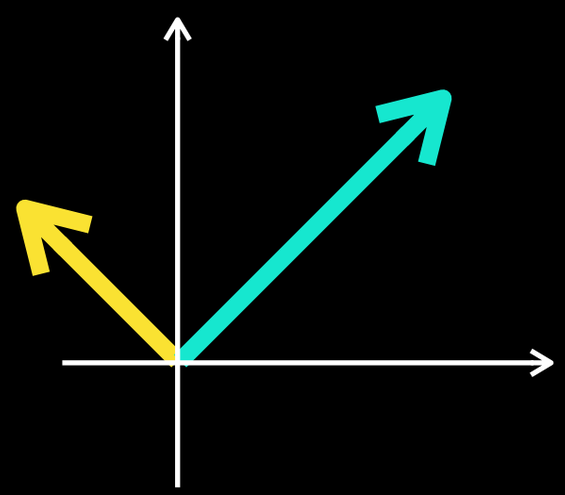
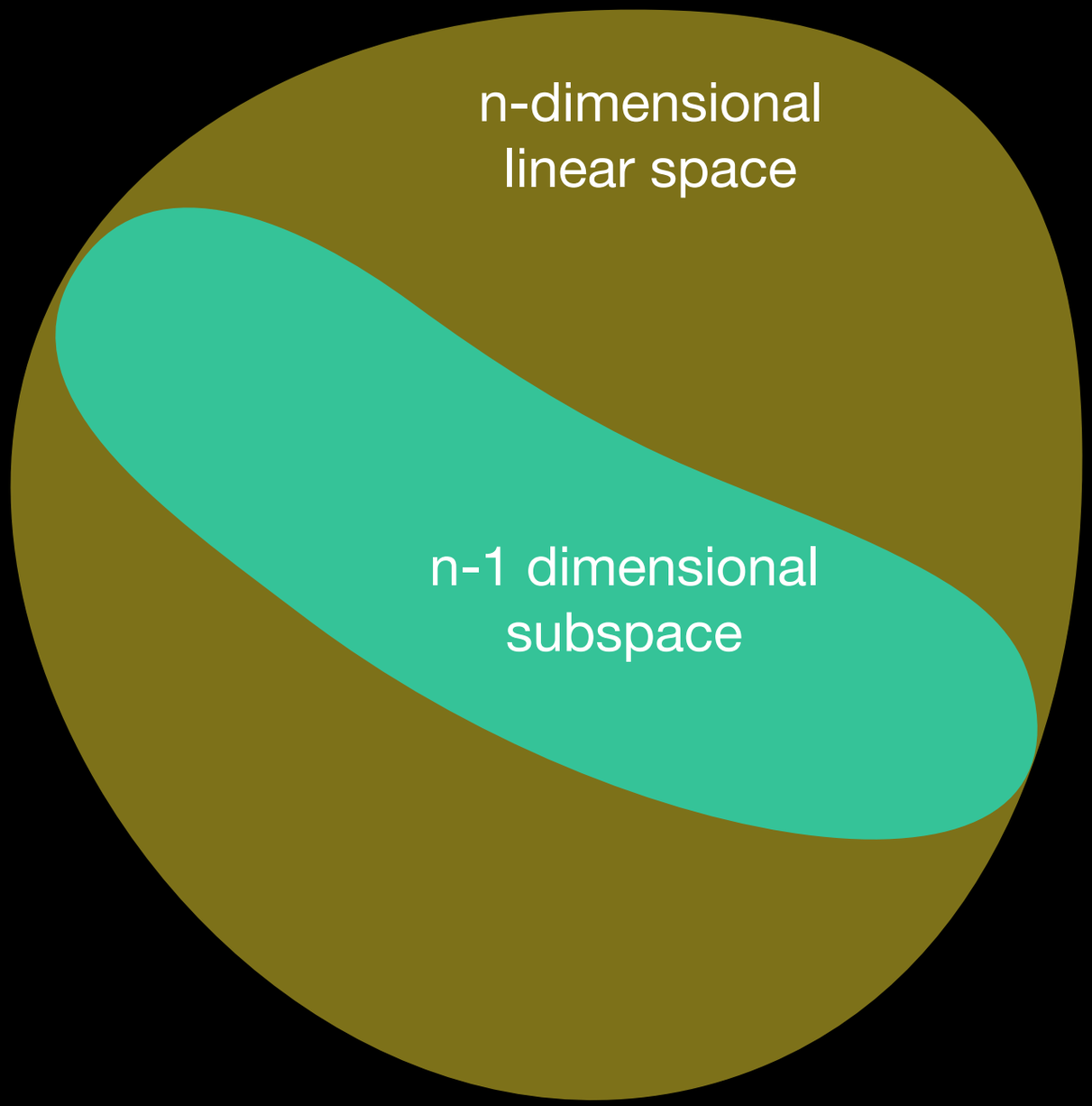
Hyperplane

