Homework 6, Anastasiia Yelchaninova

Task 1

$eq_1 := \left(x^2-1\right)\cdot y' + 2\,x\,y^2 = 0$

$$\left(x^2-1\right)\left(\frac{\mathrm{d}}{\mathrm{d}x}\,y(x)\right) + 2\,x\,y(x)^2 = 0 \tag{1}$$

$dsolve\left(eq_1\right)$

$$y(x) = \frac{1}{\ln(x-1) + \ln(x+1) + \_C1} \tag{2}$$

$dsolve\left(\left\{eq_1, y(2) = 1\right\}\right)$

$$y(x) = -\frac{1}{\ln(3) - \ln(x-1) - \ln(x+1) - 1} \tag{3}$$

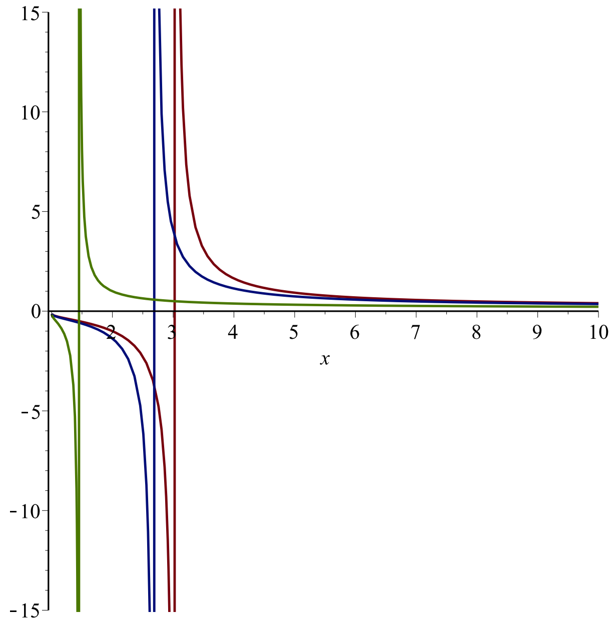
$dsolve\left(\left\{eq_1, y(3) = 4\right\}\right)$

$$y(x) = \frac{4}{1 + 4\ln(x-1) + 4\ln(x+1) - 12\ln(2)} \tag{4}$$

$dsolve\left(\left\{eq_1, y(2) = -1\right\}\right)$

$$y(x) = \frac{1}{-1 + \ln(x-1) + \ln(x+1) - \ln(3)} \tag{5}$$

$plot(\{rhs(\mathbf{(3)}), rhs(\mathbf{(4)}), rhs(\mathbf{(5)})\})$



$eq_2 := 2\,x^2\,y\,y' + y^2 = 2$

$$2\,x^2\,y(x)\,\left(\frac{\mathrm{d}}{\mathrm{d}x}\,y(x)\right) + y(x)^2 = 2 \quad \textbf{(6)}$$

$dsolve(eq_2)$

$$y(x) = \sqrt{\mathrm{e}^{\frac{1}{x}}\,\_C1 + 2},\ y(x) = -\sqrt{\mathrm{e}^{\frac{1}{x}}\,\_C1 + 2} \quad \textbf{(7)}$$

$simplify(dsolve(\{eq_2, y(2) = 1\}))$

$$y(x) = \mathrm{e}^{-\frac{1}{4}}\,\sqrt{-\mathrm{e}^{\frac{1}{x}} + 2\,\mathrm{e}^{\frac{1}{2}}} \quad \textbf{(8)}$$

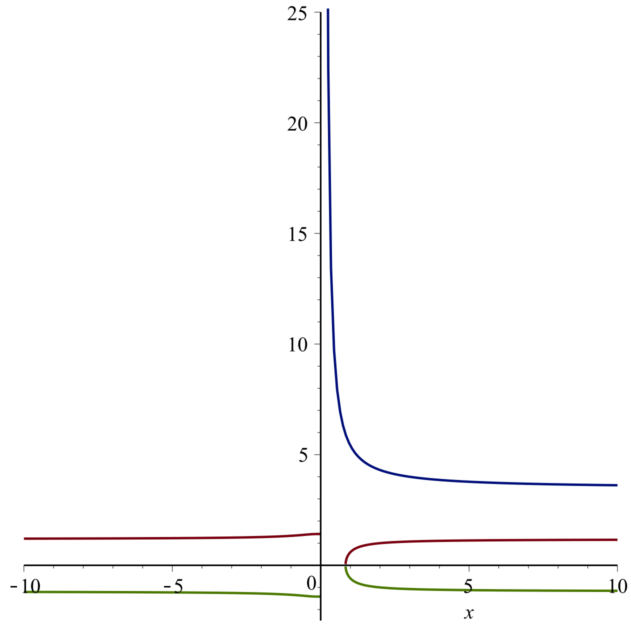
$simplify(dsolve(\{eq_2, y(3) = 4\}))$

$$y(x) = \mathrm{e}^{-\frac{1}{6}}\,\sqrt{2}\,\sqrt{\mathrm{e}^{\frac{1}{3}} + 7\,\mathrm{e}^{\frac{1}{x}}} \quad \textbf{(9)}$$

$simplify(dsolve(\{eq_2, y(2) = -1\}))$

$$y(x) = -\mathrm{e}^{-\frac{1}{4}}\,\sqrt{-\mathrm{e}^{\frac{1}{x}} + 2\,\mathrm{e}^{\frac{1}{2}}} \quad \textbf{(10)}$$

$plot(\{rhs(\textbf{(8)}), rhs(\textbf{(9)}), rhs(\textbf{(10)})\})$



$$eq_3 := y' = \frac{y+2}{x+1} + \tan\left(\frac{y-2x}{x+1}\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}x} y(x) = \frac{y(x)+2}{x+1} + \tan\left(\frac{y(x)-2x}{x+1}\right) \tag{11}$$

$$dsolve(eq_3)$$

$$y(x) = -2 + \arctan\left(\left(\left(\frac{\left(\tan(2)\ \_C1\ (x+1) + \sqrt{-(x+1)^2\ \_C1^2 + \tan(2)^2 + 1}\right)\tan(2)}{\tan(2)^2+1} - (x+1)\ \_C1\right)\left(\tan(2)^2+1\right)\right) \Big/ \left(\tan(2)\ \_C1\ (x+1) + \sqrt{-(x+1)^2\ \_C1^2 + \tan(2)^2 + 1}\right)\right)(x+1) \tag{12}$$

