# Homework 5, Anastasiia Yelchaninova

**Task 1**

**243.**

$$limit\left(\left(\frac{1}{x^2}\right)^{\frac{2x}{x+1}}, x=\infty\right)$$

$$0 \tag{1}$$

**244.**

$$limit\left(\left(\frac{x^2-2x+3}{x^2-3x+2}\right)^{\frac{\sin(x)}{x}}, x=0\right)$$

$$\frac{3}{2} \tag{2}$$

**Task 2**

**269.**

$$limit\left(\frac{x-1}{|x-1|}, x=1, left\right)$$

$$-1 \tag{3}$$

$$limit\left(\frac{x-1}{|x-1|}, x=1, right\right)$$

$$1 \tag{4}$$

**270.**

$$limit\left(\frac{x}{x-2}, x=2, left\right)$$

$$-\infty \tag{5}$$

$$limit\left(\frac{x}{x-2}, x=2, right\right)$$

$$\infty \tag{6}$$

**Task 3**

**452.**

$$diff\left(\ln\left(e^x+5\cdot\sin(x)-4\cdot\arcsin(x)\right), x\right)$$

$$\frac{e^x+5\cos(x)-\frac{4}{\sqrt{-x^2+1}}}{e^x+5\sin(x)-4\arcsin(x)} \tag{7}$$

**453.**

$$diff(\arctan(\ln(x))+\ln(\arctan(x)), x)$$

$$\frac{1}{x\left(1+\ln(x)^2\right)}+\frac{1}{\left(x^2+1\right)\arctan(x)} \tag{8}$$

Next, we have two parametrically defined functions.
As known,

$$\frac{dy(t)}{dx(t)} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y'_t}{x'_t}$$

Then:

**592.**

$y_1 := simplify\left(diff\left(\arcsin\left(\frac{t}{\sqrt{1+t^2}}\right), t\right)\right)$

$$\frac{1}{(t^2+1)^{3/2}\sqrt{\frac{1}{t^2+1}}} \tag{9}$$

$x_1 := simplify\left(diff\left(\arccos\left(\frac{t}{\sqrt{1+t^2}}\right), t\right)\right)$

$$-\frac{1}{(t^2+1)^{3/2}\sqrt{\frac{1}{t^2+1}}} \tag{10}$$

$\frac{y_1}{x_1}$

$$-1 \tag{11}$$

**591.**

$y_2 := simplify\left(diff\left(\frac{\sin^3(t)}{\sqrt{\cos(2\,t)}}, t\right)\right)$

$$\frac{\sin(t)^2\cos(t)\,\left(4\cos(t)^2-1\right)}{\cos(2\,t)^{3/2}} \tag{12}$$

$x_2 := simplify\left(diff\left(\frac{\cos^3(t)}{\sqrt{\cos(2\,t)}}, t\right)\right)$

$$-\frac{\sin(t)\cos(t)^2\,\left(4\cos(t)^2-3\right)}{\cos(2\,t)^{3/2}} \tag{13}$$

$simplify\left(combine\left(\frac{y_2}{x_2}\right)\right)$

$$-\frac{\sin(3\,t)}{\cos(3\,t)} \tag{14}$$

$convert(\mathbf{(14)}, \tan)$

$$-\tan(3\,t) \tag{15}$$

## Task 4

**535.**

$f_1 := simplify\left(diff\left(\frac{1}{3}\ln(1+x) - \frac{1}{6}\ln\left(x^2-x+1\right) + \frac{1}{\sqrt{3}}\arctan\left(\frac{2\,x-1}{\sqrt{3}}\right), x\right)\right)$

$$\frac{1}{(1+x)\,(x^2-x+1)} \tag{16}$$

$subs(x=0,\ f_1)$

$$1 \tag{17}$$

**536.**

$f_2 := simplify\left(diff\left(\frac{x\arcsin(x)}{\sqrt{1-x^2}}+\ln\left(\sqrt{1-x^2}\right), x\right)\right)$

$$\frac{\arcsin(x)}{(-x^2+1)^{3/2}} \tag{18}$$

$subs(x=0, f_2)$

$$\arcsin(0) \tag{19}$$

$\arcsin(0)$

$$0 \tag{20}$$

**Task 5**

)

$f_{51} := x \rightarrow \arcsin\left(\frac{a^2-x^2}{a^2+x^2}\right):$

$Df_{51}[left] := x \rightarrow limit\left(\frac{f_{51}(x+h)-f_{51}(x)}{h}, h=0, left\right):$

$Df_{51}[left](0)$

$$\frac{\sqrt{4}}{\sqrt{\frac{1}{a^2}\,a^2}} \tag{21}$$

$Df_{51}[right] := x \rightarrow limit\left(\frac{f_{51}(x+h)-f_{51}(x)}{h}, h=0, right\right):$