Linear Algebra Essentials



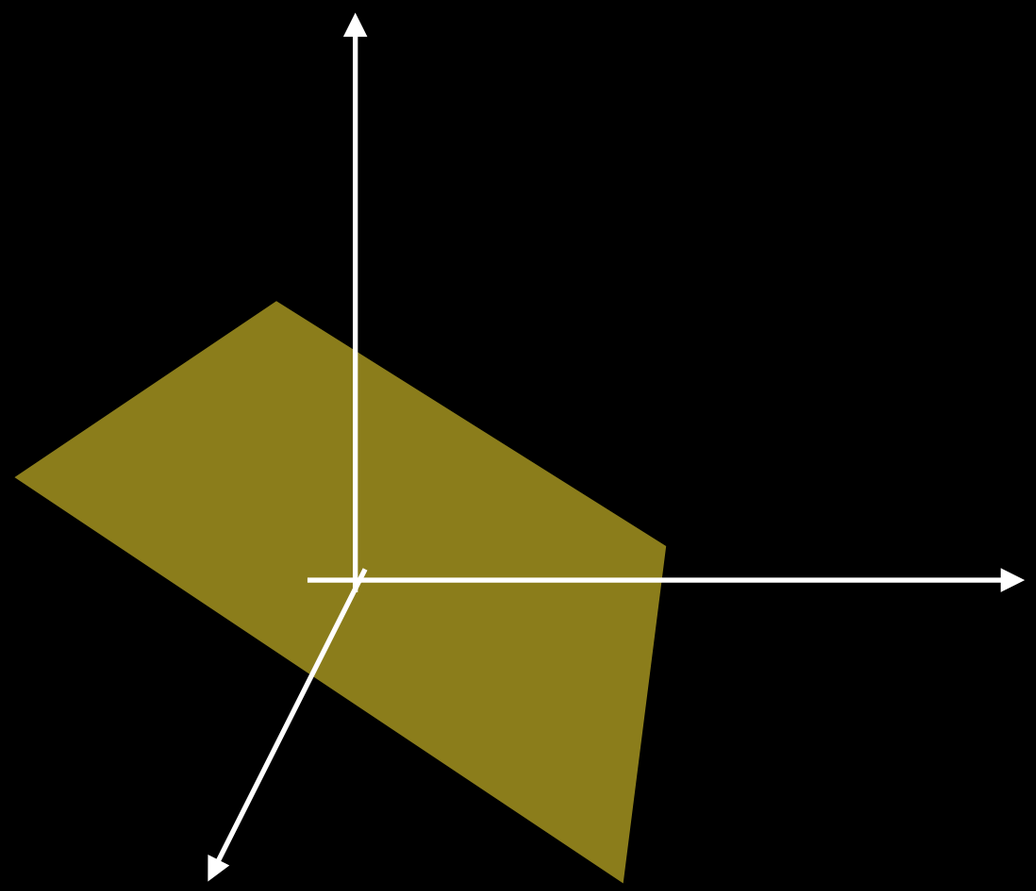
Hyperplane



Vector hyperplane

(x, n) = 0

$x = \overrightarrow{0}$ – *must be included*



Why is it (k-1)-dimensional subspace ?

