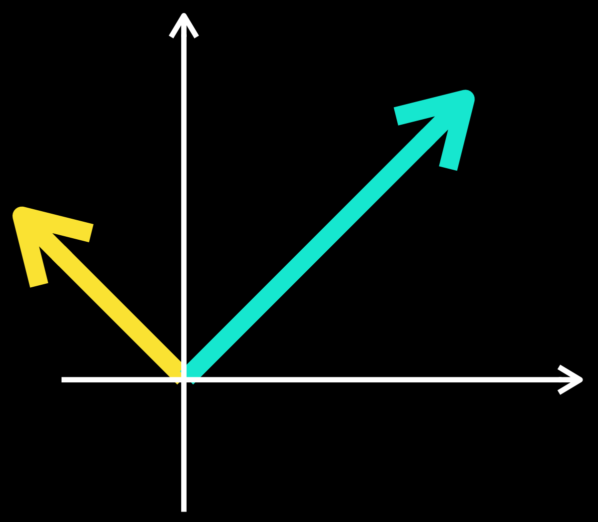
Normed Linear Spaces

Linear Algebra Essentials



California Housing Prices

https://www.kaggle.com/camnugent/california-housing-prices

```
df = pd.read_csv("housing.csv")
df.head()
```

	longitude	latitude	housing_median_age	total_rooms	total_bedrooms	population	households	median_income	median_house_value	ocean_proximity
0	-122.23	37.88	41.0	880.0	129.0	322.0	126.0	8.3252	452600.0	NEAR BAY
1	-122.22	37.86	21.0	7099.0	1106.0	2401.0	1138.0	8.3014	358500.0	NEAR BAY
2	-122.24	37.85	52.0	1467.0	190.0	496.0	177.0	7.2574	352100.0	NEAR BAY
3	-122.25	37.85	52.0	1274.0	235.0	558.0	219.0	5.6431	341300.0	NEAR BAY
4	-122.25	37.85	52.0	1627.0	280.0	565.0	259.0	3.8462	342200.0	NEAR BAY

