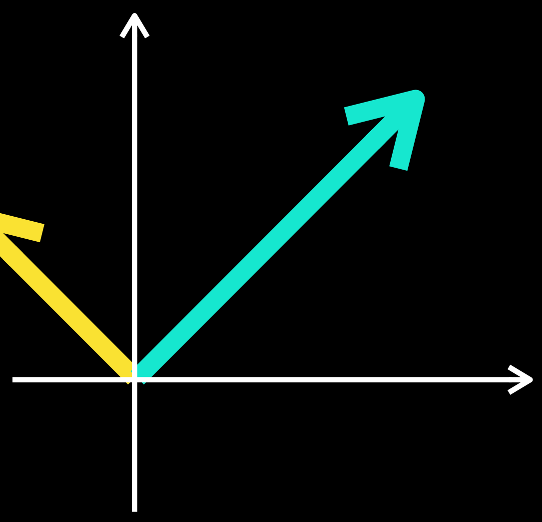
Exercise: Finding affine transformation

Linear Algebra Essentials



How many points needed

$$Aff = \begin{bmatrix} M & \begin{bmatrix} t_x \\ t_y \end{bmatrix} \\ 0\ 0 & 1 \end{bmatrix}$$

6 unknowns

$$\begin{bmatrix} m_1 & m_2 & t_x \\ m_3 & m_4 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

$$\begin{cases} m_1 x + m_2 y + t_x = u \\ m_3 x + m_4 y + t_y = v \end{cases}$$

1 point —> 2 equations

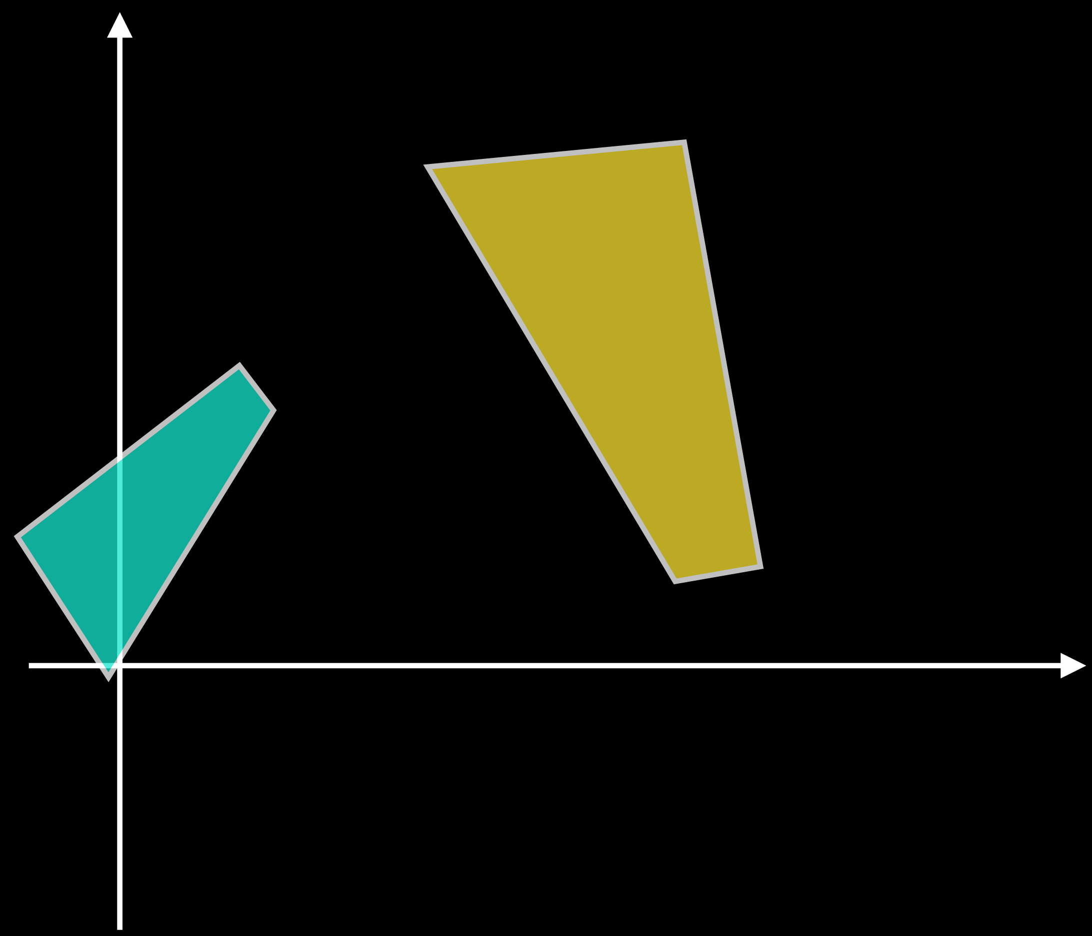
We need to know how 3 vectors (x, y) transform to 3 vectors (u, v)

