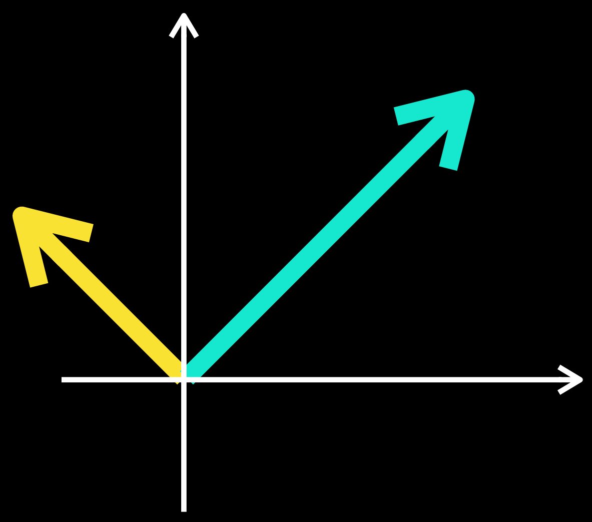
Inner product space

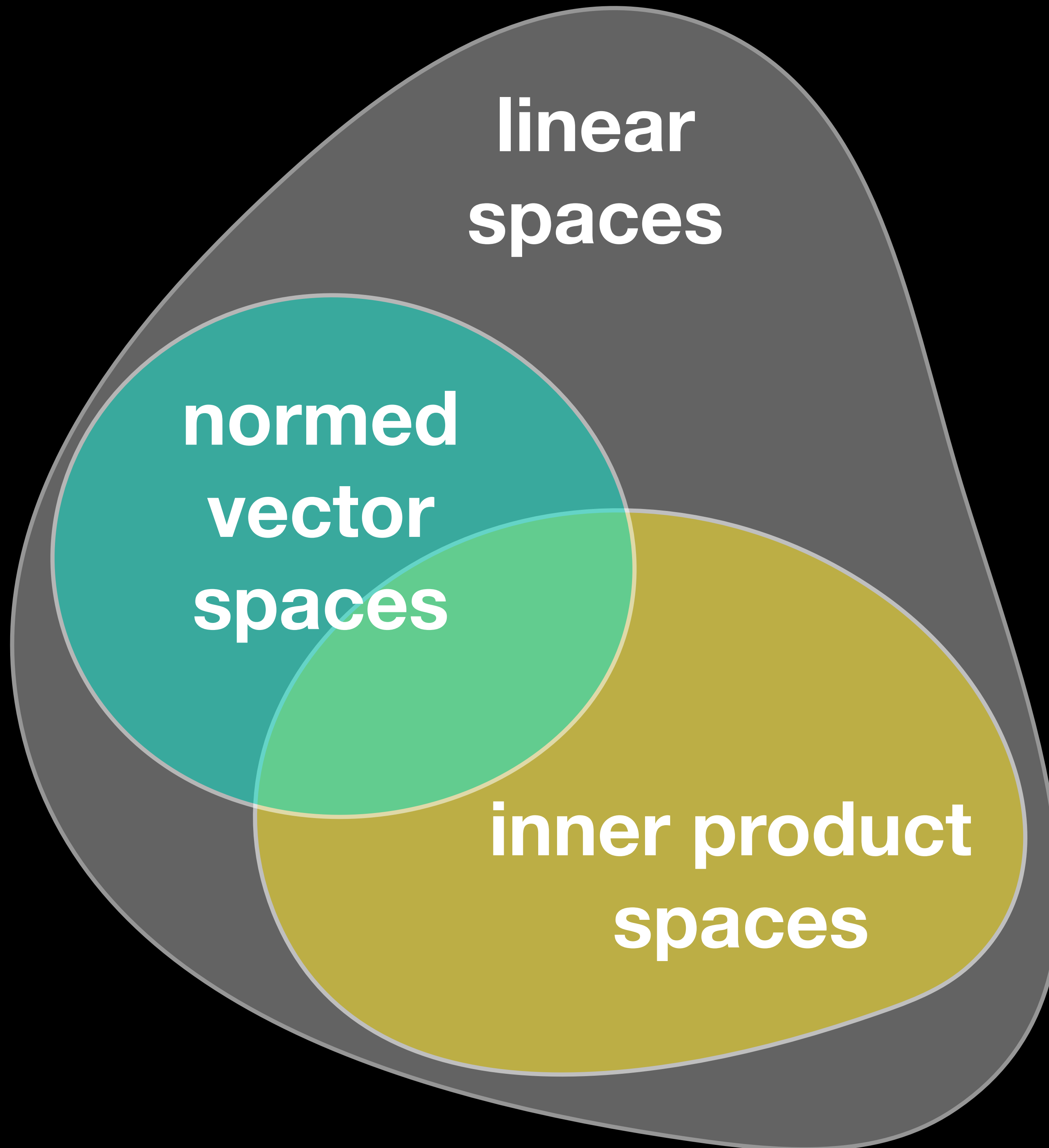
Linear Algebra Essentials



Linear Space + Inner Product

$$(\overrightarrow{a}, \overrightarrow{b}) = \sum_i a_i \cdot b_i$$

$$(a, a) = \sum a_i^2 = \left(\|a\|_{L_2}\right)^2$$



Properties

$$(a, b) = \sum_i a_i \cdot b_i$$

$$(a, b) = (b, a)$$

Commutativity

$$(a + b, c) = \sum (a_i + b_i)c_i =$$

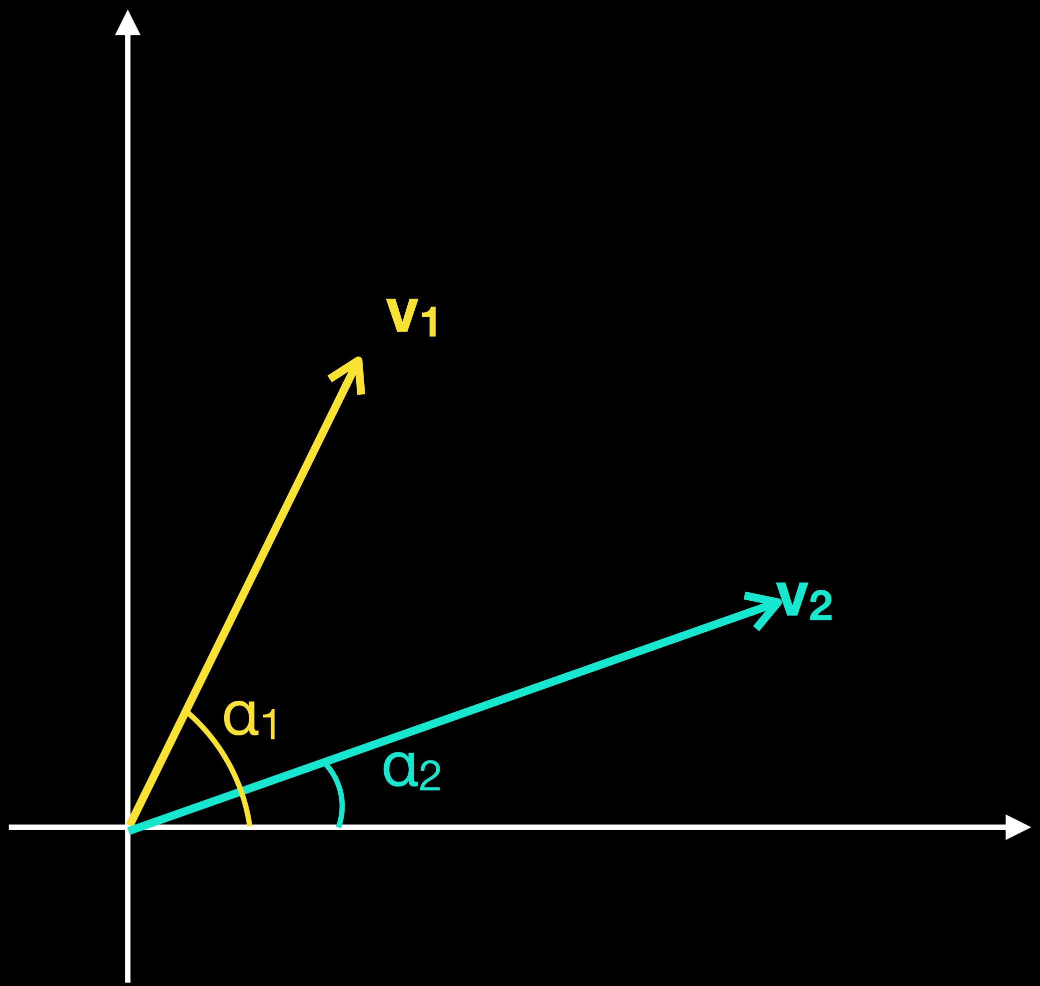
Distributivity

$$= \sum a_i c_i + \sum b_i c_i = (a, c) + (b, c)$$

$$(v_1, v_2) = \|v_1\| \cdot \|v_2\| \cdot cos(\alpha)$$

$$v_1 = \begin{vmatrix} \|v_1\| cos(\alpha_1) \\ \|v_1\| sin(\alpha_1) \end{vmatrix}$$

