# Test 2, part 1
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$with(VectorCalculus):$
$with(student):$
$with(linalg):$

### Task 1

$f_1 := \arctan\left(\frac{x+y}{1-x\,y}\right)$

$$\arctan\left(\frac{x+y}{-x\,y+1}\right) \tag{1}$$

$f_{xx} := simplify(diff(f_1, x, x))$

$$-\frac{2\,x}{\left(x^2+1\right)^2} \tag{2}$$

$f_{xy} := simplify(diff(f_1, x, y))$

$$0 \tag{3}$$

$f_{yy} := simplify(diff(f_1, y, y))$

$$-\frac{2\,y}{\left(y^2+1\right)^2} \tag{4}$$

### Task 2

$extrema\left(y^2+4\,z^2-4\,y\,z-2\,x\,z-2\,x\,y, \{2\,x^2+3\,y^2+6\,z^2=1\}, \{x, y, z\}\right)$

$$\left\{1, -\frac{1}{2}\right\} \tag{5}$$

### Task 3

$with(simplex):$
$f_3 := x+y+z$

$$x+y+z \tag{6}$$

$cond := \{x+y \le 2, z \le 1\}$

$$\{z \le 1, x+y \le 2\} \tag{7}$$

$maximize(f_3, cond)$

$$\{x=2, y=0, z=1\} \tag{8}$$

### Task 4

$Tripleint\left(\frac{\ln(z-x-y)}{(x-\mathrm{e})\cdot(x+y-\mathrm{e})}, z=\mathrm{e}..x+y+\mathrm{e}, y=0..\mathrm{e}-x-1, x=0..\mathrm{e}-1\right)$

$$\int_0^{\mathrm{e}-1}\int_0^{\mathrm{e}-x-1}\int_{\mathrm{e}}^{x+y+\mathrm{e}} \frac{\ln(z-x-y)}{(x-\mathrm{e})\,(x+y-\mathrm{e})}\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x \tag{9}$$

$value(\mathbf{(9)})$

$$2\,\mathrm{e}-5 \tag{10}$$

### Task 5

$gr := grad(x\,y - z^2, [x, y, z])$

$$\left[\begin{array}{ccc} y & x & -2\,z \end{array}\right] \tag{11}$$

The derivative of the function along the direction is equal to the scalar product of the gradient of this function on the normalized direction vector, so

$vect := \langle 1, 1, 0\rangle$

$$\left[\begin{array}{c} 1 \\ 1 \\ 0 \end{array}\right] \tag{12}$$

$vect\_bis := normalize(vect)$

$$\left[\begin{array}{ccc} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} & 0 \end{array}\right] \tag{13}$$

$dotprod(gr, vect\_bis)$

$$\frac{1}{2}\,y\sqrt{2} + \frac{1}{2}\,x\sqrt{2} \tag{14}$$

**Task 6**

$\Omega := \langle 0, 0, \omega\rangle$

$$\left[\begin{array}{c} 0 \\ 0 \\ \omega \end{array}\right] \tag{15}$$

$r := \langle x, y, z\rangle$

$$\left[\begin{array}{c} x \\ y \\ z \end{array}\right] \tag{16}$$

$V := crossprod(\Omega, r)$

$$\left[\begin{array}{ccc} -\omega\,y & \omega\,x & 0 \end{array}\right] \tag{17}$$

$w := crossprod(\Omega, V)$

$$\left[\begin{array}{ccc} -\omega^2\,x & -\omega^2\,y & 0 \end{array}\right] \tag{18}$$

$diverge(V, [x, y, z])$

$$0 \tag{19}$$

$curl(V, [x, y, z])$

$$\left[\begin{array}{ccc} 0 & 0 & 2\,\omega \end{array}\right] \tag{20}$$

$diverge(w, [x, y, z])$

$$-2\,\omega^2 \tag{21}$$

$SetCoordinates(cartesian_{x, y, z})$

$$cartesian_{x, y, z} \tag{22}$$

$V_1 := VectorField(\langle -\omega\, y, \omega\, x, 0\rangle)$

$$-\omega\, y\bar{e}_x + (\omega\, x)\bar{e}_y \tag{23}$$

$Laplacian(V)$

$$0 \tag{24}$$

**Task 7**

$u := \cos(a\,x + b\,y - \omega\,t)$

$$\cos(a\,x + b\,y - \omega\,t) \tag{25}$$

$u_{tt} := diff(u, t, t)$

$$-\cos(a\,x + b\,y - \omega\,t)\,\omega^2 \tag{26}$$

$ur := \frac{1}{c^2} \cdot u_{tt}$

$$-\frac{\cos(a\,x + b\,y - \omega\,t)\,\omega^2}{c^2} \tag{27}$$

$ul := laplacian(u, [x, y, z])$

$$-\cos(a\,x + b\,y - \omega\,t)\,a^2 - \cos(a\,x + b\,y - \omega\,t)\,b^2 \tag{28}$$

$simplify(ul = ur)$

$$-\cos(a\,x + b\,y - \omega\,t)\,(a^2 + b^2) = -\frac{\cos(a\,x + b\,y - \omega\,t)\,\omega^2}{c^2} \tag{29}$$

Assuming $\cos(a\,x + b\,y - \omega\,t)$ is not equal to 0:

$a^2 + b^2 = \frac{\omega^2}{c^2}$

$$a^2 + b^2 = \frac{\omega^2}{c^2} \tag{30}$$

$solve(\textbf{(30)}, \omega)$

$$\sqrt{a^2 + b^2}\,c, -\sqrt{a^2 + b^2}\,c \tag{31}$$

**Task 8**

$Jacobian([r\cos(\phi)\sin(\theta), r\sin(\phi)\sin(\theta), r\cos(\theta)], [r, \phi, \theta], 'determinant')$

$$\left[\begin{array}{ccc} \cos(\phi)\sin(\theta) & -r\sin(\phi)\sin(\theta) & r\cos(\phi)\cos(\theta) \\ \sin(\phi)\sin\theta & r\cos(\phi)\sin\theta & r\sin(\phi)\sin \\ \cos(\theta) & 0 & -r\sin(\theta) \end{array}\right], -\cos(\phi)^2\sin(\theta)^2 r^2 \sin\theta \tag{32}$$

$$-\cos(\phi)^2\cos(\theta)^2 r^2 \sin\theta - \sin(\phi)^2 \sin\theta\, r^2 \sin(\theta)^2 - \sin(\theta)\sin(\phi)^2\cos(\theta)\, r^2 \sin$$

**Task 9**

$sum\left(\frac{1}{n\cdot(n+1)\cdot(n+2)}, n = 1\,..infinity\right)$

$$\frac{1}{4} \tag{33}$$

$$sum\left(\frac{n!}{n^n}, n = 1..5\right)$$

$$\frac{333787}{180000} \tag{34}$$