Multiphase Flows – WS 2022/23
Problem Session 5: Interface Tracking (1/3)

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Problem Session 5: Interface Tracking (1/3)

Agenda

- Problem Session 4: Surface Tension
- Problem Session 5 (Today): Interface Tracking (1/3)
- Problem Session: Interface Tracking (2/3)

Problem Session 5: Interface Tracking (1/3)

Motivation

- Problem sessions 1-3: solution of separate sets of phase governing equations
  - ➢ Compressible fluids (flows with high velocities, e.g., injector flows)
  - ➢ No explicit interface tracking (only implicitly by advection of gas volume fraction)

- Often: flows have low Mach numbers (e.g., primary breakup)
  - ➢ Typically treated as incompressible
  - ➢ Simplified Navier-Stokes equations (compare Problem Session 1)
  - ➢ One-fluid assumption (solution of only one set of governing equations)
  - ➢ Sharp resolution of interface important

  - ➢ Interface tracking methods

Problem Session 5: Interface Tracking (1/3)

Motivation

- Today: 2-D incompressible two-phase flow solver without explicit interface tracking
  - Only "implicitly" by advection of density
  - Viscosity is considered, but simplification: both phases have equal viscosities
  - Surface tension is neglected
  - No phase change

- Next problem session: inclusion of front-tracking method
  - Advection of density replaced by explicit tracking of the interface and subsequent reconstruction of the density field

Problem Session 5: Interface Tracking (1/3)

Governing Equations

- Solution of the 2-D incompressible Navier-Stokes equations (const. viscosity):

$$\rho\left(\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial uv}{\partial y}\right)=-\frac{\partial p}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$
$$\rho\left(\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial vv}{\partial y}\right)=-\frac{\partial p}{\partial y}+\rho g_y+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

- Incompressible mass equation (divergence-free velocity field):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

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Governing Equations

- Assumption: incompressible flow
  - ➢ But: Density not constant among different locations (depending on the local phase)!
  - ➢ Only constant density assumption for each fluid particle (material derivative is zero)

➢ Advection of density field:

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho = 0$$

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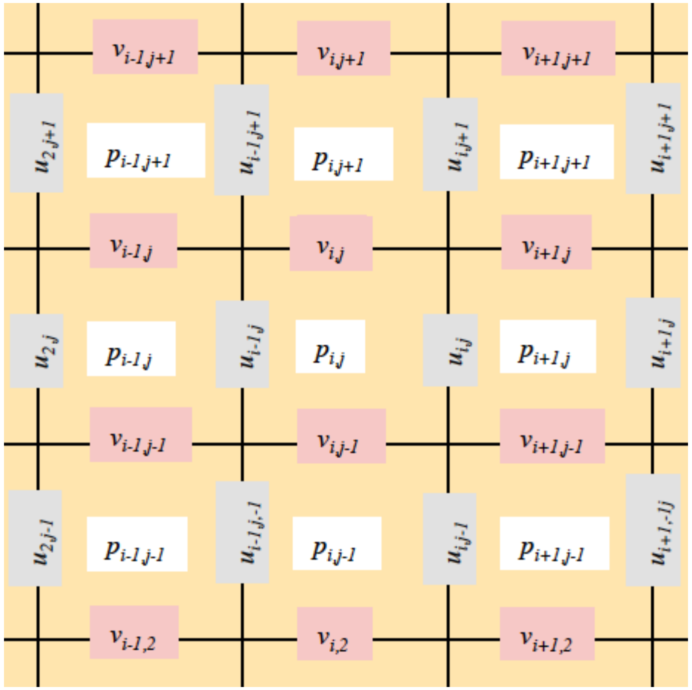
Numerical Solver

- Solution of governing equations on regular structured Cartesian grid
- ➢ Finite Volume discretization
- ➢ Staggered arrangement (advantageous for fulfilling incompressibility condition)



