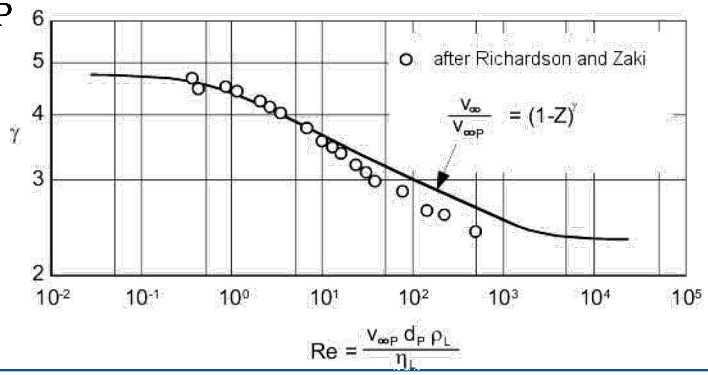
$$=> \quad \tau = 5.8 \cdot 10^{-4}\ \mathrm{s}$$

Particle Cloud Behavior

- In technical systems: particles often aggregate –> formation of particle clouds (PC)
  - → velocity smaller than velocity for single particle
  - → particles do not move any more in "pure" fluid, but in a medium with different properties due to the presence of a high particle number
- Particle volume concentration: $Z = \frac{\dot{V}_P}{\dot{V}_M} = \frac{\dot{V}_P}{\dot{V}_L + \dot{V}_P}$

➢ Cloud settling velocity [Richardson and Zaki]:

$$\frac{v_{\infty,\mathrm{PC}}}{v_{\infty,\mathrm{SP}}} = (1-Z)^{\gamma}\,, \qquad Z \geq 0.001; \qquad \frac{v_{\infty,\mathrm{PC}}}{v_{\infty,\mathrm{SP}}} = 1\,, \qquad Z < 0.001$$

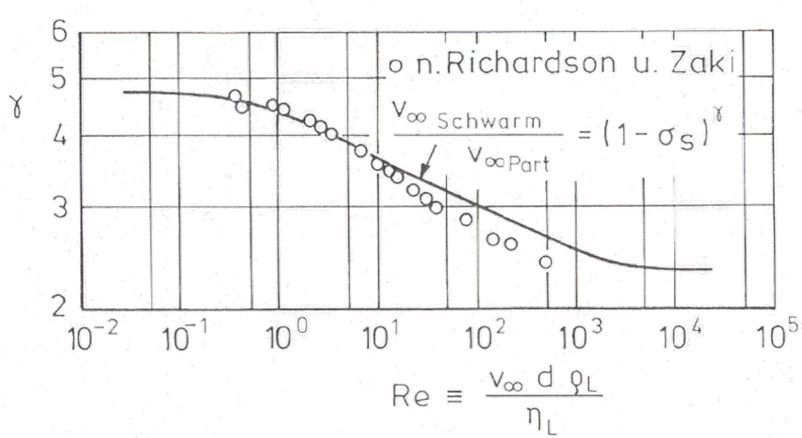


Problem 2: Sedimentation Velocity of Particle Cloud

We investigate the sedimentation of particles with diameter $d_P = 0.02\ \mathrm{mm}$*. The stationary sedimentation velocity of the single particles in water has been determined as* $v_{\infty,SP} = 9.123 \cdot 10^{-4}\ \frac{\mathrm{m}}{\mathrm{s}}$ *in Problem 1.*

Compute the particle cloud velocity of particles with this size assuming a particle mass concentration of $w_\mathrm{P} = 10\%$ *in the fluid.*

Properties: $\rho_\mathrm{P} = 5200\,\frac{\mathrm{kg}}{\mathrm{m}^3}, \rho_\mathrm{W} = 998.2\,\frac{\mathrm{kg}}{\mathrm{m}^3}, \eta_\mathrm{W} = 1.004 \cdot 10^{-3}\,\frac{\mathrm{kg}}{\mathrm{m \cdot s}}$



Problem 2: Solution

Particle volume fraction: $Z = \frac{V_P}{V_P+V_W} = \frac{\frac{m_P}{\rho_P}}{\frac{m_P}{\rho_P}+\frac{m_W}{\rho_W}} = \frac{\frac{w_P}{\rho_P}}{\frac{w_P}{\rho_P}+\frac{w_W}{\rho_W}}$

with $w_P = 0.1$ and $w_W = 1 - w_P = 0.9$
=> $Z_W = 0.0209 > 0.001$ => use of approximation formula

$Re = \frac{\rho_W v_\infty d_P}{\eta_W} = 0.018$ => from diagram: $\gamma = 4.7$

=> $v_{\infty,PC} = v_{\infty,SP}(1 - Z_W)^\gamma = 8.26 \cdot 10^{-4}\,\frac{\text{m}}{\text{s}}$

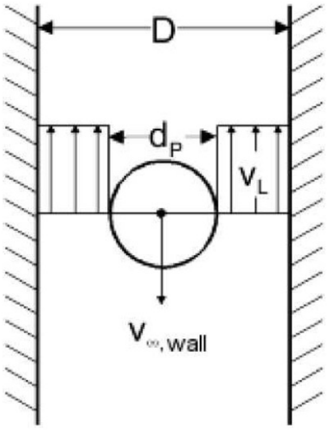
Problem 3: Sedimentation Velocity with Wall Impact

Determine the stationary sedimentation velocity $v_{\infty,\mathrm{P}}$ for a spherical particle with diameter d_P in water in a cylindric vessel with diameter D.

