

Reflections

Linear Algebra Essentials



Vertical reflection

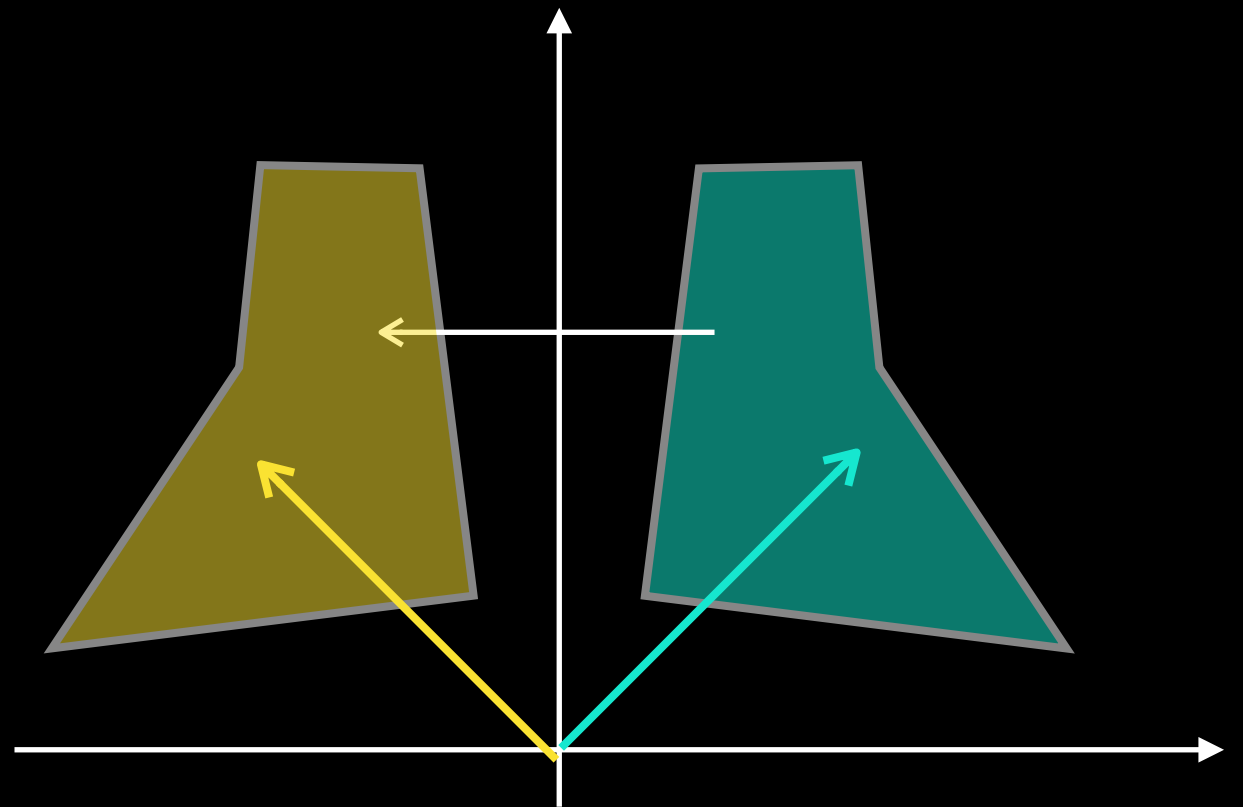
$$S_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$S_y \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ y \end{bmatrix}$$

