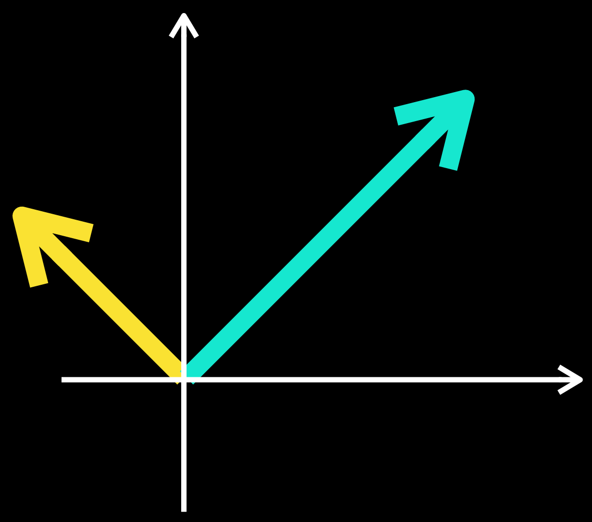
Vectors

Linear Algebra Essentials



$$5 + 7 = 12$$

$$15 / 3 = 5$$

$$23 - 9 = 14$$

vector

$$\mathbf{a} = (a_1, a_2, \ldots, a_k)$$

*k-dimensional vector **a***

Examples

1.

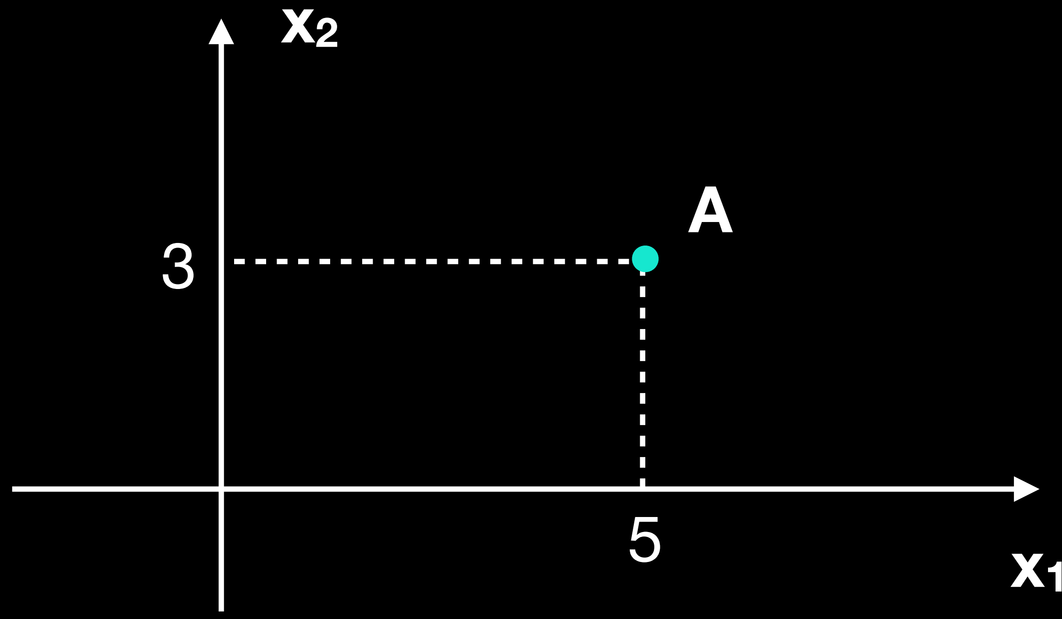
Any number can be seen as a 1-dimensional vector

$a = (5)$
$b = (7)$

2.