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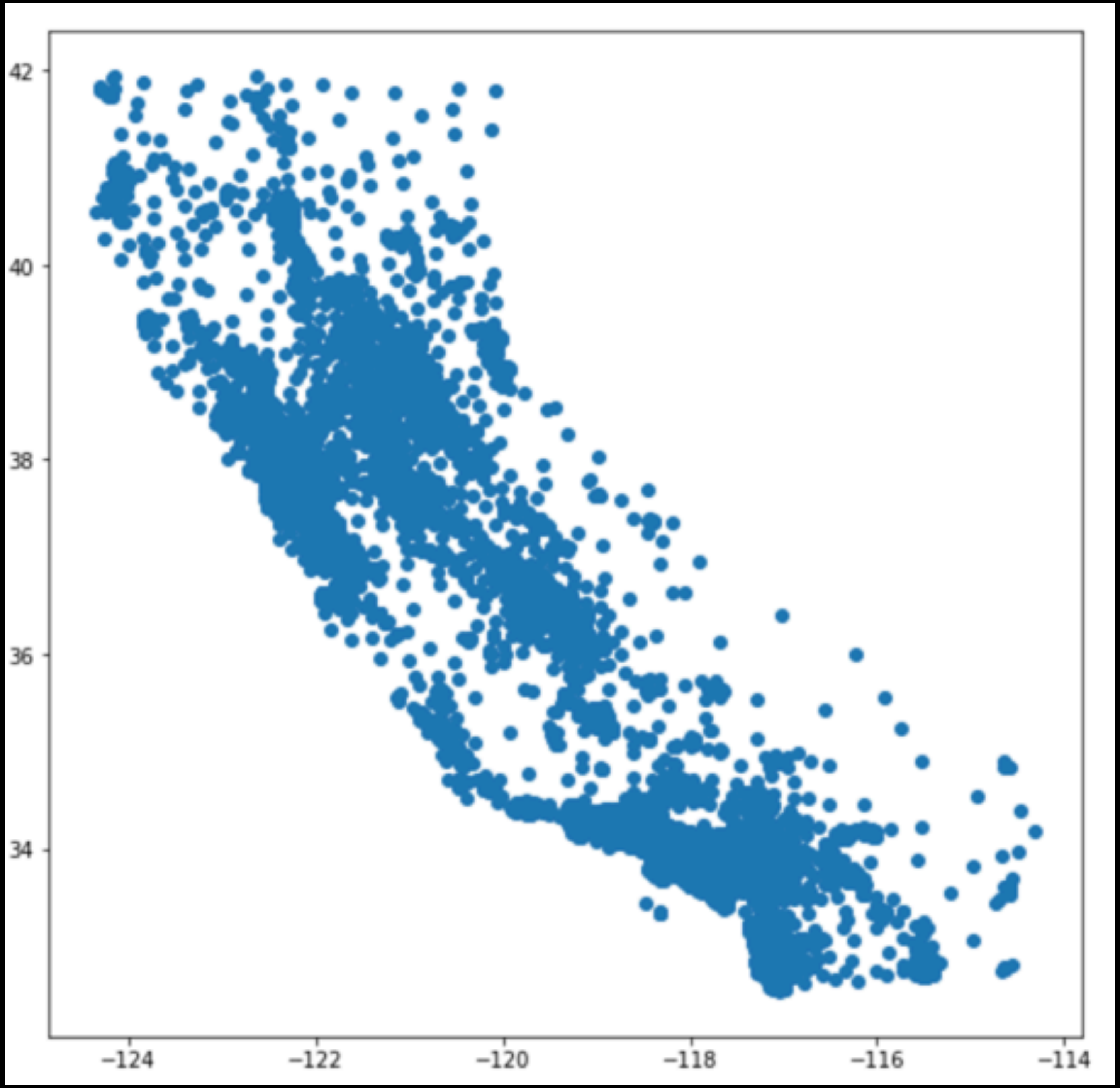
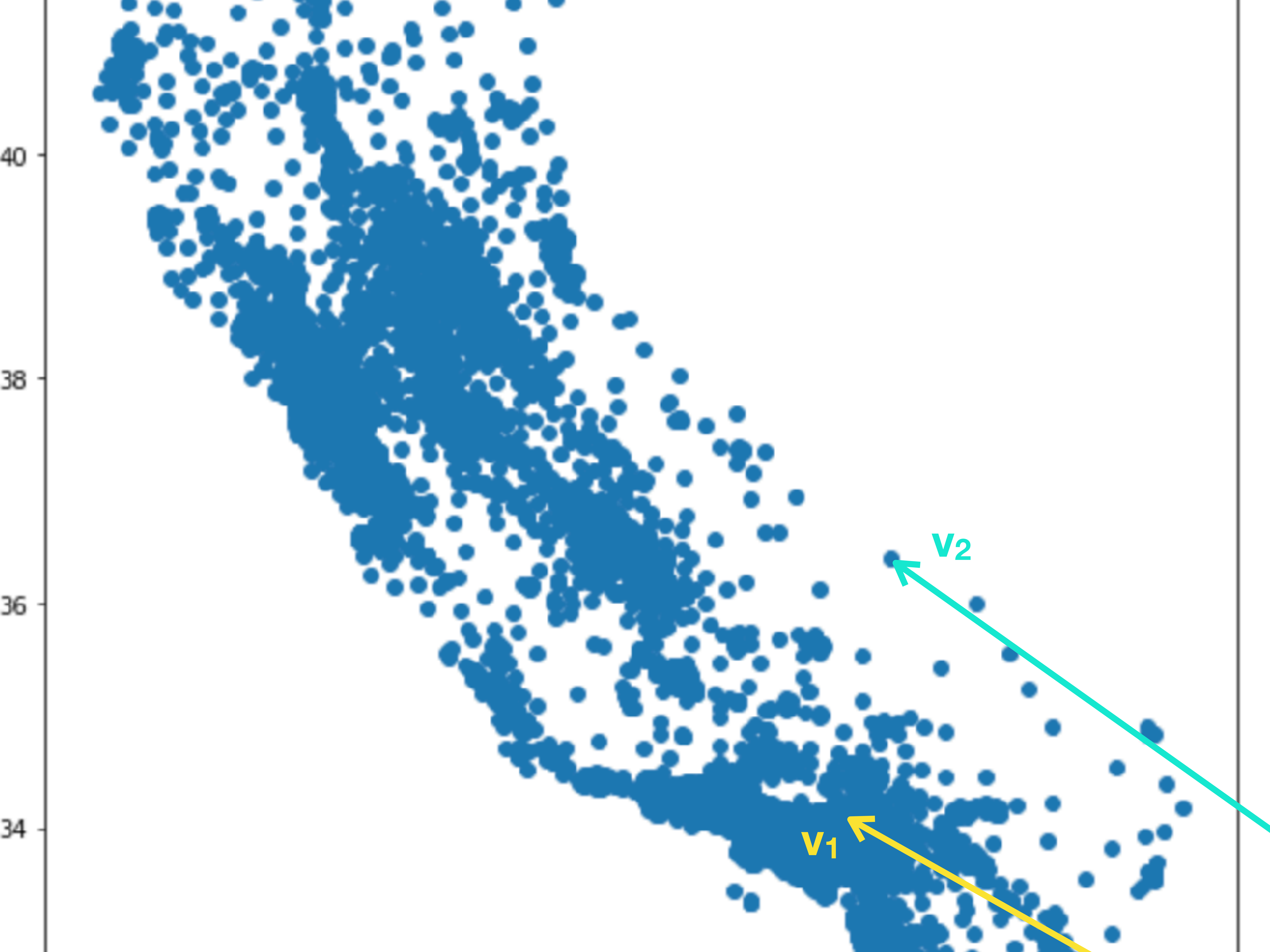
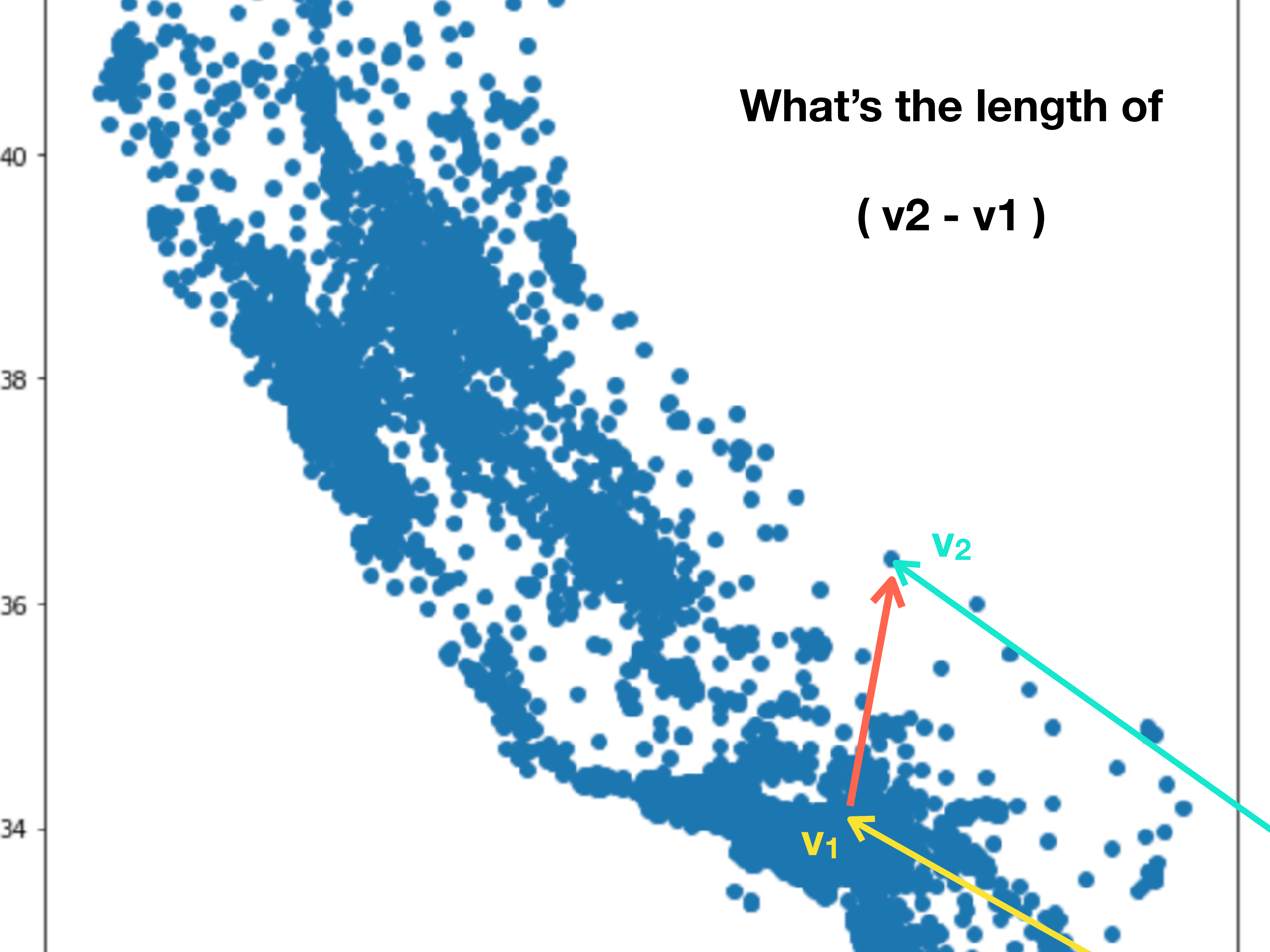
For a given location and house params, we want to find the most similar houses in the dataset