$$simplify(dsolve(\{eq_3, y(1) = 2\}))$$

$$y(x) = 2x \tag{13}$$

$$simplify(dsolve(\{eq_3, y(0) = 1\}))$$

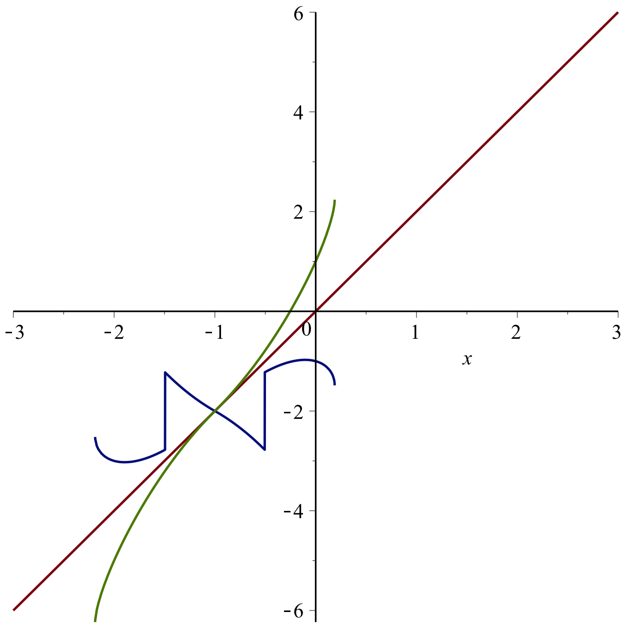
$$y(x) = \pi x \tag{14}$$

$$+\arctan\Big(\Big(\Big(4\cos(1)^3+\sqrt{(x+1)^2\left(\cos(1)^2x^2+2\cos(1)^2x+\cos(1)^2-x^2-2x\right)}$$
$$-2\cos(1)\Big)\sin(1)\Big)\Big/\left(4\cos(1)^4+\cos(1)^2x^2+2\cos(1)^2x-3\cos(1)^2-x^2-2x\right)\Big)x$$
$$+\pi$$
$$+\arctan\Big(\Big(\Big(4\cos(1)^3+\sqrt{(x+1)^2\left(\cos(1)^2x^2+2\cos(1)^2x+\cos(1)^2-x^2-2x\right)}$$
$$-2\cos(1)\Big)\sin(1)\Big)\Big/\left(4\cos(1)^4+\cos(1)^2x^2+2\cos(1)^2x-3\cos(1)^2-x^2-2x\right)\Big)-2$$

$simplify\big(dsolve(\{eq_3, y(0)=-1\})\big)$

$$y(x)=-2 \tag{15}$$
$$+\arctan\Big(\Big(\Big(4\cos(1)^3-2\cos(1)$$
$$-\sqrt{(x+1)^2\left(\cos(1)^2x^2+2\cos(1)^2x+\cos(1)^2-x^2-2x\right)}\Big)\sin(1)\Big)\Big/\Big(4\cos(1)^4$$
$$+\cos(1)^2x^2+2\cos(1)^2x-3\cos(1)^2-x^2-2x\Big)\Big)x$$
$$+\arctan\Big(\Big(\Big(4\cos(1)^3-2\cos(1)$$
$$-\sqrt{(x+1)^2\left(\cos(1)^2x^2+2\cos(1)^2x+\cos(1)^2-x^2-2x\right)}\Big)\sin(1)\Big)\Big/\Big(4\cos(1)^4$$
$$+\cos(1)^2x^2+2\cos(1)^2x-3\cos(1)^2-x^2-2x\Big)\Big)$$

$plot(\{rhs(\mathbf{(13)}), rhs(\mathbf{(14)}), rhs(\mathbf{(15)})\}, x=-3..3)$



$eq_4 := y'^3 + \left(y'^2 - 2\,y'\right)\cdot x = 3\,y' - y$

$$\left(\frac{d}{dx}\,y(x)\right)^3 + \left(\left(\frac{d}{dx}\,y(x)\right)^2 - 2\left(\frac{d}{dx}\,y(x)\right)\right)x = 3\left(\frac{d}{dx}\,y(x)\right) - y(x) \tag{16}$$

$dsolve\left(eq_4\right)$

$$y(x) = x + 2,\ y(x) = \Bigg(\tag{17}$$

$$-\Bigg(\frac{1}{6}\,\Big(108\,\_C1 - 8\,x^3 - 36\,x^2 - 54\,x - 27$$

$$+ 6\sqrt{-48\,\_C1\,x^3 - 216\,\_C1\,x^2 + 324\,\_C1^2 - 324\,\_C1\,x - 162\,\_C1}\Big)^{1/3} + \frac{1}{6}\,(2\,x$$

$$+ 3)^2 \Big/ \Big(108\,\_C1 - 8\,x^3 - 36\,x^2 - 54\,x - 27$$

$+ 6 \sqrt{-48 \_C1 x^3 - 216 \_C1 x^2 + 324 \_C1^2 - 324 \_C1 x - 162 \_C1}\Big)^{1/3} - \frac{1}{3} x - \frac{1}{2}\Big)^2$

$+ \frac{1}{3} \Big(108 \_C1 - 8 x^3 - 36 x^2 - 54 x - 27$

$+ 6 \sqrt{-48 \_C1 x^3 - 216 \_C1 x^2 + 324 \_C1^2 - 324 \_C1 x - 162 \_C1}\Big)^{1/3} + \frac{1}{3} (2 x$

$+ 3)^2 \Big/ \Big(108 \_C1 - 8 x^3 - 36 x^2 - 54 x - 27$

$+ 6 \sqrt{-48 \_C1 x^3 - 216 \_C1 x^2 + 324 \_C1^2 - 324 \_C1 x - 162 \_C1}\Big)^{1/3} - \frac{2}{3} x - 1\Big) x$

$- \Bigg(\frac{1}{6} \Big(108 \_C1 - 8 x^3 - 36 x^2 - 54 x - 27$

$+ 6 \sqrt{-48 \_C1 x^3 - 216 \_C1 x^2 + 324 \_C1^2 - 324 \_C1 x - 162 \_C1}\Big)^{1/3} + \frac{1}{6} (2 x$

$+ 3)^2 \Big/ \Big(108 \_C1 - 8 x^3 - 36 x^2 - 54 x - 27$

$+ 6 \sqrt{-48 \_C1 x^3 - 216 \_C1 x^2 + 324 \_C1^2 - 324 \_C1 x - 162 \_C1}\Big)^{1/3} - \frac{1}{3} x - \frac{1}{2}\Big)^3$

$+ \frac{1}{2} \Big(108 \_C1 - 8 x^3 - 36 x^2 - 54 x - 27$

$+ 6 \sqrt{-48 \_C1 x^3 - 216 \_C1 x^2 + 324 \_C1^2 - 324 \_C1 x - 162 \_C1}\Big)^{1/3} + \frac{1}{2} (2 x$

$+ 3)^2 \Big/ \Big(108 \_C1 - 8 x^3 - 36 x^2 - 54 x - 27$

$+ 6 \sqrt{-48 \_C1 x^3 - 216 \_C1 x^2 + 324 \_C1^2 - 324 \_C1 x - 162 \_C1}\Big)^{1/3} - x - \frac{3}{2}, y(x)$

$= \Bigg(- \Bigg(- \frac{1}{12} \Big(108 \_C1 - 8 x^3 - 36 x^2 - 54 x - 27$

$+ 6 \sqrt{-48 \_C1 x^3 - 216 \_C1 x^2 + 324 \_C1^2 - 324 \_C1 x - 162 \_C1}\Big)^{1/3} - \frac{1}{12} (2 x$

