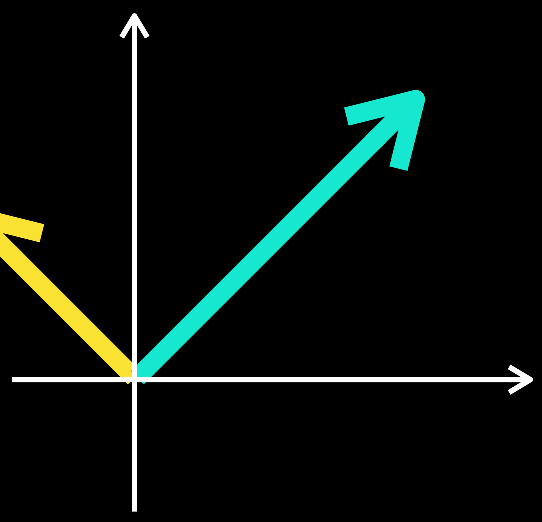
Basis

Linear Algebra Essentials

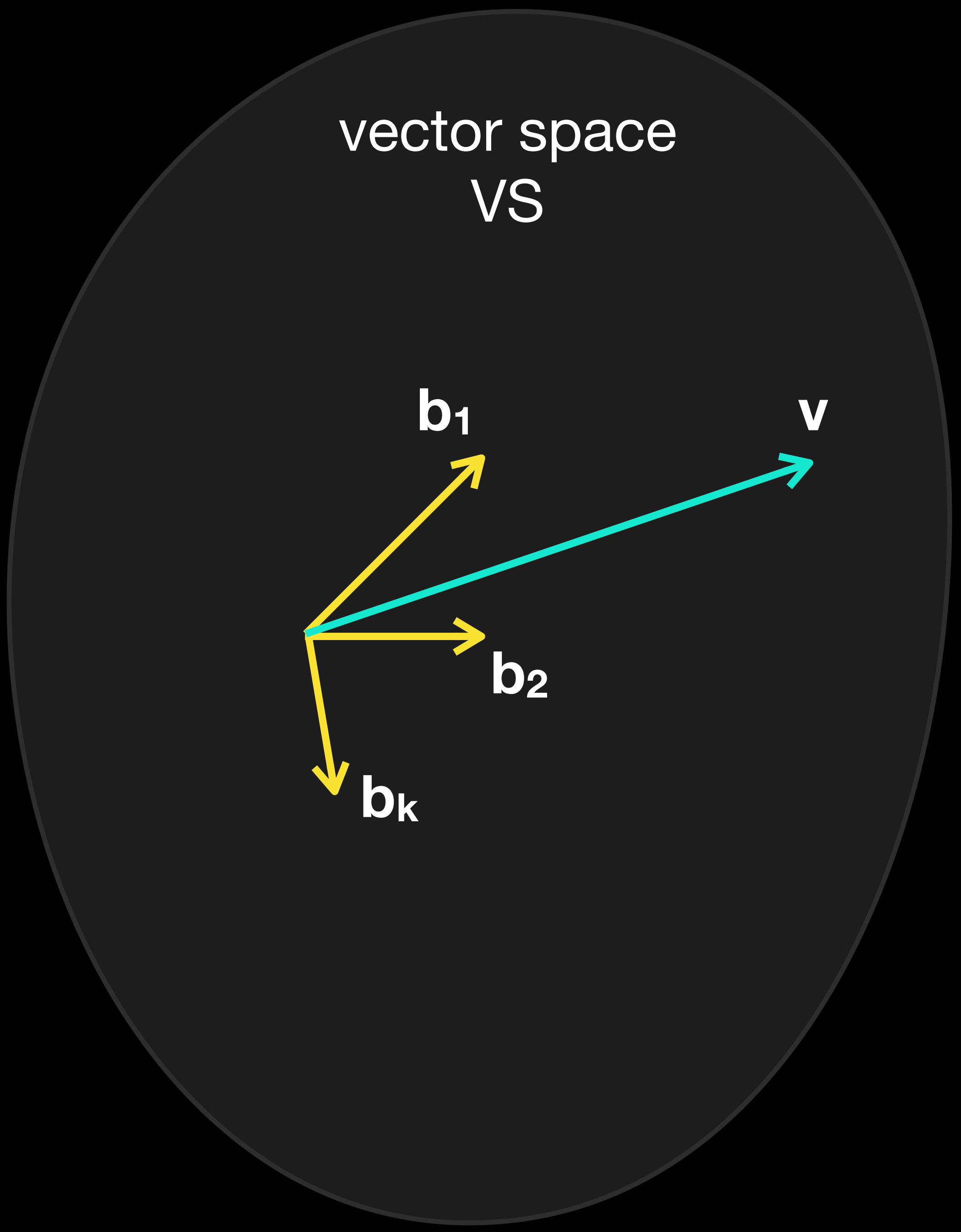


Basis = { b_1, b_2, …, b_k}

1. b_i - linearly independent

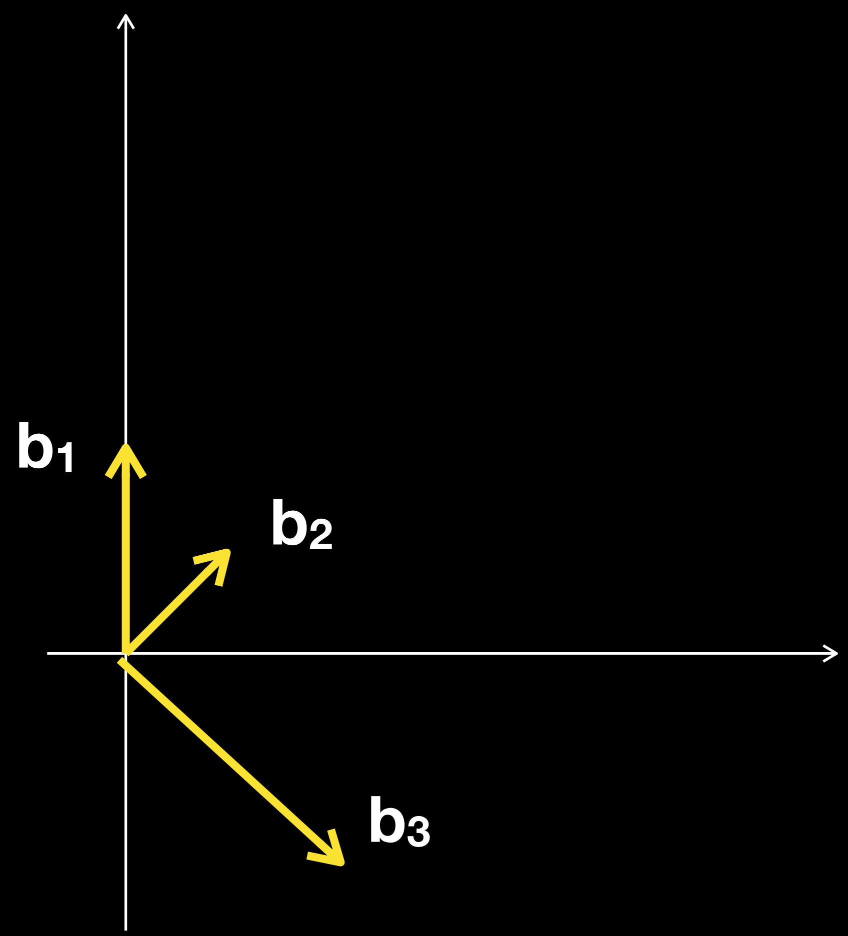
2. Any vector **v** from VS can be represented as

$$\vec{v} = \sum_{i}^{k} t_i \cdot \vec{b}_i$$



Independence property

$\mathbf{b}_1 = (0, 2)$ $\mathbf{b}_2 = (1, 1)$ $\mathbf{b}_3 = (3, -3)$



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$\mathbf{b}_1 = (0, 2)$ $\mathbf{b}_2 = (1, 1)$ $\mathbf{b}_3 = (3, -3)$

