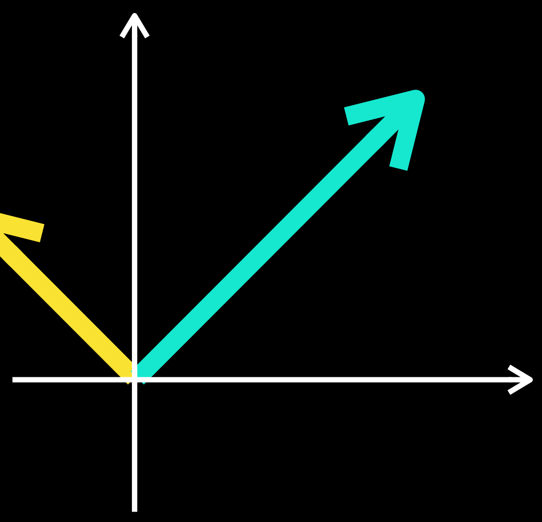
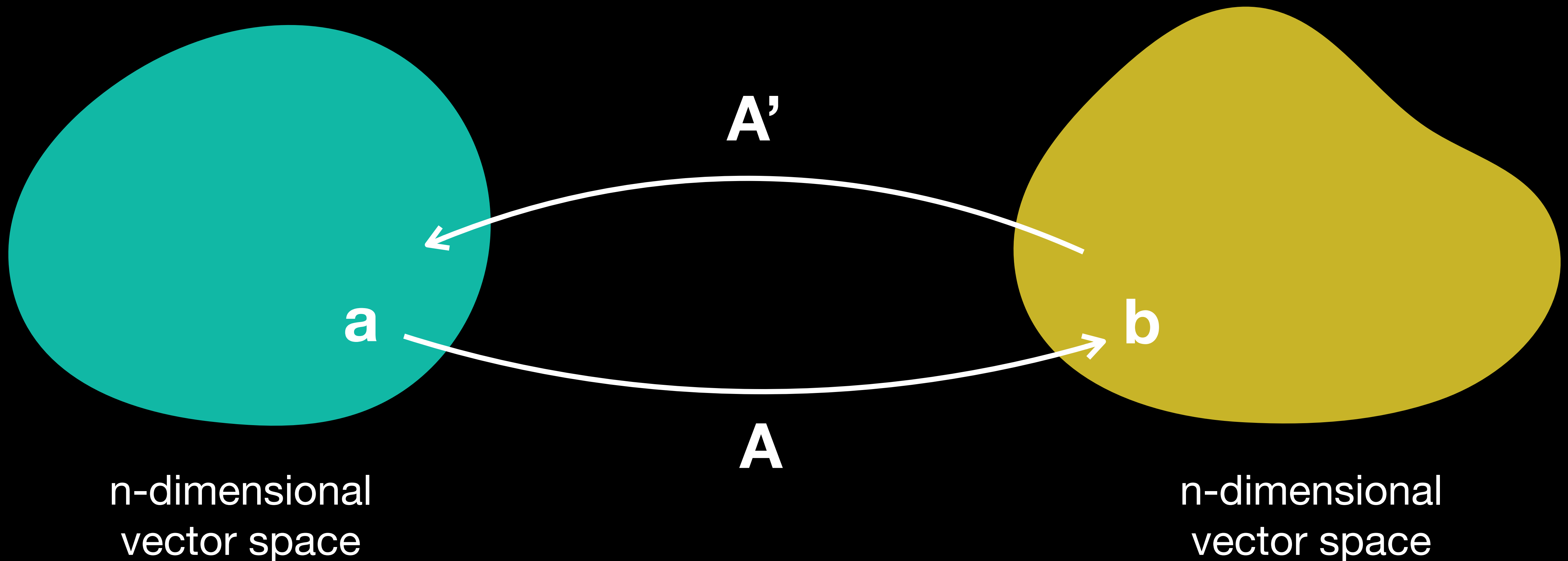
Matrix Rank

