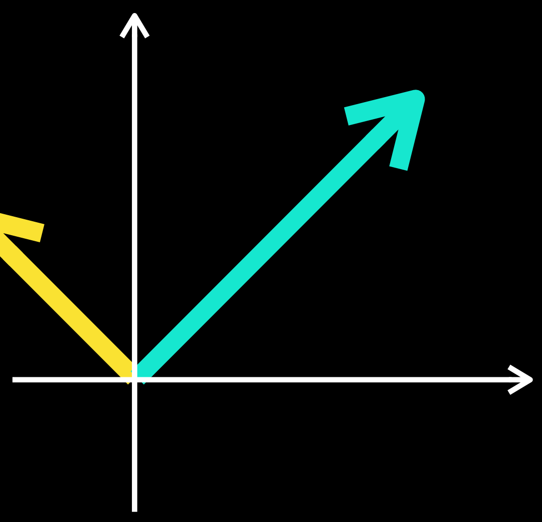
Changing reference frame

Linear Algebra Essentials



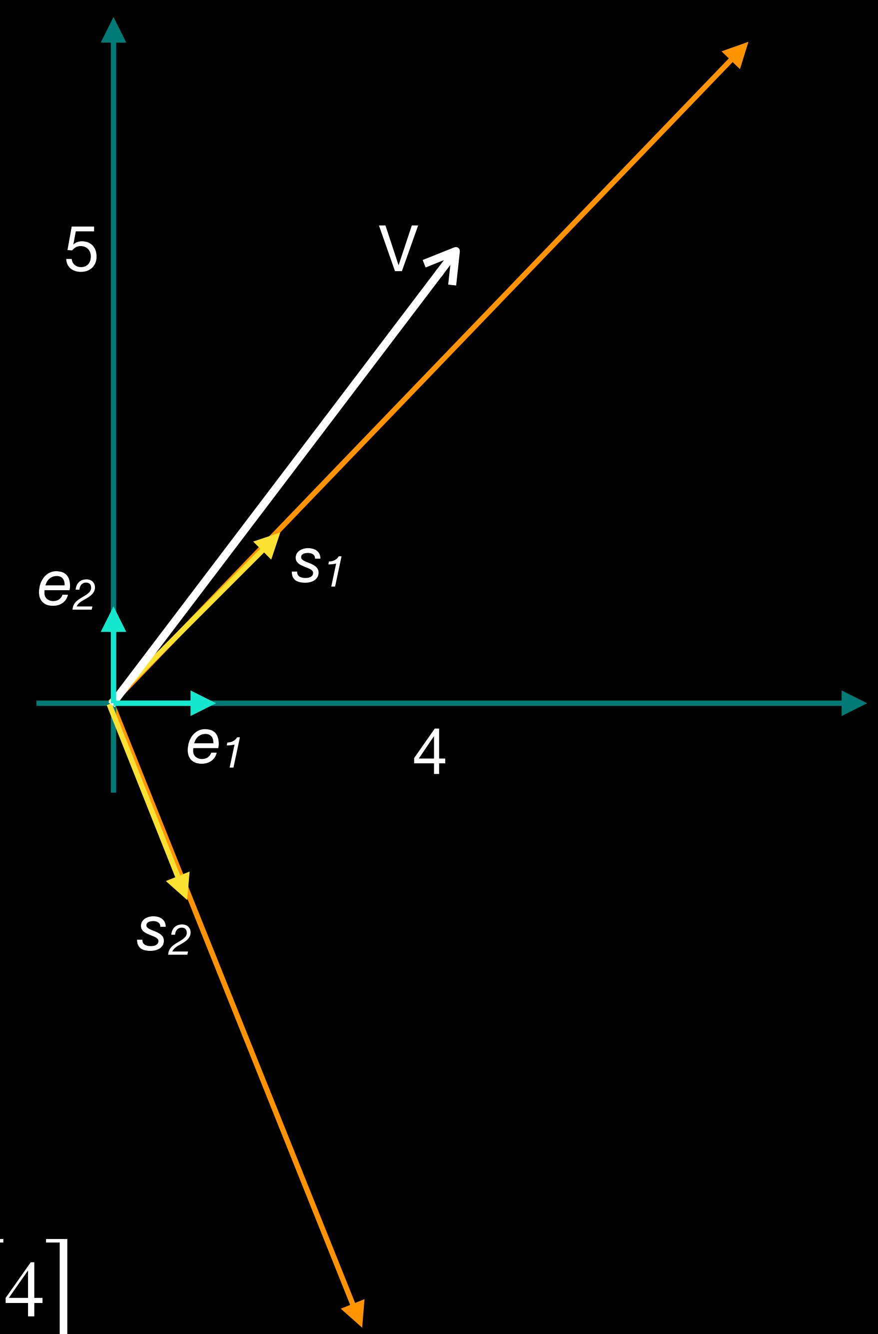
$$v = \begin{bmatrix} 4 \\ 5 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 5 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 4\vec{e}_1 + 5\vec{e}_2$$

$$s_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \qquad s_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$ *- new basis*

$$v = a_1\vec{s}_1 + a_2\vec{s}_2$$

$$= \begin{bmatrix} 2 \\ 2 \end{bmatrix} a_1 + \begin{bmatrix} 1 \\ -2 \end{bmatrix} a_2 = \begin{bmatrix} 2 & 1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = v \qquad \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = S^{-1}v = \begin{bmatrix} 1/3 & 1/6 \\ 1/3 & -1/3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$



