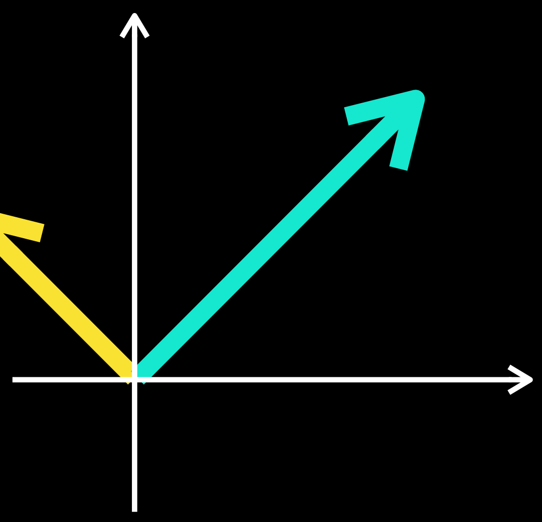
Singular value decomposition (SVD)

Linear Algebra Essentials



$$A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$$

$$Ax = y$$

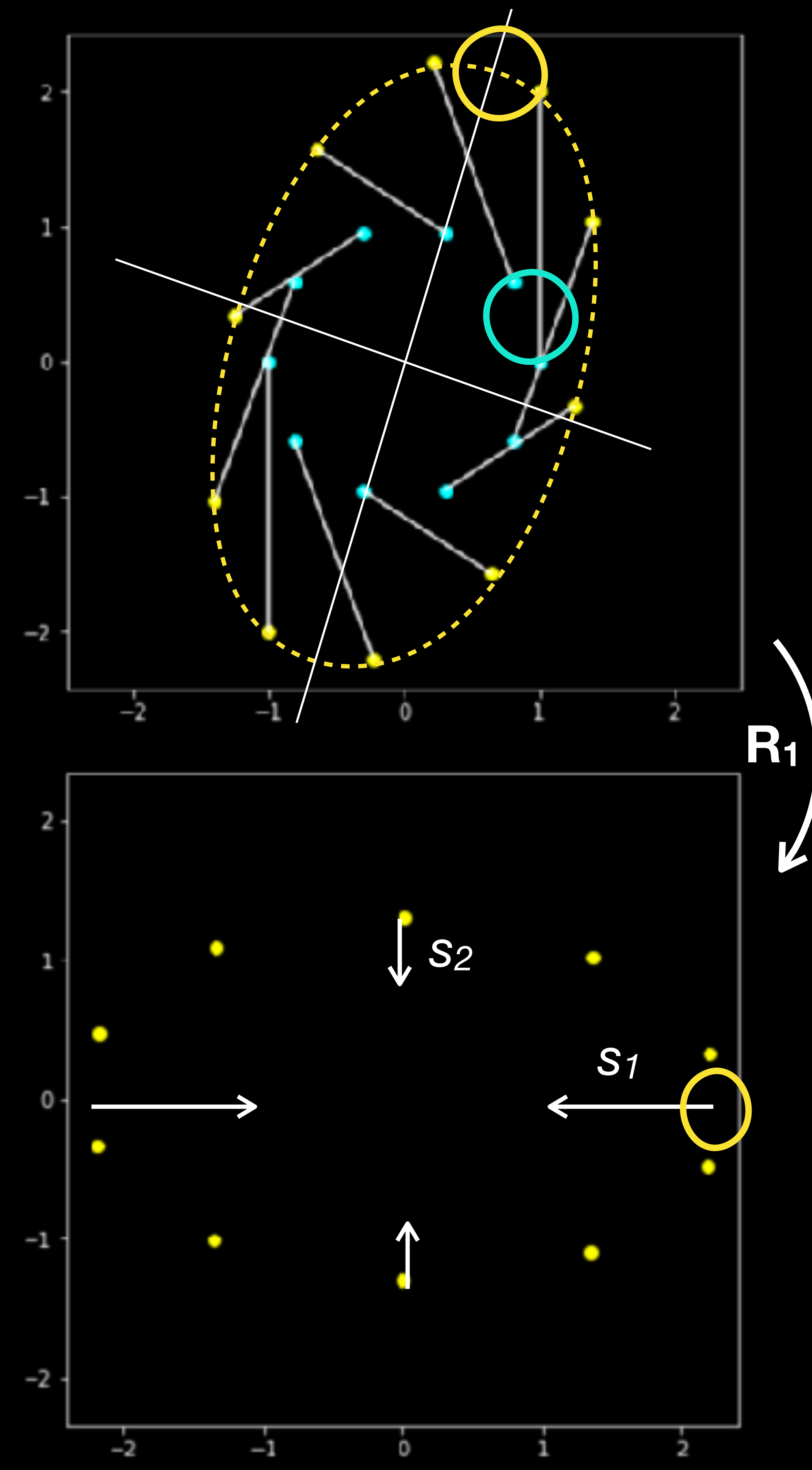
$$R_1 A x = R_1 y \qquad \alpha_1 = -1.32$$