Linear Algebra Essentials



Singular matrix



if A^{-1} does not exist, A is called a *singular* matrix

$$M = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

M^{-1} does not exist

M is singular

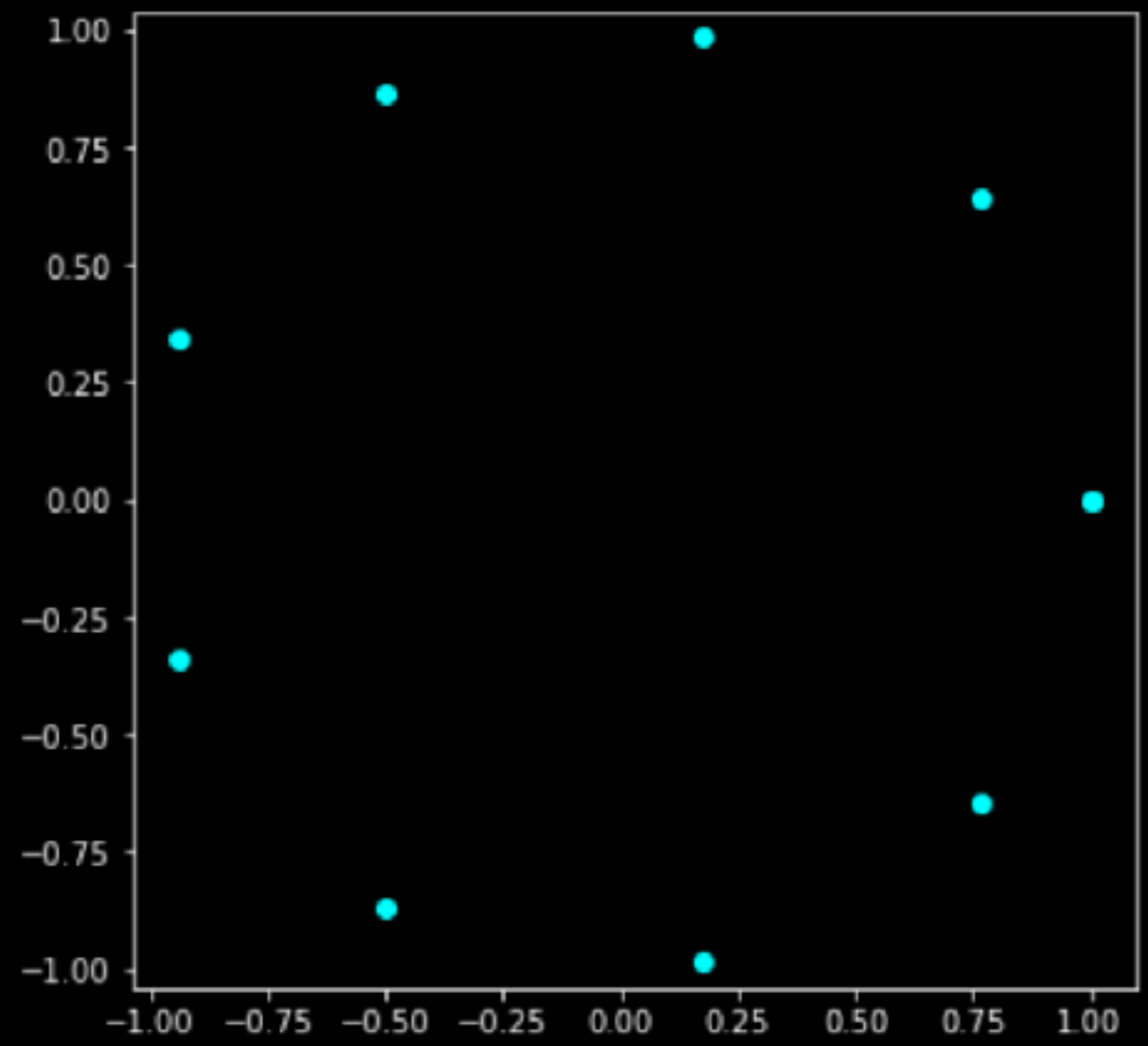
```
M = np.array([[1/2, -1/2],
              [-1/2, 1/2]])
```

```
np.linalg.inv(M)
```

```
---------------------------------------------------------------------
LinAlgError                                                        Trac
<ipython-input-19-41dc54aeb38d> in <module>
----> 1 np.linalg.inv(M)

<__array_function__ internals> in inv(*args, *

~/opt/anaconda3/envs/net/lib/python3.8/site-pa
    545     signature = 'D->D' if isComplexTyp
    546     extobj = get_linalg_error_extobj(_
--> 547     ainv = _umath_linalg.inv(a, signat
    548     return wrap(ainv.astype(result_t,
    549

~/opt/anaconda3/envs/net/lib/python3.8/site-pa
     95
     96 def _raise_linalgerror_singular(err, f
---> 97     raise LinAlgError("Singular matrix
     98
     99 def _raise_linalgerror_nonposdef(err,

LinAlgError: Singular matrix
```