$\mathbf{b}_2 - 1/3\ \mathbf{b}_3 = (1, 1) - (1, -1) = (0, 2) = b_1$

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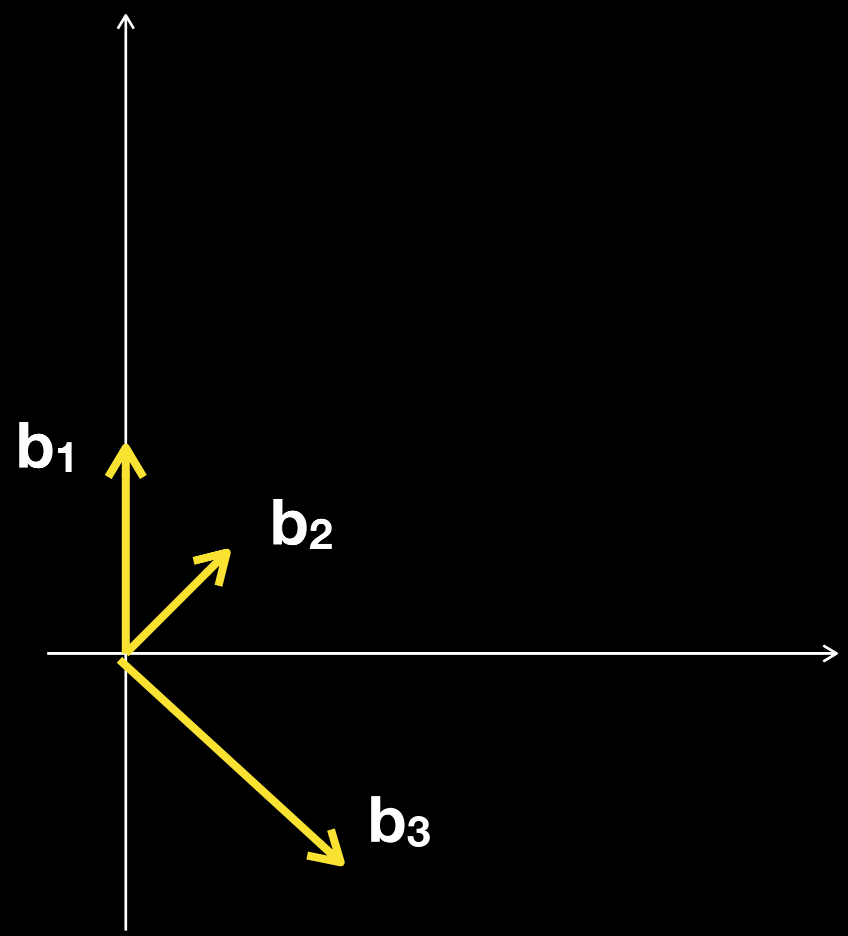

