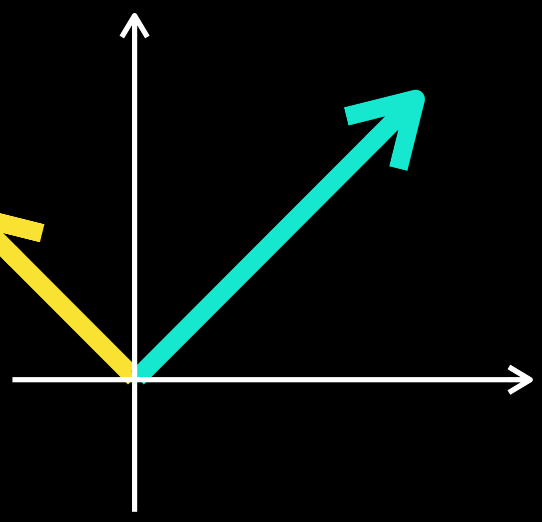
Affine transformations

Linear Algebra Essentials



Vector translation

$\mathbf{v} \in S_V$ $\qquad$ $\mathbf{v'} = \mathbf{v} + \mathbf{t_R}$

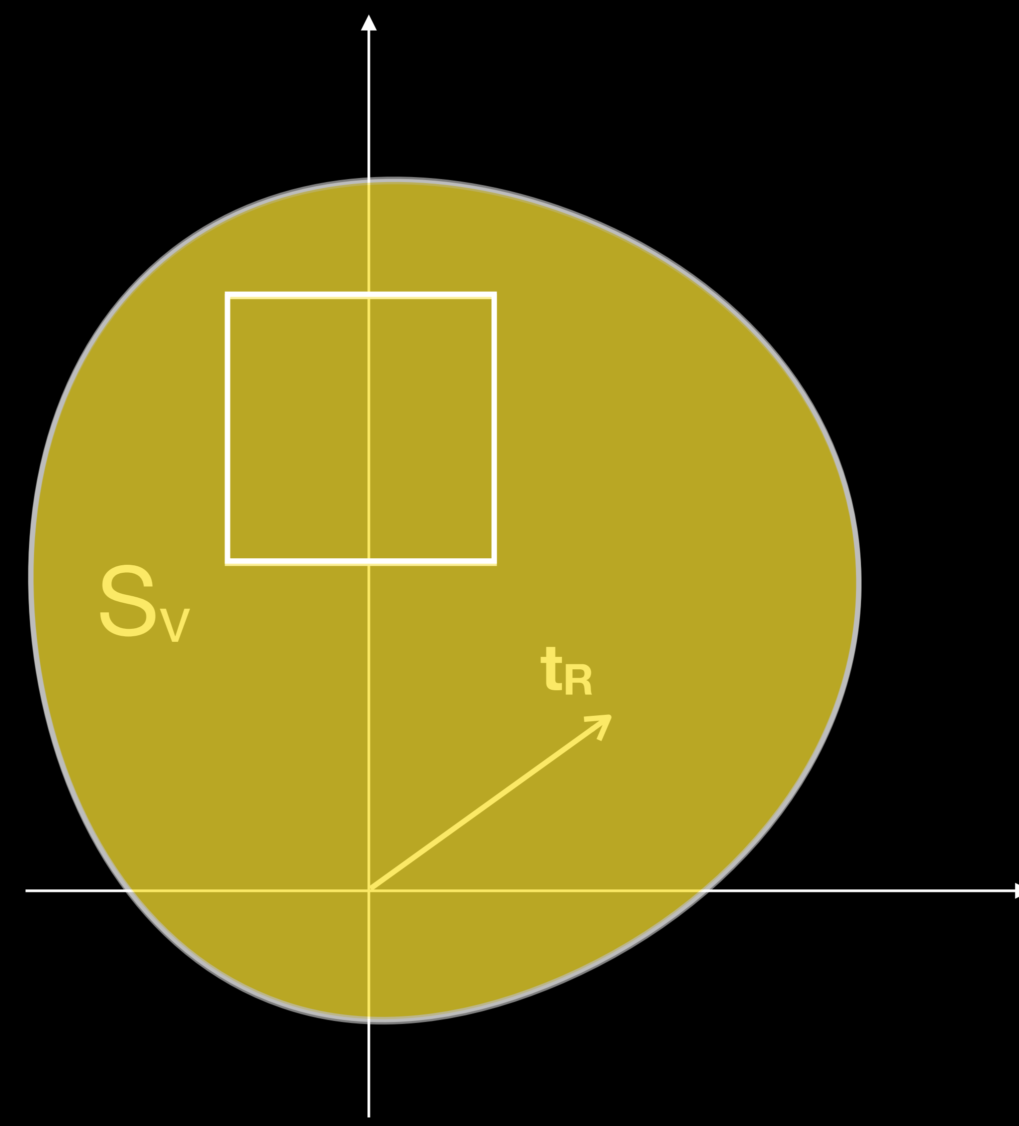
$\mathbf{t_R} = [2, 1]$