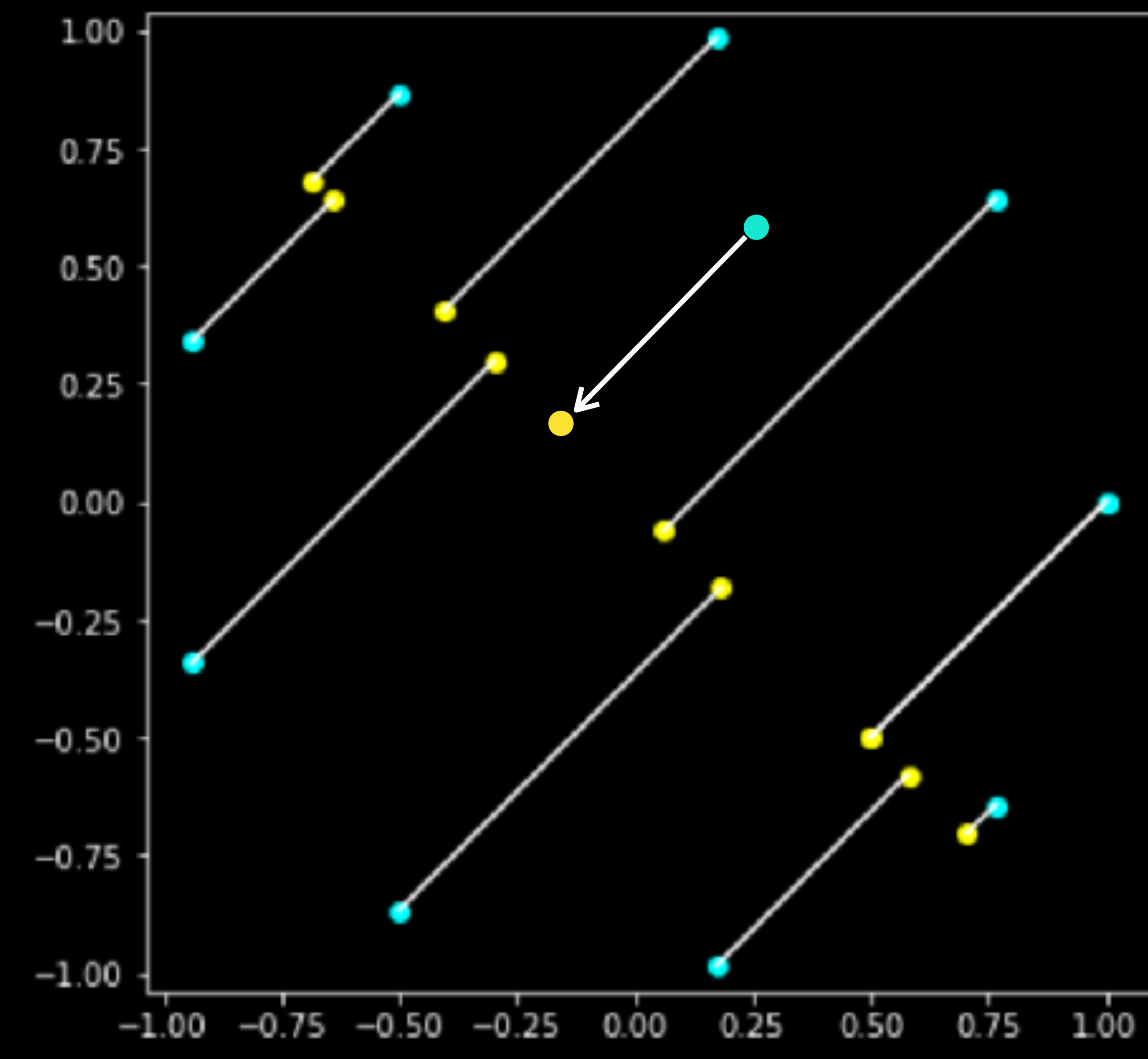
```python
a = np.linspace(0, 2*np.pi, 10)
V = np.array([np.cos(a), np.sin(a)]).T
V
```

```
array([[ 1.00000000e+00,  0.00000000e+00],
       [ 7.66044443e-01,  6.42787610e-01],
       [ 1.73648178e-01,  9.84807753e-01],
       [-5.00000000e-01,  8.66025404e-01],
       [-9.39692621e-01,  3.42020143e-01],
       [-9.39692621e-01, -3.42020143e-01],
       [-5.00000000e-01, -8.66025404e-01],
       [ 1.73648178e-01, -9.84807753e-01],
       [ 7.66044443e-01, -6.42787610e-01],
       [ 1.00000000e+00, -2.44929360e-16]])
```



```
V1 = M.dot(V.T).T
V1
```

```
array([[ 0.5       , -0.5       ],
       [ 0.06162842, -0.06162842],
       [-0.40557979,  0.40557979],
       [-0.6830127 ,  0.6830127 ],
       [-0.64085638,  0.64085638],
       [-0.29883624,  0.29883624],
       [ 0.1830127 , -0.1830127 ],
       [ 0.57922797, -0.57922797],
       [ 0.70441603, -0.70441603],
       [ 0.5       , -0.5       ]])
```