$$v_{new} = \begin{bmatrix} 13/6 \\ -1/3 \end{bmatrix}$$

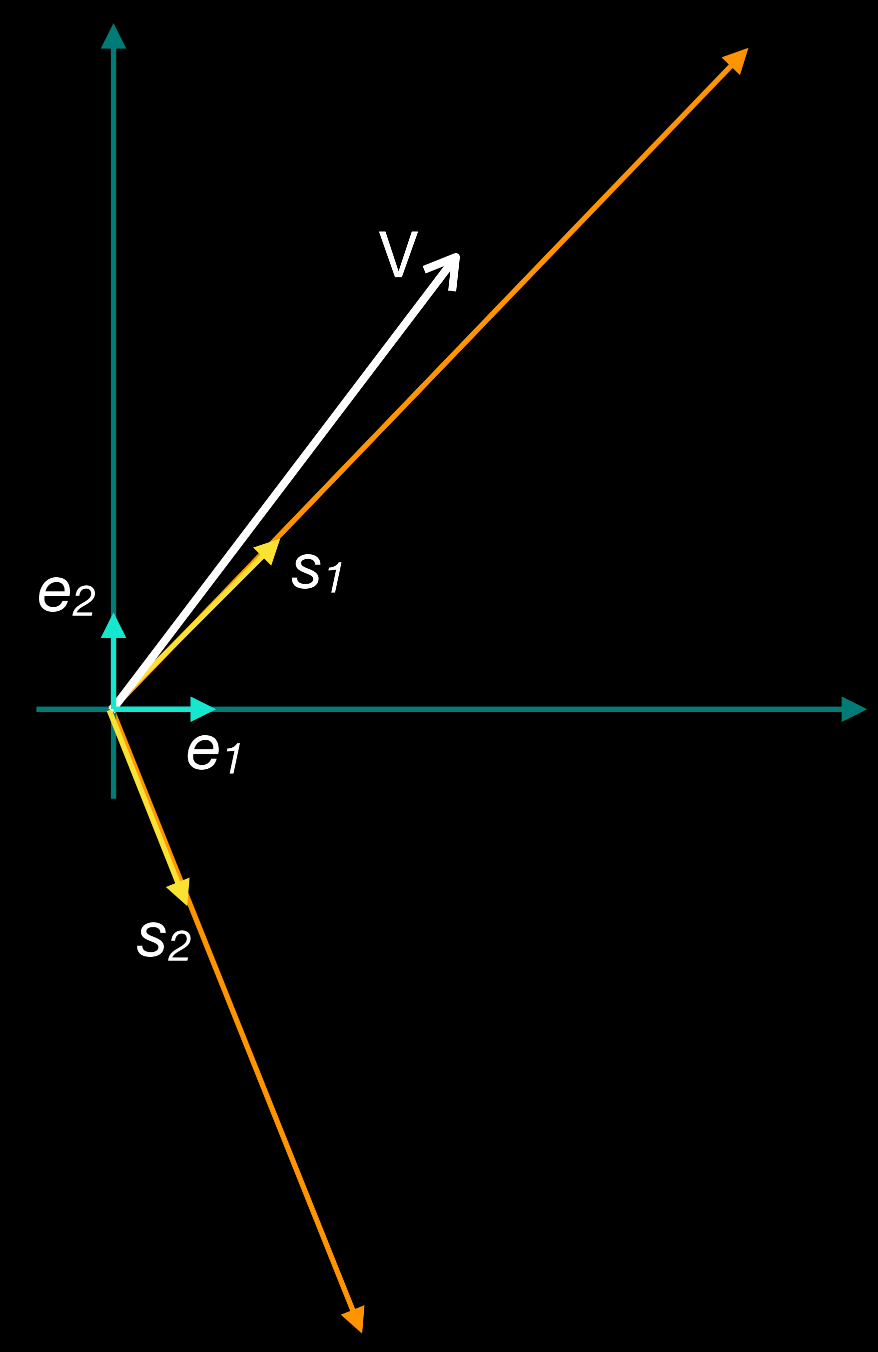
Old reference frame: {e_1, e_2, … , e_n}

New basis: {s_1, s_2, … , s_n}

$$S = \begin{bmatrix} s_1 & s_2 & \ldots & s_n \end{bmatrix}$$

$$v_{old} = a_1 e_1 + \ldots + a_n e_n \;=\; \begin{bmatrix} a_1 \\ \cdots \\ a_n \end{bmatrix}$$

$$v_{new} = S^{-1} v_{old} = \begin{bmatrix} b_1 \\ \cdots \\ b_n \end{bmatrix}$$



$$s_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \qquad s_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$ *- new basis*

$$t = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$ *V' = V - t - in the old ref. frame*

$$v'_{new} = S^{-1} v'_{old}$$

$$v'_{new} = S^{-1} v_{old} - S^{-1} t$$

$$v'_{new} = \left[\; \underbrace{S^{-1} \;\; \text{-}S^{-1} t}_{(2\text{ x }3)} \right] \begin{bmatrix} v_{old} \\ 1 \end{bmatrix}$$

$$v'_{new} = \begin{bmatrix} 1/3 & 1/6 & -1/6 \\ 1/3 & -1/3 & -5/3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$