Test 1, Anastasiia Yelchaninova

$with(LinearAlgebra):$
$with(linalg):$

Task 1

$a_1 := \langle 1, 2, 2, 3 \rangle$

$$\begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix} \tag{1}$$

$b_1 := \langle 3, 1, 5, 1 \rangle$

$$\begin{bmatrix} 3 \\ 1 \\ 5 \\ 1 \end{bmatrix} \tag{2}$$

$DotProduct(a_1, b_1)$

$$18 \tag{3}$$

$VectorAngle(a_1, b_1)$

$$\frac{1}{4}\pi \tag{4}$$

Task 2

$a_2 := \langle 2, -3, 2 \rangle$

$$\begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix} \tag{5}$$

$b_2 := \langle -3, 1, 2 \rangle$

$$\begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix} \tag{6}$$

$c_2 := \langle 1, 2, 3 \rangle$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \tag{7}$$

$p_1 := CrossProduct(a_2, CrossProduct(b_2, c_2))$

$$\begin{bmatrix} -1 \\ 12 \\ 19 \end{bmatrix} \quad (8)$$

$p_2 := CrossProduct(CrossProduct(a_2, b_2), c_2)$

$$\begin{bmatrix} -16 \\ 17 \\ -6 \end{bmatrix} \quad (9)$$

$p_3 := CrossProduct(CrossProduct(b_2, c_2), a_2)$

$$\begin{bmatrix} 1 \\ -12 \\ -19 \end{bmatrix} \quad (10)$$

$p_4 := CrossProduct(CrossProduct(c_2, a_2), b_2)$

$$\begin{bmatrix} 15 \\ -5 \\ 25 \end{bmatrix}$$

$p_2 + p_3 + p_4$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (12)$$

Task 3

$a_1 := \langle 2, 1, 3, -1 \rangle$

$$\begin{bmatrix} 2 \\ 1 \\ 3 \\ -1 \end{bmatrix} \quad (13)$$

$a_2 := \langle 7, 4, 3, -3 \rangle$

$$\begin{bmatrix} 7 \\ 4 \\ 3 \\ -3 \end{bmatrix} \quad (14)$$

$a_3 := \langle 1, 1, -6, 0 \rangle$

$$\begin{bmatrix} 1 \\ 1 \\ -6 \\ 0 \end{bmatrix} \tag{15}$$

$a_4 := \langle 5, 3, 0, 4 \rangle$

$$\begin{bmatrix} 5 \\ 3 \\ 0 \\ 4 \end{bmatrix} \tag{16}$$

$b := \mathit{Basis}([a_1, a_2, a_3, a_4])$

$$\left[\begin{bmatrix} 2 \\ 1 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 7 \\ 4 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 0 \\ 4 \end{bmatrix}\right] \tag{17}$$

As you can see, the system of vectors {_1,_2,_3,_4} is not a basis. The basis is the system of vectors {_1,_2,_4}.

$\mathit{GramSchmidt}([a_1, a_2, a_4], \mathit{normalized})$

$$\left[\begin{bmatrix} \frac{2}{15}\sqrt{15} \\ \frac{1}{15}\sqrt{15} \\ \frac{1}{5}\sqrt{15} \\ -\frac{1}{15}\sqrt{15} \end{bmatrix}, \begin{bmatrix} \frac{3}{23}\sqrt{23} \\ \frac{2}{23}\sqrt{23} \\ -\frac{3}{23}\sqrt{23} \\ -\frac{1}{23}\sqrt{23} \end{bmatrix}, \begin{bmatrix} \frac{91}{105915}\sqrt{105915} \\ \frac{53}{105915}\sqrt{105915} \\ \frac{8}{35305}\sqrt{105915} \\ \frac{1}{345}\sqrt{105915} \end{bmatrix}\right] \tag{18}$$

Task 4

$A := \mathit{Matrix}(4, 4, [[5, 7, -3, -4], [7, 6, -4, -5], [6, 4, -3, -2], [8, 5, -6, -1]])$

$$\begin{bmatrix} 5 & 7 & -3 & -4 \\ 7 & 6 & -4 & -5 \\ 6 & 4 & -3 & -2 \\ 8 & 5 & -6 & -1 \end{bmatrix}$$

