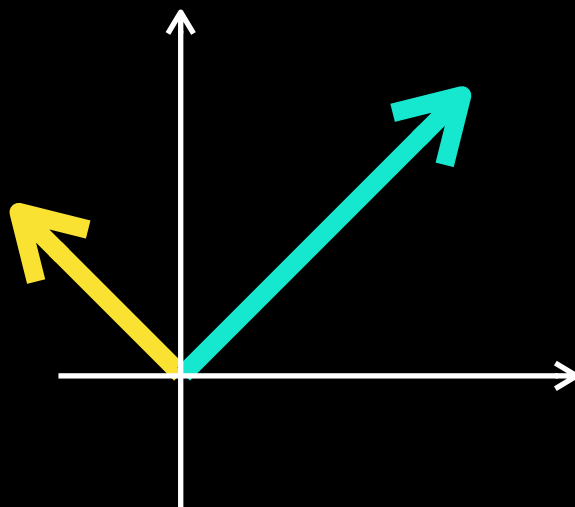
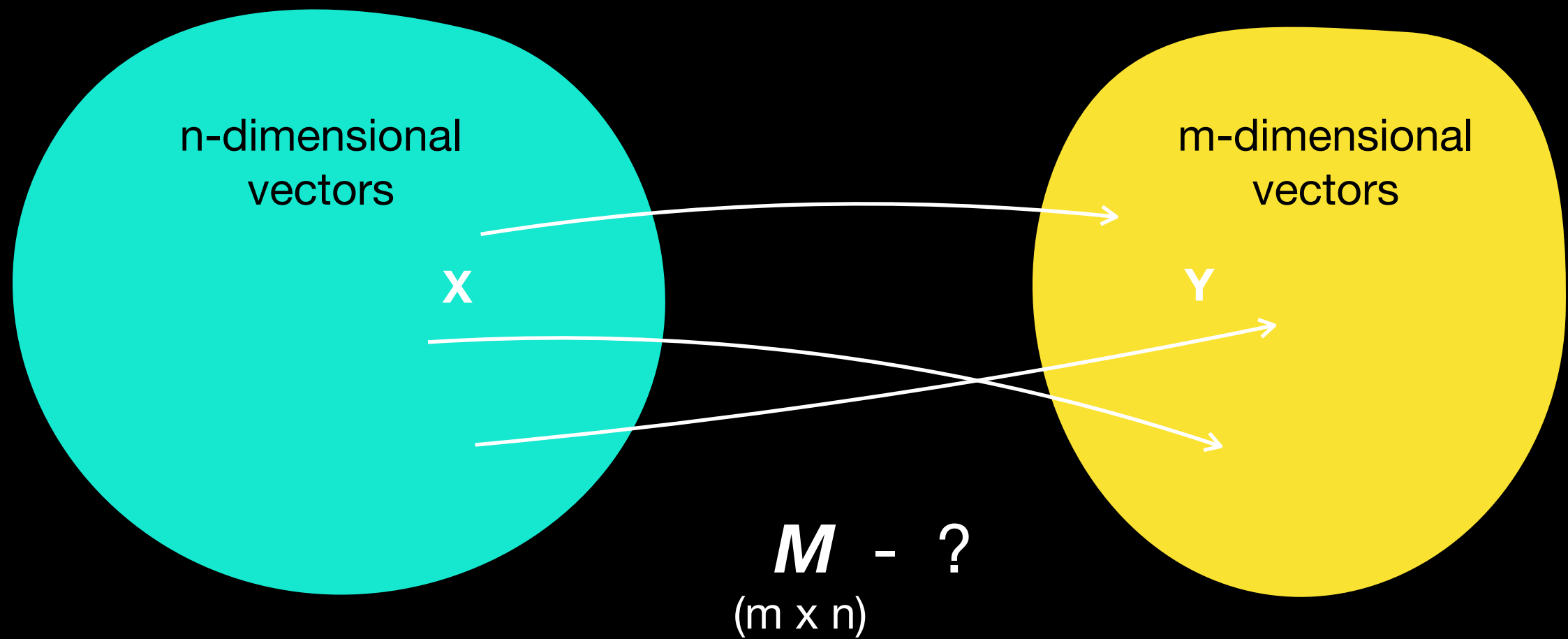


Linear regression

Linear Algebra Essentials



General view



$$Mx = \hat{y}, \quad \hat{y} \approx y \quad \| \hat{y} - y \| \rightarrow \min$$