Note that the velocity $v_{\infty,\mathrm{P}}$ is influenced by the displaced fluid. Assume Stokes flow and a "piston profile" for the fluid velocity around the particle.

Properties: $\rho_P = 5200\ \frac{\mathrm{kg}}{\mathrm{m}^3},\ \rho_W = 998.2\ \frac{\mathrm{kg}}{\mathrm{m}^3},\ \eta_W = 1.004 \cdot 10^{-3}\ \frac{\mathrm{kg}}{\mathrm{m \cdot s}}$

Geometry: $d_P = 2\ \mathrm{mm},\ \ D = 10\ \mathrm{mm}$



Wall influence

- Simplified investigation
  - Difference to the velocity $v_{\infty,wall}$ results in a displacement flow $\dot{V}_L$ which is created by the sphere

$$\dot{V}_L = \frac{\pi}{4} d_P^2 \cdot v_{\infty,wall}$$

  - Fluid velocity

$$v_L = \frac{\dot{V}_L}{\frac{\pi}{4}(D^2 - d_P^2)} = v_{\infty,\mathrm{wall}} \cdot \frac{1}{\frac{D^2}{d_P^2} - 1}$$

  - Relative velocity

$$v_r = v_\infty = v_L + v_{\infty,\mathrm{wall}} = v_{\infty,wall} \cdot \frac{1}{1 - \frac{d_P^2}{D^2}}$$

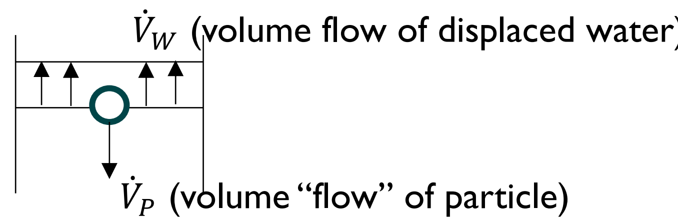
- Simplified investigation
  - Difference to the velocity $v_{\infty,wall}$ results in a displacement flow $\dot{V}_L$ which is created by the sphere
  - Fluid velocity
  - Relative velocity

Modelling of the wall influence

Problem 3: Solution

As in problem 1: $v_\infty = \frac{d_P^2 g(\rho_P - \rho_W)}{18\eta_W}$

=> v_∞ is relative velocity between particle and fluid: $v_\infty = v_{\infty,P} + v_W$ (1)



Assumption: "piston profile"

=> $\dot{V}_W = \dot{V}_P$

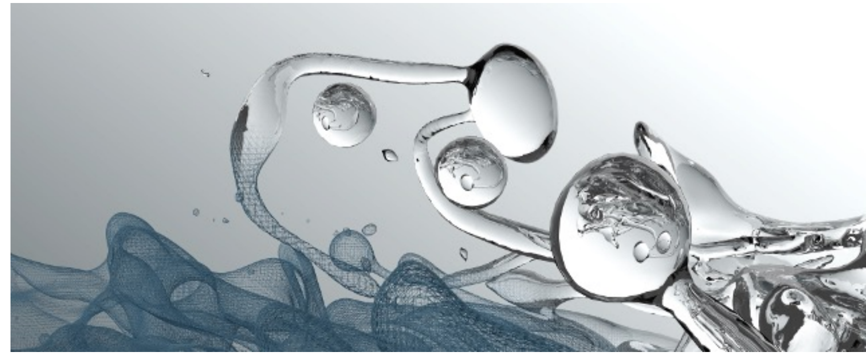
=> $\left(\frac{\pi}{4}D^2 - \frac{\pi}{4}d_P^2\right) v_w = \frac{\pi}{4} d_P^2 v_{\infty,P}$

=> $v_W = v_{\infty,P} \frac{d^2}{D^2 - d^2}$ (2)

(2) in (1): $v_{\infty,P} = v_\infty \left(1 - \frac{d_P^2}{D^2}\right)$

=> without wall: $v_\infty = 9.12 \frac{\mathrm{m}}{\mathrm{s}}$

=> with wall: $v_{\infty,P} = 8.76 \frac{\mathrm{m}}{\mathrm{s}}$



Thank you for your attention

Aranya Dan, M. Tech.

Institute for Combustion Technology
RWTH Aachen University

http://www.itv.rwth-aachen.de