$+ 3)^2 \Big/ \Big(108 \_C1 - 8 x^3 - 36 x^2 - 54 x - 27$

$+ 6 \sqrt{-48 \_C1 x^3 - 216 \_C1 x^2 + 324 \_C1^2 - 324 \_C1 x - 162 \_C1}\Big)^{1/3} - \frac{1}{3} x - \frac{1}{2}$

$$- \frac{1}{2} \mathrm{I} \sqrt{3} \Big(\frac{1}{6} (108 \_C1 - 8 x^3 - 36 x^2 - 54 x - 27$$

$$+ 6 \sqrt{-48 \_C1 x^3 - 216 \_C1 x^2 + 324 \_C1^2 - 324 \_C1 x - 162 \_C1})^{1/3} - \frac{1}{6} (2 x$$

$$+ 3)^2 \Big/ (108 \_C1 - 8 x^3 - 36 x^2 - 54 x - 27$$

$$+ 6 \sqrt{-48 \_C1 x^3 - 216 \_C1 x^2 + 324 \_C1^2 - 324 \_C1 x - 162 \_C1})^{1/3}) \Big)^2$$

$$- \frac{1}{6} (108 \_C1 - 8 x^3 - 36 x^2 - 54 x - 27$$

$$+ 6 \sqrt{-48 \_C1 x^3 - 216 \_C1 x^2 + 324 \_C1^2 - 324 \_C1 x - 162 \_C1})^{1/3} - \frac{1}{6} (2 x$$

$$+ 3)^2 \Big/ (108 \_C1 - 8 x^3 - 36 x^2 - 54 x - 27$$

$$+ 6 \sqrt{-48 \_C1 x^3 - 216 \_C1 x^2 + 324 \_C1^2 - 324 \_C1 x - 162 \_C1})^{1/3} - \frac{2}{3} x - 1$$

$$- \mathrm{I} \sqrt{3} \Big(\frac{1}{6} (108 \_C1 - 8 x^3 - 36 x^2 - 54 x - 27$$

$$+ 6 \sqrt{-48 \_C1 x^3 - 216 \_C1 x^2 + 324 \_C1^2 - 324 \_C1 x - 162 \_C1})^{1/3} - \frac{1}{6} (2 x$$

$$+ 3)^2 \Big/ (108 \_C1 - 8 x^3 - 36 x^2 - 54 x - 27$$

$$+ 6 \sqrt{-48 \_C1 x^3 - 216 \_C1 x^2 + 324 \_C1^2 - 324 \_C1 x - 162 \_C1})^{1/3}) \Big) x - \Big($$

$$- \frac{1}{12} (108 \_C1 - 8 x^3 - 36 x^2 - 54 x - 27$$

$$+ 6 \sqrt{-48 \_C1 x^3 - 216 \_C1 x^2 + 324 \_C1^2 - 324 \_C1 x - 162 \_C1})^{1/3} - \frac{1}{12} (2 x$$

$$+ 3)^2 \Big/ (108 \_C1 - 8 x^3 - 36 x^2 - 54 x - 27$$

$$+ 6 \sqrt{-48\,\_C1\,x^3 - 216\,\_C1\,x^2 + 324\,\_C1^2 - 324\,\_C1\,x - 162\,\_C1}\,)^{1/3} - \frac{1}{3}\,x - \frac{1}{2}$$

$$- \frac{1}{2}\,\mathrm{I}\sqrt{3}\,\Bigg(\frac{1}{6}\,\big(108\,\_C1 - 8\,x^3 - 36\,x^2 - 54\,x - 27$$

$$+ 6 \sqrt{-48\,\_C1\,x^3 - 216\,\_C1\,x^2 + 324\,\_C1^2 - 324\,\_C1\,x - 162\,\_C1}\,)^{1/3} - \frac{1}{6}\,(2\,x$$

$$+ 3)^2 \Big/ \big(108\,\_C1 - 8\,x^3 - 36\,x^2 - 54\,x - 27$$

$$+ 6 \sqrt{-48\,\_C1\,x^3 - 216\,\_C1\,x^2 + 324\,\_C1^2 - 324\,\_C1\,x - 162\,\_C1}\,)^{1/3}\big)\Big)^3$$

$$- \frac{1}{4}\,\big(108\,\_C1 - 8\,x^3 - 36\,x^2 - 54\,x - 27$$

$$+ 6 \sqrt{-48\,\_C1\,x^3 - 216\,\_C1\,x^2 + 324\,\_C1^2 - 324\,\_C1\,x - 162\,\_C1}\,)^{1/3} - \frac{1}{4}\,(2\,x$$

$$+ 3)^2 \Big/ \big(108\,\_C1 - 8\,x^3 - 36\,x^2 - 54\,x - 27$$

$$+ 6 \sqrt{-48\,\_C1\,x^3 - 216\,\_C1\,x^2 + 324\,\_C1^2 - 324\,\_C1\,x - 162\,\_C1}\,)^{1/3} - x - \frac{3}{2}$$

$$- \frac{3}{2}\,\mathrm{I}\sqrt{3}\,\Bigg(\frac{1}{6}\,\big(108\,\_C1 - 8\,x^3 - 36\,x^2 - 54\,x - 27$$

$$+ 6 \sqrt{-48\,\_C1\,x^3 - 216\,\_C1\,x^2 + 324\,\_C1^2 - 324\,\_C1\,x - 162\,\_C1}\,)^{1/3} - \frac{1}{6}\,(2\,x$$

$$+ 3)^2 \Big/ \big(108\,\_C1 - 8\,x^3 - 36\,x^2 - 54\,x - 27$$

$$+ 6 \sqrt{-48\,\_C1\,x^3 - 216\,\_C1\,x^2 + 324\,\_C1^2 - 324\,\_C1\,x - 162\,\_C1}\,)^{1/3}\big),\ y(x) = \Bigg($$

$$-\Bigg(-\frac{1}{12}\,\big(108\,\_C1 - 8\,x^3 - 36\,x^2 - 54\,x - 27$$

$$+ 6 \sqrt{-48\,\_C1\,x^3 - 216\,\_C1\,x^2 + 324\,\_C1^2 - 324\,\_C1\,x - 162\,\_C1}\,)^{1/3} - \frac{1}{12}\,(2\,x$$

$$+ 3)^2 \Big/ \big(108\,\_C1 - 8\,x^3 - 36\,x^2 - 54\,x - 27$$

$$+ 6 \sqrt{-48\,\_C1\,x^3 - 216\,\_C1\,x^2 + 324\,\_C1^2 - 324\,\_C1\,x - 162\,\_C1}\,)^{1/3} - \frac{1}{3}\,x - \frac{1}{2}$$

$$+\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{6}\left(108\,\_C1-8\,x^3-36\,x^2-54\,x-27\right.\right.$$

$$\left.+6\sqrt{-48\,\_C1\,x^3-216\,\_C1\,x^2+324\,\_C1^2-324\,\_C1\,x-162\,\_C1}\right)^{1/3}-\frac{1}{6}\,(2\,x$$

$$+3)^2\Big/\left(108\,\_C1-8\,x^3-36\,x^2-54\,x-27\right.$$

$$\left.\left.\left.+6\sqrt{-48\,\_C1\,x^3-216\,\_C1\,x^2+324\,\_C1^2-324\,\_C1\,x-162\,\_C1}\right)^{1/3}\right)\right)^2$$

$$-\frac{1}{6}\left(108\,\_C1-8\,x^3-36\,x^2-54\,x-27\right.$$

$$\left.+6\sqrt{-48\,\_C1\,x^3-216\,\_C1\,x^2+324\,\_C1^2-324\,\_C1\,x-162\,\_C1}\right)^{1/3}-\frac{1}{6}\,(2\,x$$