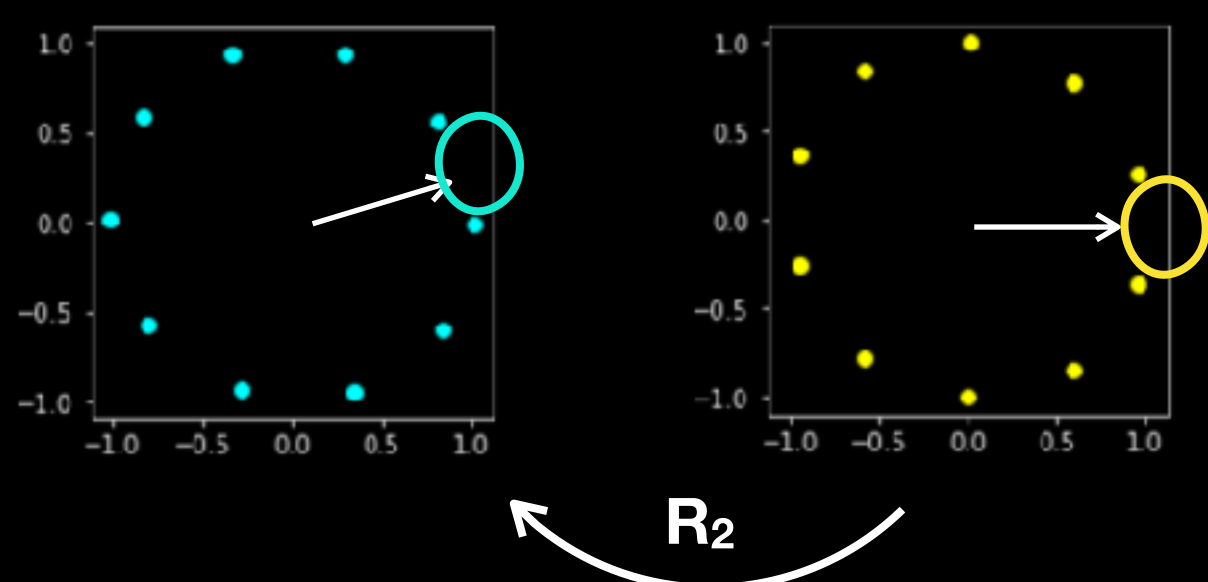
$$S R_1 A x = S R_1 y \qquad s_{!} = 1/2.3 \quad s_2 = 1/1.3$$

