Test 2, part 2
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with(student):
with(VectorCalculus):
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Task 1

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Sum(n·(n+1)·x^n, n=1..infinity) = sum(n·(n+1)·x^n, n=1..infinity)
```

$$\sum_{n=1}^{\infty} n\,(n+1)\,x^n = -\frac{2x}{(x-1)^3}$$

```
Sum(n/x^n, n=1..infinity) = sum(n/x^n, n=1..infinity)
```

$$\sum_{n=1}^{\infty} \frac{n}{x^n} = \frac{x}{(x-1)^2} \tag{2}$$

Task 2

```
series(arcsin(x), x=0, 9)
```

$$x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \frac{5}{112}x^7 + O(x^9) \tag{3}$$

Task 3

```
readlib(mtaylor):
mtaylor(arctan((x-y)/(1+xy)), [x=0, y=0], 6)
```

$$\frac{x}{1+xy} - \frac{y}{1+xy} - \frac{1}{3}\frac{x^3}{(1+xy)^3} + \frac{yx^2}{(1+xy)^3} - \frac{y^2x}{(1+xy)^3} + \frac{1}{3}\frac{y^3}{(1+xy)^3} + \frac{1}{5}\frac{x^5}{(1+xy)^5} - \frac{yx^4}{(1+xy)^5} + \frac{2y^2x^3}{(1+xy)^5} - \frac{2y^3x^2}{(1+xy)^5} + \frac{y^4x}{(1+xy)^5} - \frac{1}{5}\frac{y^5}{(1+xy)^5} \tag{4}$$

Task 4

```
fourierseries := proc(f, x, x1, x2, n) local k, l,
 a, b, s;
 l := (x2-x1)/2;
 a[0] := int(f, x=x1..x2)/l;
 a[k] := int(f·cos(k·π·x/l), x=x1..x2)/l;
 b[k] := int(f·sin(k·π·x/l), x=x1..x2)/l;
 s := a[0]/2 + sum(a[k]·cos(k·π·x/l) + b[k]·sin(k·π·x/l), k=1..n);
end;
```

```
proc(f, x, x1, x2, n)
    local k, l, a, b, s;
    l := 1/2*x2 - 1/2*x1;
    a[0] := int(f, x = x1..x2)/l;
    a[k] := int(f*cos(k*Pi*x/l), x = x1..x2)/l;
    b[k] := int(f*sin(k*Pi*x/l), x = x1..x2)/l;
    s := 1/2*a[0] + sum(a[k]*cos(k*Pi*x/l) + b[k]*sin(k*Pi*x/l), k = 1..n)
end proc
```

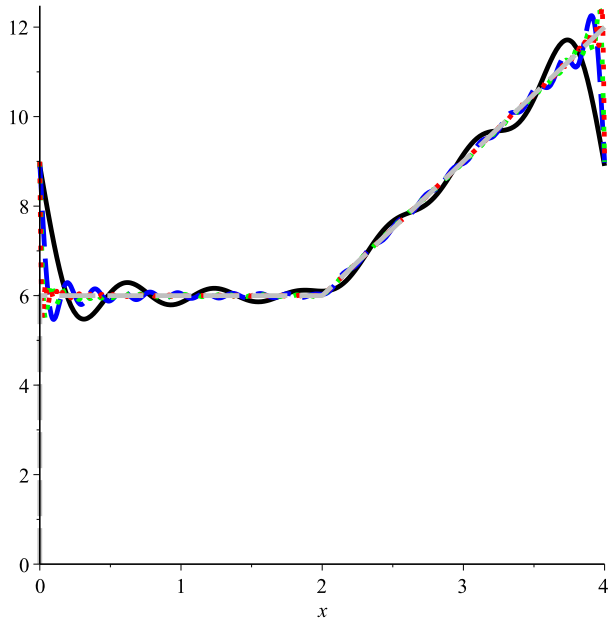
(5)