40
38
36
34
v2
v1

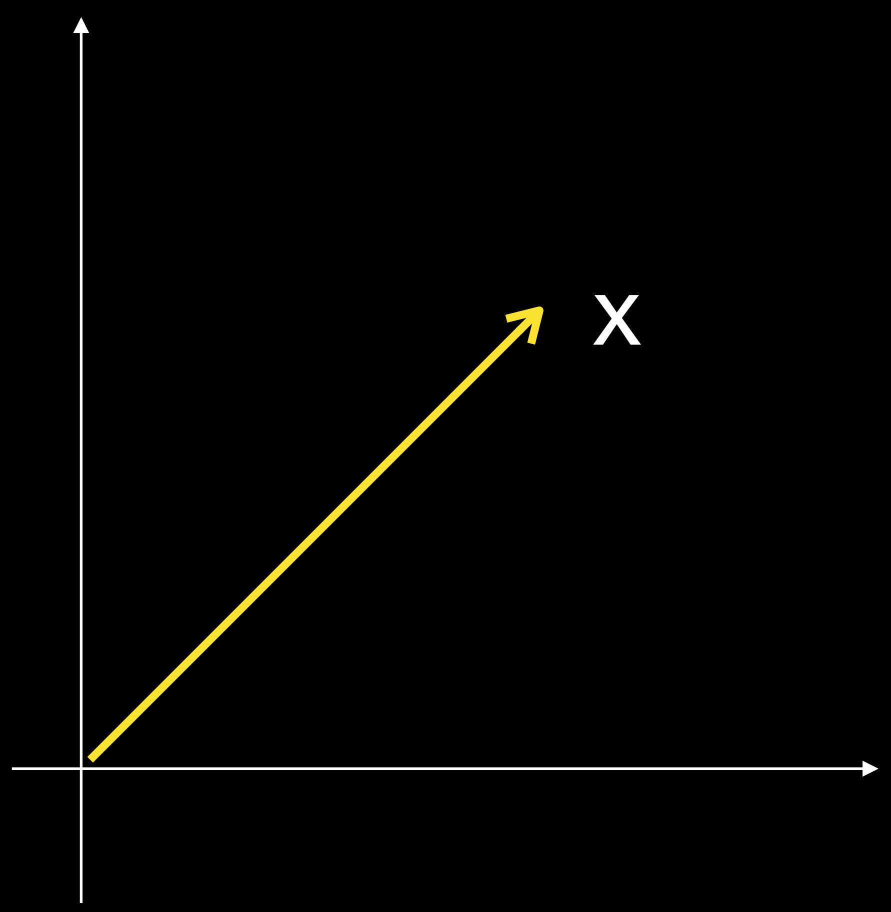

What's the length of
(v2 - v1)
v2
v1
40
38
36
34

Normed vector spaces

A norm is a function:

$$f(x) \mapsto R$$

$$\begin{cases} 1.\ f(x) = 0,\ if\ and\ only\ if\ x = \overrightarrow{0} \\ 2.\ f(x) > 0,\ if\ x \neq \overrightarrow{0} \\ 3.\ f(\lambda \overrightarrow{x}) = |\lambda| \cdot f(x) \\ 4.\ f(a+b) \leqslant f(a) + f(b) \end{cases}$$