$\mathbf{a} = [-3, 1]$

$\mathbf{b} = [3, -1]$ $\qquad$ $\mathbf{a} + \mathbf{b} = \mathbf{0}$

$\mathbf{a'} = [-1, 2]$

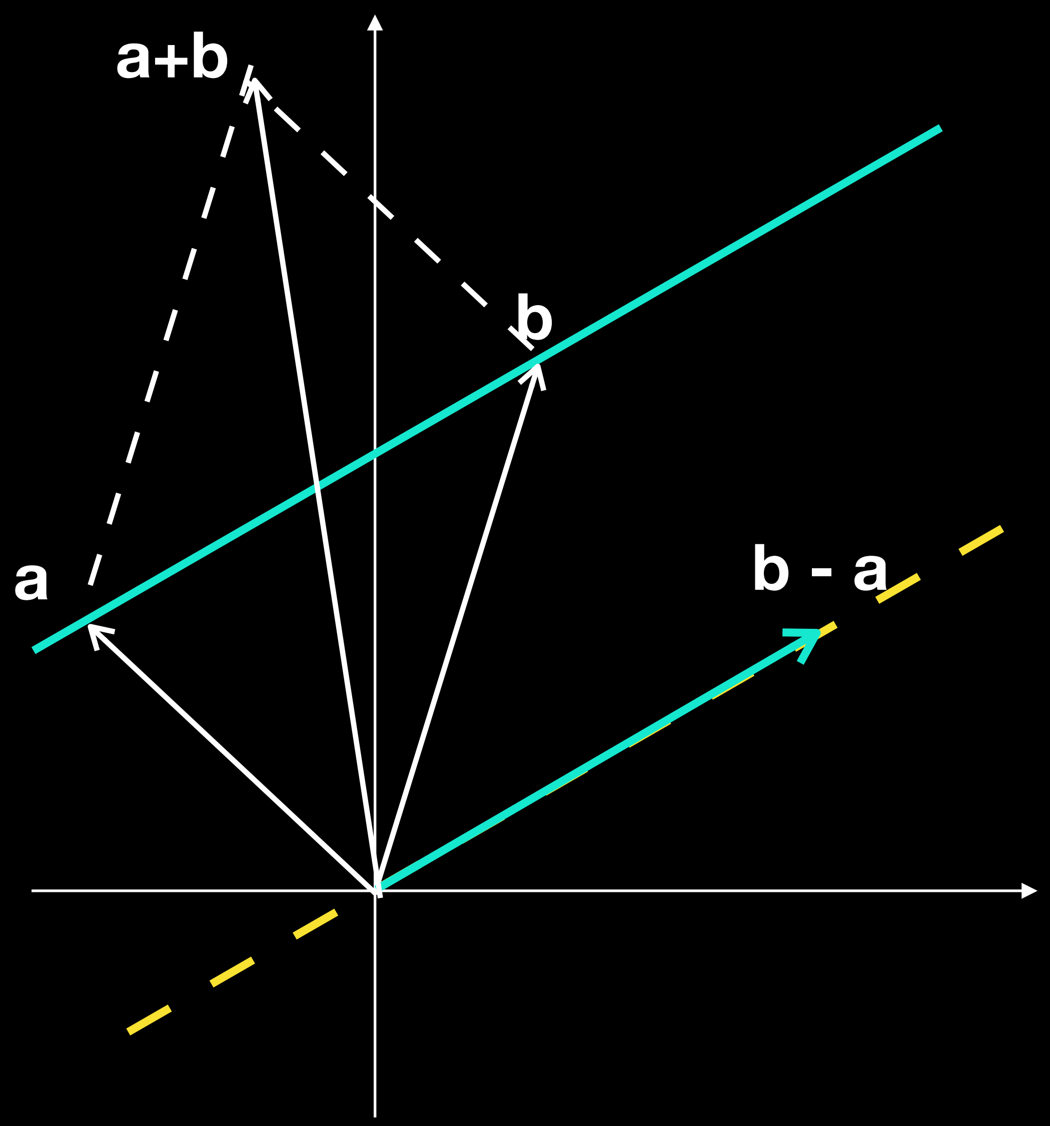
$\mathbf{b'} = [5, 0]$ $\qquad$ $\mathbf{a'} + \mathbf{b'} = [4, 2] \neq \mathbf{0'}$