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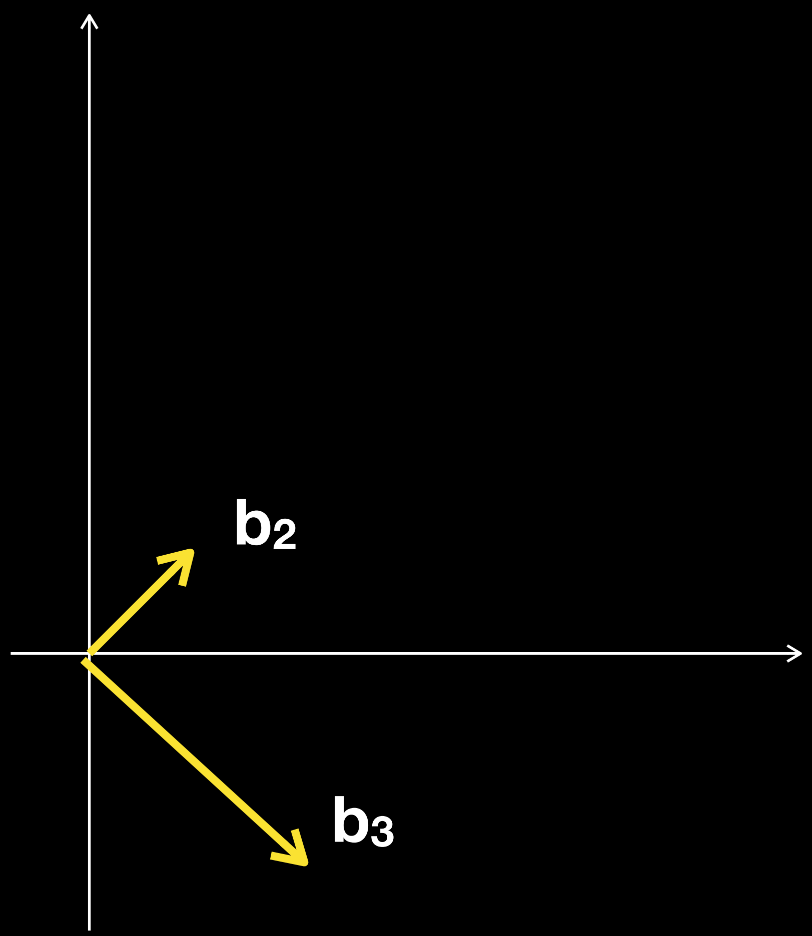
$\mathbf{b_2 = t\ b_3}$



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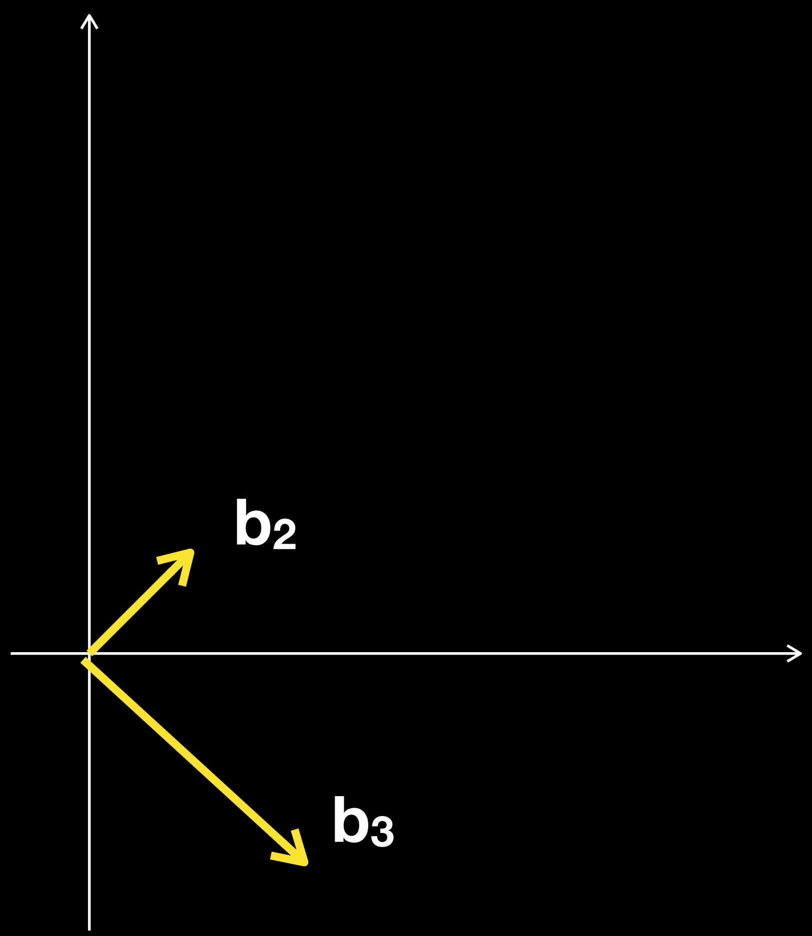
$\mathbf{b}_2 - 1/3\ \mathbf{b}_3 = b_1$

