Homework 3, Anastasiia Yelchaninova

Task 1

$convert(10101100011111011100, decimal, binary)$

$$706524 \quad \textbf{(1)}$$

$convert(\textbf{(1)}, hex)$

$$AC7DC \quad \textbf{(2)}$$

Task 2

$$combine\left(\left(\sqrt[4]{32\sqrt[3]{4}} + \sqrt[4]{\frac{64}{\sqrt[3]{2}}} - 3\cdot\sqrt[3]{2\sqrt[4]{2}}\right)\cdot 3\sqrt[12]{2}\right)$$

$$3\sqrt{2} \quad \textbf{(3)}$$

Task 3

$$subs\left(x = \sqrt{\frac{3}{2}}, \tan\left(\frac{x^7 + 6x^5}{\log_3(x)}\right)\right)$$

$$\tan\left(\frac{135}{16}\ \frac{\sqrt{6}\ \ln(3)}{\ln\left(\frac{1}{2}\sqrt{6}\right)}\right) \quad \textbf{(4)}$$

$evalf_5(\textbf{(4)})$

$$-1.9614 \quad \textbf{(5)}$$

Task 4

$$convert\left(\frac{x^3 + 2x - 2}{x^3 - 2x^2 + x}, parfrac\right)$$

$$1 - \frac{2}{x} + \frac{1}{(x-1)^2} + \frac{4}{x-1} \quad \textbf{(6)}$$

Task 5

$subs(\{\ln(y) = 2, \ln(z) = 3\}, \log_y(z) + \log_z(y))$

$$\frac{13}{6} \quad \textbf{(7)}$$

Task 6

$a1 := \langle 2, 3, -1, 2\rangle :$
$b1 := \langle 3, -2, 1, 2\rangle :$
$with(LinearAlgebra) :$
$VectorAngle(a1, b1)$

$$\arccos\left(\frac{1}{6}\right) \quad \textbf{(8)}$$

Task 7

$a2 := \langle 2, -3, 1\rangle :$
$b2 := \langle -3, 1, 2\rangle :$
$c2 := \langle 1, 2, 3\rangle :$

$CrossProduct(CrossProduct(a2, b2), c2)$

$$\begin{bmatrix} -7 \\ 14 \\ -7 \end{bmatrix} \tag{9}$$

$CrossProduct(a2, CrossProduct(b2, c2))$

$$\begin{bmatrix} 10 \\ 13 \\ 19 \end{bmatrix} \tag{10}$$

Task 8 -

$a3 := \langle x1, x2, x3 \rangle$:
$b3 := \langle y1, y2, y3 \rangle$:
$c3 := \langle z1, z2, z3 \rangle$:
$p1 := CrossProduct(a3, CrossProduct(b3, c3))$

$$\begin{bmatrix} x2\,(y1\,z2 - y2\,z1) - x3\,(-y1\,z3 + y3\,z1) \\ -x1\,(y1\,z2 - y2\,z1) + x3\,(y2\,z3 - y3\,z2) \\ x1\,(-y1\,z3 + y3\,z1) - x2\,(y2\,z3 - y3\,z2) \end{bmatrix} \tag{11}$$

$p2 := CrossProduct(b3, CrossProduct(c3, a3))$

$$\begin{bmatrix} y2\,(-x1\,z2 + x2\,z1) - y3\,(x1\,z3 - x3\,z1) \\ -y1\,(-x1\,z2 + x2\,z1) + y3\,(-x2\,z3 + x3\,z2) \\ y1\,(x1\,z3 - x3\,z1) - y2\,(-x2\,z3 + x3\,z2) \end{bmatrix}$$

$p3 := CrossProduct(c3, CrossProduct(a3, b3))$

$$\begin{bmatrix} z2\,(x1\,y2 - x2\,y1) - z3\,(-x1\,y3 + x3\,y1) \\ -z1\,(x1\,y2 - x2\,y1) + z3\,(x2\,y3 - x3\,y2) \\ z1\,(-x1\,y3 + x3\,y1) - z2\,(x2\,y3 - x3\,y2) \end{bmatrix}$$

$p1 + p2 + p3$

$$\begin{aligned} &[[x2\,(y1\,z2 - y2\,z1) - x3\,(-y1\,z3 + y3\,z1) + y2\,(-x1\,z2 + x2\,z1) - y3\,(x1\,z3 - x3\,z1) \\ &\quad + z2\,(x1\,y2 - x2\,y1) - z3\,(-x1\,y3 + x3\,y1)], \\ &\quad [-x1\,(y1\,z2 - y2\,z1) + x3\,(y2\,z3 - y3\,z2) - y1\,(-x1\,z2 + x2\,z1) + y3\,(-x2\,z3 + x3\,z2) \\ &\quad - z1\,(x1\,y2 - x2\,y1) + z3\,(x2\,y3 - x3\,y2)], \\ &\quad [x1\,(-y1\,z3 + y3\,z1) - x2\,(y2\,z3 - y3\,z2) + y1\,(x1\,z3 - x3\,z1) - y2\,(-x2\,z3 + x3\,z2) \\ &\quad + z1\,(-x1\,y3 + x3\,y1) - z2\,(x2\,y3 - x3\,y2)]] \end{aligned} \tag{14}$$