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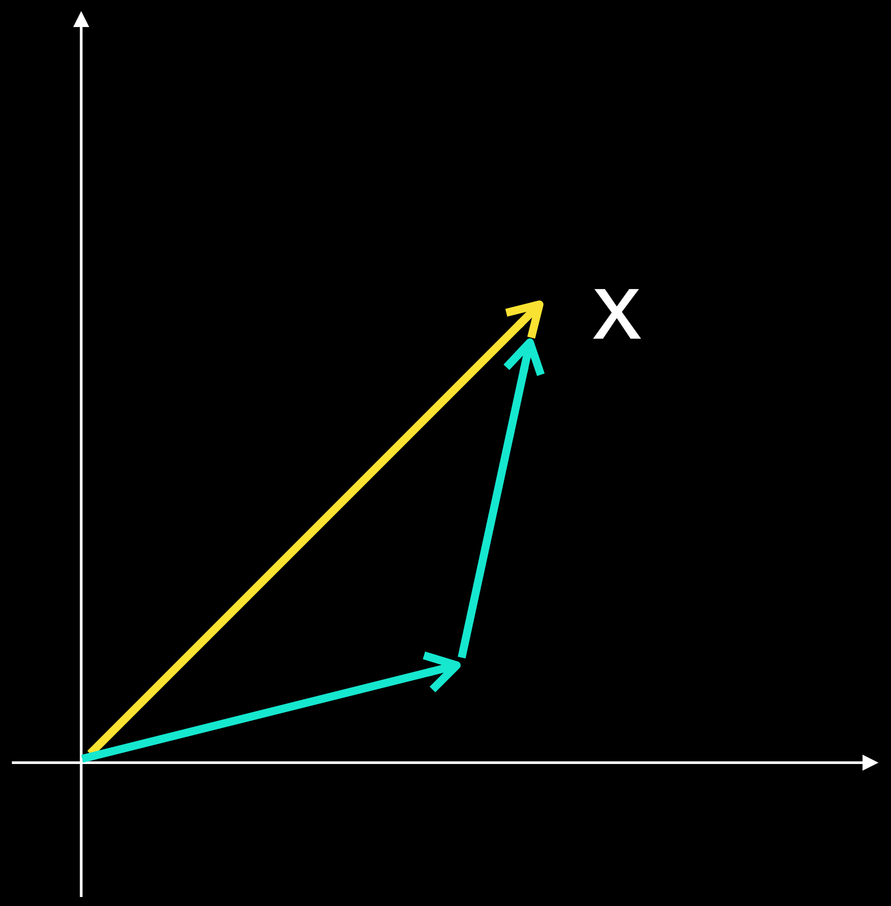
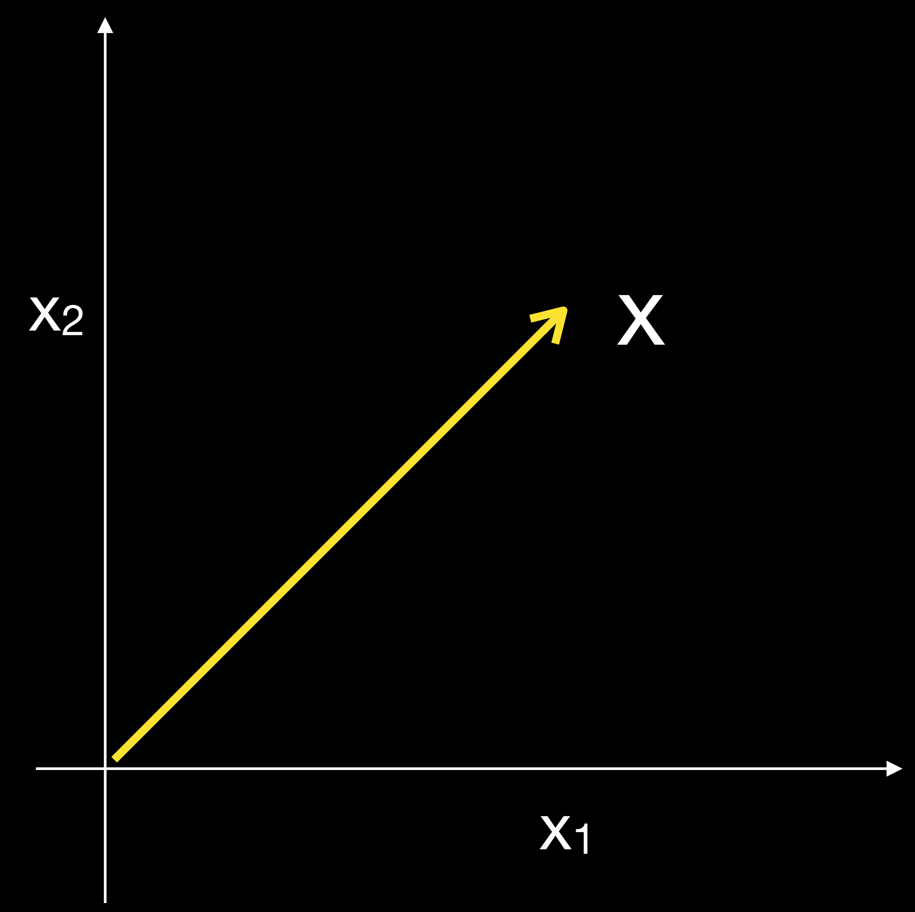