$$R_2 S R_1 A x = R_2 S R_1 y = x \qquad \alpha_2 = 0.35$$

$$A^{-1} = R_2 S R_1$$

$$A = (R_2 S R_1)^{-1} = R_1^T S^{-1} R_2^T$$





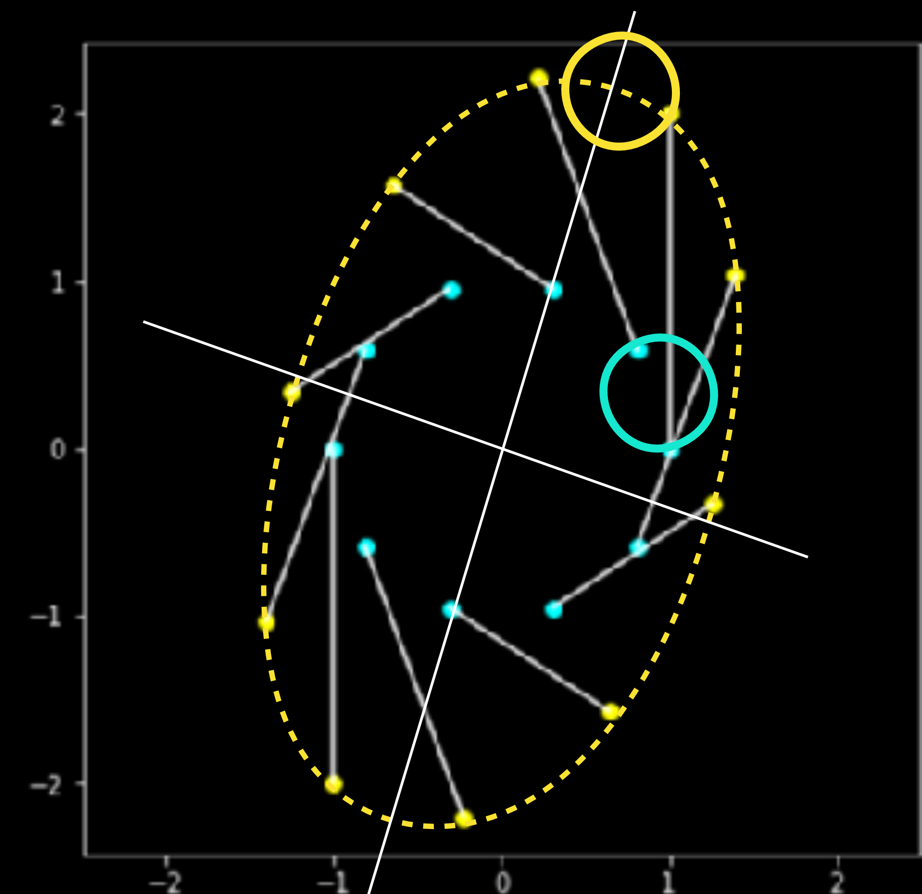
$$A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$$