$simplify(\mathbf{(14)})$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

Task 9

with(*VectorCalculus*) :

r1 := $\langle e^t, e^t, \sqrt{2}\,t \rangle$

$$\begin{bmatrix} e^t \\ e^t \\ \sqrt{2}\,t \end{bmatrix} \tag{16}$$

c := *Curvature*(*r1*, *t*)

$$\frac{\sqrt{2\left(-\frac{2\,e^t e^{2t}}{(2+2e^{2t})^{3/2}}+\frac{e^t}{\sqrt{2+2e^{2t}}}\right)^2+\frac{8\,(e^{2t})^2}{(2+2e^{2t})^3}}}{\sqrt{2+2e^{2t}}} \tag{17}$$

simplify(*c*) assuming *t* :: *real*

$$\frac{e^t}{(1+e^{2t})\sqrt{2+2e^{2t}}} \tag{18}$$

subs(*t* = 0, **(18)**)

$$\frac{e^0}{(1+e^0)\sqrt{2+2e^0}} \tag{19}$$

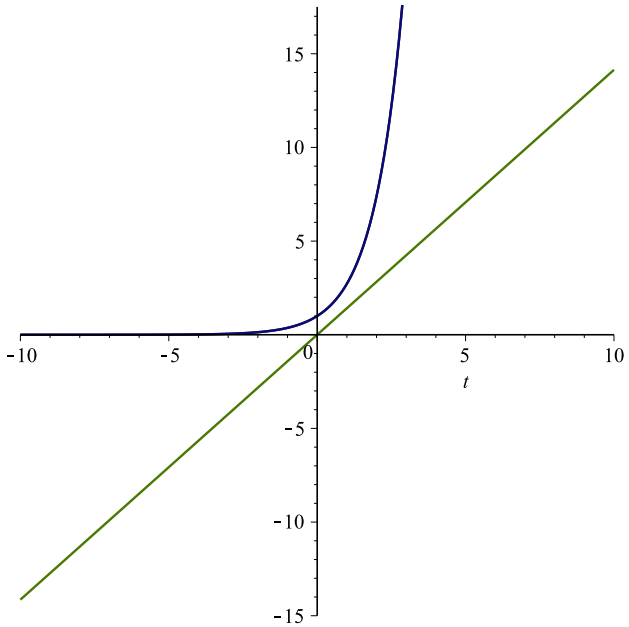
tor := *Torsion*(*r1*, *t*)

$$0 \tag{20}$$

subs(*t* = 0, *tor*)

$$0 \tag{21}$$

plot(*r1*)



Task 10

$r2 := \langle a \cdot \cos(t), a \cdot \sin(t), t \rangle$

$$(a\cos(t))e_x + (a\sin(t))e_y + (t)e_z \quad \textbf{(22)}$$

$d1 := diff(r2, t)$

$$-a\sin(t)e_x + (a\cos(t))e_y + e_z \quad \textbf{(23)}$$

$d2 := diff(diff(r2, t), t)$

$$-a\cos(t)e_x - a\sin(t)e_y \quad \textbf{(24)}$$

$CrossProduct(d1, d2)$

$$(a\sin(t))e_x - a\cos(t)e_y + (a^2\sin(t)^2 + a^2\cos(t)^2)e_z \quad \textbf{(25)}$$

$CrossProduct(CrossProduct(d1, d2), d1)$

$$(-a\cos(t) - (a^2\sin(t)^2 + a^2\cos(t)^2)\,a\cos(t))e_x + (-a\sin(t) - (a^2\sin(t)^2 + a^2\cos(t)^2)\,a\sin(t))e_y \quad \textbf{(26)}$$

$simplify(\textbf{(26)})$

$$-a\cos(t)\,(a^2+1)e_x - a\sin(t)\,(a^2+1)e_y \quad \textbf{(27)}$$

$\alpha := \langle 1, -1, 3 \rangle$

$$e_x - e_y + 3e_z \tag{28}$$

DotProduct(**(27)**, α)

$$-a\cos(t)\left(a^2+1\right) + a\sin(t)\left(a^2+1\right) \tag{29}$$

solve(**(29)** = 0)

$$\{a=0,\ t=t\},\ \left\{a=a,\ t=\frac{1}{4}\,\pi\right\},\ \{a=\mathrm{I},\ t=t\},\ \{a=-\mathrm{I},\ t=t\} \tag{30}$$

subs($\{a=0,\ t=t\}$, *r2*)

$$(t)e_z \tag{31}$$

subs($\{a=I,\ t=t\}$, *r2*)

$$(\mathrm{I}\cos(t))e_x + (\mathrm{I}\sin(t))e_y + (t)e_z \tag{32}$$

subs($\{a=-I,\ t=t\}$, *r2*)

$$-\mathrm{I}\cos(t)e_x - \mathrm{I}\sin(t)e_y + (t)e_z \tag{33}$$

subs$\left(\left\{a=a,\ t=\frac{\pi}{4}\right\},\ r2\right)$

$$\frac{1}{2}\,a\sqrt{2}\,e_x + \frac{1}{2}\,a\sqrt{2}\,e_y + \frac{1}{4}\,\pi e_z \tag{34}$$