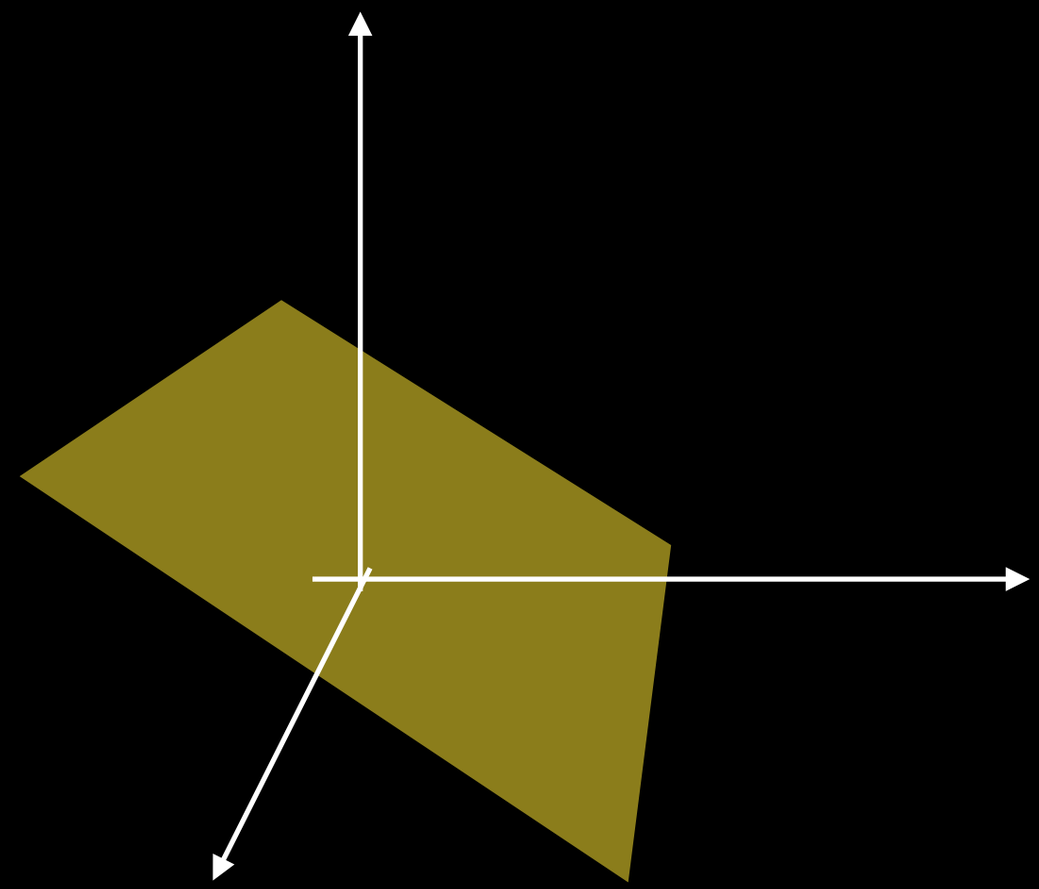
$$\underbrace{x_1 n_1 + x_2 n_2 + \ldots + x_{k-1} n_{k-1}}_{= b} + x_k n_k = 0$$