Affine line

$\mathbf{v} = \gamma\, \mathbf{a} + (1-\gamma)\, \mathbf{b}$

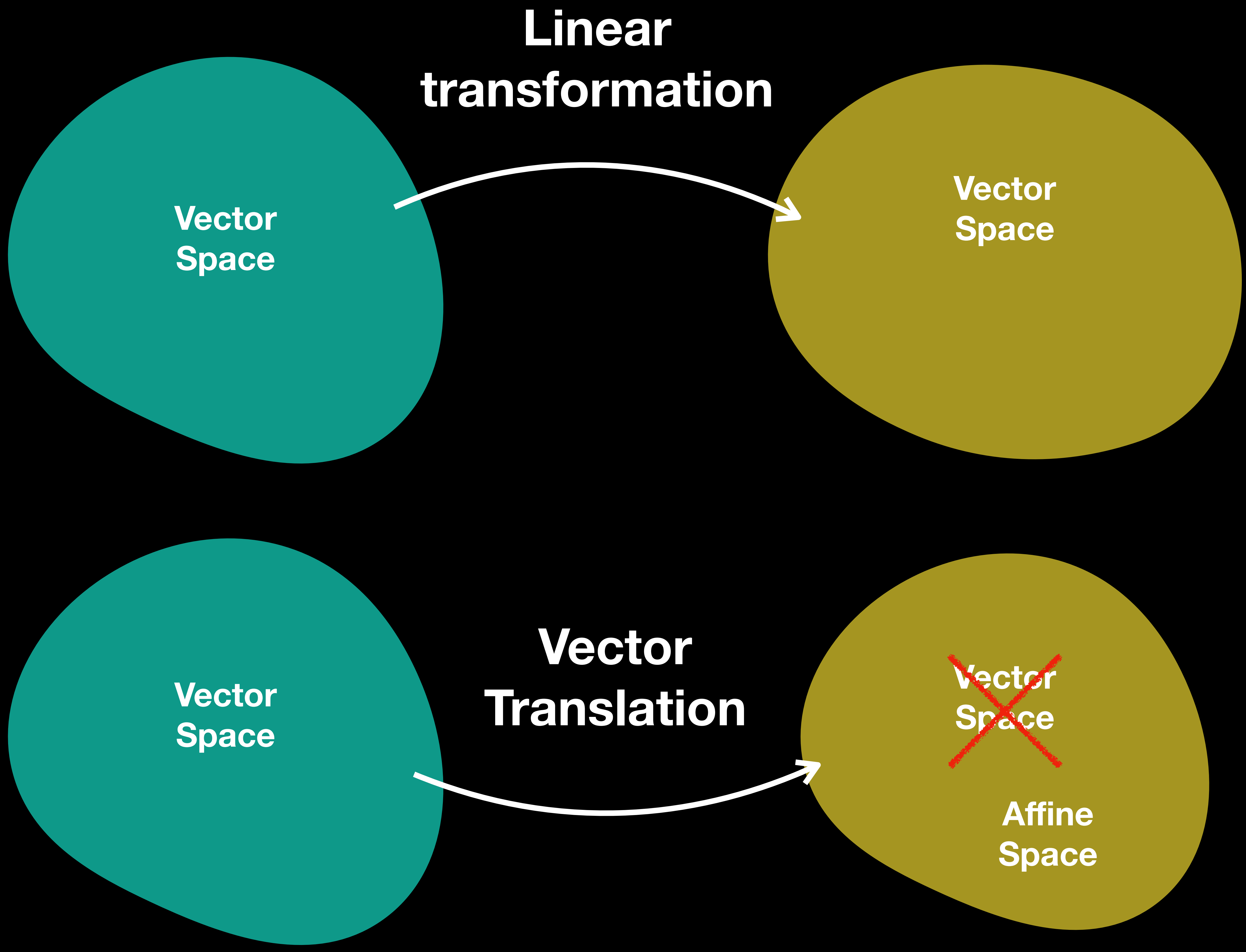
$\mathbf{v} \in$ Affine space