Euclidean distance (norm)

$$\|x\| = \sqrt{x_1^2 + x_2^2}$$



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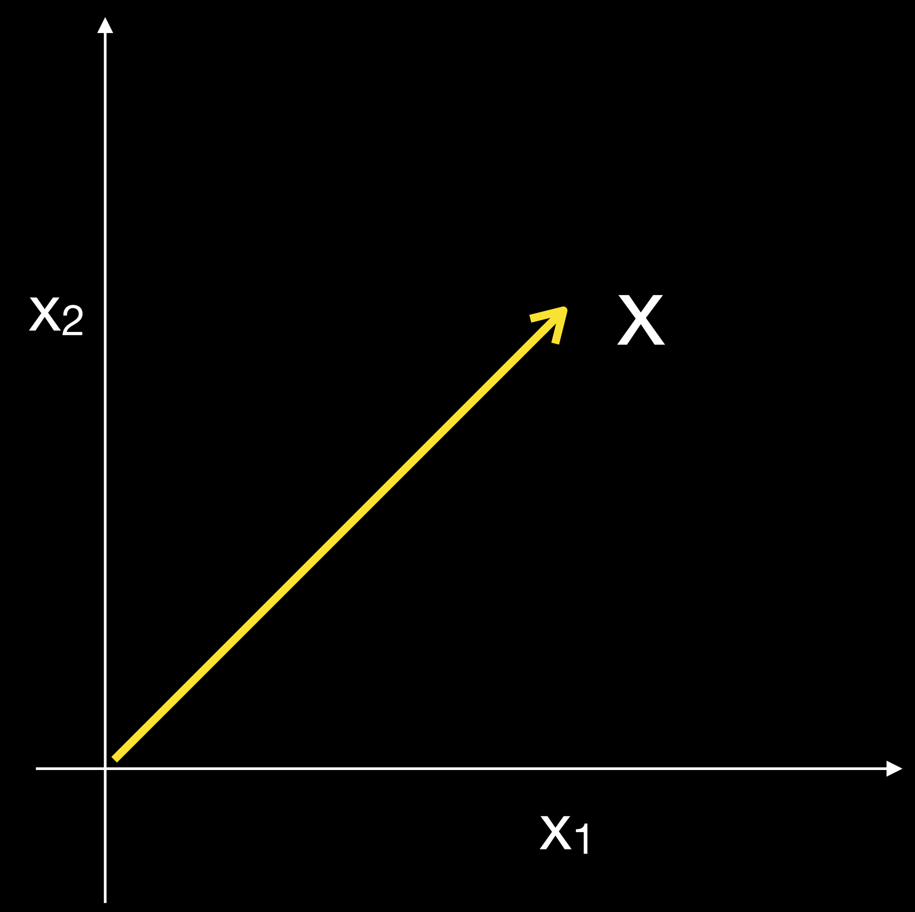
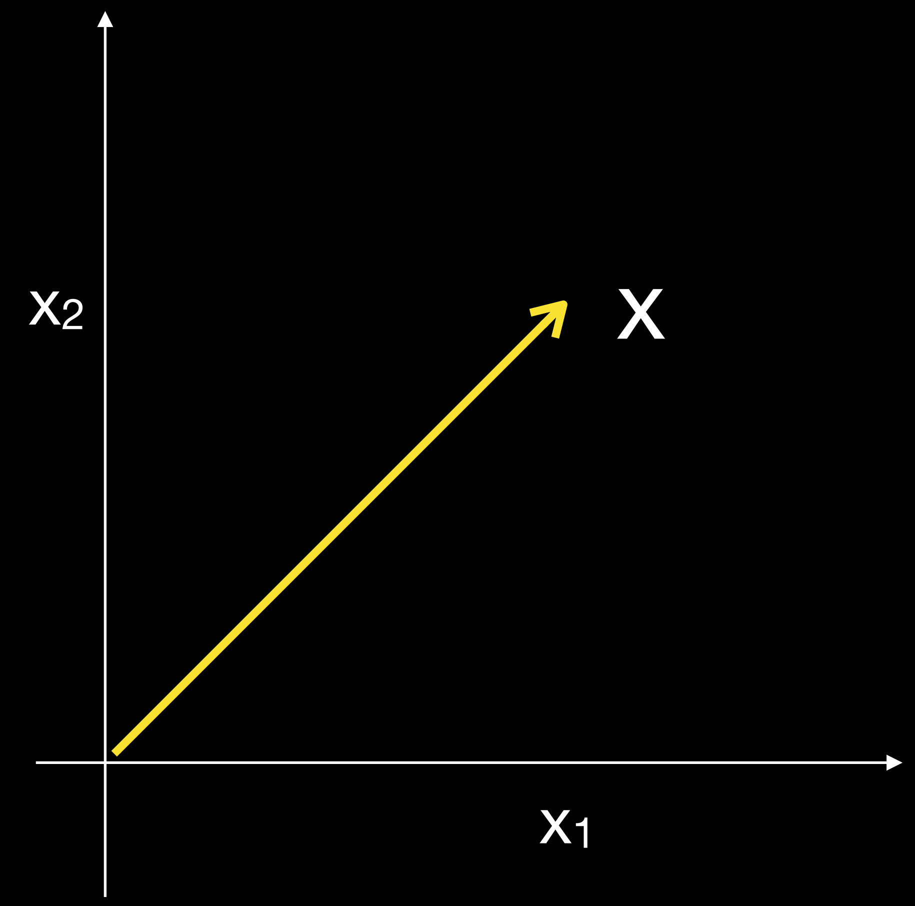
1.2. $\|x\| \geqslant 0$



Euclidean distance (L_2 norm)

$$\|x\| = \sqrt{x_1^2 + x_2^2}$$



$$1.2.\ \|x\| \geqslant 0$$

$$3.\ \|\lambda x\| = \sqrt{(\lambda x_1)^2 + (\lambda x_2)^2}$$

$$= |\lambda| \cdot \sqrt{x_1^2 + x_2^2} = |\lambda| \cdot \|x\|$$

Euclidean distance (L_2 norm)