$Df_{51}[right](0)$

$$-\frac{\sqrt{4}}{\sqrt{\frac{1}{a^2}\,a^2}} \tag{22}$$

)

$f_{52} := x \rightarrow x \cdot \sin\left(\frac{1}{x}\right)$

$$x \rightarrow x\sin\left(\frac{1}{x}\right) \tag{23}$$

$Df_{52}[left] := x \rightarrow limit\left(\frac{f_{52}(x+h)-f_{52}(x)}{h}, h=0, left\right)$

$$x \rightarrow \lim_{h\rightarrow 0^-}\frac{f_{52}(x+h)-f_{52}(x)}{h} \tag{24}$$

$Df_{52}[left](0)$

Error, (in f[52]) numeric exception: division by zero

$Df_{52}[right] := x \to limit\left(\frac{f_{52}(x+h) - f_{52}(x)}{h}, h = 0, left\right)$

$$x \to \lim_{h \to 0^-} \frac{f_{52}(x+h) - f_{52}(x)}{h} \qquad \textbf{(25)}$$

$Df_{52}[right](0)$

Error, (in f[52]) numeric exception: division by zero

$f_{22} := diff\left(x \sin\left(\frac{1}{x}\right), x, right\right)$ assuming $x \neq 0, f_{22}(0) = 0$

$$0 \qquad \textbf{(26)}$$

**3)**

$y_{53} := \frac{1}{1 + x + \ln(x)}$

$$\frac{1}{1 + x + \ln(x)} \qquad \textbf{(27)}$$

$y_{531} := diff\left(\frac{1}{1 + x + \ln(x)}, x\right)$

$$-\frac{1 + \frac{1}{x}}{(1 + x + \ln(x))^2} \qquad \textbf{(28)}$$

$simplify(y_{531})$

$$-\frac{1 + x}{(1 + x + \ln(x))^2 x} \qquad \textbf{(29)}$$

Let's check:

$$xy' = y(y \cdot \ln(x) - 1)$$
$$y' = \frac{y(y \cdot \ln(x) - 1)}{x}$$

$y_{lhs} := \frac{y_{53} \cdot (y_{53} \cdot \ln(x) - 1)}{x}$

$$\frac{\frac{\ln(x)}{1 + x + \ln(x)} - 1}{(1 + x + \ln(x))\, x} \qquad \textbf{(30)}$$

$simplify(numer(\textbf{(30)}))$

$$-1 - x \qquad \textbf{(31)}$$

$y_{lhs1} := \frac{\textbf{(31)}}{denom(\textbf{(30)})}$

$$\frac{-1 - x}{(1 + x + \ln(x))^2 x} \qquad \textbf{(32)}$$

$simplify(y_{lhs1})$

$$-\frac{1 + x}{(1 + x + \ln(x))^2 x} \qquad \textbf{(33)}$$

So,

$$y_{531} = y_{lhs1}$$

## Task 6

For an implicit function F(x,y):

$$y' = -\frac{F'_x}{F'_y}$$

Then:

**609.**

$f_{61} := a\cos^2(x+y) = b$

$$a\cos(x+y)^2 = b \tag{34}$$

$F_{1x} := diff\left(a\cos^2(x+y) - b, x\right)$

$$-2\,a\cos(x+y)\sin(x+y) \tag{35}$$

$F_{1y} := diff\left(a\cos^2(x+y) - b, y\right)$

$$-2\,a\cos(x+y)\sin(x+y) \tag{36}$$

$y_{111} := -\frac{F_{1x}}{F_{1y}}$

$$-1 \tag{37}$$

**618.**

$F_{2x} := diff\left(x^y - y^x, x\right)$

$$\frac{x^y\,y}{x} - y^x\ln(y) \tag{38}$$

$F_{2y} := diff\left(x^y - y^x, y\right)$

$$x^y\ln(x) - \frac{y^x\,x}{y} \tag{39}$$

$y_{222} := simplify\left(-\frac{F_{2x}}{F_{2y}}\right)$

$$\frac{y\,\left(x^{y-1}\,y - y^x\ln(y)\right)}{-x^y\ln(x)\,y + y^x\,x} \tag{40}$$

