

# Multiphase Flows – WS 2021/22

## Problem Session 2 - **Solution:** Multiphase Shock Tube



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# Problem Session 2: Multiphase Shock Tube

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## Problem 1: 4-Equation Model

*The 1-D single-phase Euler system is written as*

$$\frac{\partial \mathbf{W}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{U})}{\partial x} = 0$$

with  $\mathbf{W} = \begin{pmatrix} \rho \\ \rho u \\ E \end{pmatrix}$  and  $\mathbf{F} = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ (E + p)u \end{pmatrix}$

*In the following, a 4-equation model describing two phases of vapor (“v”) and liquid (“l”) shall be used to describe a two-phase problem.*

- a) *Extend the 3-equation single-phase Euler system by an additional conservative equation for the vapor mass density  $\rho Y_v$ . Include a source term for inter-phase mass transfer.*

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## **Problem I: Mixture Stiffened-Gas Equation of State**

*In the 4-equation model, only mixture momentum and energy equations are solved. This is possible based on the equilibrium assumptions  $u_v = u_l = u$ ,  $p_v = p_l = p$ , and  $T_v = T_l = T$ .*

- b) *Derive an expression  $T = T(p, \rho, Y_v)$ .  
(Hint: Start from the expression for mixture specific volume  $v = Y_v v_v + Y_l v_l$  and use the single-phase stiffened-gas EOS for vapor and liquid.)*
- c) *Derive an expression  $p = p(\rho, e, Y_v)$ .  
(Hint: Use the expression for temperature derived in (a) and the expression for mixture internal energy  $e = Y_v e_v + Y_l e_l$ .)*

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## **Problem 1: Solution**

a) Additional conservation equation needed

Conserved quantity:  $\rho Y_v$

$$\Rightarrow \frac{\partial \rho Y_v}{\partial t} + \frac{\partial \rho Y_v u}{\partial x} = \dot{m}$$

Mass transfer from liquid to vapor  
phase (assumed to be zero for  
Problem 2 of Problem Session 2)

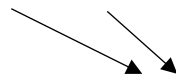
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## Problem 1: Solution

b) Equilibrium assumptions:  $p_v = p_l = p$ ,  $T_v = T_l = T$ ,  $u_v = u_l = u$

Needed:  $T = T(p, \rho, Y_v)$

Specific volume (mixture):  $v = Y_v v_v + Y_l v_l = \frac{Y_v}{\rho_v} + \frac{Y_l}{\rho_l} = \frac{1}{\rho}$


$$\rho_k(p, T) = \frac{p + p_{\infty, k}}{c_k(\gamma_k - 1)T}$$

Solve for  $T$  (with  $Y_l = 1 - Y_v$ ):

$$T(p, \rho, Y_v) = \frac{1}{\rho \left( Y_v \frac{(\gamma_v - 1)c_v}{p + p_{\infty, v}} + (1 - Y_v) \frac{(\gamma_l - 1)c_l}{p + p_{\infty, l}} \right)}$$

# Problem Session 2: Multiphase Shock Tube

## Problem 1: Solution

c) Needed:  $p = p(\rho, e, Y_v)$

Mixture internal energy:  $e = Y_v e_v + Y_l e_l$

$$e_k(p, T) = \frac{(p + \gamma_k p_{\infty, k}) c_k T}{(p + p_{\infty, k})} + q_k$$

Insert expression for  $T$  from (b)

➤ Quadratic equation in  $p$  – to be solved for  $p$ :

$$p(\rho, e, Y_v, Y_l) = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \quad (\text{with } Y_l = 1 - Y_v)$$

$$A = Y_v c_v + Y_l c_l$$

$$B = \rho [Y_v (\gamma_v - 1) c_v + Y_l (\gamma_l - 1) c_l] (-e + Y_v q_v + Y_l q_l) \\ + Y_v c_v (p_{\infty, l} + \gamma_v p_{\infty, v}) + Y_l c_l (p_{\infty, v} + \gamma_l p_{\infty, l})$$

$$C = \rho [Y_v (\gamma_v - 1) c_v p_{\infty, l} + Y_l (\gamma_l - 1) c_l p_{\infty, v}] (-e + Y_v q_v + Y_l q_l) \\ + Y_v c_v \gamma_v p_{\infty, v} p_{\infty, l} + Y_l c_l \gamma_l p_{\infty, v} p_{\infty, l}$$

# Problem Session 2: Multiphase Shock Tube

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## ***Problem 2: Multiphase Shock Tube (in MATLAB)***

The solution to the Matlab implementation of the 2-Phase shock tube problem of Problem Session 2 is equivalent to the template provided for solution of Problem Session 3.



**Thank you for your attention**

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