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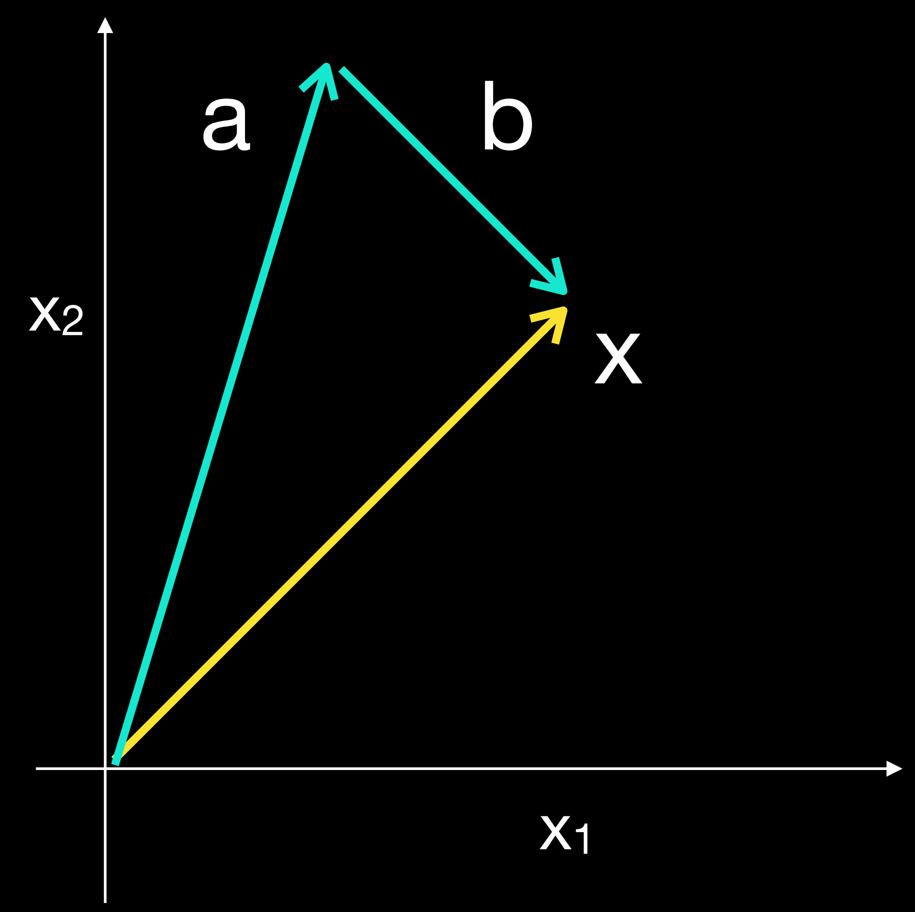


1. $\|x\| \geqslant 0$

2. $\|\lambda x\| = \sqrt{(\lambda x_1)^2 + (\lambda x_2)^2}$

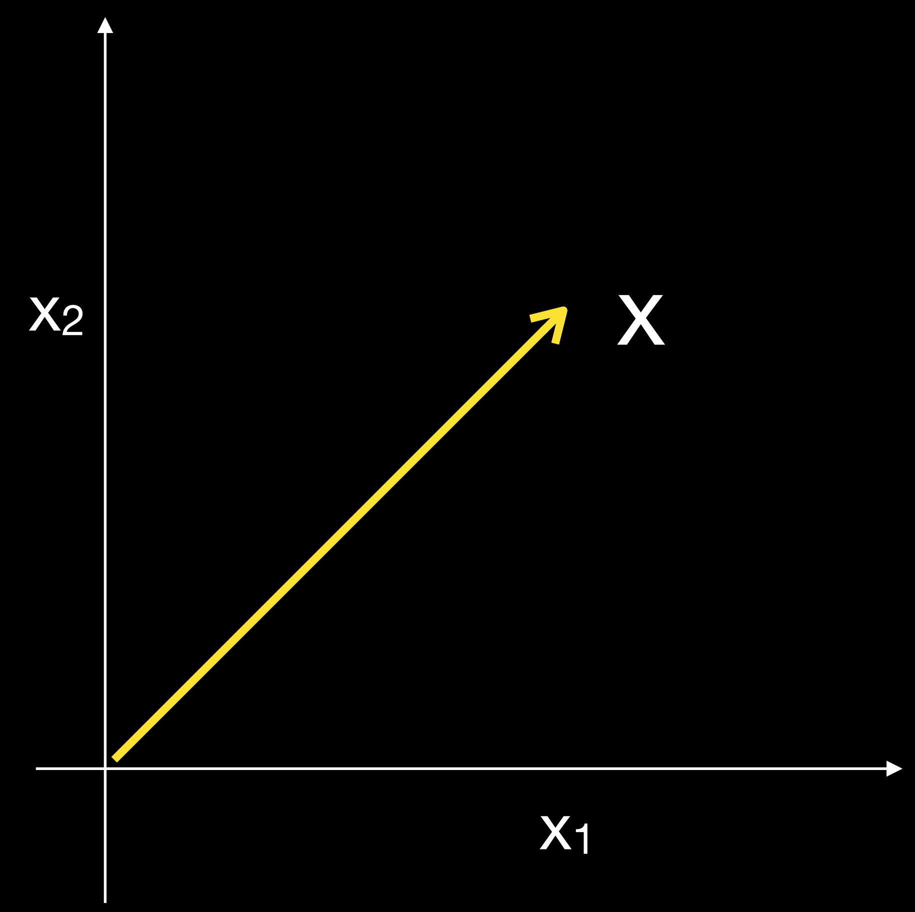
   $= |\lambda| \cdot \sqrt{x_1^2 + x_2^2} = |\lambda| \cdot \|x\|$