A point on a plane is a 2-dimensional vector

$A = (5, 3)$



Operations

1. Summation

$$\overrightarrow{a} + \overrightarrow{b} = \overrightarrow{c}$$

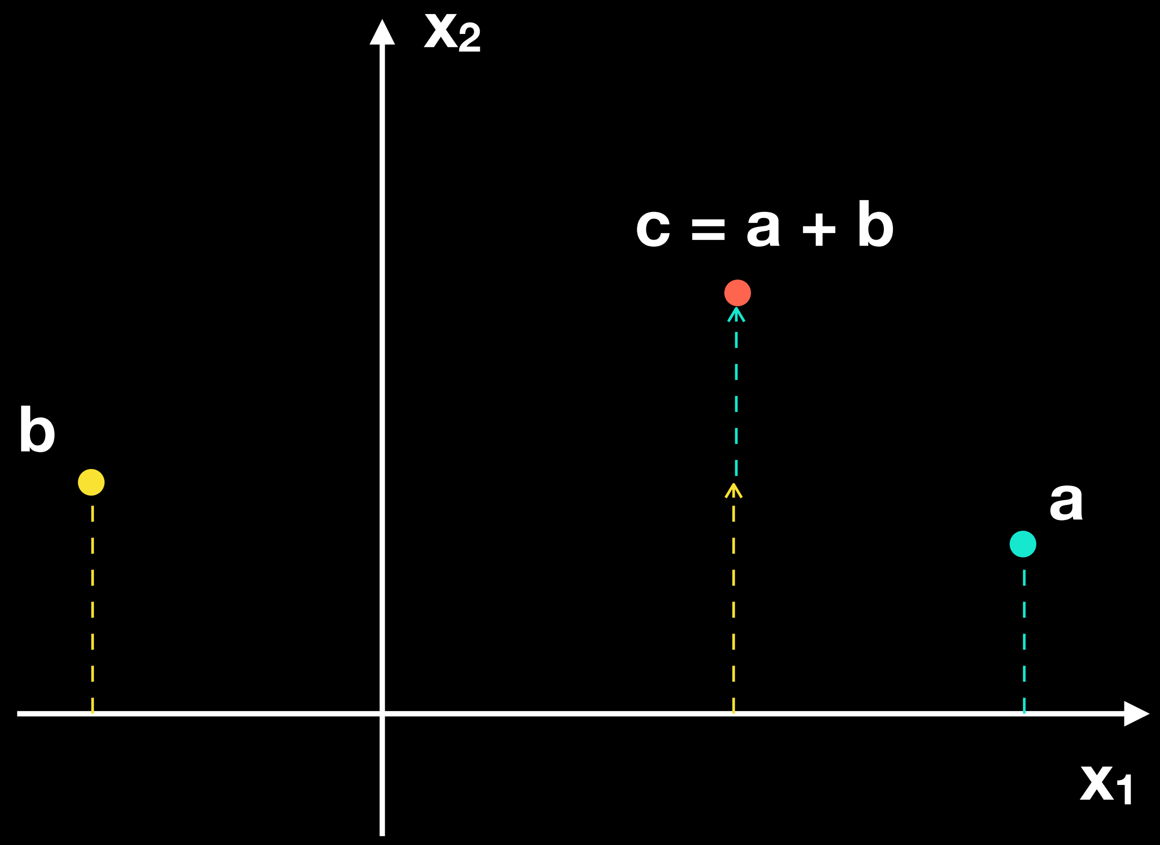
$$c = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \vdots \\ a_m + b_m \end{bmatrix}$$

2. Scaling

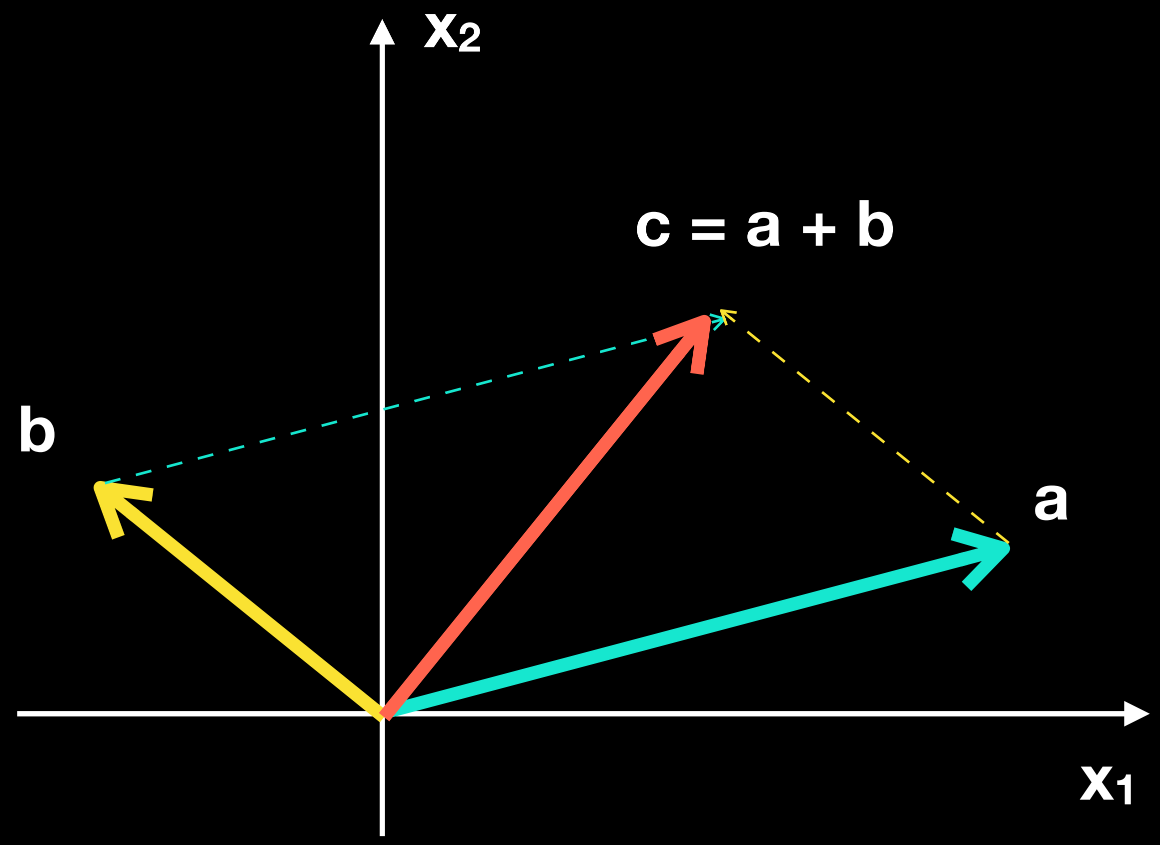
$$\lambda \overrightarrow{b} = \overrightarrow{d}$$