Rotation, scaling, translation

$$Aff = \begin{bmatrix} \lambda R & \begin{bmatrix} t_x \\ t_y \end{bmatrix} \\ 0\ 0 & 1 \end{bmatrix}$$

$$c = \lambda \cdot cos(\beta), \quad s = \lambda \cdot sin(\beta)$$

$$\lambda R = \begin{bmatrix} c & -s \\ s & c \end{bmatrix}$$



We have 4 unknowns, so we need just 2 points

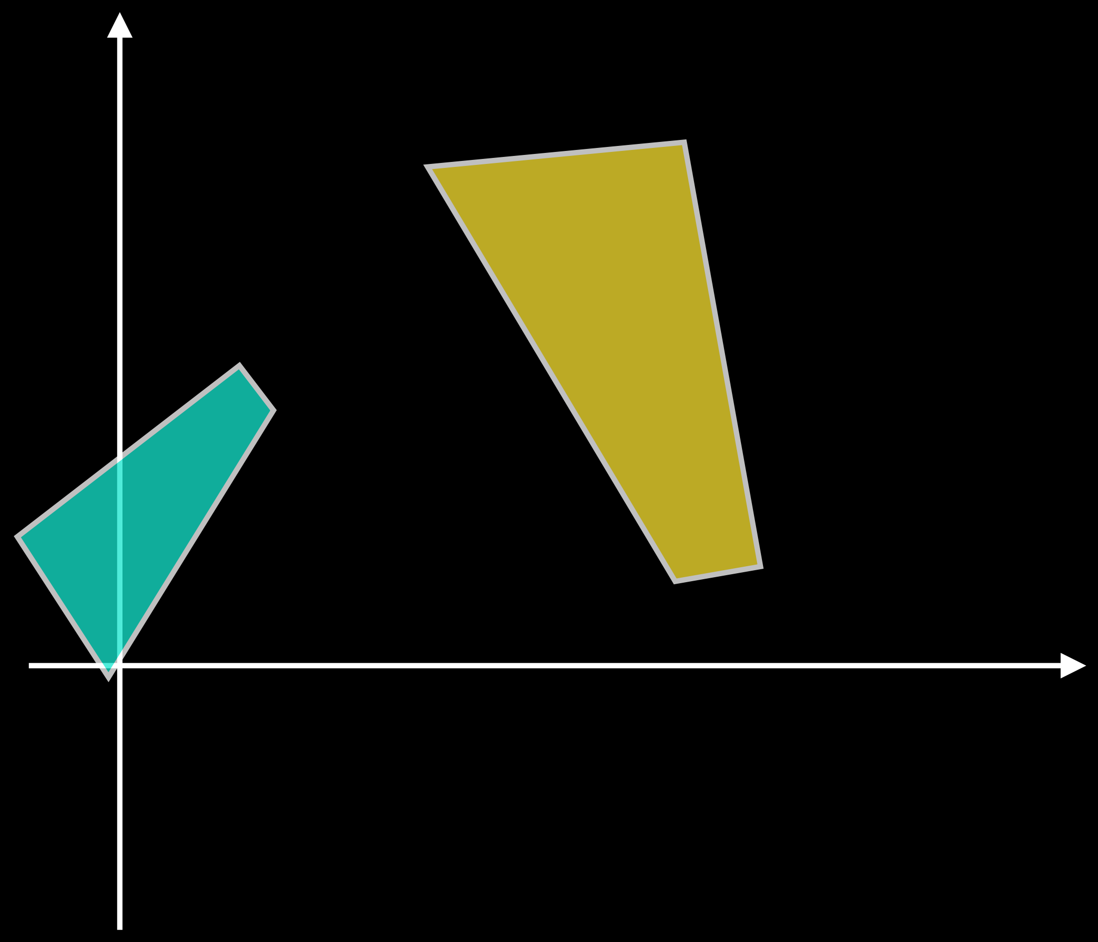
Rotation, scaling, translation

$$\begin{bmatrix} c & -s & t_x \\ s & c & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

$$\begin{cases} cx_1 - sy_1 + t_x = u_1 \\ sx_1 + cy_1 + t_y = v_1 \end{cases}$$

$$\begin{cases} cx_2 - sy_2 + t_x = u_2 \\ sx_2 + cy_2 + t_y = v_2 \end{cases}$$

$$\begin{bmatrix} x_1 & -y_1 & 1 & 0 \\ y_1 & x_1 & 0 & 1 \\ x_2 & -y_2 & 1 & 0 \\ y_2 & x_2 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} c \\ s \\ t_x \\ t_y \end{bmatrix} = \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix}$$