**711.a)**

$f := x^2 + 2\,x\,y + y^2 - 4\,x + 2\,y - 2 = 0$

$$x^2 + 2\,x\,y + y^2 - 4\,x + 2\,y - 2 = 0 \tag{41}$$

$f_{y1} := implicitdiff(f, y, x)$

$$-\frac{x+y-2}{x+y+1} \tag{42}$$

$f_{y2} := implicitdiff\left(f_{y1}, y, x\right)$

$$-1 \tag{43}$$

$implicitdiff\left(f_{y2}, y, x\right)$

$$0 \tag{44}$$

$subs(\{x = 1, y = 1\}, \mathbf{(44)})$

$$0 \tag{45}$$

## Task 7

$F_{3x} := diff\left(x + \ln\left(\frac{y}{x}\right) - y^2, x\right)$

$$1 - \frac{1}{x} \tag{46}$$

$F_{3y} := diff\left(x + \ln\left(\frac{y}{x}\right) - y^2, y\right)$

$$\frac{1}{y} - 2y$$

$y_{333} := -\frac{F_{3x}}{F_{3y}}$

$$-\frac{1 - \frac{1}{x}}{\frac{1}{y} - 2y}$$

$subs(\{x=1, y=1\}, y_{333})$

$$0 \tag{49}$$

## Task 8

**1)**

$f_8 := diff((2x-3)^5, x\$3)$

$$480\,(2x-3)^2 \tag{50}$$

$subs(x=3, f_8)$

$$4320 \tag{51}$$

**2)**

$y_{84} := diff(\mathrm{e}^{-x} \cdot \cos(x), x\$4)$

$$-4\,\mathrm{e}^{-x}\cos(x) \tag{52}$$

$combine\left(y_{84} + 2\,\mathrm{e}^{-x} \cdot \cos(x)\right)$

$$-2\,\mathrm{e}^{-x}\cos(x) \tag{53}$$

//equality is not met

$combine\left(y_{84} + 4\,\mathrm{e}^{-x} \cdot \cos(x)\right)$

$$0 \tag{54}$$

**3)**

Next, we have parametrically defined function.
As known,

$$y''_{xx} = \frac{y''_{tt}x'_t - y'_t x''_{tt}}{(x'_t)^3}$$

Then:

$x_{81} := diff(a \cdot (\sin(t) - t \cdot \cos(t)), t)$

$$a\,t\sin(t) \tag{55}$$

$x_{82} := diff(x_{81}, t)$

$$a\sin(t) + a\,t\cos(t) \tag{56}$$

$y_{81} := diff(a\cdot(\cos(t) + t\cdot\sin(t)), t)$

$$a\,t\cos(t) \tag{57}$$

$y_{82} := diff\left(y_{81}, t\right)$

$$a\cos(t) - a\,t\sin(t) \tag{58}$$

$simplify\left(\dfrac{y_{82}\cdot x_{81} - y_{81}\cdot x_{82}}{\left(x_{81}\right)^3}\right)$