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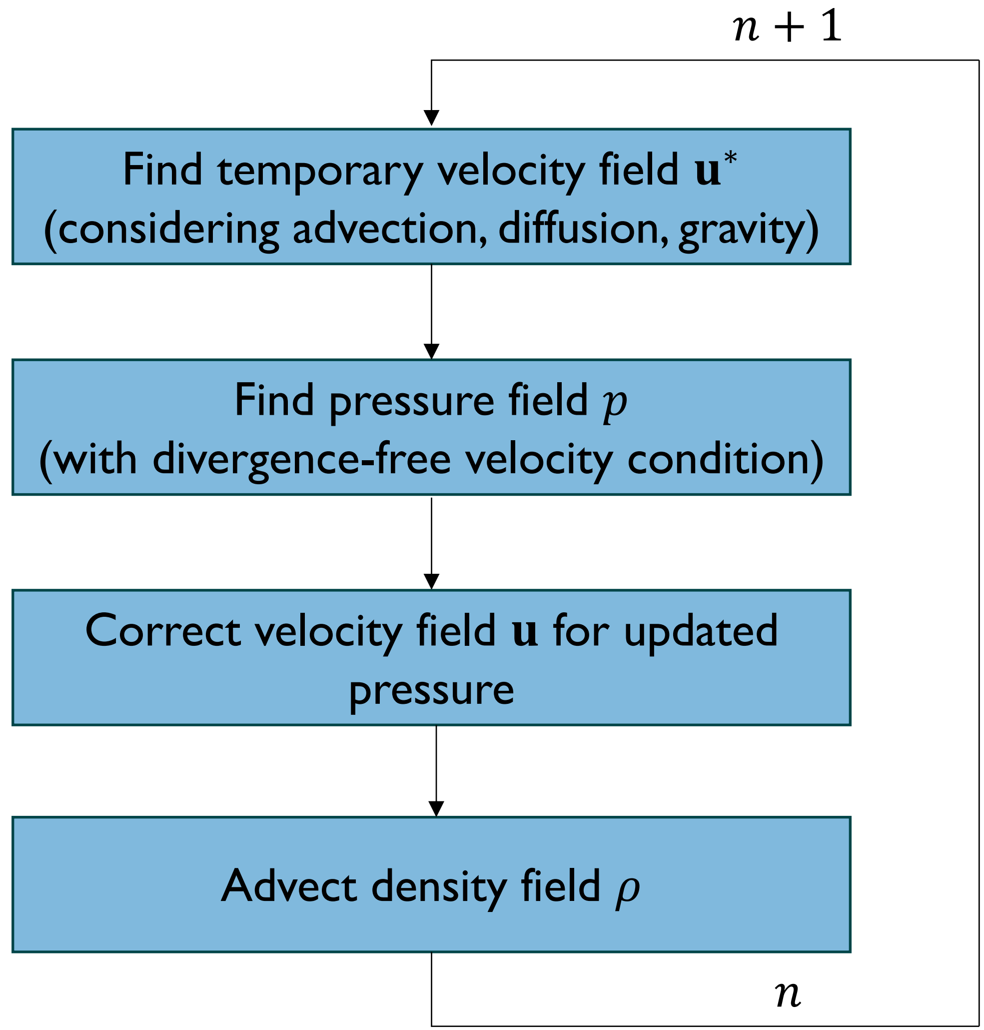
Numerical Solver

- Splitting approach based on projection method
  - First: Solution of momentum equations with advection, diffusion, and gravity terms
  - Then: Addition of pressure term (to reach finally divergence-free velocity field)



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Problem 1: Discrete Density Advection Equation

For good mass conservation in the numerical solver, we rewrite the density advection equation as $\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}) + \mu_0 \nabla^2 \rho$*. Here, a numerical diffusion term has been added in order to allow for a stable solution.*

Discretize the density advection equation in order to compute the new density $\rho_{i,j}^{n}$ *at time step* $(n+1)$ *and at location* i, j*!*

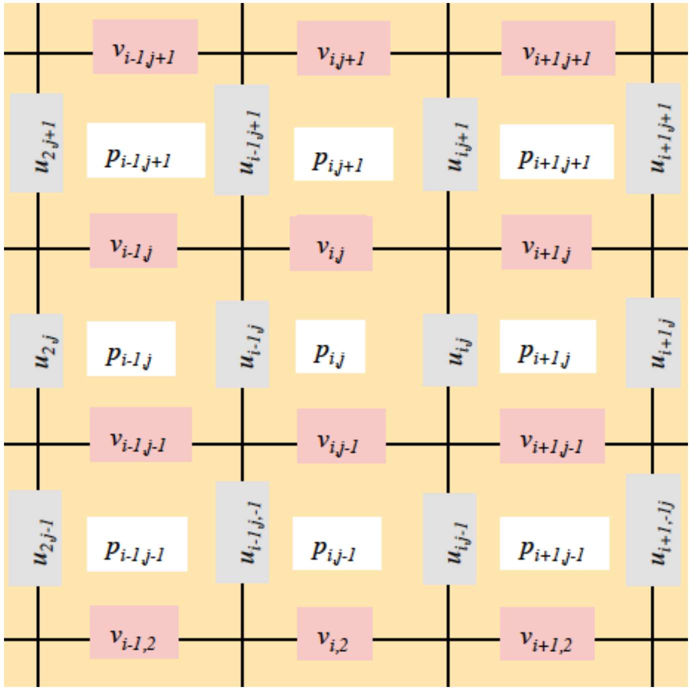
Use an explicit forward time discretization and centered difference spatial discretization. Approximate the density at cell interfaces by interpolating between the values at the cell centers.

Problem 2 (in MATLAB): Flow Solver

Extend the provided Flow Solver by the discretized density advection equation.

Please note the staggered arrangement of cells for the velocity field compared to those of the pressure and density fields in the flow solver! The spatial indices of u and v are thus shifted by ½ compared to those of p and ρ!

Also note that a row of ghost cells is arranged behind each boundary!



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Problem 3 (in MATLAB): Simulation of Falling Drop

a) *Use your implementation to simulate a falling drop initially located in the middle of a square domain. Evaluate how well the interface motion is captured.*

b) *Increase the simulation time until the drop reaches the ground of the square domain. Is the simulated density field realistic?*

c) *Repeat the simulations with different grid resolutions and evaluate the impact of the grid resolution. If necessary, decrease the time step to obtain stable simulations.*

d) *Compute the location of the center of mass vs. time. Evaluate how it depends on the resolution.*

Thank you for your attention

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