$$A = R_1^T S^{-1} R_2^T$$

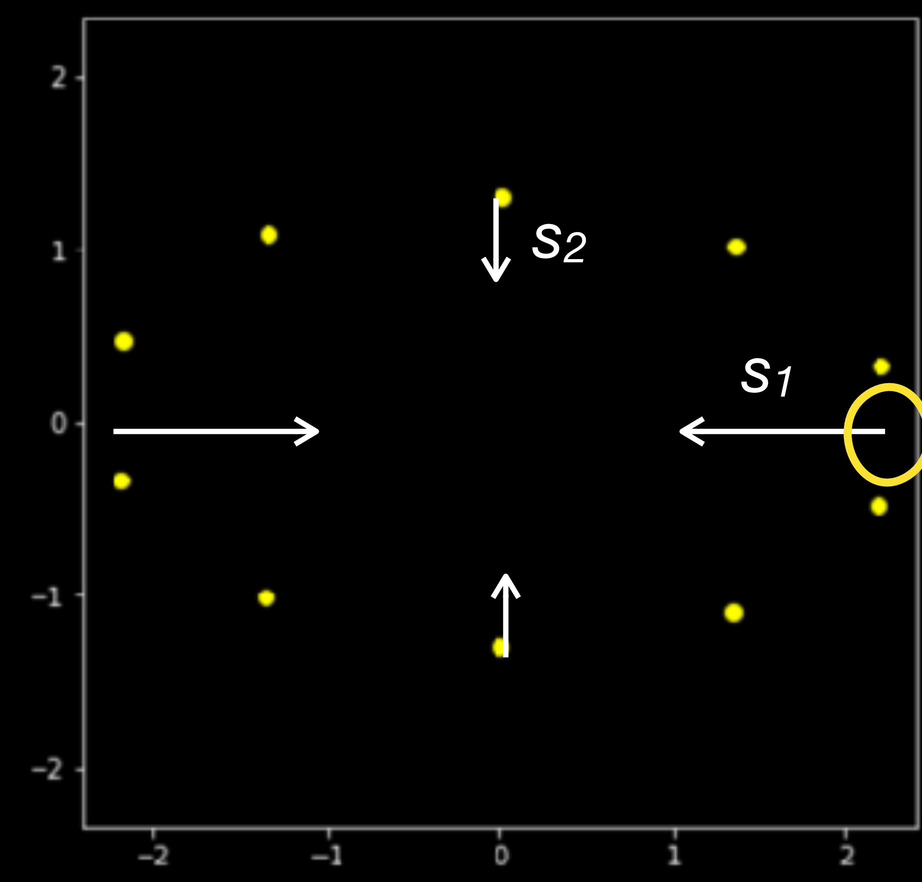
$$\alpha_1 = -1.32$$

$$s_! = 1/2.3 \quad s_2 = 1/1.3$$

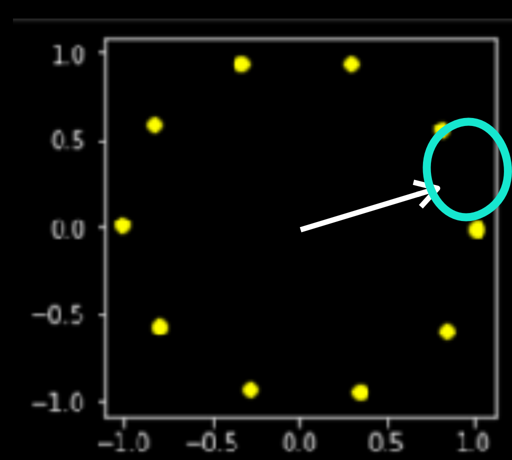
$$\alpha_2 = 0.35$$

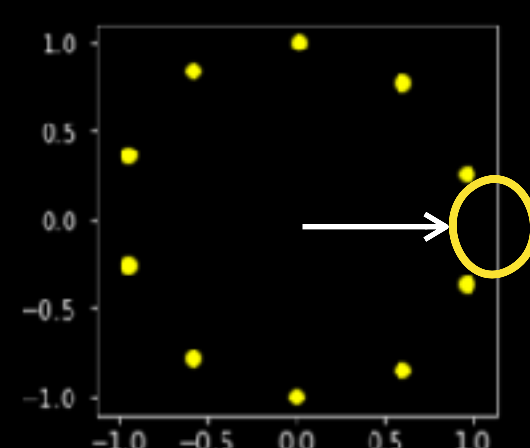


R1



$$\begin{bmatrix} 0.25 & -0.97 \\ 0.97 & 0.25 \end{bmatrix} \cdot \begin{bmatrix} 2.3 & 0 \\ 0 & 1.3 \end{bmatrix} \cdot \begin{bmatrix} 0.94 & -0.34 \\ 0.34 & 0.94 \end{bmatrix} = \begin{bmatrix} 0.97 & -0.99 \\ 1.98 & 1.06 \end{bmatrix}$$





R2

$$A^{-1} = R_2 S R_1$$

$$A = \underbrace{R_1^T}\underbrace{S^{-1}}\underbrace{R_2^T}$$

```
U, S, V = np.linalg.svd(A)
U
```