3. $\|a + b\| \leqslant \|a\| + \|b\| -$ *triangle inequality*

L_1 norm

$$\|x\| = |x_1| + |x_2|$$

Check whether it satisfies all conditions for a norm

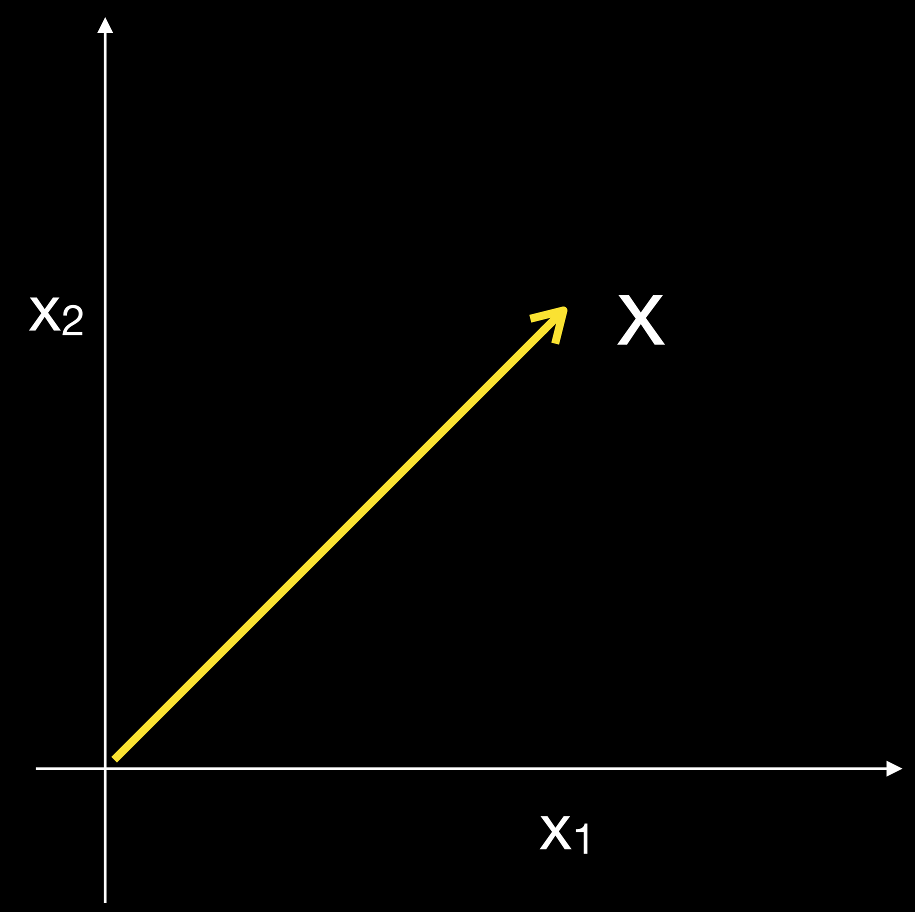


L_1 norm

$$\|x\| = |x_1| + |x_2|$$

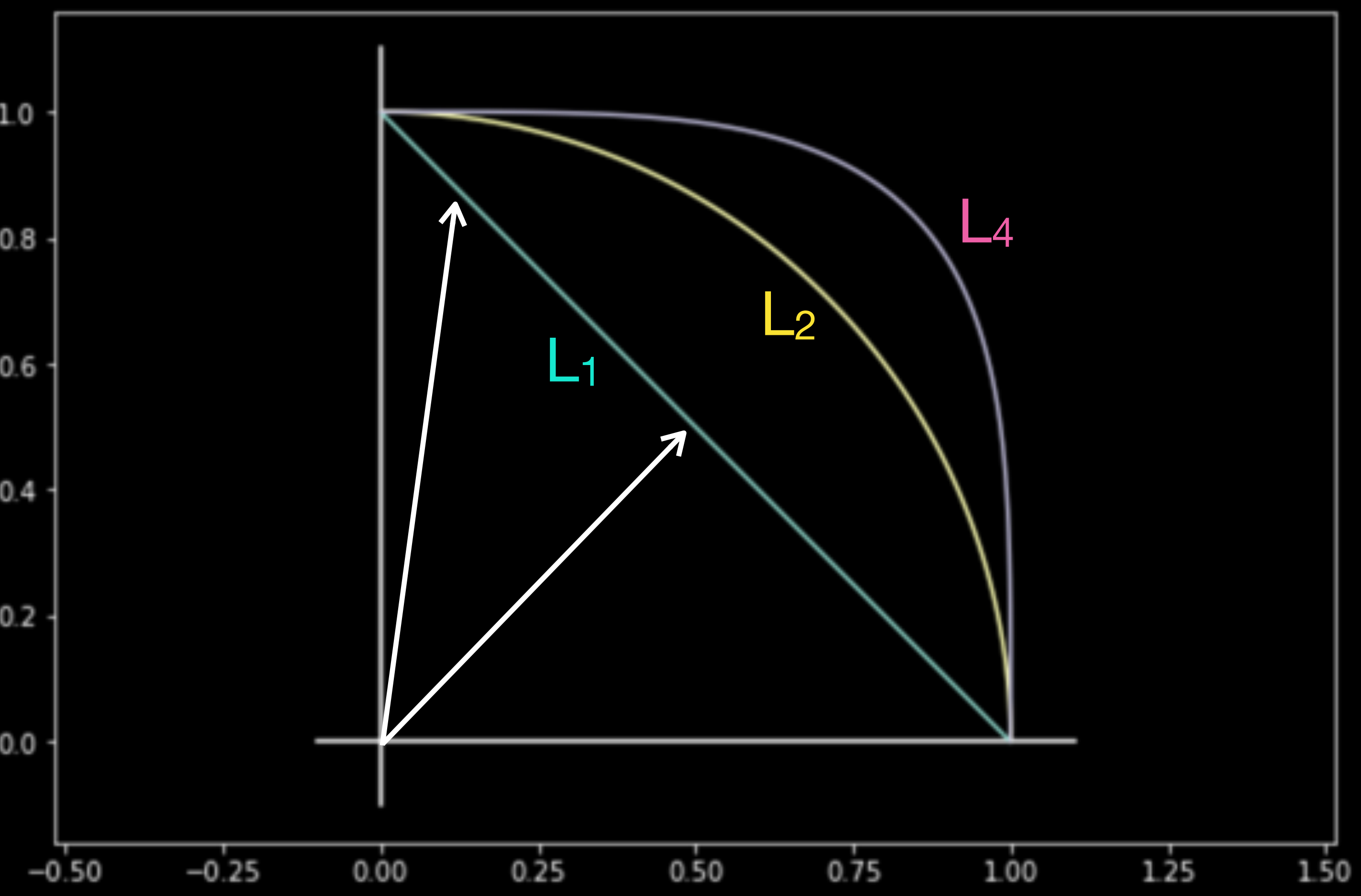
Check whether it satisfies all conditions for a norm