$$+3)^2\Big/\left(108\,\_C1-8\,x^3-36\,x^2-54\,x-27\right.$$

$$\left.+6\sqrt{-48\,\_C1\,x^3-216\,\_C1\,x^2+324\,\_C1^2-324\,\_C1\,x-162\,\_C1}\right)^{1/3}-\frac{2}{3}\,x-1$$

$$+\mathrm{I}\sqrt{3}\left(\frac{1}{6}\left(108\,\_C1-8\,x^3-36\,x^2-54\,x-27\right.\right.$$

$$\left.+6\sqrt{-48\,\_C1\,x^3-216\,\_C1\,x^2+324\,\_C1^2-324\,\_C1\,x-162\,\_C1}\right)^{1/3}-\frac{1}{6}\,(2\,x$$

$$+3)^2\Big/\left(108\,\_C1-8\,x^3-36\,x^2-54\,x-27\right.$$

$$\left.\left.+6\sqrt{-48\,\_C1\,x^3-216\,\_C1\,x^2+324\,\_C1^2-324\,\_C1\,x-162\,\_C1}\right)^{1/3}\right)\right)x-\Bigg($$

$$-\frac{1}{12}\left(108\,\_C1-8\,x^3-36\,x^2-54\,x-27\right.$$

$$\left.+6\sqrt{-48\,\_C1\,x^3-216\,\_C1\,x^2+324\,\_C1^2-324\,\_C1\,x-162\,\_C1}\right)^{1/3}-\frac{1}{12}\,(2\,x$$

$$+3)^2\Big/\left(108\,\_C1-8\,x^3-36\,x^2-54\,x-27\right.$$

$$+6\sqrt{-48\,\_C1\,x^3-216\,\_C1\,x^2+324\,\_C1^2-324\,\_C1\,x-162\,\_C1}\Big)^{1/3}-\frac{1}{3}x-\frac{1}{2}$$

$$+\frac{1}{2}\mathrm{I}\sqrt{3}\Bigg(\frac{1}{6}\Big(108\,\_C1-8x^3-36x^2-54x-27$$

$$+6\sqrt{-48\,\_C1\,x^3-216\,\_C1\,x^2+324\,\_C1^2-324\,\_C1\,x-162\,\_C1}\Big)^{1/3}-\frac{1}{6}(2x$$

$$+3)^2\Big/\Big(108\,\_C1-8x^3-36x^2-54x-27$$

$$+6\sqrt{-48\,\_C1\,x^3-216\,\_C1\,x^2+324\,\_C1^2-324\,\_C1\,x-162\,\_C1}\Big)^{1/3}\Big)\Bigg)^3$$

$$-\frac{1}{4}\Big(108\,\_C1-8x^3-36x^2-54x-27$$

$$+6\sqrt{-48\,\_C1\,x^3-216\,\_C1\,x^2+324\,\_C1^2-324\,\_C1\,x-162\,\_C1}\Big)^{1/3}-\frac{1}{4}(2x$$

$$+3)^2\Big/\Big(108\,\_C1-8x^3-36x^2-54x-27$$

$$+6\sqrt{-48\,\_C1\,x^3-216\,\_C1\,x^2+324\,\_C1^2-324\,\_C1\,x-162\,\_C1}\Big)^{1/3}-x-\frac{3}{2}$$

$$+\frac{3}{2}\mathrm{I}\sqrt{3}\Bigg(\frac{1}{6}\Big(108\,\_C1-8x^3-36x^2-54x-27$$

$$+6\sqrt{-48\,\_C1\,x^3-216\,\_C1\,x^2+324\,\_C1^2-324\,\_C1\,x-162\,\_C1}\Big)^{1/3}-\frac{1}{6}(2x$$

$$+3)^2\Big/\Big(108\,\_C1-8x^3-36x^2-54x-27$$

$$+6\sqrt{-48\,\_C1\,x^3-216\,\_C1\,x^2+324\,\_C1^2-324\,\_C1\,x-162\,\_C1}\Big)^{1/3}\Bigg)$$

$simplify(dsolve(\{eq_4, y(0)=1\}))$

$$y(x)=x+2 \qquad \textbf{(18)}$$

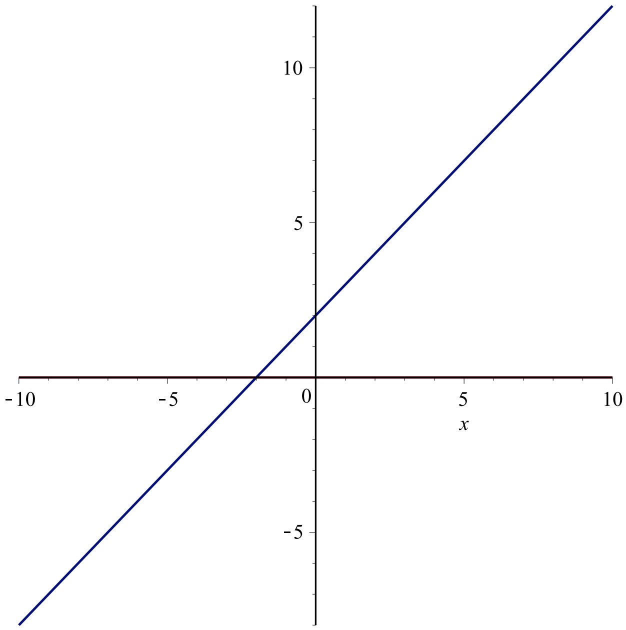
$simplify(dsolve(\{eq_4, y(1)=0\}))$

$$y(x)=0 \qquad \textbf{(19)}$$

$simplify(dsolve(\{eq_4, y(2)=5\}))$

$$y(x)=x+2 \qquad \textbf{(20)}$$

$plot(\{rhs(\textbf{(18)}), rhs(\textbf{(19)}), rhs(\textbf{(20)})\})$



$eq_5 := y^2 \cdot \left(y' \cdot diff\left(y(x), x\$3\right) - 2\, diff\left(y(x), x\$2\right)^2\right) = y\, y'^2\, diff\left(y(x), x\$2\right) + 2\, y'^4$

$$y(x)^2 \left(\left(\frac{d}{dx} y(x)\right)\left(\frac{d^3}{dx^3} y(x)\right) - 2\left(\frac{d^2}{dx^2} y(x)\right)^2\right) = y(x)\left(\frac{d}{dx} y(x)\right)^2 \left(\frac{d^2}{dx^2} y(x)\right) + 2\left(\frac{d}{dx} y(x)\right)^4 \quad \textbf{(21)}$$