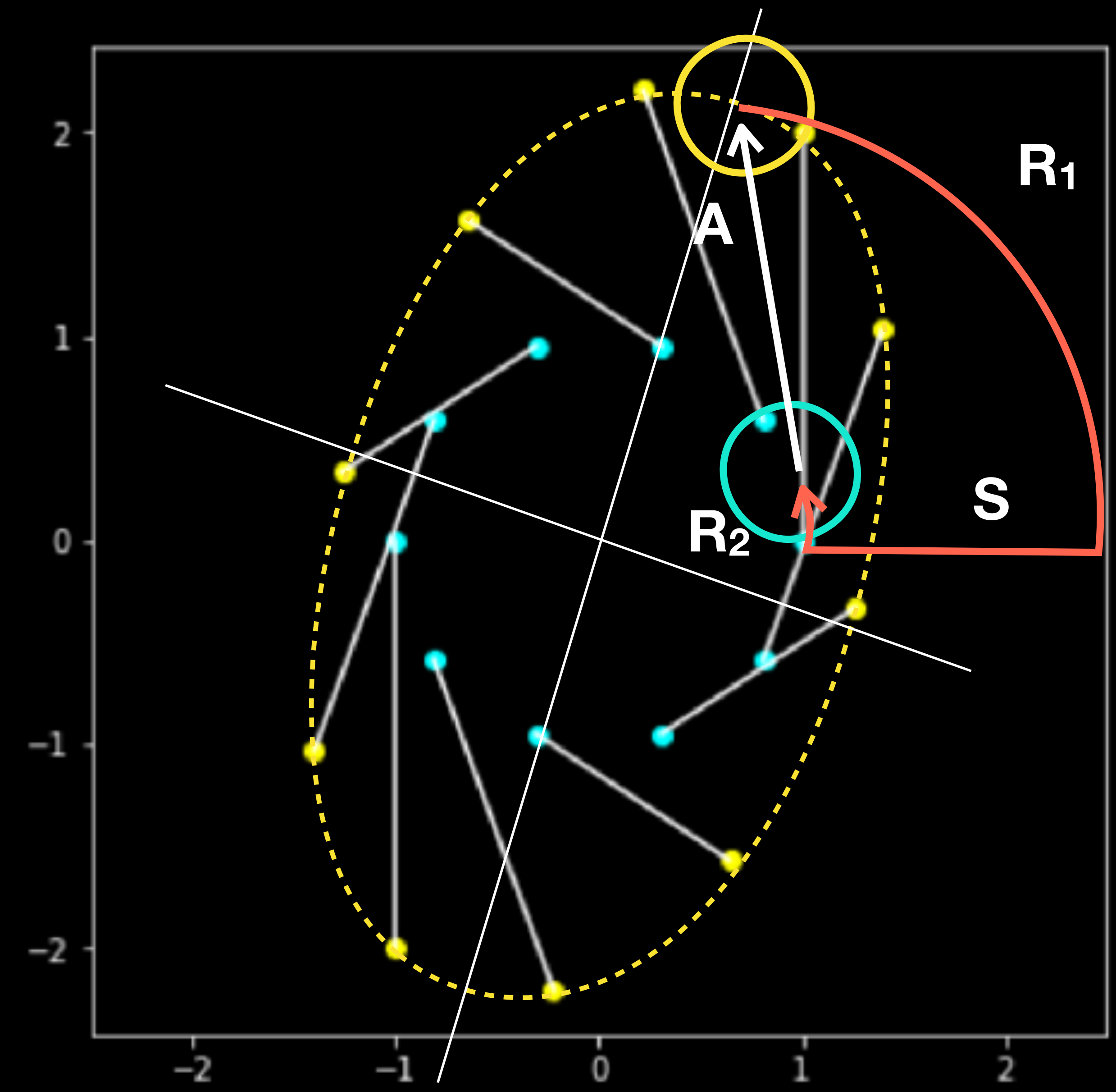
```
array([[-0.28978415, -0.95709203],
       [-0.95709203,  0.28978415]])
```

```
V
```

```
array([[-0.95709203, -0.28978415],
       [-0.28978415,  0.95709203]])
```

```
S
```

```
array([2.30277564, 1.30277564])
```



$$A = U \cdot S \cdot V$$