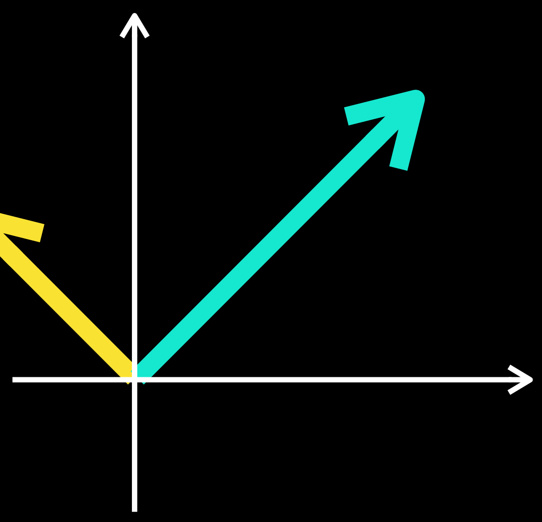
Matrix Determinant

Linear Algebra Essentials



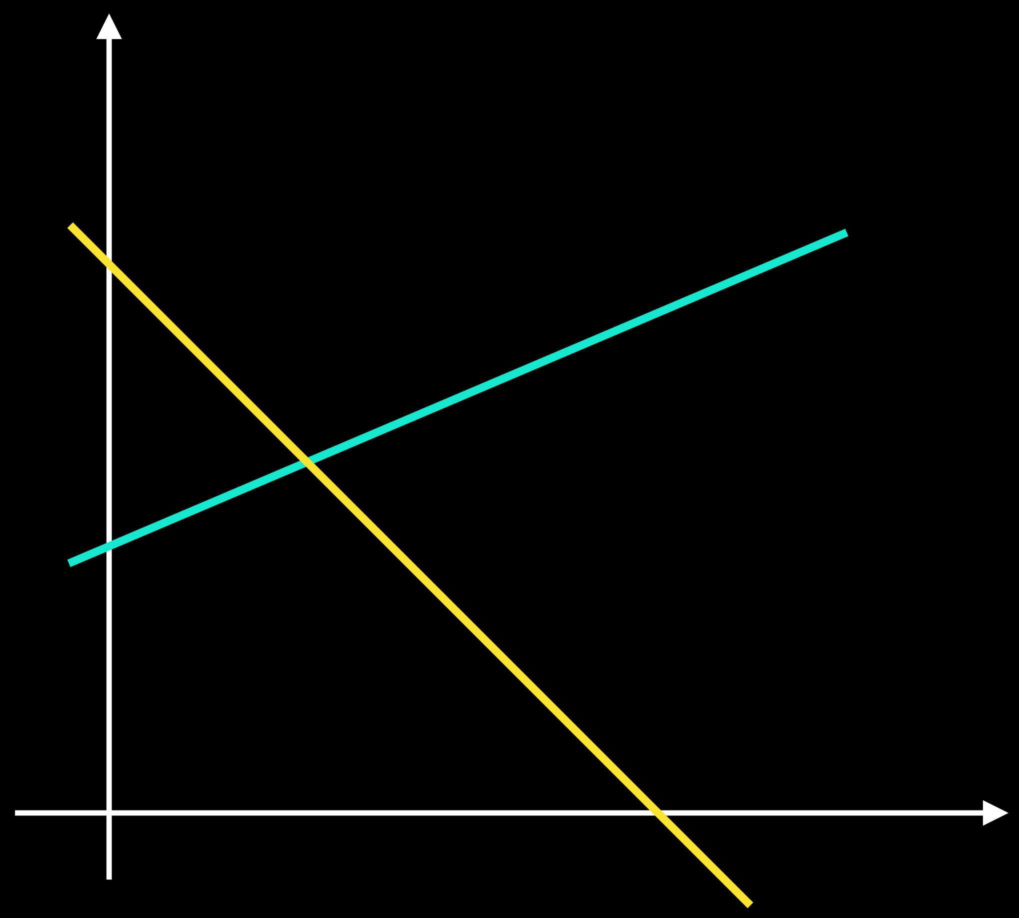
$$\begin{cases} ax + by = e \\ cx + dy = f \end{cases}$$

$$\begin{cases} y = \frac{e}{b} - \frac{a}{b}x \\ y = \frac{f}{d} - \frac{c}{d}x \end{cases}$$

$$\frac{a}{b} \neq \frac{c}{d} \; - \; not\ parallel$$

$$ad \neq bc$$

$$det = ad - bc \neq 0$$



$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$$

$$det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) \neq 0$$

System of linear equations

$A\,\mathbf{x} = \mathbf{b}$

$A^{-1} A\,\mathbf{x} = A^{-1}\,\mathbf{b}$

$\mathbf{x} = A^{-1}\,\mathbf{b}$

if $\det(A) \neq 0$,
then A^{-1} exists

otherwise, A is singular

```python
A = np.array([
    [1, -1],
    [-2, 2]
])
```

```python
np.linalg.det(A)
```

```
0.0
```