$dsolve(eq_5)$

$$y(x) = \sqrt{\ln(-\_C1\ \_C2 - \_C1\ x) - 1}\ \_C3 \quad \textbf{(22)}$$

$dsolve(\{eq_5, y(0) = 0\})$

$$y(x) = 0 \quad \textbf{(23)}$$

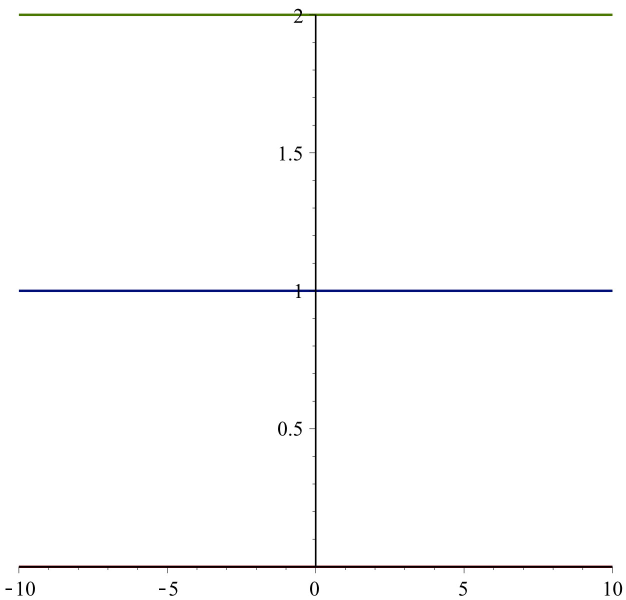
$dsolve(\{eq_5, y(1) = 1\})$

$$y(x) = e^{\frac{1}{2}\ln(\ln(-\_C1\ \_C2 - \_C1\ x) - 1) - \frac{1}{2}\ln(\ln(-\_C1\ \_C2 - \_C1) - 1)}, y(x) = 1 \quad \textbf{(24)}$$

$dsolve(\{eq_5, y(1) = 2\})$

$$y(x) = \frac{2}{e^{-\frac{1}{2}\ln(\ln(-\_C1\ \_C2 - \_C1\ x) - 1) + \frac{1}{2}\ln(\ln(-\_C1\ \_C2 - \_C1) - 1)}}, y(x) = 2 \quad \textbf{(25)}$$

$plot(\{rhs((\mathbf{23})), rhs((\mathbf{24})_2), rhs((\mathbf{25})_2)\})$



$eq_6 := x^2\ diff(y(x), x\$2) - 3\,x\,y' = \frac{6\,y^2}{x^2} - 4\,y$

$$x^2 \left(\frac{d^2}{dx^2}\,y(x)\right) - 3\left(\frac{d}{dx}\,y(x)\right)x = \frac{6\,y(x)^2}{x^2} - 4\,y(x) \tag{26}$$

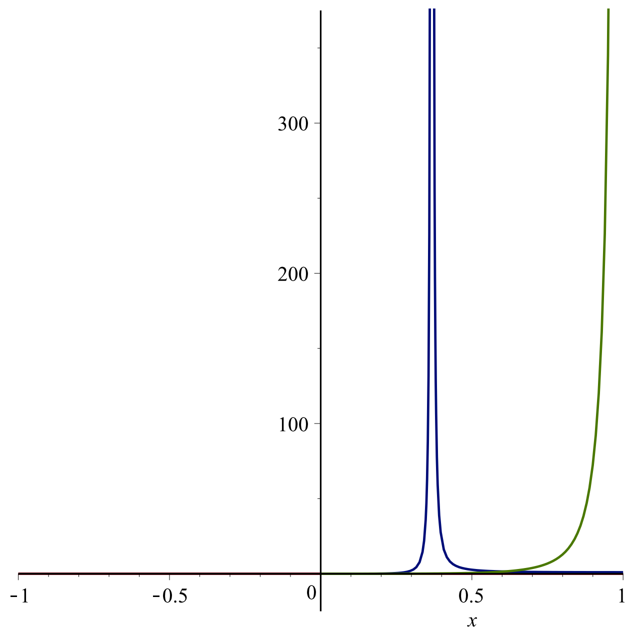
$dsolve(eq_6)$

$$y(x) = \mathrm{WeierstrassP}(\ln(x) + \_C1, 0, \_C2)\,x^2 \tag{27}$$

$dsolve(\{eq_6, y(0) = 0\})$

$$y(x) = 0, y(x) = \frac{x^2}{\ln(x)^2}, y(x) = \frac{x^2}{(\ln(x) + \_C1)^2} \tag{28}$$

$plot(\{rhs((\mathbf{28})_1), rhs((\mathbf{28})_2), subs(\_C1 = 1, rhs((\mathbf{28})_3))\}, x = -1..1)$



$eq_7 := diff(y(x), x\$3) - 2\, diff(y(x), x\$2) + 4\, y' - 8\, y = e^{2x} \cdot \sin(2\,x) + 2\,x^2$

$$\frac{d^3}{dx^3} y(x) - 2 \left(\frac{d^2}{dx^2} y(x)\right) + 4 \left(\frac{d}{dx} y(x)\right) - 8\, y(x) = e^{2x} \sin(2\,x) + 2\,x^2 \tag{29}$$

$dsolve(eq_7)$

$$y(x) = \left(-\frac{1}{40} e^{2x} + \frac{1}{16}\right) \cos(2\,x) + \left(-\frac{1}{16} - \frac{1}{20} e^{2x}\right) \sin(2\,x) - \frac{1}{4} x^2 - \frac{1}{16} e^{2x} - \frac{1}{4} x + \_C1 \cos(2\,x) + \_C2\, e^{2x} + \_C3 \sin(2\,x) \tag{30}$$

$dsolve(\{eq_7, y(0) = 0\})$

$$y(x) = \left(-\frac{1}{40} e^{2x} + \frac{1}{16}\right) \cos(2\,x) + \left(-\frac{1}{16} - \frac{1}{20} e^{2x}\right) \sin(2\,x) - \frac{1}{4} x^2 - \frac{1}{16} e^{2x} - \frac{1}{4} x + \left(\frac{1}{40} - \_C2\right) \cos(2\,x) + \_C2\, e^{2x} + \_C3 \sin(2\,x) \tag{31}$$

$dsolve(\{eq_7, y(1) = 0\})$

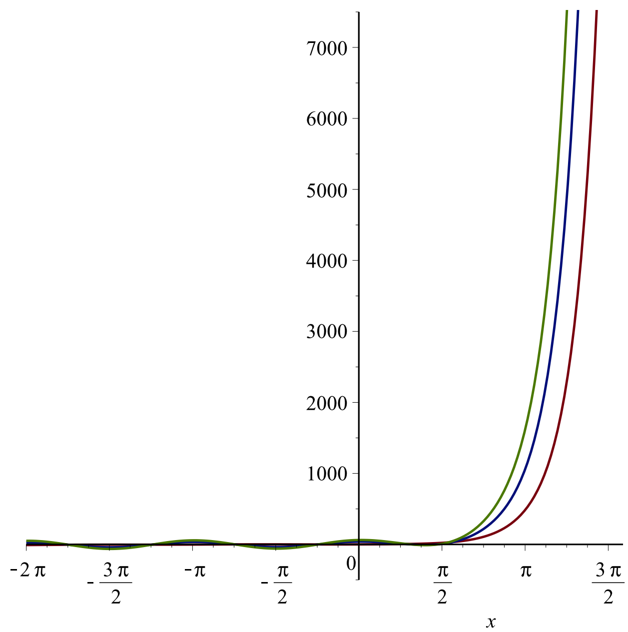
$$y(x) = \left(-\frac{1}{40} e^{2x} + \frac{1}{16}\right) \cos(2\,x) + \left(-\frac{1}{16} - \frac{1}{20} e^{2x}\right) \sin(2\,x) - \frac{1}{4} x^2 - \frac{1}{16} e^{2x} - \frac{1}{4} x \tag{32}$$