$$M = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

M: $\quad \mathbf{x}=(x_1, x_2) \rightarrow \mathbf{y}=(x_1-x_2, x_2-x_1)$

$(x_1-x_2) = t \qquad \mathbf{y} = (t, -t)$

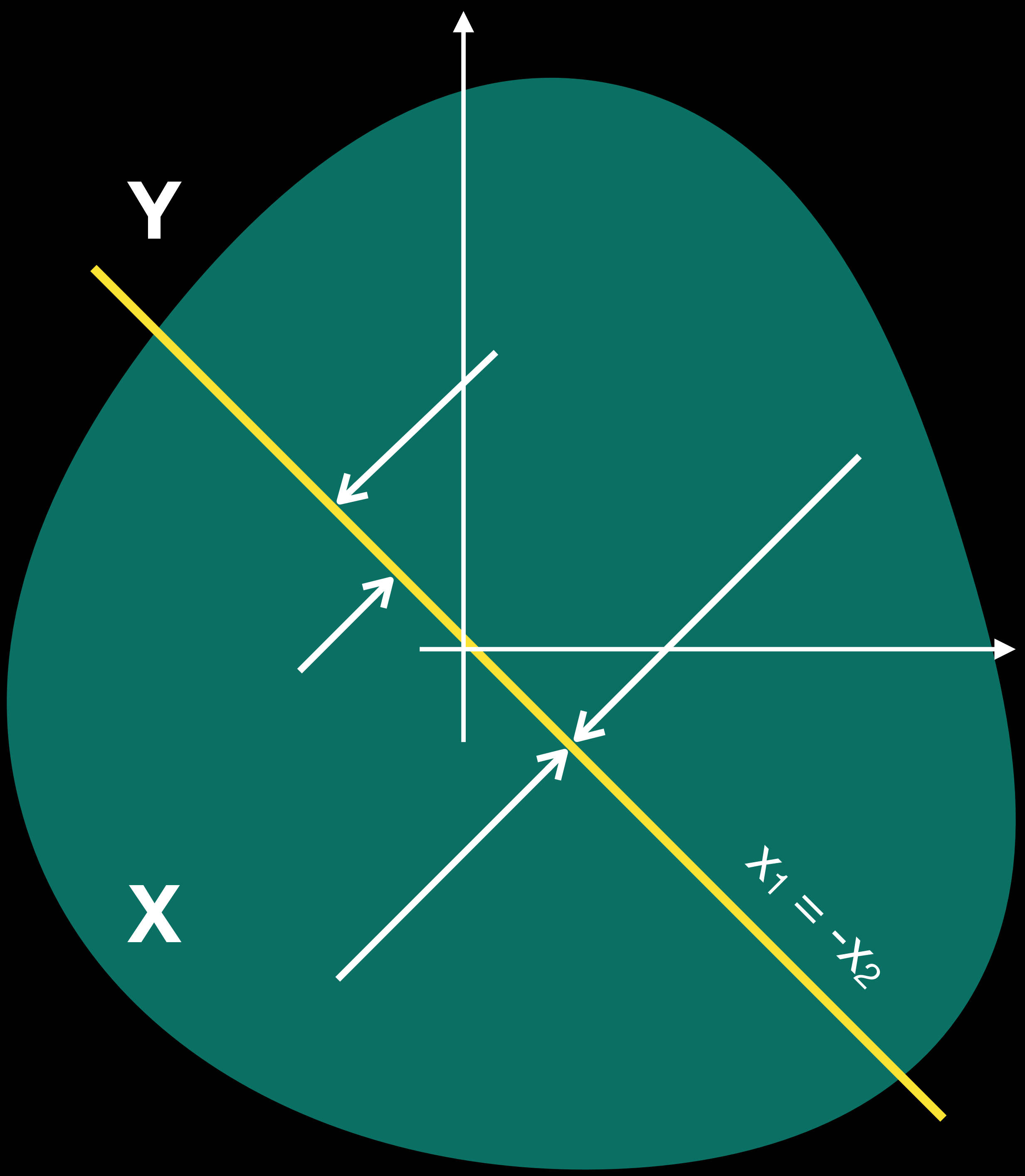
$$M = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$y = \boldsymbol{M}\, x$

rank($\boldsymbol{M}$) - is a number of linearly independent column-vectors of matrix $\boldsymbol{M}$

$\boldsymbol{M} = [v_1, v_2] = [v_1, -v_1]$

rank($\boldsymbol{M}$) = 1



Conclusion

if $\boldsymbol{M}$ is a square (n x n) matrix

and rank($\boldsymbol{M}$) = n

then $\boldsymbol{M}$ is invertible

otherwise, $\boldsymbol{M}$ is singular

if $\boldsymbol{M}$ is (n x m), then rank($\boldsymbol{M}$) $\leq$ min(n, m)