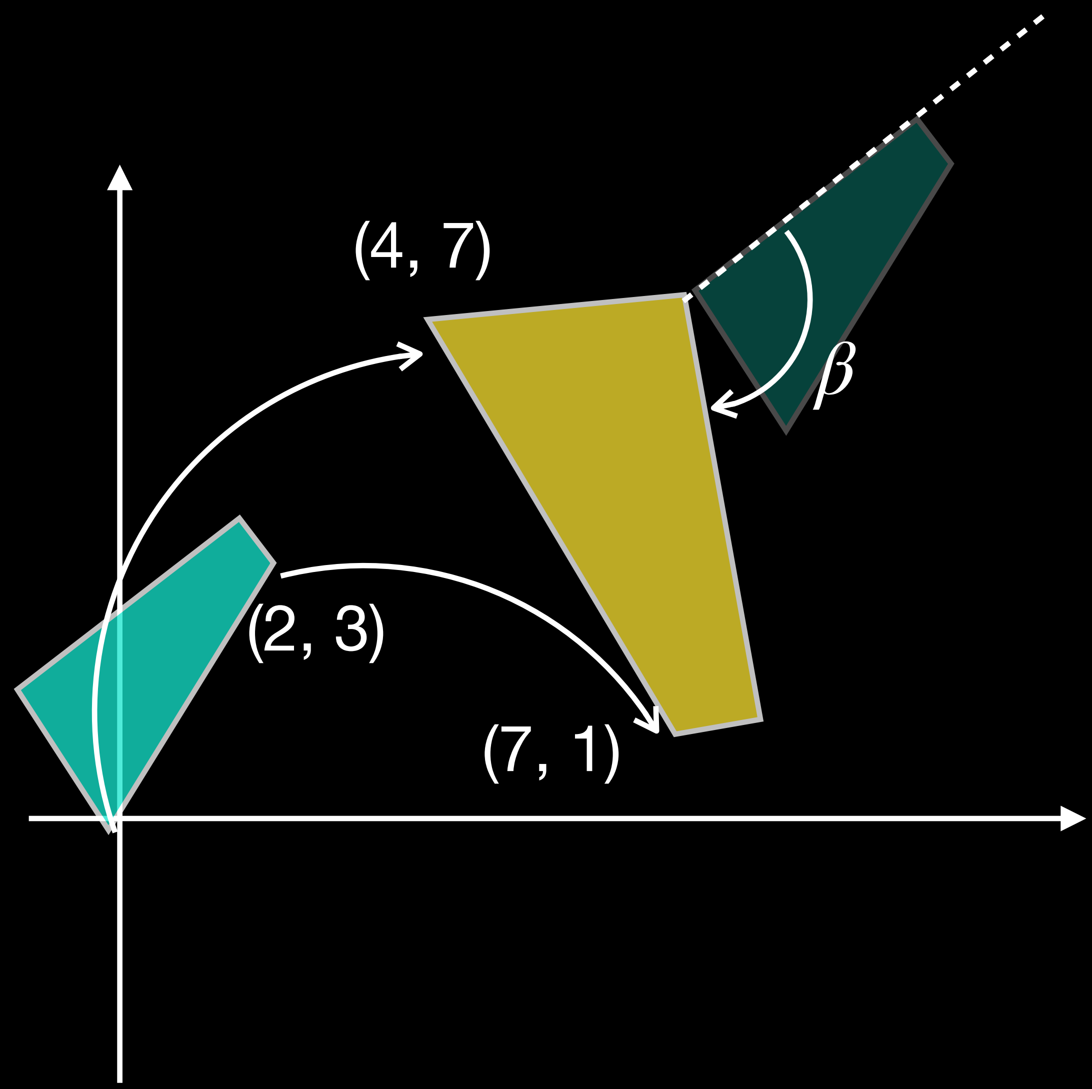


Rotation, scaling, translation

$$\begin{bmatrix} c \\ s \\ t_x \\ t_y \end{bmatrix} = \begin{bmatrix} x_1 & -y_1 & 1 & 0 \\ y_1 & x_1 & 0 & 1 \\ x_2 & -y_2 & 1 & 0 \\ y_2 & x_2 & 0 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix}$$

$xy_1 = (0, 0)$ —> $uv_1 = (4, 7)$
$xy_2 = (2, 3)$ —> $uv_2 = (7, 1)$

$$\begin{bmatrix} c \\ s \\ t_x \\ t_y \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & -3 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 4 \\ 7 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.92 \\ -1.62 \\ 4 \\ 7 \end{bmatrix}$$



$$\lambda = \sqrt{c^2 + s^2} = 1.86$$

$$\beta = arccos(\frac{c}{\lambda}) = -2.09$$

Rotation, scaling, translation

Given:
$xy_1 = (0, 0)$ —> $uv_1 = (4, 7)$
$xy_2 = (2, 3)$ —> $uv_2 = (7, 1)$

- Data

$$Aff = \begin{bmatrix} \lambda R & \begin{bmatrix} t_x \\ t_y \end{bmatrix} \\ 0\ 0 & 1 \end{bmatrix}$$

- Model

$$\lambda = 1.86$$

$$\beta = -2.09$$

$$t = (4,7)$$

- Model params