$$S_y \cdot S_y = I$$

$$(S_y)^{-1} = S_y = S_y^T$$

$$\det(S_y) = -1$$



Horizontal reflection

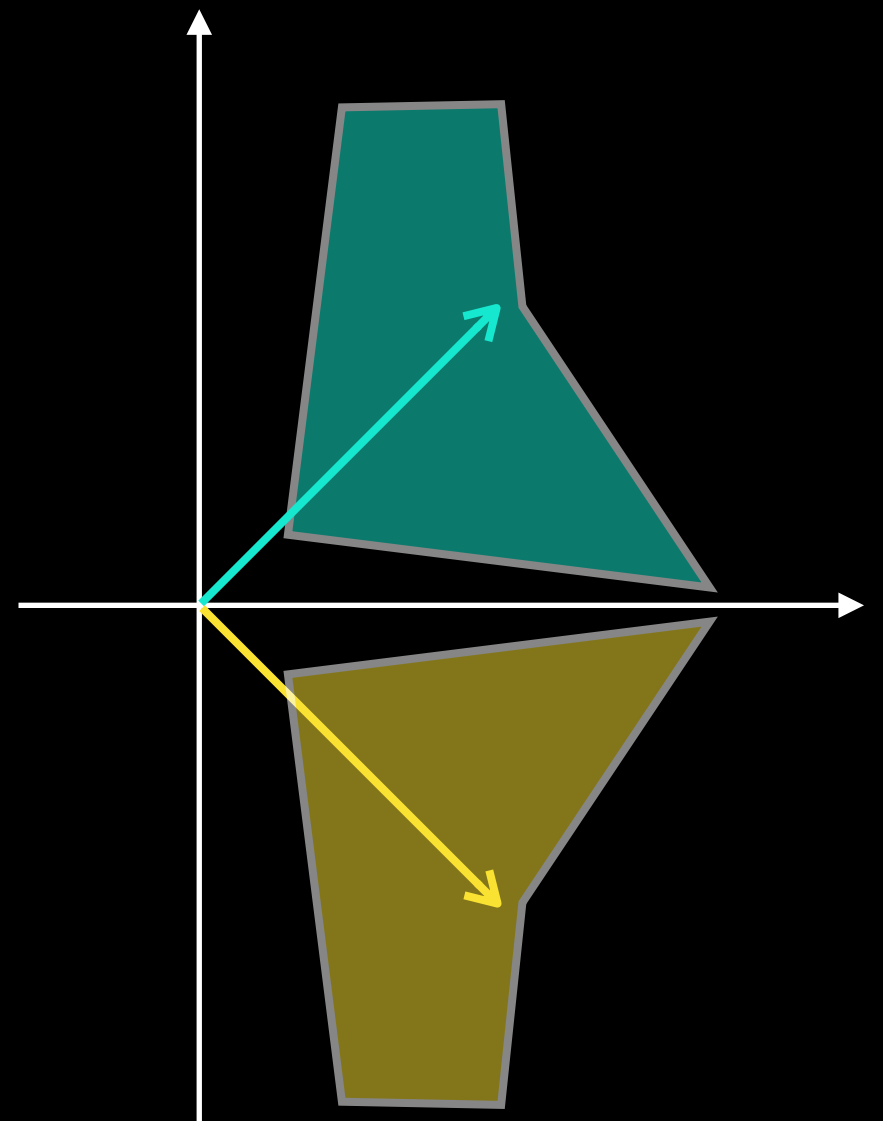
$$S_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$S_x \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix}$$