$$d = \begin{bmatrix} \lambda b_1 \\ \lambda b_2 \\ \vdots \\ \lambda b_m \end{bmatrix}$$

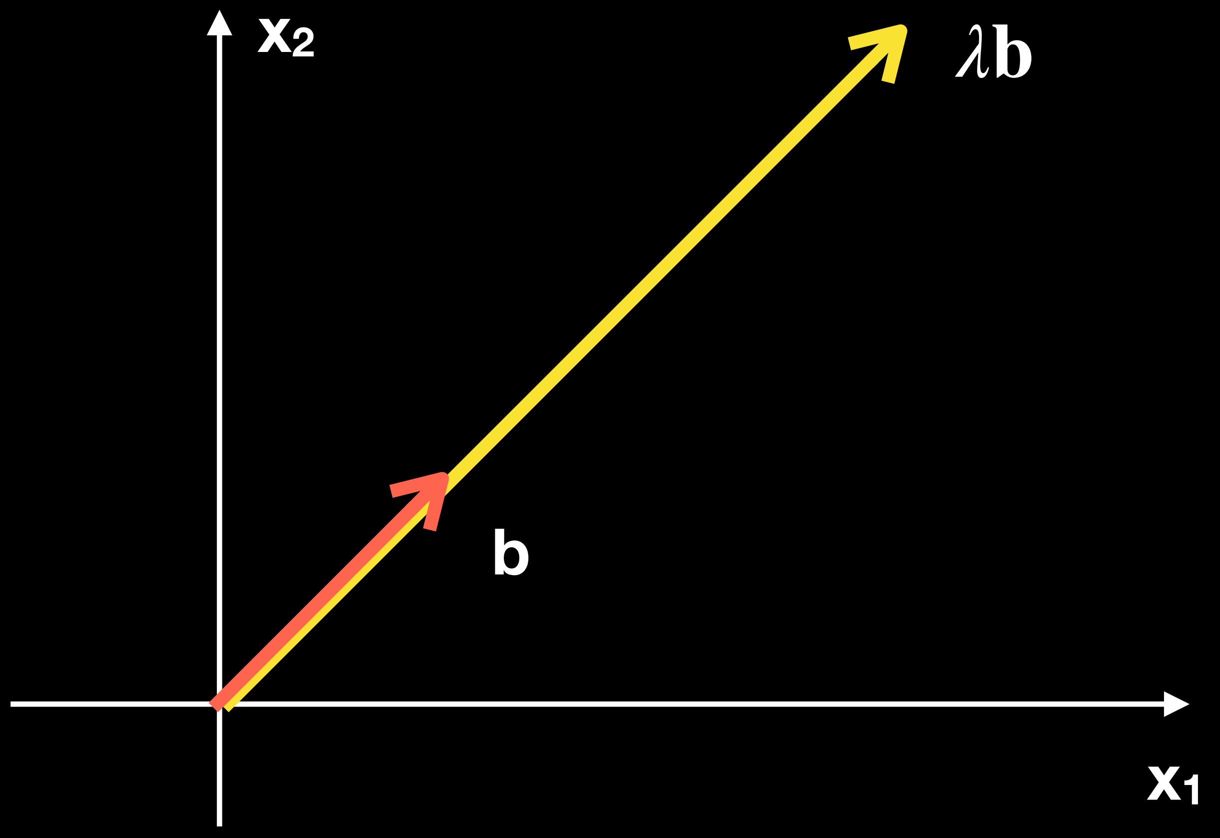
$$\overrightarrow{c} = \overrightarrow{a} + \overrightarrow{b}$$



$$\vec{c} = \vec{a} + \vec{b}$$



$$\overrightarrow{d} = \lambda \cdot \overrightarrow{b}$$



Vector space

1. **a** + b = b + a - commutativity
2. a + (b + c) = (a + b) + c - associativity
3. a + 0 = a - identity element
4. for a, -a must exist, a + (-a) = 0
5. p(q a) = pq a - compatibility
6. 1 a = a - identity for scalar multiplication
7. ƛ (a + b) = ƛa + ƛb - distributivity of scalar w.r.t. vectors
8. (m + n) a = ma + na - distributivity w.r.t field addition

Vectors - Summary

$\mathbf{a} = (a_1, a_2, \ldots, a_k)$

Operations: (1) Summation, (2) Scaling

Vector Space: 8 Axioms