$$\| Mx - y \| \rightarrow \min$$

$$M X = Y$$

$$X^T M^T = Y^T$$

 X^T
 M^T
 Y^T

 \cdot

 $=$

 a
 y

$$X^T a = y$$

Simple case

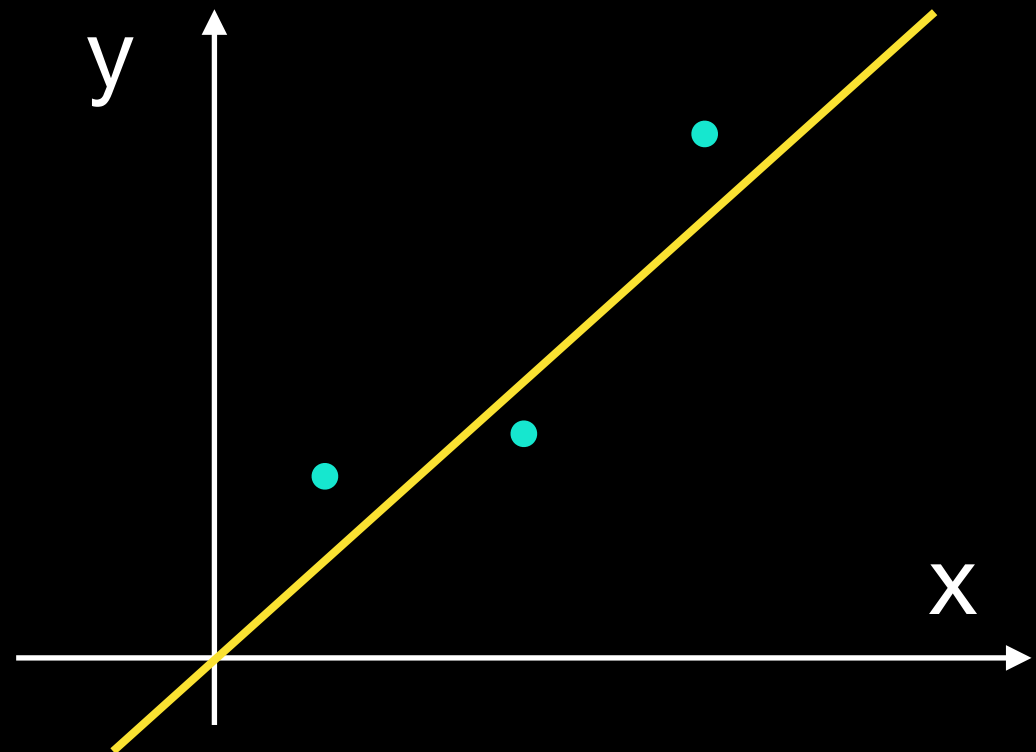
$$\{ (\mathbf{x}_1, y_1), \dots, (\mathbf{x}_k, y_k) \}$$

$$(\mathbf{a}, \mathbf{x}_j) \approx y_j$$

$$\begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{k1} & x_{k2} & \dots & x_{kn} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_k \end{bmatrix}$$

if $n = k$

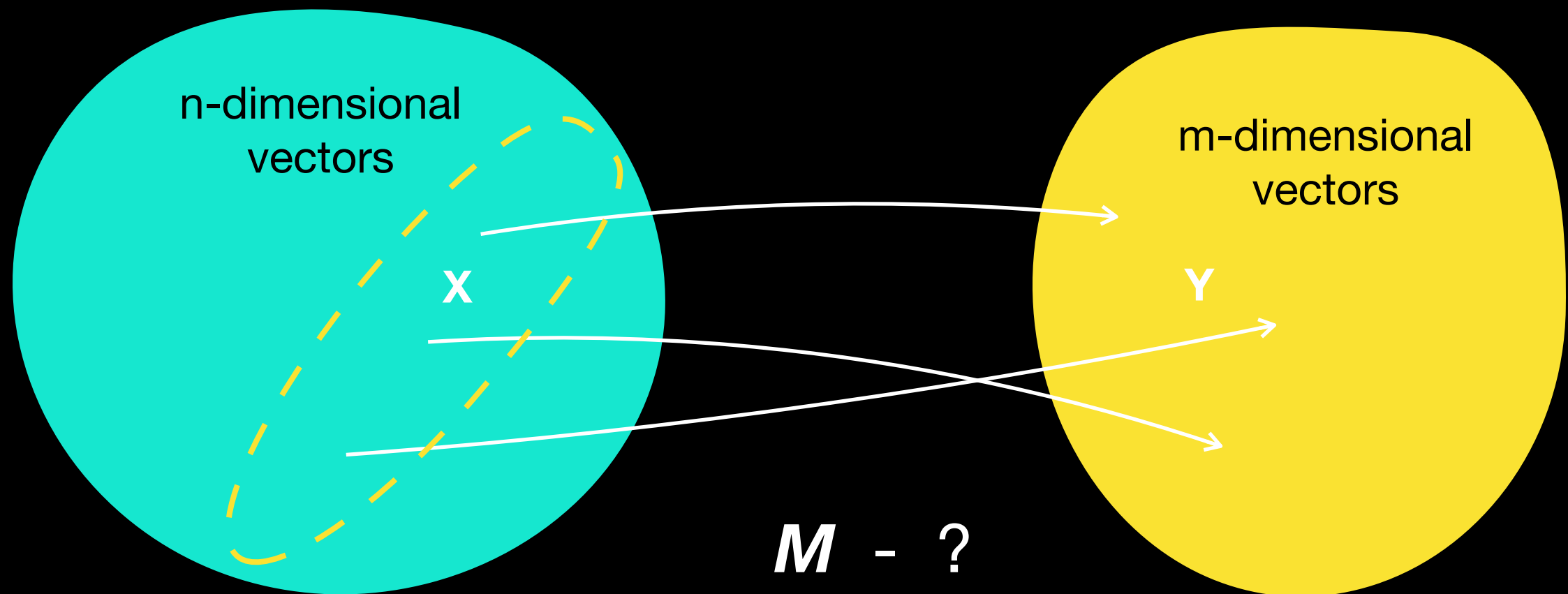
$$\mathbf{a} = (\mathbf{X}^T)^{-1} \mathbf{y}$$



if $n < k$

*overdetermined system
of linear equations*

Dimensionality reduction



$\{ x \}$ - *n-dimensional vectors*

$$M x \approx (y_1, \dots, y_m, \underbrace{0, 0, \dots})$$

$$n \rightarrow m$$