L_1 norm $\quad \|x\| = \sum_i |x_i|$



L_k norm

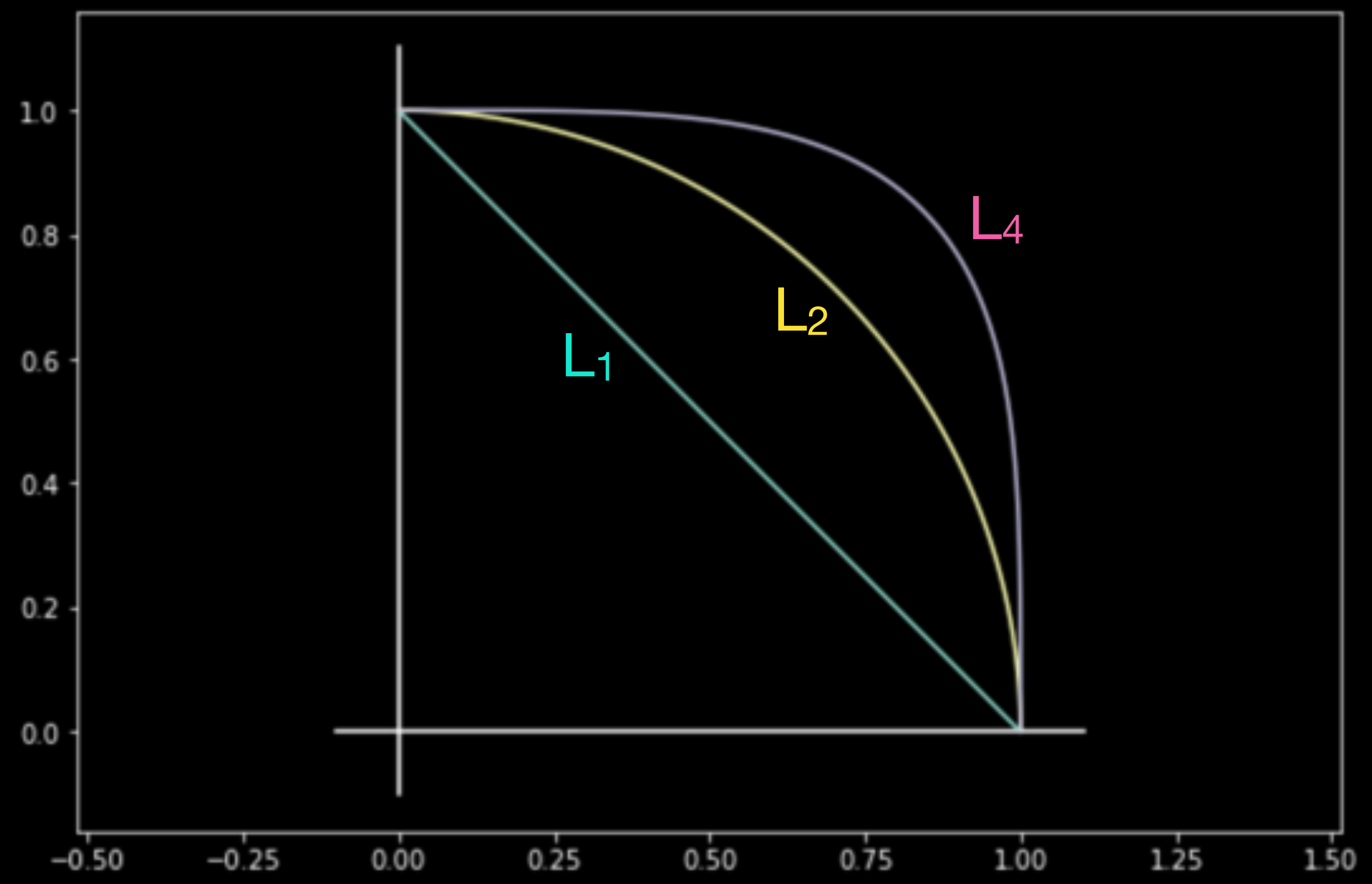
$$\|x\|_k = \left(\sum_i |x_i|^k\right)^{\frac{1}{k}}$$



L_k norm

$$L_1 : \|(3,4)\|_1 \geqslant \|(5,1)\|_1$$

$$L_2 : \|(3,4)\|_2 \leqslant \|(5,1)\|_2$$



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