$\mathbf{b}_2 = \mathbf{t}\ \mathbf{b}_3$

$$\begin{cases} 1 = 3t \\ 1 = -3t \end{cases}$$

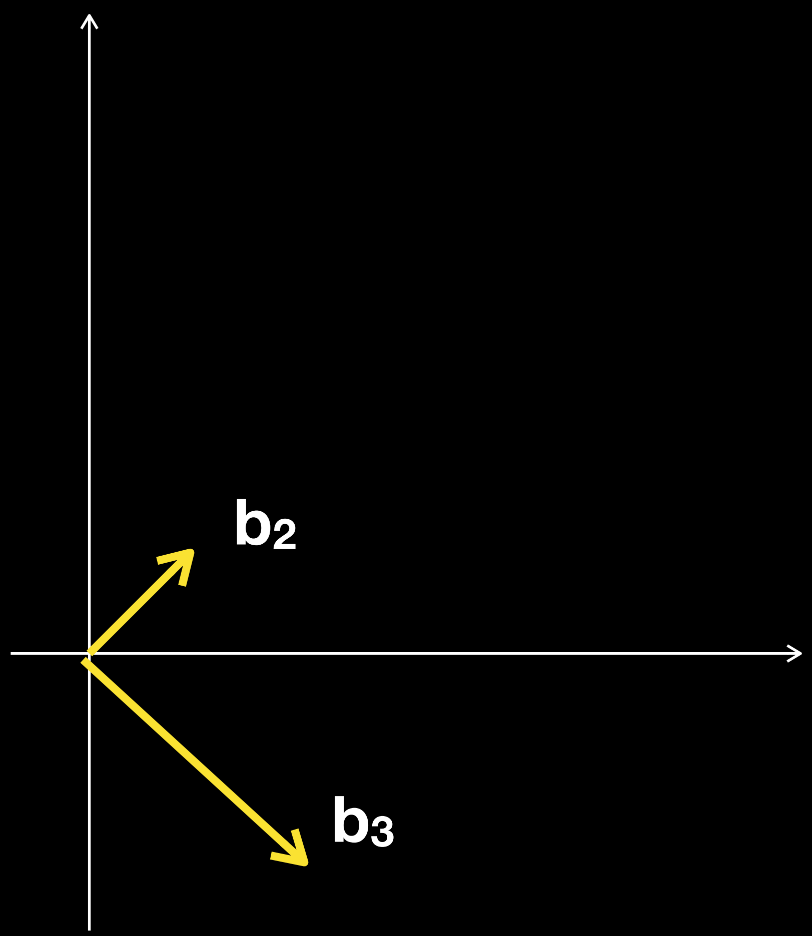
Linearly independent



Spanning property

b$_2$ = (1, 1) **b**$_3$ = (3, -3)

$$\overrightarrow{v} = \sum_{i}^{k} t_i \cdot \overrightarrow{b}_i$$



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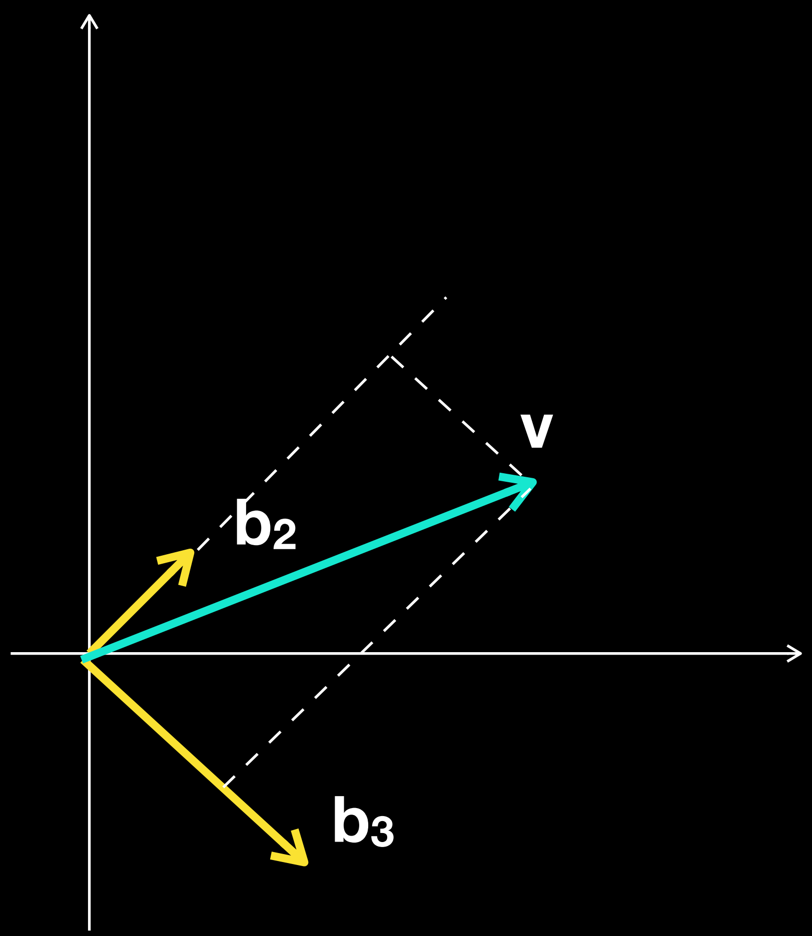