$$x_k = -\frac{b}{n_k}$$

Vector hyperplane

(x, n) = 0

$\overrightarrow{0} \in subspace$



if a, b ∈ subspace, then

a + b ∈ subspace

ƛa ∈ subspace

(a + b, n) = 0

(a + b, n) = (a, n) + (b, n) = 0 + 0 = 0

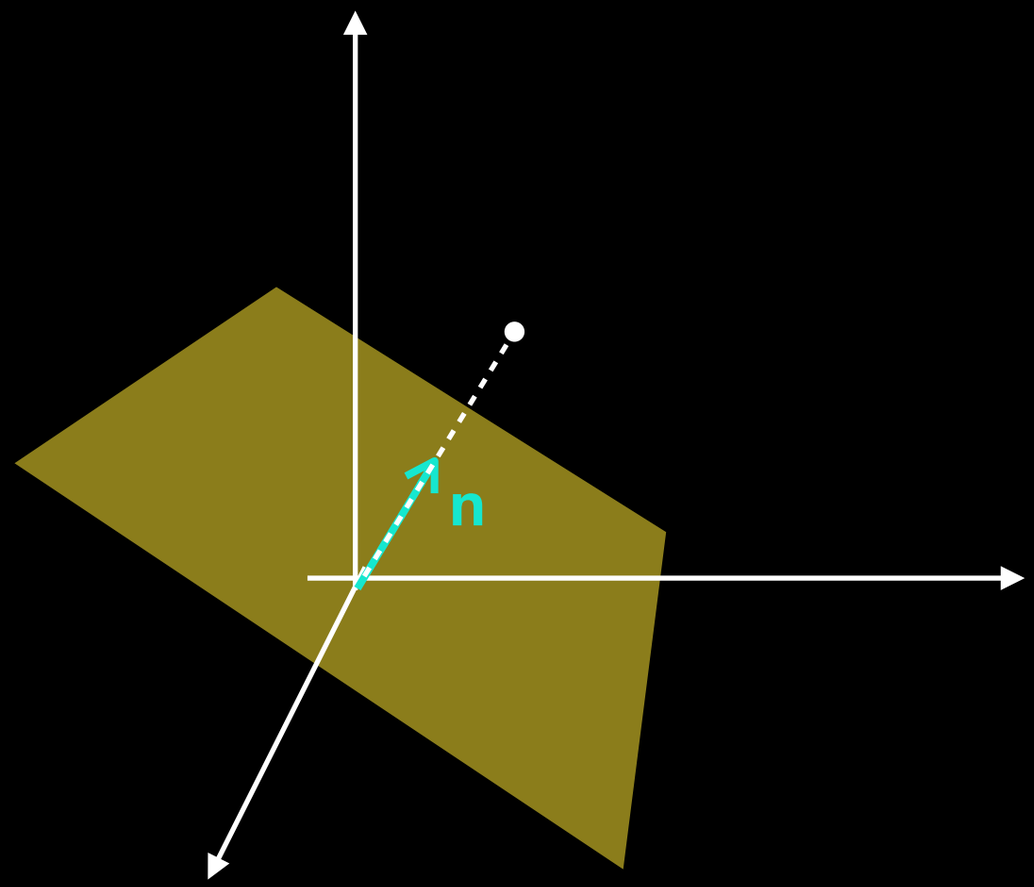
Affine hyperplane

(x, n) = d