$$-\frac{1}{a\,t\sin(t)^3} \tag{59}$$

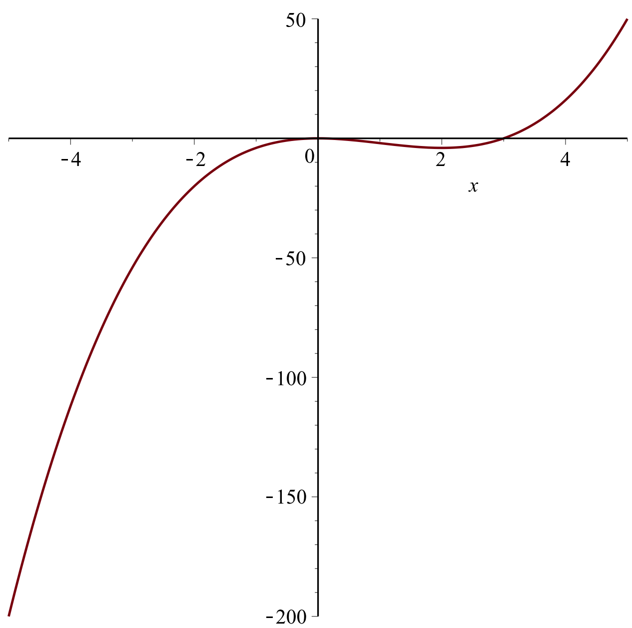
**Task 9**

**814.**

$y_{91} := x^2\cdot(x-3)$

$$x^2\,(x-3) \tag{60}$$

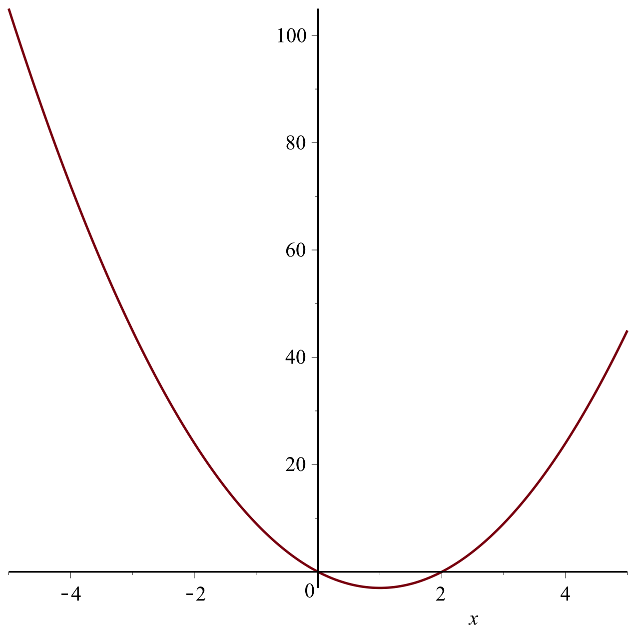
$plot\left(y_{91}, x=-5..5\right)$




$y_{911} := diff\left(y_{91}, x\right)$

$$2\,x\,(x-3) + x^2 \tag{61}$$

$plot(y_{911}, x=-5..5)$




$solve(y_{911}=0)$

$$0, 2 \quad (62)$$

$subs(x=-1, y_{911})$

$$9 \quad (63)$$

$subs(x=1, y_{911})$

$$-3$$

$subs(x=3, y_{911})$

$$9 \quad (65)$$

In conclusion,
intervals of increase of function y_91 are (-∞; 0) and (2;+∞);
interval of decrease of function y_91 is (0,2)

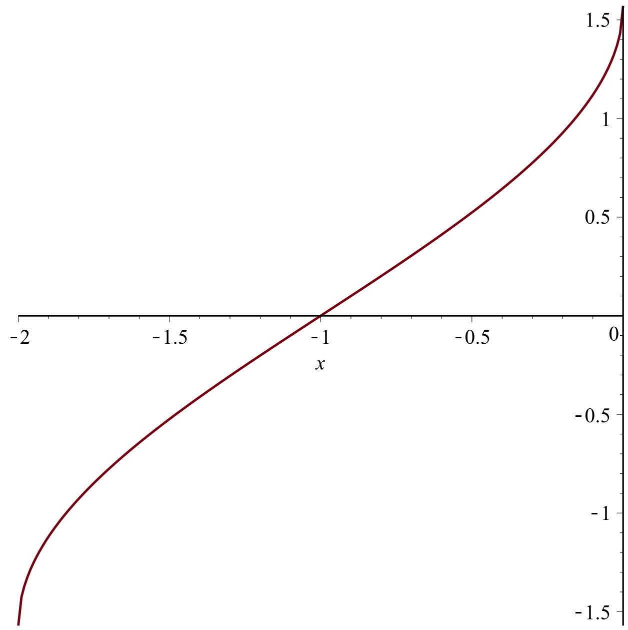
**822.**

$y_{92} := \arcsin(1+x)$

$$\arcsin(1+x) \quad (66)$$

As known, D(arcsin(x))=[-1,1], so D(arcsin(1+x))=[-2,0]

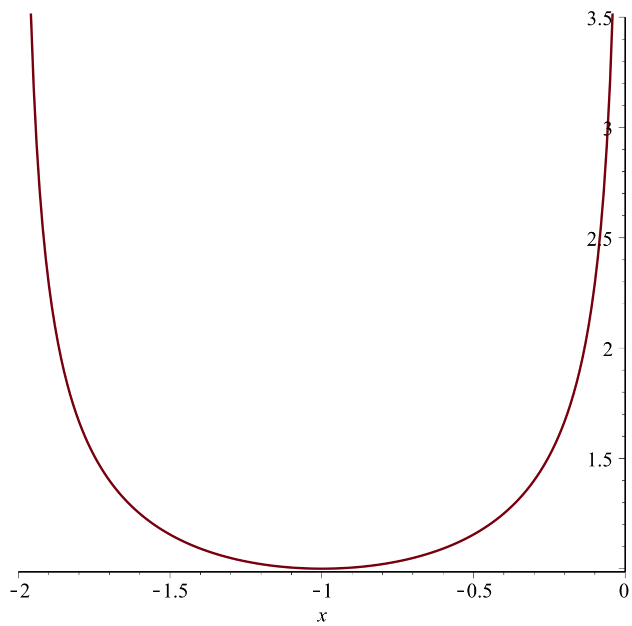
$plot(y_{92}, x=-2..0)$




$y_{921} := diff\left(y_{92}, x\right)$

$$\frac{1}{\sqrt{1-(1+x)^2}} \tag{67}$$

$plot\left(y_{921}, x=-2\,..0\right)$




$solve(y_{921} = 0)$

No zeros of derivative.

$subs(x = -1, y_{921})$

$$1 \tag{68}$$

In conclusion,
function y_92 increases monotonically over [-2,0]