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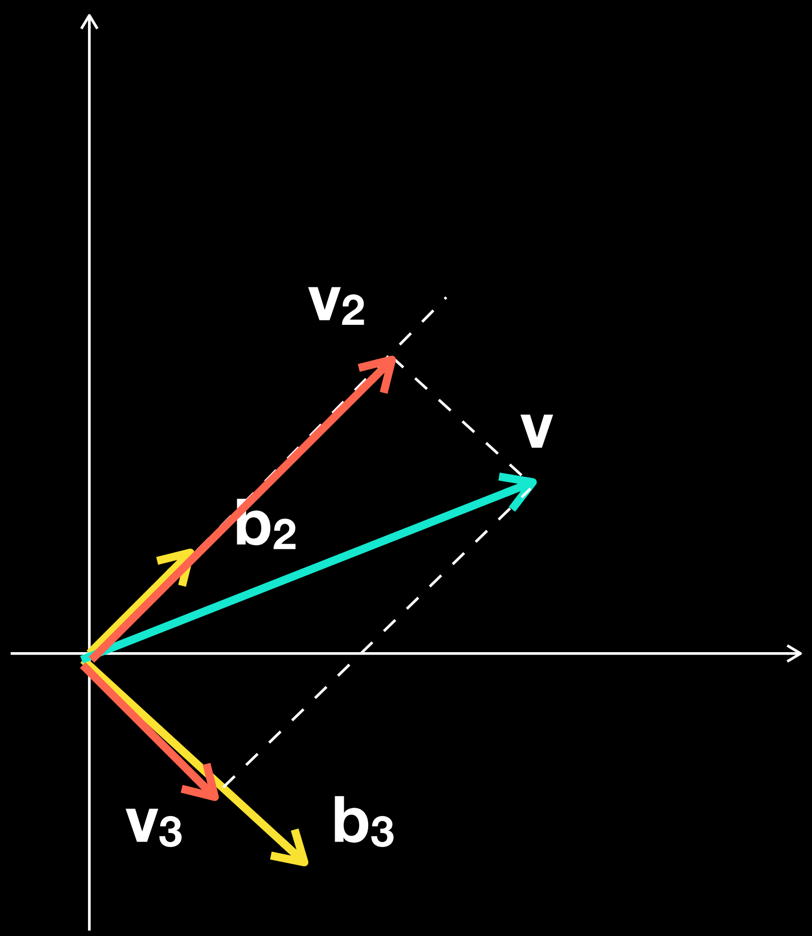

$\mathbf{v} = \mathbf{v}_2 + \mathbf{v}_3$

$= t_2\, \mathbf{b}_2 + t_3\, \mathbf{b}_3$

Basis

k-dimensional vector space

$$\vec{v} = \sum_{i}^{k} t_i \cdot \vec{b}_i$$