

Homework 4, Anastasiia Yelchaninova

with(LinearAlgebra) :

with(linalg) :

Task 1

$$a1 := \langle 2, 1, 3, -1 \rangle$$

$$\begin{bmatrix} 2 \\ 1 \\ 3 \\ -1 \end{bmatrix} \quad (1)$$

$$a2 := \langle 7, 4, 3, -3 \rangle$$

$$\begin{bmatrix} 7 \\ 4 \\ 3 \\ -3 \end{bmatrix} \quad (2)$$

$$a3 := \langle 1, 1, -6, 0 \rangle$$

$$\begin{bmatrix} 1 \\ 1 \\ -6 \\ 0 \end{bmatrix} \quad (3)$$

$$a4 := \langle 5, 3, 0, 4 \rangle$$

$$\begin{bmatrix} 5 \\ 3 \\ 0 \\ 4 \end{bmatrix} \quad (4)$$

$$b := \text{Basis}([a1, a2, a3, a4])$$

$$\left[\begin{bmatrix} 2 \\ 1 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 7 \\ 4 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 0 \\ 4 \end{bmatrix} \right] \quad (5)$$

, $\{1, 2, 3, 4\}$. $\{1, 2, 4\}$.
GramSchmidt($[a1, a2, a4], \text{normalized}$)

$$\left[\left[\begin{array}{c} \frac{2}{15} \sqrt{15} \\ \frac{1}{15} \sqrt{15} \\ \frac{1}{5} \sqrt{15} \\ -\frac{1}{15} \sqrt{15} \end{array} \right], \left[\begin{array}{c} \frac{3}{23} \sqrt{23} \\ \frac{2}{23} \sqrt{23} \\ -\frac{3}{23} \sqrt{23} \\ -\frac{1}{23} \sqrt{23} \end{array} \right], \left[\begin{array}{c} \frac{91}{105915} \sqrt{105915} \\ \frac{53}{105915} \sqrt{105915} \\ \frac{8}{35305} \sqrt{105915} \\ \frac{1}{345} \sqrt{105915} \end{array} \right] \right] \quad (6)$$

Task 2

$$A := \text{Matrix}(4, 4, [[5, 7, -3, -4], [7, 6, -4, -5], [6, 4, -3, -2], [8, 5, -6, -1]])$$

$$\begin{bmatrix} 5 & 7 & -3 & -4 \\ 7 & 6 & -4 & -5 \\ 6 & 4 & -3 & -2 \\ 8 & 5 & -6 & -1 \end{bmatrix}$$

$$B := \text{Matrix}(4, 4, [[1, 2, 3, 4], [2, 3, 4, 5], [1, 3, 5, 7], [2, 4, 6, 8]])$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \end{bmatrix}$$

(8)

$$\text{multiply}(A, B)$$

$$\begin{bmatrix} 8 & 6 & 4 & 2 \\ 5 & 0 & -5 & -10 \\ 7 & 7 & 7 & 7 \\ 10 & 9 & 8 & 7 \end{bmatrix}$$

(9)

$$\text{multiply}(B, A)$$

$$\begin{bmatrix} 69 & 51 & -44 & -24 \\ 95 & 73 & -60 & -36 \\ 112 & 80 & -72 & -36 \\ 138 & 102 & -88 & -48 \end{bmatrix}$$

(10)

$$\det(A)$$

$$-92$$

(11)

$$\det(B)$$

$$0 \quad (12)$$

Task 3

$$A := \text{Matrix}(4, 4, [[1, 2, 3, 4], [2, 3, 1, 2], [1, 1, 1, -1], [1, 0, -2, -6]])$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 2 \\ 1 & 1 & 1 & -1 \\ 1 & 0 & -2 & -6 \end{bmatrix} \quad (13)$$

$$\det(A)$$

$$-1 \quad (14)$$

$$\text{MatrixInverse}(A)$$

$$\begin{bmatrix} 22 & -6 & -26 & 17 \\ -17 & 5 & 20 & -13 \\ -1 & 0 & 2 & -1 \\ 4 & -1 & -5 & 3 \end{bmatrix} \quad (15)$$

$$\text{minor}(A, 3, 2)$$

$$\begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 1 & -2 & -6 \end{bmatrix} \quad (16)$$

$$\text{Minor}(A, 3, 2)$$

$$20 \quad (17)$$

Task 4

$$At := \text{Transpose}(A)$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 3 & 1 & 0 \\ 3 & 1 & 1 & -2 \\ 4 & 2 & -1 & -6 \end{bmatrix} \quad (18)$$