$$+\left(-\frac{e^2\,\_C2}{\cos(2)}-\frac{\sin(2)\,\_C3}{\cos(2)}\right.$$
$$\left.+\frac{1}{80}\,\frac{2\cos(2)\,e^2+4\sin(2)\,e^2-5\cos(2)+5\,e^2+5\sin(2)+40}{\cos(2)}\right)\cos(2x)+\_C2\,e^{2x}$$
$$+\_C3\sin(2x)$$

dsolve$(\{eq_7, y(1)=-1\})$

$$y(x)=\left(-\frac{1}{40}\,e^{2x}+\frac{1}{16}\right)\cos(2x)+\left(-\frac{1}{16}-\frac{1}{20}\,e^{2x}\right)\sin(2x)-\frac{1}{4}\,x^2-\frac{1}{16}\,e^{2x}-\frac{1}{4}\,x \qquad \textbf{(33)}$$
$$+\left(-\frac{e^2\,\_C2}{\cos(2)}-\frac{\sin(2)\,\_C3}{\cos(2)}\right.$$
$$\left.+\frac{1}{80}\,\frac{2\cos(2)\,e^2+4\sin(2)\,e^2-5\cos(2)+5\,e^2+5\sin(2)-40}{\cos(2)}\right)\cos(2x)+\_C2\,e^{2x}$$
$$+\_C3\sin(2x)$$

plot$(\{subs(\{\_C2=1, \_C3=1\}, rhs(\mathbf{(31)})), subs(\{\_C2=2, \_C3=2\}, rhs(\mathbf{(32)})), subs(\{\_C2=3, \_C3=3\}, rhs(\mathbf{(33)}))\})$



$eq_8 := x^2\cdot diff(y(x), x\$2) + x\,y' + 4\,y = 10\,x$

$$x^2 \left(\frac{d^2}{dx^2} y(x) \right) + \left(\frac{d}{dx} y(x) \right) x + 4\, y(x) = 10\, x \tag{34}$$

$dsolve(eq_8)$

$$y(x) = \sin(2 \ln(x))\ \_C2 + \cos(2 \ln(x))\ \_C1 + 2\, x \tag{35}$$

$dsolve(\{eq_8, y(1) = 0\})$

$$y(x) = \sin(2 \ln(x))\ \_C2 - 2 \cos(2 \ln(x)) + 2\, x \tag{36}$$

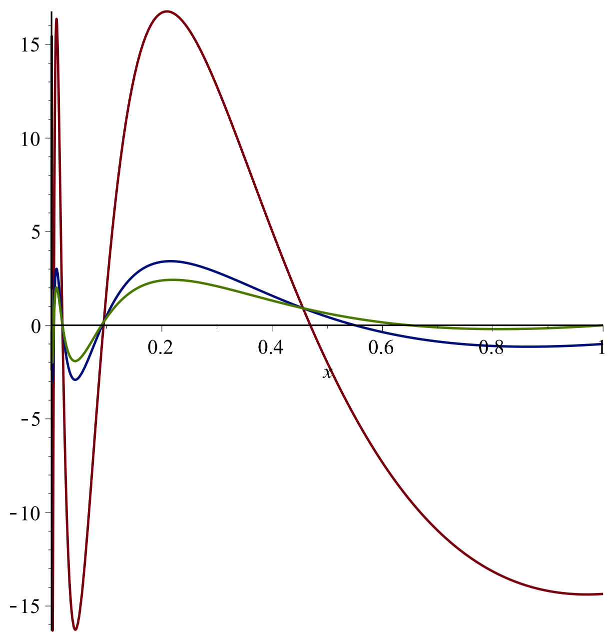
$dsolve(\{eq_8, y(1) = -1\})$

$$y(x) = \sin(2 \ln(x))\ \_C2 - 3 \cos(2 \ln(x)) + 2\, x \tag{37}$$

$dsolve(\{eq_8, y(2) = 1\})$

$$y(x) = \sin(2 \ln(x))\ \_C2 + \cos(2 \ln(x)) \left(-\frac{\sin(2 \ln(2))\ \_C2}{\cos(2 \ln(2))} - \frac{3}{\cos(2 \ln(2))} \right) + 2\, x \tag{38}$$

$plot(\{subs(\_C2 = 0, rhs(\mathbf{(36)})), subs(\_C2 = 0, rhs(\mathbf{(37)})), subs(\_C2 = 0, rhs(\mathbf{(38)}))\}, x = 0..1)$



$eq_9 := (3\, x^3 + x) \cdot diff(y(x), x\$2) + 2\, y' - 6\, x\, y = 4 - 12\, x^2$

$$(3\, x^3 + x) \left(\frac{d^2}{dx^2} y(x) \right) + 2 \left(\frac{d}{dx} y(x) \right) - 6\, x\, y(x) = -12\, x^2 + 4 \tag{39}$$

$dsolve(eq_9)$

$$y(x) = \frac{\_C2}{x} + (x^2+1)\ \_C1 + 2\,x \tag{40}$$

$dsolve(\{eq_9, y(0) = 0\})$

$$y(x) = (x^2+1)\ \_C1 + 2\,x \tag{41}$$

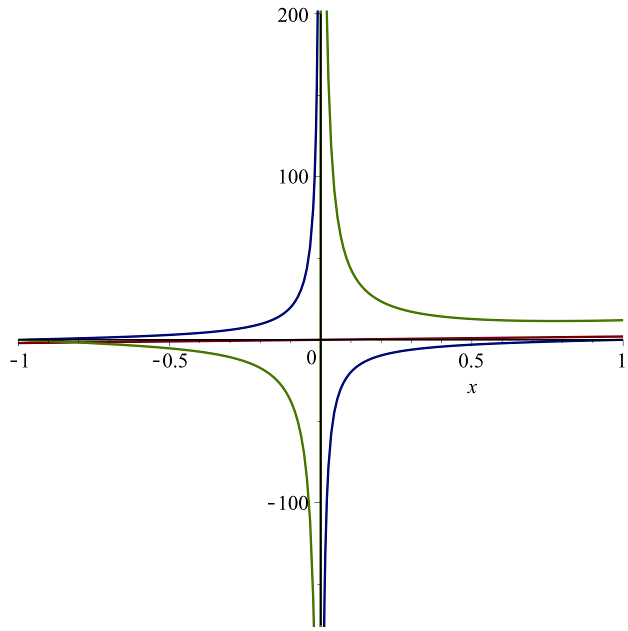
$dsolve(\{eq_9, y(1) = 0\})$

$$y(x) = \frac{-2 - 2\ \_C1}{x} + (x^2+1)\ \_C1 + 2\,x \tag{42}$$

$dsolve(\{eq_9, y(-1) = 0\})$

$$y(x) = \frac{-2 + 2\ \_C1}{x} + (x^2+1)\ \_C1 + 2\,x \tag{43}$$

$plot(\{subs(\_C1 = 0, rhs(\mathbf{(41)})), subs(\_C1 = 0, rhs(\mathbf{(42)})), subs(\_C1 = 3, rhs(\mathbf{(43)}))\}, x = -1..1)$



Task 2

$eq_{21} := diff(y(x), x\$2) + y(x) = 2\,x - \pi$

$$\frac{d^2}{dx^2}\,y(x) + y(x) = 2\,x - \pi \tag{44}$$

$dsolve\left(\left\{eq_{21}, y(0)=0, y(\pi)=0\right\}\right)$

$$y(x)=\sin(x)\ \_C2+\cos(x)\ \pi+2\ x-\pi \tag{45}$$

$eq_{22} := diff(y(x), x\$2)-2\ \mathrm{i}\ y(x)=0$

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\ y(x)-2\ \mathrm{I}\ y(x)=0 \tag{46}$$