(x, n) > d

(x, n) < d

- half spaces

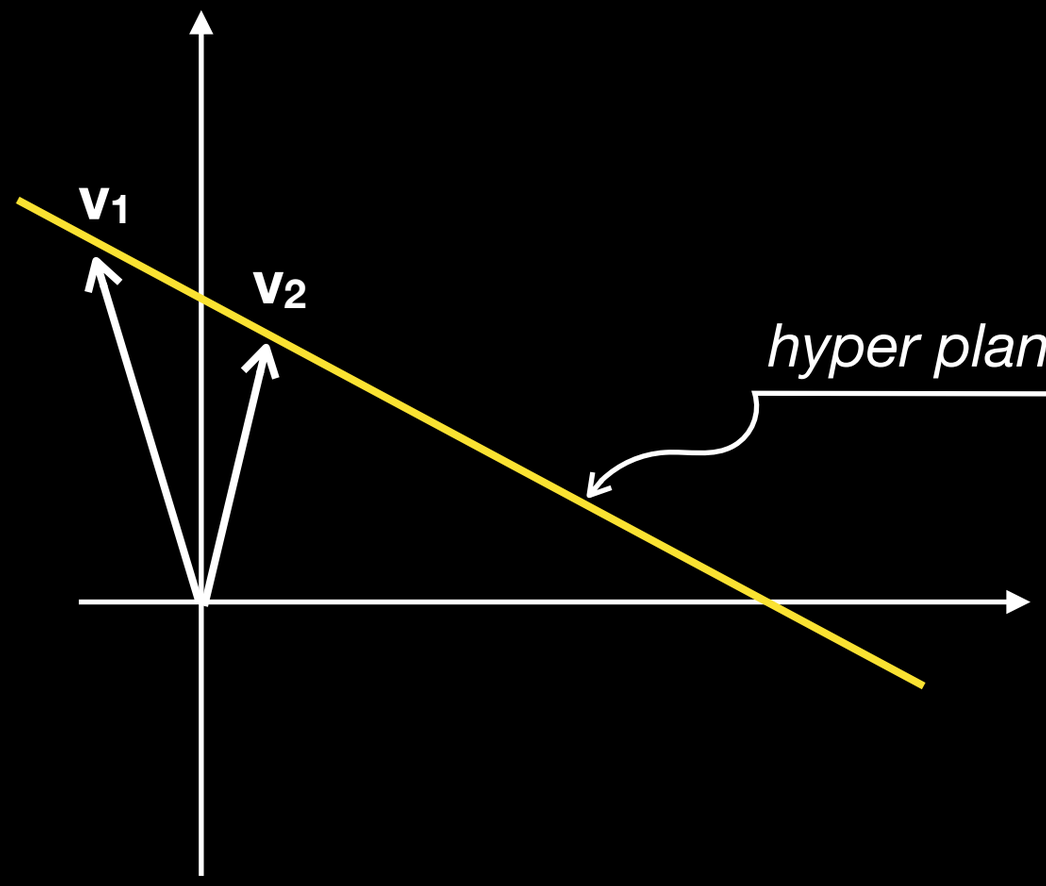


Hyperplane in 2-D

(v, n) = d

$$v_2 - v_1 = \begin{vmatrix} v_x \\ v_y \end{vmatrix}$$

$$n = \frac{1}{\sqrt{v_x^2 + v_y^2}} \begin{vmatrix} v_y \\ -v_x \end{vmatrix} \qquad \|n\| = 1$$