Task 5

$$C := \text{Matrix}(5, 5, [[-6, 4, 8, -1, 6], [-5, 2, 4, 1, 3], [7, 2, 4, 1, 3], [2, 4, 8, -7, 6], [3, 2, 4, -5, 3]])$$

$$\begin{bmatrix} -6 & 4 & 8 & -1 & 6 \\ -5 & 2 & 4 & 1 & 3 \\ 7 & 2 & 4 & 1 & 3 \\ 2 & 4 & 8 & -7 & 6 \\ 3 & 2 & 4 & -5 & 3 \end{bmatrix}$$

ffgausselim(C)

$$\begin{bmatrix} -6 & 4 & 8 & -1 & 6 \\ 0 & 8 & 16 & -11 & 12 \\ 0 & 0 & 0 & 72 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(20)

gaussjord(C)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & \frac{3}{2} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(21)

Task 6

$A := \text{Matrix}(5, 5, [[5, 4, 3, 2, 1], [4, 8, 6, 4, 2], [3, 6, 9, 6, 3], [2, 4, 6, 8, 4], [1, 2, 3, 4, 5]])$

$$\begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ 4 & 8 & 6 & 4 & 2 \\ 3 & 6 & 9 & 6 & 3 \\ 2 & 4 & 6 & 8 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$

eigenvalues(A)

$$6, 2, 3, 12 - 6\sqrt{3}, 12 + 6\sqrt{3}$$

(23)

$PA := \text{charpoly}(A, \lambda)$

$$\lambda^5 - 35\lambda^4 + 336\lambda^3 - 1296\lambda^2 + 2160\lambda - 1296$$

(24)

simplify(*combine*(*subs*($\lambda=A, PA$)))

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (25)$$

Task 7

$$T := \text{Matrix}(3, 3, [[4, 2, -5], [6, 4, -9], [5, 3, -7]])$$

$$\begin{bmatrix} 4 & 2 & -5 \\ 6 & 4 & -9 \\ 5 & 3 & -7 \end{bmatrix} \quad (26)$$

$$et := \text{exponential}(T)$$

$$\begin{bmatrix} -1 + 3e & e & -3e + 1 \\ 3e & 3 + e & -3e - 3 \\ -1 + 3e & e + 1 & -3e \end{bmatrix} \quad (27)$$

$$\det(et)$$

$$e \quad (28)$$

$$\text{eigenvalues}(et)$$

$$e, 1, 1 \quad (29)$$

$$\text{eigenvectors}(et)$$

$$[e, 1, \{r\}], [1, 2, \{r\}] \quad (30)$$

$$\text{kernel}(T)$$

$$\left\{ \begin{bmatrix} 1 & 3 & 2 \end{bmatrix} \right\} \quad (31)$$

Task 8

$$U := \text{Matrix}(4, 4, [[3, -4, 0, 2], [4, -5, -2, 4], [0, 0, 3, -2], [0, 0, 2, -1]])$$

$$\begin{bmatrix} 3 & -4 & 0 & 2 \\ 4 & -5 & -2 & 4 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 2 & -1 \end{bmatrix}$$

$$\text{eigenvectors}(U)$$

$$[-1, 2, \left\{ \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} \right\}], [1, 2, \left\{ \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \right\}] \quad (33)$$

$$\text{eigenvalues}(U)$$

$$\text{charpoly}(U, \lambda) \quad 1, -1, 1, -1 \quad (34)$$

$$\text{minpoly}(U, \lambda) \quad \lambda^4 - 2\lambda^2 + 1 \quad (35)$$

$$\text{JordanForm}(\langle V1|V2|V3|V4 \rangle) \quad \lambda^4 - 2\lambda^2 + 1 \quad (36)$$

$$\begin{aligned} V1 &:= \langle 3, -4, 0, 2 \rangle : \\ V2 &:= \langle 4, -5, -2, 4 \rangle : \\ V3 &:= \langle 0, 0, 3, -2 \rangle : \\ V4 &:= \langle 0, 0, 2, -1 \rangle : \\ \text{JordanForm}(\langle V1|V2|V3|V4 \rangle) & \end{aligned} \quad \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (37)$$

Task 9

$$A := \text{Matrix}(3, 3, [[1, 2, -3], [3, 2, -4], [2, -1, 0]]) \quad \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{bmatrix} \quad (38)$$

$$B := \text{Matrix}(3, 3, [[1, -3, 0], [10, 2, 7], [10, 7, 8]]) \quad \begin{bmatrix} 1 & -3 & 0 \\ 10 & 2 & 7 \\ 10 & 7 & 8 \end{bmatrix} \quad (39)$$

$$\text{LinearSolve}(A, B) \quad \begin{bmatrix} 6 & 4 & 5 \\ 2 & 1 & 2 \\ 3 & 3 & 3 \end{bmatrix} \quad (40)$$