$$v_2 = \begin{vmatrix} \|v_2\| cos(\alpha_2) \\ \|v_2\| sin(\alpha_2) \end{vmatrix}$$



$$(v_1, v_2) = \|v_1\| \|v_2\| cos(\alpha_1) cos(\alpha_2) +$$

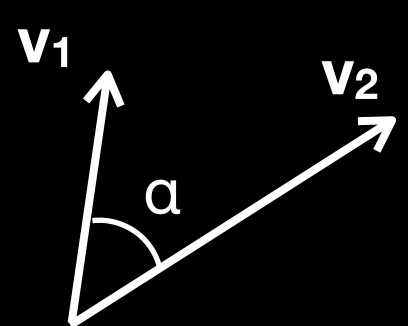
$$+ \|v_1\| \|v_2\| sin(\alpha_1) sin(\alpha_2) = \|v_1\| \|v_2\| cos(\alpha_1 - \alpha_2)$$

$$(v_1, v_2) = \|v_1\| \cdot \|v_2\| \cdot cos(\alpha)$$

Summary

$$(a, b) = \sum_i a_i \cdot b_i$$

$$(v_1, v_2) = \|v_1\| \cdot \|v_2\| \cdot cos(\alpha)$$



$$\alpha = 0 : (v_1, v_2) = \|v_1\| \cdot \|v_2\|$$

$$\alpha < \pi/2 : (v_1, v_2) > 0$$

$$\alpha > \pi/2 : (v_1, v_2) < 0$$

$$\alpha = \pi/2 : (v_1, v_2) = 0$$