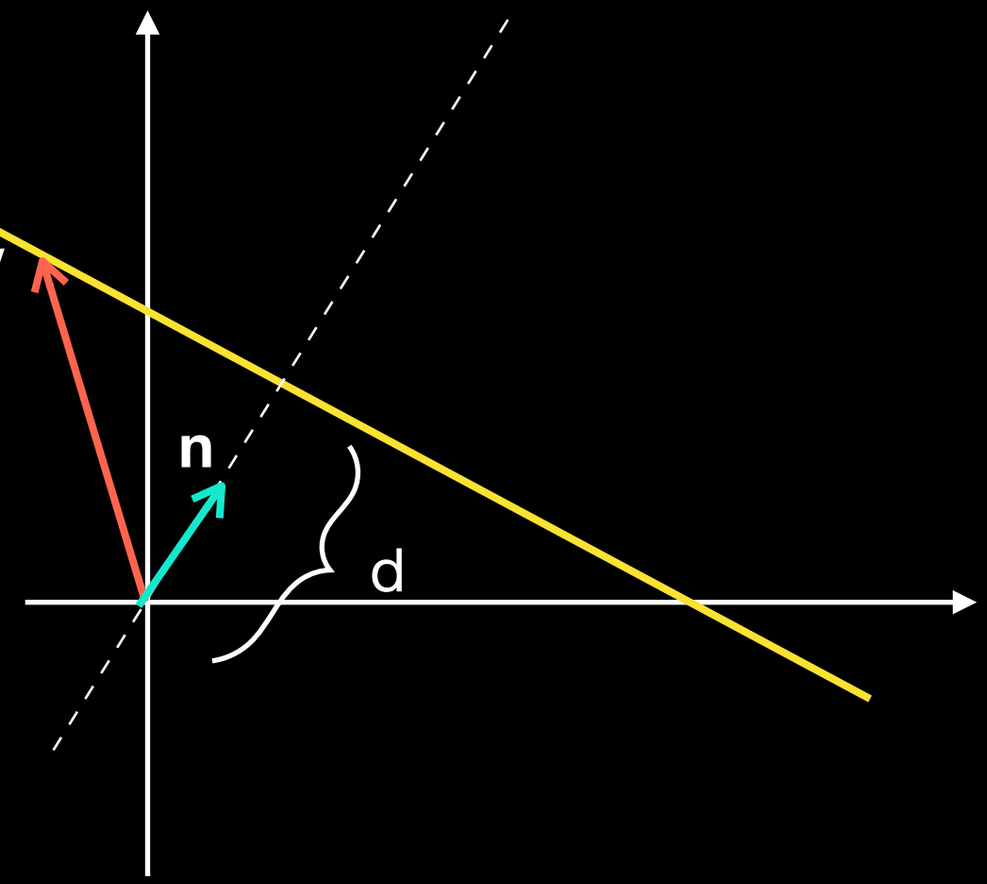
v = a v_1 + (1 - a) v_2

[a v_1 + (1 - a) v_2] · n = a(v_1 - v_2) · n + v_2 · n = v_2 · n = d

= 0

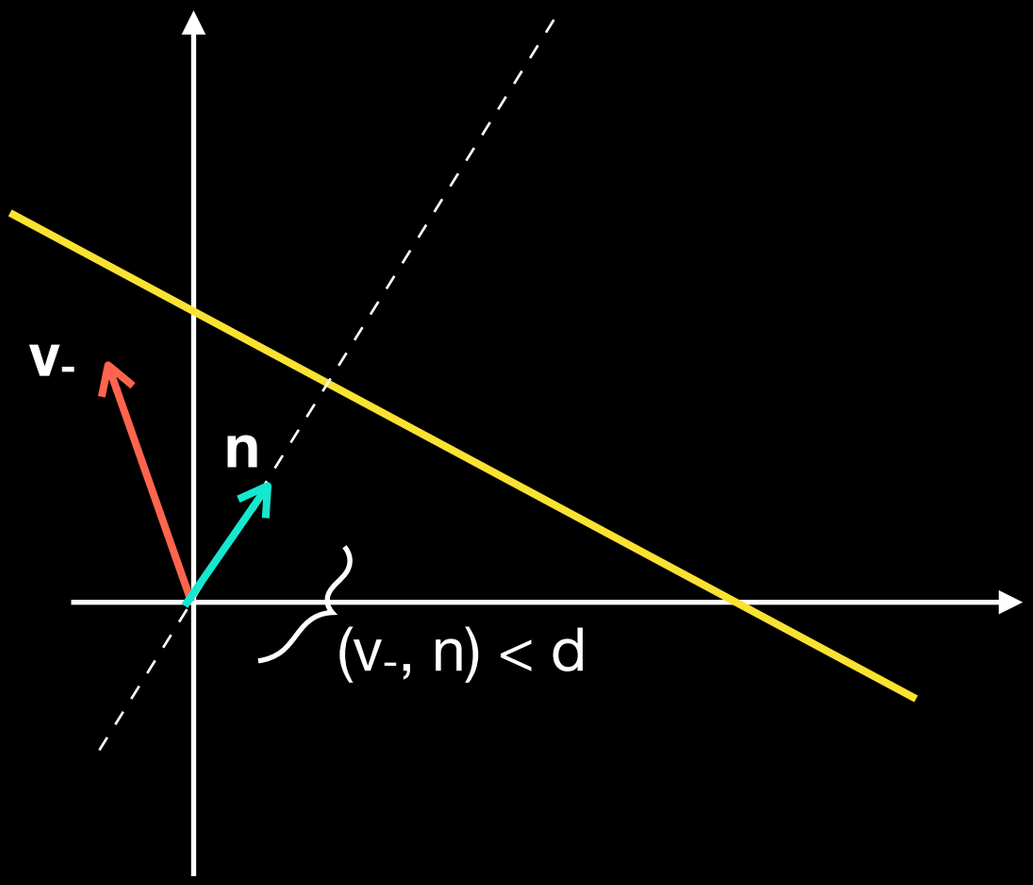
Hyperplane in 2-D

(v, n) = d



Hyperplane in 2-D

$(v_-, n) < d$



Hyperplane in 2-D

$(v_+, n) > d$