$dsolve\left(\left\{eq_{22}, y(0)=-1, y(+\infty)=0\right\}\right)$

Task 3

$Sys_1 := diff(x(t), t)-5\ x(t)-3\ y(t)=0, diff(y(t), t)+3\ x(t)+y(t)=0$

$$\frac{\mathrm{d}}{\mathrm{d}t}\ x(t)-5\ x(t)-3\ y(t)=0,\ \frac{\mathrm{d}}{\mathrm{d}t}\ y(t)+3\ x(t)+y(t)=0 \tag{47}$$

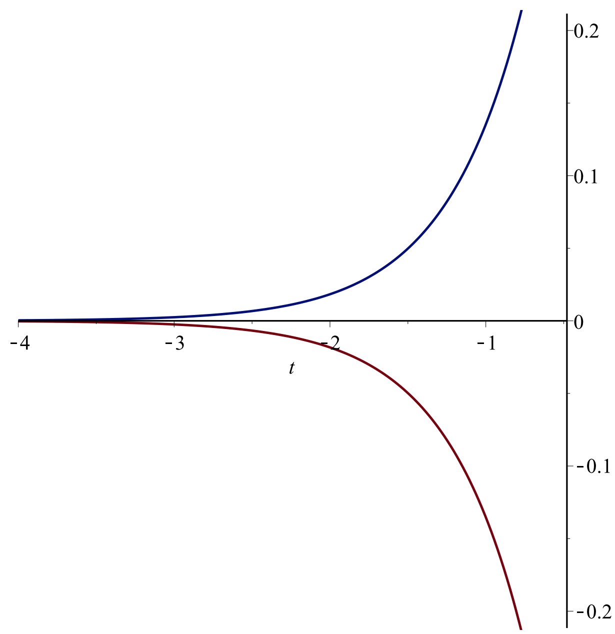
$dsolve\left(\left\{Sys_1\right\}\right)$

$$\left\{x(t)=\mathrm{e}^{2\,t}\ (\_C2\ t+\_C1), y(t)=-\frac{1}{3}\ \mathrm{e}^{2\,t}\ (3\ \_C2\ t+3\ \_C1-\_C2)\right\} \tag{48}$$

$dsolve\left(\left\{Sys_1, x(0)=1, y(0)=-1\right\}\right)$

$$\left\{x(t)=\mathrm{e}^{2\,t}, y(t)=-\mathrm{e}^{2\,t}\right\} \tag{49}$$

$plot\left(\left\{rhs\left(\mathbf{(49)}_1\right), rhs\left(\mathbf{(49)}_2\right)\right\}, t=-4..0\right)$



$$Sys_2 := diff(x(t), t) = 2\,y(t) - x(t),\ diff(y(t), t) = 4\,y(t) - 3\,x(t) + \frac{e^{3t}}{e^{2t}+1}$$

$$\frac{d}{dt}\,x(t) = 2\,y(t) - x(t),\ \frac{d}{dt}\,y(t) = 4\,y(t) - 3\,x(t) + \frac{e^{3t}}{e^{2t}+1} \tag{50}$$

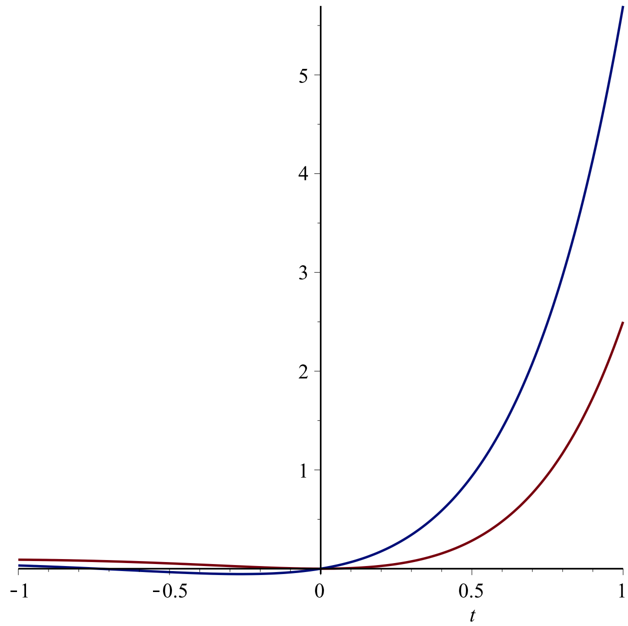
$simplify\big(dsolve(\{Sys_2\})\big)$

$$\Big\{x(t) = 2\,e^{2t}\arctan(e^{t}) + \_C1\,e^{2t} - e^{t}\ln(e^{2t}+1) + e^{t}\,\_C2,\ y(t) = 3\,e^{2t}\arctan(e^{t}) + \frac{3}{2}\,\_C1\,e^{2t} - e^{t}\ln(e^{2t}+1) + e^{t}\,\_C2\Big\} \tag{51}$$

$dsolve(\{Sys_2, x(0) = 0, y(0) = 0\})$

$$\Big\{x(t) = 2\,e^{2t}\arctan(e^{t}) - \frac{1}{2}\,\pi\,e^{2t} - e^{t}\ln(e^{2t}+1) + e^{t}\ln(2),\ y(t) = 3\,e^{2t}\arctan(e^{t}) - \frac{3}{4}\,\pi\,e^{2t} - e^{t}\ln(e^{2t}+1) + e^{t}\ln(2)\Big\} \tag{52}$$

$plot\big(\{rhs(\textbf{(52)}_1), rhs(\textbf{(52)}_2)\}, t = -1\,..1\big)$



$Sys_3 := diff(x(t), t\$2) = 2\,x(t) - 3\,y(t), diff(y(t), t\$2) = x(t) - 2\,y(t)$

$$\frac{d^2}{dt^2}\,x(t) = 2\,x(t) - 3\,y(t), \frac{d^2}{dt^2}\,y(t) = x(t) - 2\,y(t) \quad \textbf{(53)}$$

$dsolve(\{Sys_3\})$

$$\left\{x(t) = \_C1\,e^{-t} + e^{t}\,\_C2 + \_C3\,\sin(t) + \_C4\,\cos(t), y(t) = \frac{1}{3}\,\_C1\,e^{-t} + \frac{1}{3}\,e^{t}\,\_C2 + \_C3\,\sin(t) + \_C4\,\cos(t)\right\} \quad \textbf{(54)}$$