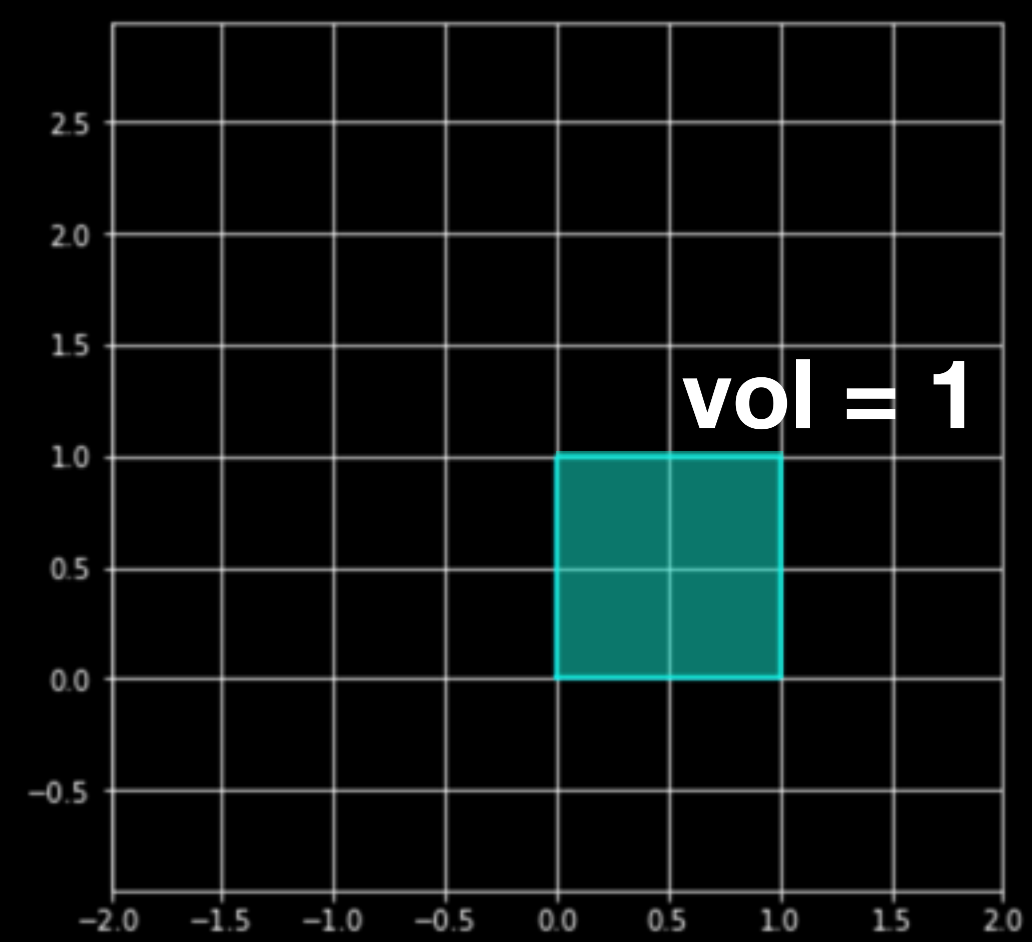
```python
np.linalg.matrix_rank(A)
```

```
1
```