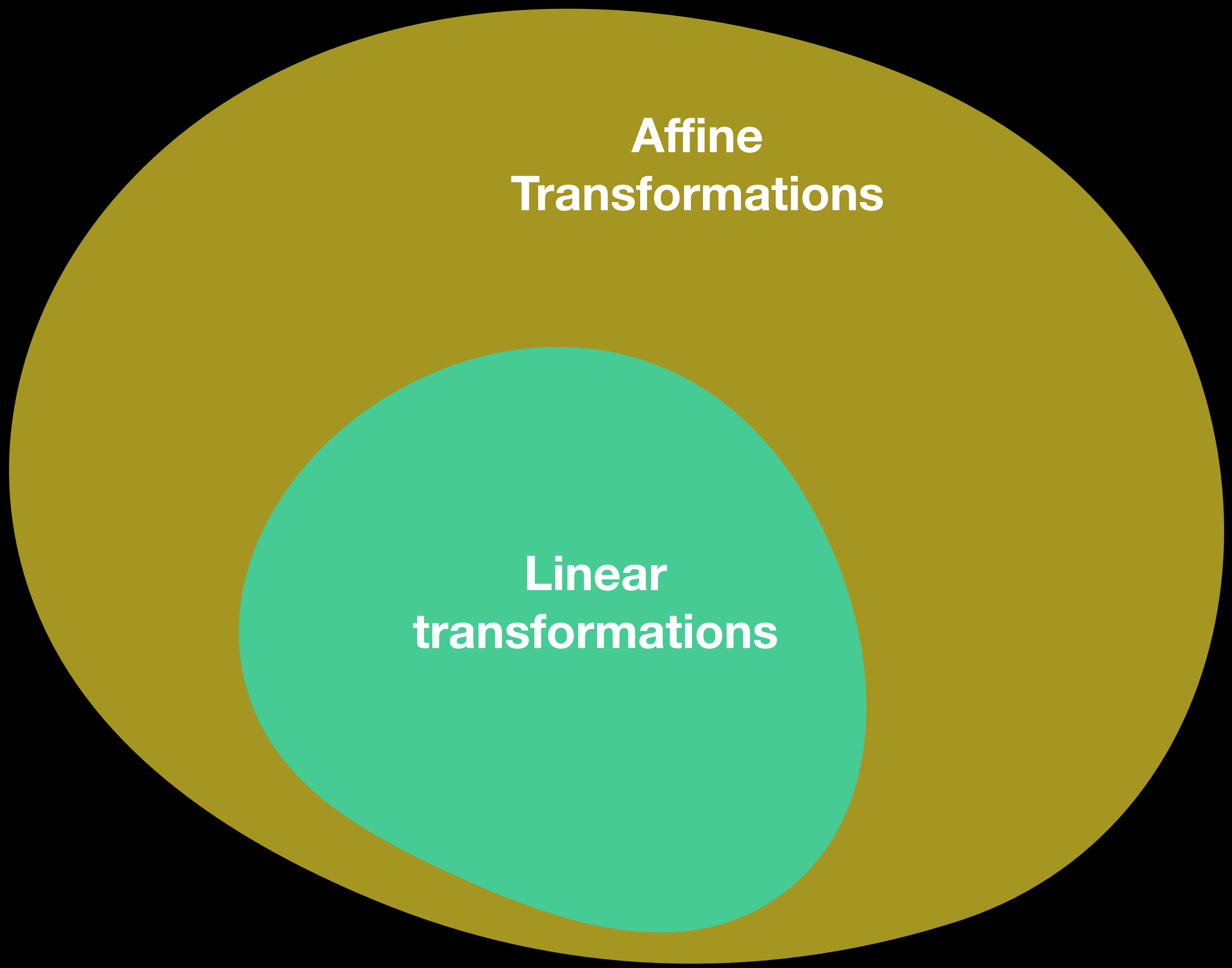
$\mathbf{v} = \gamma\, \mathbf{a} + \mathbf{b} - \gamma\, \mathbf{b} = \mathbf{b} \underbrace{- \gamma\, (\mathbf{b} - \mathbf{a})}$