$B := \mathit{Matrix}(4, 4, [[1, 2, 3, 4], [2, 3, 4, 5], [1, 3, 5, 7], [2, 4, 6, 8]])$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \end{bmatrix} \quad \mathbf{(20)}$$

multiply(A, B)

$$\begin{bmatrix} 8 & 6 & 4 & 2 \\ 5 & 0 & -5 & -10 \\ 7 & 7 & 7 & 7 \\ 10 & 9 & 8 & 7 \end{bmatrix} \quad \mathbf{(21)}$$

multiply(B, A)

$$\begin{bmatrix} 69 & 51 & -44 & -24 \\ 95 & 73 & -60 & -36 \\ 112 & 80 & -72 & -36 \\ 138 & 102 & -88 & -48 \end{bmatrix} \quad \mathbf{(22)}$$

det(A)

$$-92 \quad \mathbf{(23)}$$

det(B)

$$0 \quad \mathbf{(24)}$$

Task 5

$A := Matrix(4, 4, [[1, 2, 3, 4], [2, 3, 1, 2], [1, 1, 1, -1], [1, 0, -2, -6]])$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 2 \\ 1 & 1 & 1 & -1 \\ 1 & 0 & -2 & -6 \end{bmatrix} \quad \mathbf{(25)}$$

det(A)

$$-1 \quad \mathbf{(26)}$$

MatrixInverse(A)

$$\begin{bmatrix} 22 & -6 & -26 & 17 \\ -17 & 5 & 20 & -13 \\ -1 & 0 & 2 & -1 \\ 4 & -1 & -5 & 3 \end{bmatrix} \quad \mathbf{(27)}$$

minor$(A, 3, 2)$

$$\begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 1 & -2 & -6 \end{bmatrix} \quad \mathbf{(28)}$$

$Minor(A, 3, 2)$

$$20 \tag{29}$$

$At := Transpose(A)$

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 3 & 1 & 0 \\ 3 & 1 & 1 & -2 \\ 4 & 2 & -1 & -6 \end{bmatrix} \tag{30}$$

Task 6

$C := Matrix(5, 5, [[-6, 4, 8, -1, 6], [-5, 2, 4, 1, 3], [7, 2, 4, 1, 3], [2, 4, 8, -7, 6], [3, 2, 4, -5, 3]])$

$$\begin{bmatrix} -6 & 4 & 8 & -1 & 6 \\ -5 & 2 & 4 & 1 & 3 \\ 7 & 2 & 4 & 1 & 3 \\ 2 & 4 & 8 & -7 & 6 \\ 3 & 2 & 4 & -5 & 3 \end{bmatrix}$$

$Rank(C)$

$$3 \tag{32}$$

$ffgausselim(C)$

$$\begin{bmatrix} -6 & 4 & 8 & -1 & 6 \\ 0 & 8 & 16 & -11 & 12 \\ 0 & 0 & 0 & 72 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{33}$$

$gaussjord(C)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & \frac{3}{2} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{34}$$

Task 7

$A := Matrix(5, 5, [[5, 4, 3, 2, 1], [4, 8, 6, 4, 2], [3, 6, 9, 6, 3], [2, 4, 6, 8, 4], [1, 2, 3, 4, 5]])$

$$\begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ 4 & 8 & 6 & 4 & 2 \\ 3 & 6 & 9 & 6 & 3 \\ 2 & 4 & 6 & 8 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$

$eigenvalues(A)$

$$6, 2, 3, 12 - 6\sqrt{3}, 12 + 6\sqrt{3} \tag{36}$$

$PA := charpoly(A, \lambda)$

$$\lambda^5 - 35\lambda^4 + 336\lambda^3 - 1296\lambda^2 + 2160\lambda - 1296 \tag{37}$$

$simplify(combine(subs(\lambda = A, PA)))$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{38}$$

Task 8

$T := Matrix(3, 3, [[4, 2, -5], [6, 4, -9], [5, 3, -7]])$

$$\begin{bmatrix} 4 & 2 & -5 \\ 6 & 4 & -9 \\ 5 & 3 & -7 \end{bmatrix} \tag{39}$$

$et := exponential(T)$

$$\begin{bmatrix} -1 + 3\,e & e & -3\,e + 1 \\ 3\,e & 3 + e & -3\,e - 3 \\ -1 + 3\,e & e + 1 & -3\,e \end{bmatrix} \tag{40}$$