Determinant meaning

$$A = \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}$$

det(A) = 2



Determinant meaning

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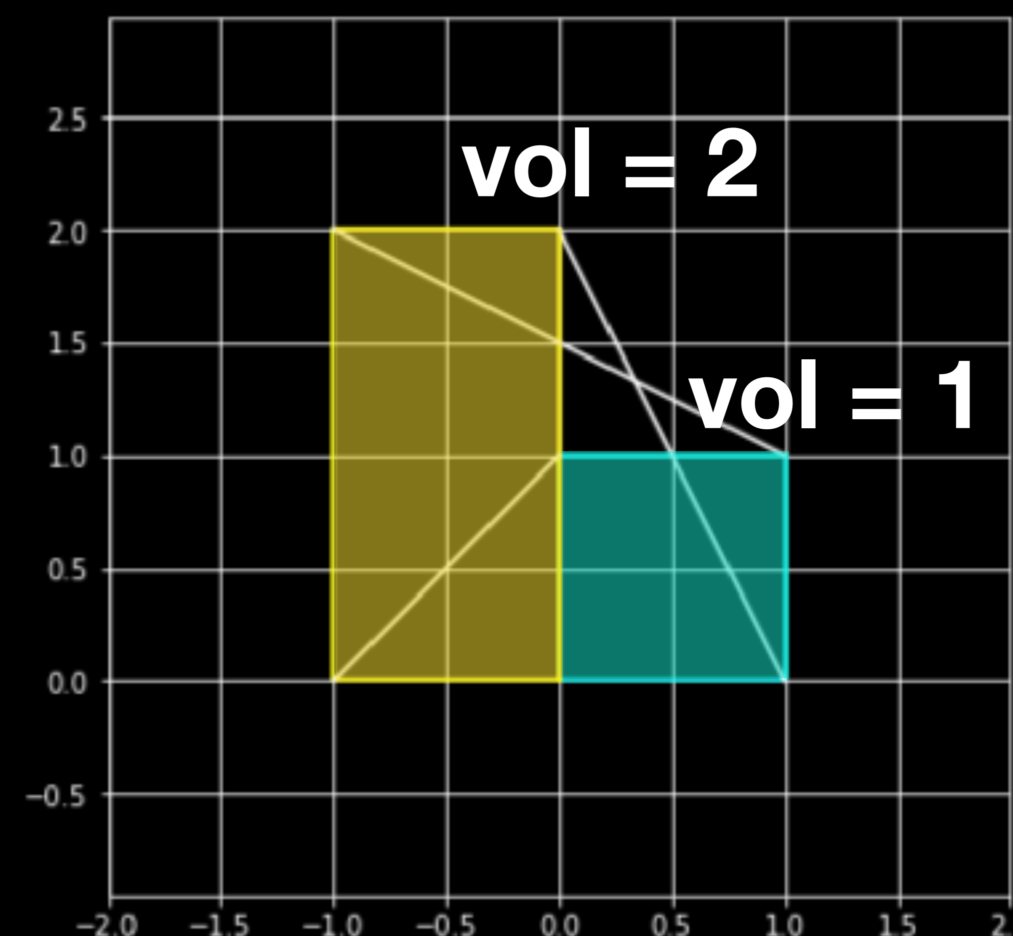
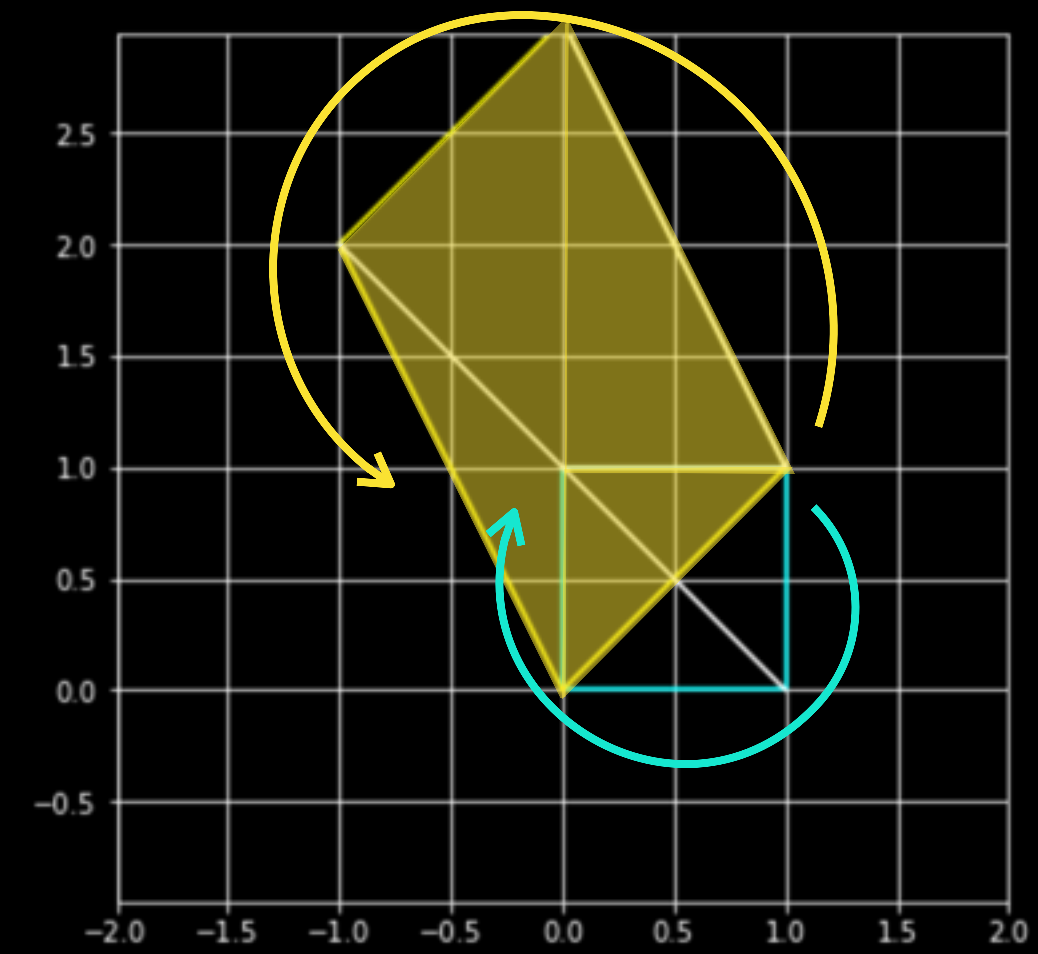


Determinant meaning

$$A = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$$

det(A) = -3

vol = 3



Conclusion

- If matrix determinant is 0, then the matrix is singular
- Matrix determinant shows how the matrix stretches the space