$$S_x \cdot S_x = I$$

$$(S_x)^{-1} = S_x = S_x^T$$

$$\det(S_x) = -1$$



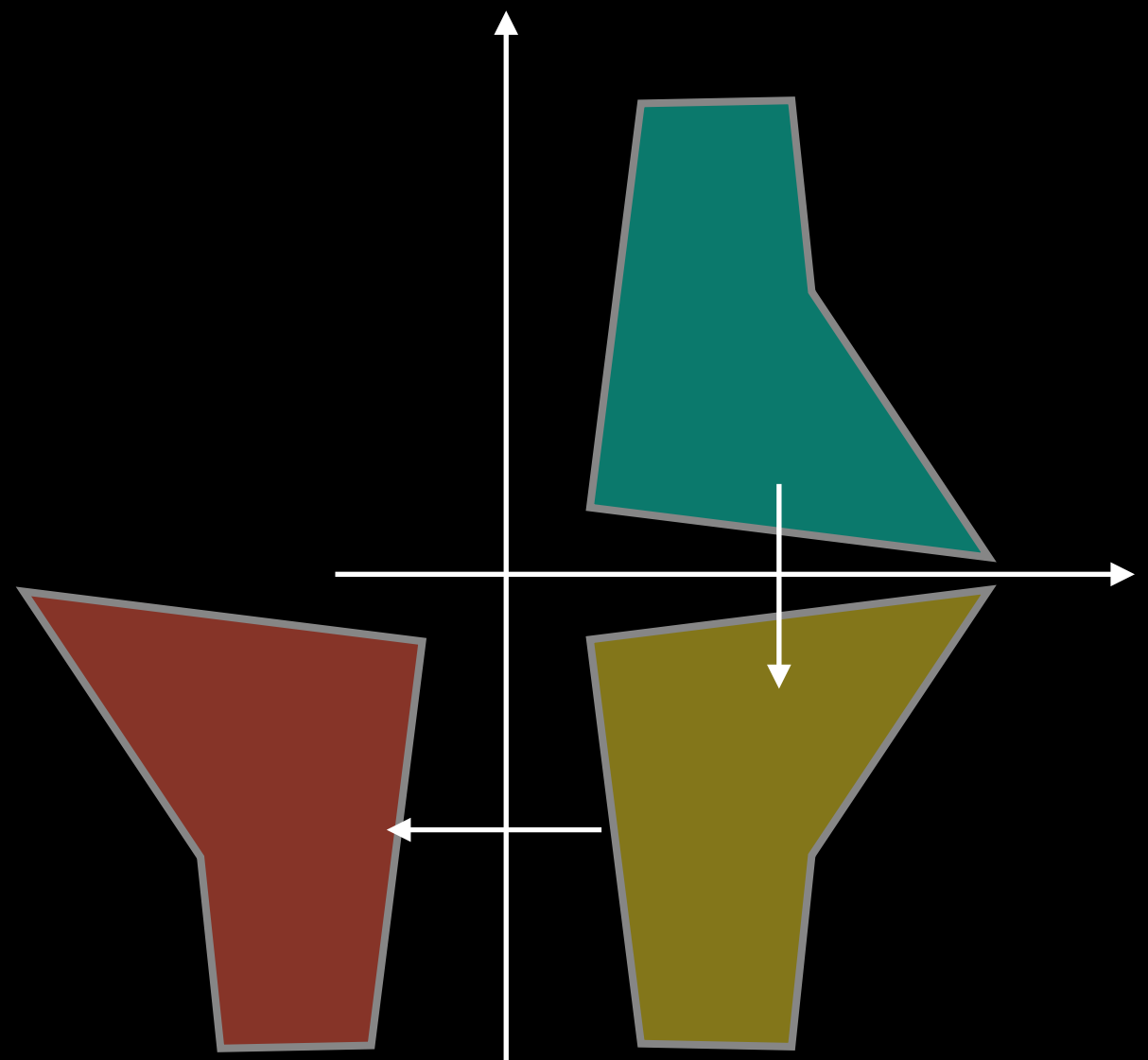
H-flip + V-flip

$$S_y \cdot S_x = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I$$

$$S_y \cdot S_x = R(\pi)$$

$$S_x \cdot S_y = S_y \cdot S_x$$

*Generally, reflections
are not commutative*



Improper rotation

$$S_a x = \underbrace{R_a} \cdot \underbrace{S_x} \cdot \underbrace{R_{-a}} x$$

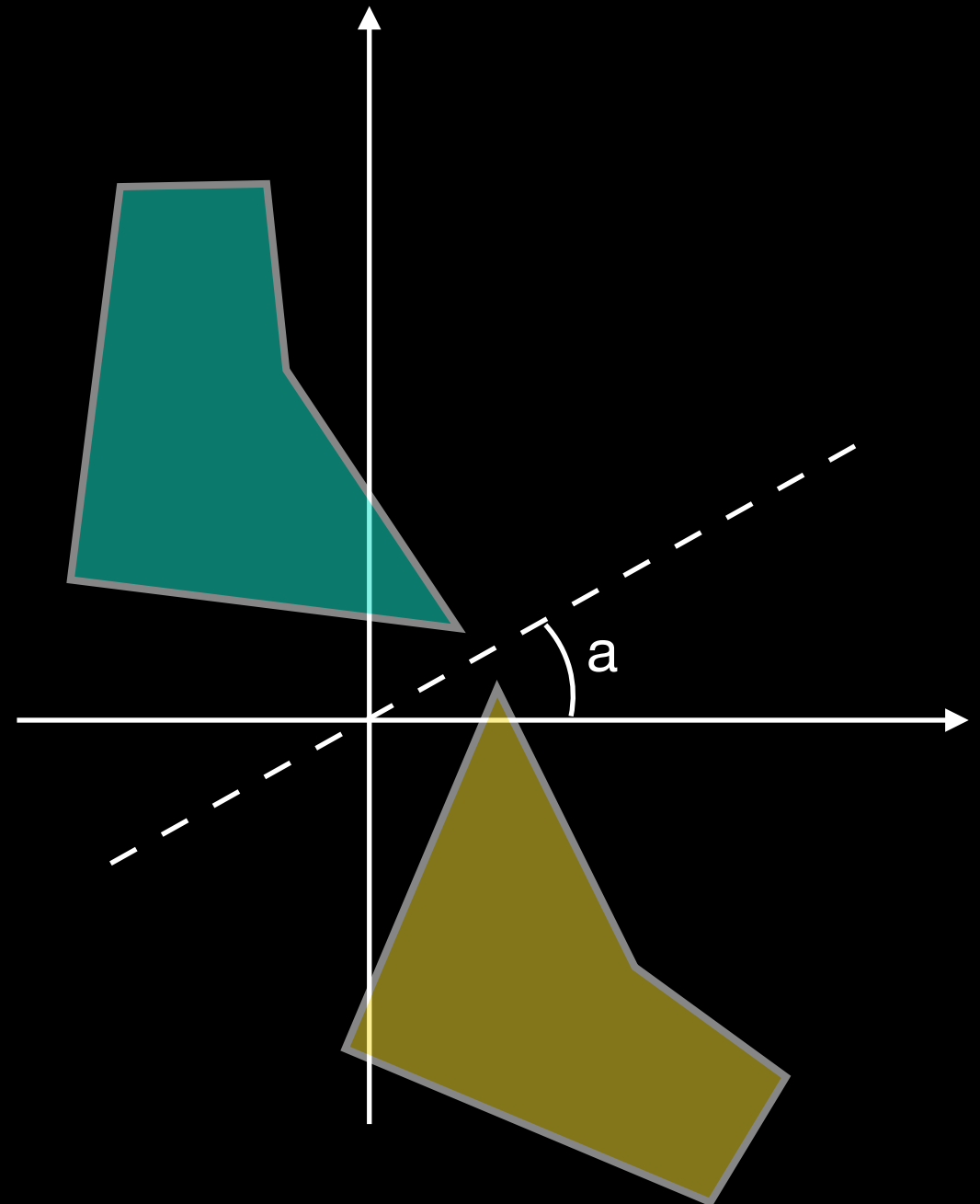
$$= \begin{bmatrix} \cos(a) & -\sin(a) \\ \sin(a) & \cos(a) \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \cos(a) & \sin(a) \\ -\sin(a) & \cos(a) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(a) & \sin(a) \\ \sin(a) & -\cos(a) \end{bmatrix} \cdot \begin{bmatrix} \cos(a) & \sin(a) \\ -\sin(a) & \cos(a) \end{bmatrix} =$$

$$= \begin{bmatrix} \cos(2a) & \sin(2a) \\ \sin(2a) & -\cos(2a) \end{bmatrix}$$

$$S_x = S_0$$

$$S_y = S_{\frac{\pi}{2}}$$



Orthogonal transformations

Rotations

$$R_a = \begin{bmatrix} \cos(a) & -\sin(a) \\ \sin(a) & \cos(a) \end{bmatrix}$$

$$\det = 1$$

$$R_a R_a = R_{2a}$$

Commutative

Improper rotations

$$S_a = \begin{bmatrix} \cos(2a) & \sin(2a) \\ \sin(2a) & -\cos(2a) \end{bmatrix}$$

$$Q^{-1} = Q^T$$

$$\det = -1$$

$$S_a S_a = I$$

Noncommutative