$- \gamma\, (\mathbf{b} - \mathbf{a})$ - vector space

$\mathbf{b} - \gamma\, (\mathbf{b} - \mathbf{a}) = v$ - affine space


Linear transformation
Vector Space
Vector Space
Vector Translation
Vector Space
~~Vector Space~~
Affine Space

Affine transformations

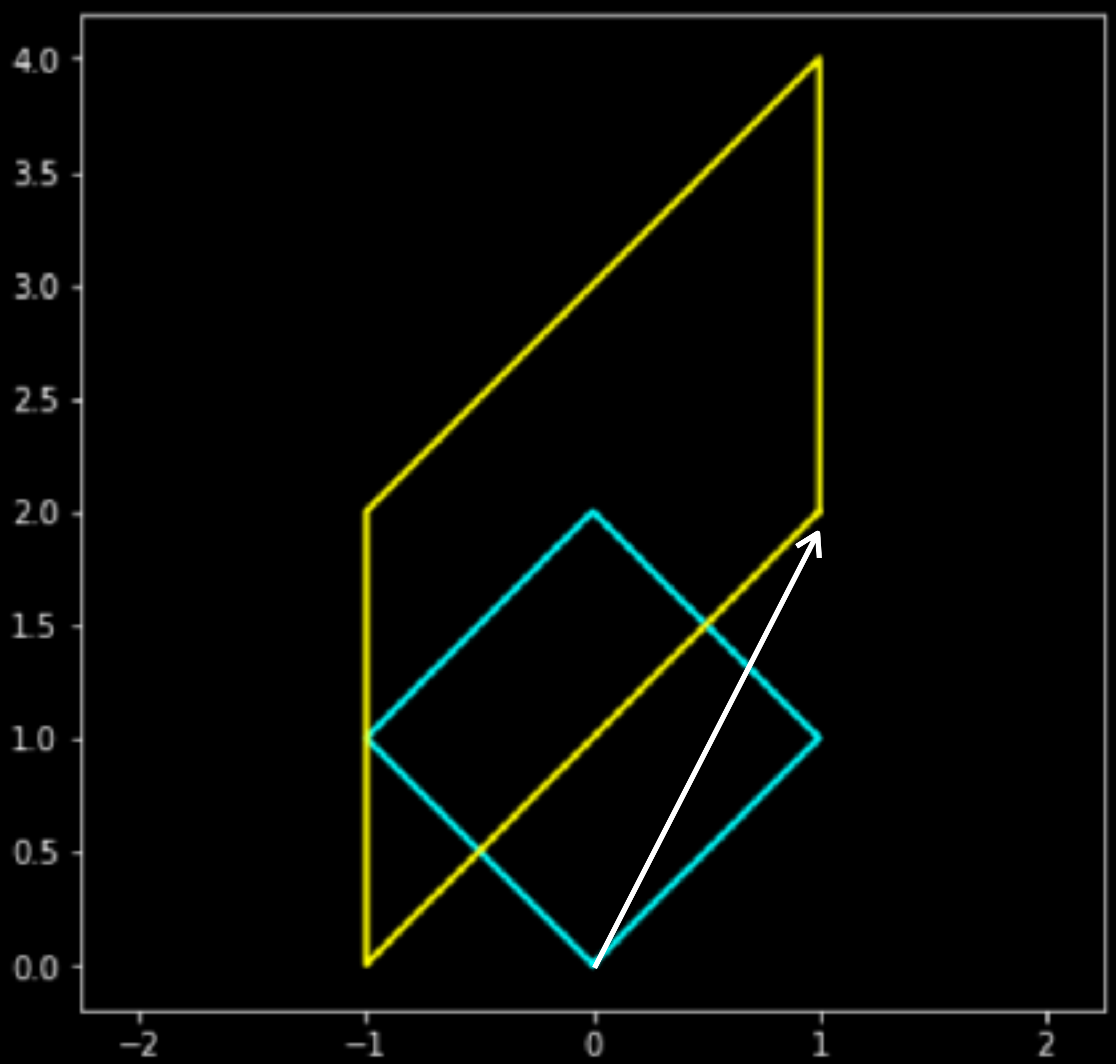


Affine transformation

$$M = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} \qquad t = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$y = M \cdot x + t$$

$$\begin{bmatrix} y \\ 1 \end{bmatrix} = \begin{bmatrix} M & \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} \\ 0\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ 1 \end{bmatrix}$$



$$y' = M' \cdot x'$$ ***M’ -** affine transformation matrix*