```
f := piecewise(x < 2 and x > 0, 6, 2 <= x and x <= 4, 3*x) : x1 := 0 : x2 := 4 :
fr := fourierseries(f, x, x1, x2, 6);
```

$$\frac{15}{2} + \frac{12\cos\left(\frac{1}{2}\pi x\right)}{\pi^2} - \frac{6\sin\left(\frac{1}{2}\pi x\right)}{\pi} - \frac{3\sin(\pi x)}{\pi} + \frac{4}{3}\,\frac{\cos\left(\frac{3}{2}\pi x\right)}{\pi^2} - \frac{2\sin\left(\frac{3}{2}\pi x\right)}{\pi} - \frac{3}{2}\,\frac{\sin(2\pi x)}{\pi} + \frac{12}{25}\,\frac{\cos\left(\frac{5}{2}\pi x\right)}{\pi^2} - \frac{6}{5}\,\frac{\sin\left(\frac{5}{2}\pi x\right)}{\pi} - \frac{\sin(3\pi x)}{\pi}$$

(6)

```
fr1 := fourierseries(f, x, x1, x2, 6) :
fr2 := fourierseries(f, x, x1, x2, 20) :
fr3 := fourierseries(f, x, x1, x2, 50) :
fr4 := fourierseries(f, x, x1, x2, 100) :
plot({f, fr1, fr2, fr3, fr4}, x = x1..x2, color = [black, blue, green, red, gray], thickness = 3, linestyle = [1,
    3, 2, 2, 3])
```



Task 5

$$limit\left(\frac{\frac{(n+1)!}{(n+1)^{n+1}}}{\frac{n!}{n^n}}, n = infinity\right)$$

$$e^{-1} \quad \textbf{(7)}$$

Limit equals e^(-1), what is less than 1, so integral converges.

$$limit\left(\frac{\frac{3^{n+1}\cdot(n+1)!}{(n+1)^{n+1}}}{\frac{3^n\cdot n!}{n^n}}, n = infinity\right)$$

$$3\,e^{-1} \quad \textbf{(8)}$$

Limit equals e^(-1), what is greater than 1, so integral doesn't converge.

Task 6

Since the given power series contains the powers x^n, the radius of convergence is given by:

$$a(n) = \frac{1}{n\,2^n} \xrightarrow{\text{assign as function}} a$$

$$R = \lim_{n \to \infty} \frac{a(n)}{a(n+1)} \xrightarrow{\text{assign}}$$

$$R = 2$$

At the right endpoint x=R=2, the given power series becomes Σ 1/n, which is the divergent harmonic series:

$$a(n) \cdot R^n = \frac{1}{n}$$

At the left endpoint x=-R=-2, the given power series becomes the negative of the alternating harmonic series Σ (-1)^n/n, which converges conditionally to -ln(2):

$$a(n) \cdot (-R)^n = \frac{(-2)^n}{n\,2^n} \xrightarrow{\text{assuming integer}} \frac{(-1)^n}{n}$$

Hence, the interval of convergence is [-R,R)=[-2,2)

Task 7

$$Int\left(x^3 e^{-x^2}, x = 0..infinity\right) = int\left(x^3 e^{-x^2}, x = 0..infinity\right)$$

$$\int_0^{\infty} x^3\, e^{-x^2}\, dx = \frac{1}{2} \qquad \textbf{(9)}$$

Task 8

$$Doubleint\left(e^{\frac{x}{y}}, x = 0..y^2, y = 0..1\right)$$

$$\int_0^1 \int_0^{y^2} e^{\frac{x}{y}}\, dx\, dy \qquad \textbf{(10)}$$

$value(\textbf{(10)})$

$$-\frac{1}{2}\,\frac{\ln(e)^2 - 2\,e\ln(e) + 2\,e - 2}{\ln(e)^3} \qquad \textbf{(11)}$$

$$Tripleint\left(x\,y^2\,z^3, z = 0..x\,y, y = 0..x, x = 0..1\right)$$

$$\int_0^1 \int_0^x \int_0^{xy} x\,y^2\,z^3\, dz\, dy\, dx \qquad \textbf{(12)}$$

$value(\textbf{(12)})$

$$\frac{1}{364} \qquad \textbf{(13)}$$

Task 9

Task 10

$$with(Student[VectorCalculus]):$$

$$LineInt\left(VectorField\left(\langle 2x - y^2, 8y\rangle\right), Path\left(\langle 2\cos(t), \sin(t)\rangle, t = \mathrm{Pi}..0\right)\right)$$

Error, (in Student:-VectorCalculus:-*) exponentiation operation not defined for Vectors