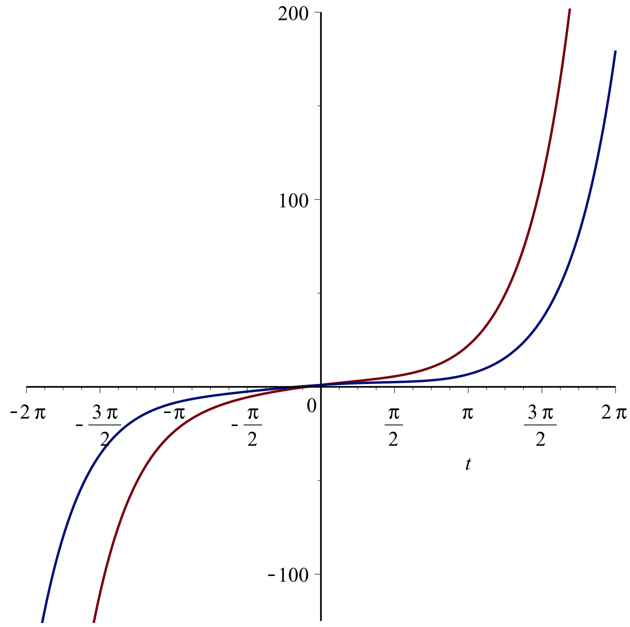
$dsolve(\{Sys_3, x(0) = 1, y(0) = 1\})$

$$\left\{x(t) = -\_C2\,e^{-t} + e^{t}\,\_C2 + \_C3\,\sin(t) + \cos(t), y(t) = -\frac{1}{3}\,\_C2\,e^{-t} + \frac{1}{3}\,e^{t}\,\_C2 + \_C3\,\sin(t) + \cos(t)\right\} \quad \textbf{(55)}$$

$subs(\{\_C2 = 1, \_C3 = 1\}, \textbf{(55)})$

$$\left\{x(t) = -e^{-t} + e^{t} + \sin(t) + \cos(t), y(t) = -\frac{1}{3}\,e^{-t} + \frac{1}{3}\,e^{t} + \sin(t) + \cos(t)\right\} \quad \textbf{(56)}$$

$plot(\{rhs(\textbf{(56)}_1), rhs(\textbf{(56)}_2)\})$



Task 4

$Sys_4 := diff(x(t), t) = 4\, y(t) - 2\, z(t) - 3\, x(t), diff(y(t), t) = z(t) + x(t), diff(z(t), t) = 6\, x(t) - 6\, y(t) + 5\, z(t)$

$$\frac{\mathrm{d}}{\mathrm{d}t}\, x(t) = 4\, y(t) - 2\, z(t) - 3\, x(t), \frac{\mathrm{d}}{\mathrm{d}t}\, y(t) = z(t) + x(t), \frac{\mathrm{d}}{\mathrm{d}t}\, z(t) = 6\, x(t) - 6\, y(t) + 5\, z(t) \qquad \mathbf{(57)}$$

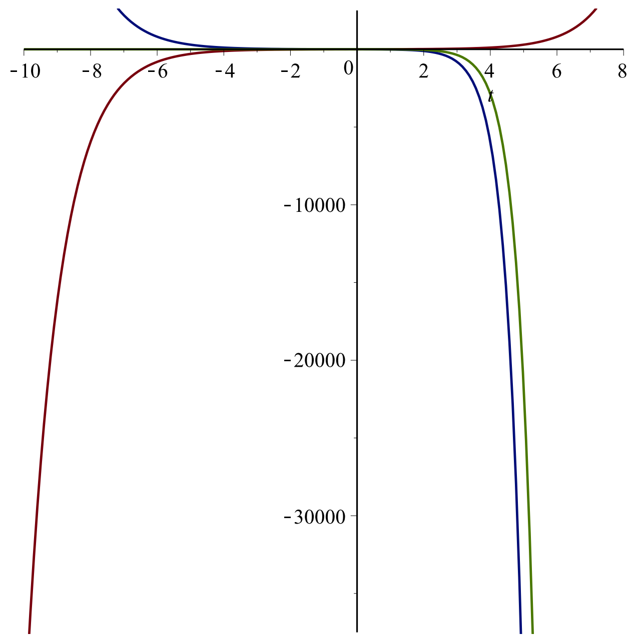
$dsolve(\{Sys_4\})$

$$\{x(t) = \_C3\, \mathrm{e}^{t} + \_C1\, \mathrm{e}^{-t}, y(t) = \_C2\, \mathrm{e}^{2t} + \_C3\, \mathrm{e}^{t}, z(t) = 2\, \_C2\, \mathrm{e}^{2t} - \_C1\, \mathrm{e}^{-t}\} \qquad \mathbf{(58)}$$

$dsolve(\{Sys_4, x(0) = 0, y(0) = 1, z(0) = 0\})$

$$\{x(t) = 2\, \mathrm{e}^{t} - 2\, \mathrm{e}^{-t}, y(t) = -\mathrm{e}^{2t} + 2\, \mathrm{e}^{t}, z(t) = -2\, \mathrm{e}^{2t} + 2\, \mathrm{e}^{-t}\} \qquad \mathbf{(59)}$$

$plot(\{rhs(\mathbf{(59)}_1), rhs(\mathbf{(59)}_2), rhs(\mathbf{(59)}_3)\})$



$Sys_5 := diff(x(t), t) = 2\,x(t) - y(t) - z(t), diff(y(t), t) = 3\,x(t) - 2\,y(t) - 3\,z(t), diff(z(t), t)$
$= 2\,z(t) - x(t) + y(t)$

$$\frac{d}{dt}\,x(t) = 2\,x(t) - y(t) - z(t), \frac{d}{dt}\,y(t) = 3\,x(t) - 2\,y(t) - 3\,z(t), \frac{d}{dt}\,z(t) = 2\,z(t) - x(t) + y(t) \quad \textbf{(60)}$$

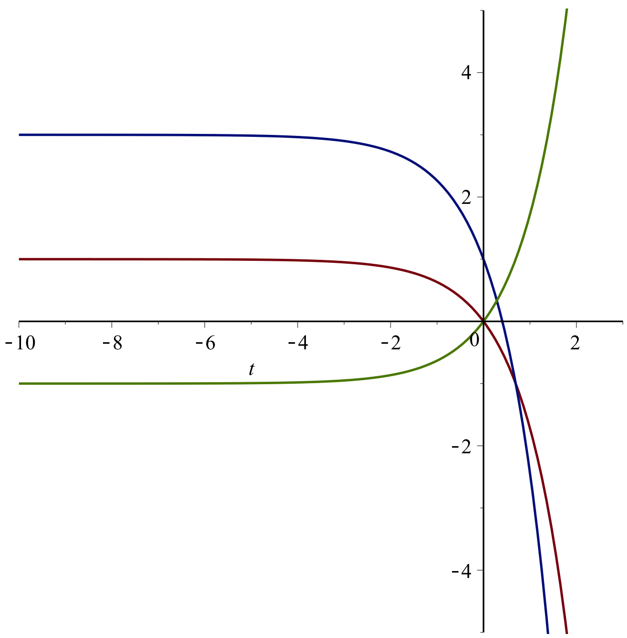
$dsolve(\{Sys_5\})$

$$\{x(t) = \_C2 + \_C3\,e^t, y(t) = 3\,\_C2 + 3\,\_C3\,e^t + e^t\,\_C1, z(t) = -2\,\_C3\,e^t - \_C2 - e^t\,\_C1\} \quad \textbf{(61)}$$

$dsolve(\{Sys_5, x(0) = 0, y(0) = 1, z(0) = 0\})$

$$\{x(t) = 1 - e^t, y(t) = 3 - 2\,e^t, z(t) = e^t - 1\} \quad \textbf{(62)}$$

$plot(\{rhs(\textbf{(62)}_1), rhs(\textbf{(62)}_2), rhs(\textbf{(62)}_3)\})$


4
2
-2
-4
-10
-8
-6
-4
-2
0
2
t