$det(et)$

$$e \tag{41}$$

$eigenvalues(et)$

$$e, 1, 1 \tag{42}$$

$eigenvectors(et)$

$$[e, 1, \{r\}], [1, 2, \{r\}] \tag{43}$$

$kernel(T)$

$$\left\{\begin{bmatrix} 1 & 3 & 2 \end{bmatrix}\right\} \tag{44}$$

Task 9

$U := Matrix(4, 4, [[3, -4, 0, 2], [4, -5, -2, 4], [0, 0, 3, -2], [0, 0, 2, -1]])$

$$\begin{bmatrix} 3 & -4 & 0 & 2 \\ 4 & -5 & -2 & 4 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 2 & -1 \end{bmatrix}$$

eigenvectors(U)

$$\left[-1, 2, \left\{\left[\begin{array}{cccc} 1 & 1 & 0 & 0 \end{array}\right]\right\}\right], \left[1, 2, \left\{\left[\begin{array}{cccc} 1 & 1 & 1 & 1 \end{array}\right]\right\}\right] \tag{46}$$

eigenvalues(U)

$$1, -1, 1, -1 \tag{47}$$

charpoly(U, λ)

$$\lambda^4 - 2\lambda^2 + 1 \tag{48}$$

minpoly(U, λ)

$$\lambda^4 - 2\lambda^2 + 1 \tag{49}$$

$V1 := \langle 3, -4, 0, 2 \rangle$:
$V2 := \langle 4, -5, -2, 4 \rangle$:
$V3 := \langle 0, 0, 3, -2 \rangle$:
$V4 := \langle 0, 0, 2, -1 \rangle$:
JordanForm$(\langle V1 | V2 | V3 | V4 \rangle)$

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{50}$$

Task 10

$A := Matrix(3, 3, [[1, 2, -3], [3, 2, -4], [2, -1, 0]])$

$$\begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{bmatrix} \tag{51}$$

$B := Matrix(3, 3, [[1, -3, 0], [10, 2, 7], [10, 7, 8]])$

$$\begin{bmatrix} 1 & -3 & 0 \\ 10 & 2 & 7 \\ 10 & 7 & 8 \end{bmatrix} \tag{52}$$

LinearSolve(A, B)

$$\begin{bmatrix} 6 & 4 & 5 \\ 2 & 1 & 2 \\ 3 & 3 & 3 \end{bmatrix} \tag{53}$$

Task 11

$a_1 := \langle 1, 2, 0, 1\rangle$

$$\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \tag{54}$$

$a_2 := \langle 1, 1, 1, 0\rangle$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \tag{55}$$

$b_1 := \langle 1, 0, 1, 0\rangle$

$$\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \tag{56}$$

$b_2 := \langle 1, 3, 0, 1\rangle$

$$\begin{bmatrix} 1 \\ 3 \\ 0 \\ 1 \end{bmatrix} \tag{57}$$

$\mathit{SumBasis}([[a_1, a_2], [b_1, b_2]])$

$$\left[\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}\right] \tag{58}$$

$\mathit{IntersectionBasis}([[a_1, a_2], [b_1, b_2]])$

$$\left[\begin{bmatrix} 2 \\ 3 \\ 1 \\ 1 \end{bmatrix}\right] \tag{59}$$

Task 12

$A := \mathit{Matrix}(2, 3, [[1, 2, 3], [2, -1, 1]])$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \end{bmatrix} \tag{60}$$

$B := Matrix(2, 3, [[0, 1, -2], [3, -1, 5]])$

$$\begin{bmatrix} 0 & 1 & -2 \\ 3 & -1 & 5 \end{bmatrix} \tag{61}$$

$\langle A|B \rangle$

$$\begin{bmatrix} 1 & 2 & 3 & 0 & 1 & -2 \\ 2 & -1 & 1 & 3 & -1 & 5 \end{bmatrix} \tag{62}$$

$\langle B|A \rangle$

$$\begin{bmatrix} 0 & 1 & -2 & 1 & 2 & 3 \\ 3 & -1 & 5 & 2 & -1 & 1 \end{bmatrix} \tag{63}$$

$\langle A, B \rangle$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 0 & 1 & -2 \\ 3 & -1 & 5 \end{bmatrix} \tag{64}$$

$\langle B, A \rangle$

$$\begin{bmatrix} 0 & 1 & -2 \\ 3 & -1 & 5 \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{bmatrix} \tag{65}$$