Homework 2, Anastasiia Yelchaninova

Task 1

$f := (x, y) \to \left(\frac{\arctan(x+y)}{\arctan(x-y)} \right)^2$

$$(x, y) \to \frac{\arctan(x+y)^2}{\arctan(x-y)^2} \tag{1}$$

$f(1, 0)$

$$1 \tag{2}$$

$f\left(\frac{1+\sqrt{3}}{2}, \frac{1-\sqrt{3}}{2} \right)$

$$\frac{9}{16} \tag{3}$$

Task 2

$f := \frac{x^3 y^2 - x^2 y^3}{(x\, y)^5}$

$$\frac{x^3 y^2 - x^2 y^3}{x^5 y^5} \tag{4}$$

$subs\left(\left\{ x = a, y = \frac{1}{a} \right\}, f \right)$

$$a - \frac{1}{a} \tag{5}$$

Task 3

$\_EnvExplicit := true :$

$s := solve\left(\left\{ x^2 - 5\, x\, y + 6\, y^2 = 0,\ x^2 + y^2 = 10 \right\}, \{x, y\} \right)$

$$\left\{ x = 2\sqrt{2}, y = \sqrt{2} \right\}, \left\{ x = -2\sqrt{2}, y = -\sqrt{2} \right\}, \{ x = 3, y = 1 \}, \{ x = -3, y = -1 \} \tag{6}$$

Task 4

$\_EnvAllSolutions := true :$

$solve\left(\sin(x)^4 - \cos(x)^4 = \frac{1}{2}, x \right)$

$$\frac{2}{3}\pi + 2\pi\_Z1\sim,\ -\frac{2}{3}\pi + 2\pi\_Z1\sim,\ \frac{1}{3}\pi + 2\pi\_Z2\sim,\ -\frac{1}{3}\pi + 2\pi\_Z2\sim \tag{7}$$

Task 5

$solve\left(e^x = 2(1-x)^2, x \right)$

$$-2\,\mathrm{LambertW}\left(\_Z19\sim, -\frac{1}{4}\sqrt{2}\, e^{\frac{1}{2}} \right) + 1,\ -2\,\mathrm{LambertW}\left(\_Z20\sim, \frac{1}{4}\sqrt{2}\, e^{\frac{1}{2}} \right) + 1 \tag{8}$$

$fsolve\left(e^x = 2(1-x)^2, x \right)$

$$0.2133086343 \quad (9)$$

Task 6

$exp\_l := solve\left(2\ln(x)^2 - \ln(x) < 1, \ln(x)\right)$

$$\left\{-\frac{1}{2} < \ln(x), \ln(x) < 1\right\}$$

$solve(exp\_l, \{x\});$

$$\left\{x < \mathrm{e}, \frac{1}{\mathrm{e}^{\frac{1}{2}}} < x\right\} \quad (11)$$

Task 7

$exp7 := \{5\,x\,y = 5, 4\,x + y = -9\}$

$$\{5\,x\,y = 5, 4\,x + y = -9\} \quad (12)$$

$convert(\{solve(exp7)\}, radical)$

$$\left\{\left\{x = -\frac{9}{8} - \frac{1}{8}\sqrt{65}, y = -\frac{9}{2} + \frac{1}{2}\sqrt{65}\right\}, \left\{x = -\frac{9}{8} + \frac{1}{8}\sqrt{65}, y = -\frac{9}{2} - \frac{1}{2}\sqrt{65}\right\}\right\} \quad (13)$$

Task 8

$exp8 := \dfrac{1}{x^4+1} = \dfrac{a\,x+b}{x^2 - x\cdot\sqrt{2} + 1} + \dfrac{c\,x+d}{x^2 + x\cdot\sqrt{2} + 1}$

$$\frac{1}{x^4+1} = \frac{a\,x+b}{x^2 - x\sqrt{2} + 1} + \frac{c\,x+d}{x^2 + x\sqrt{2} + 1} \quad (14)$$

$c1 := numer(op(1, op(2, exp8)))$

$$-a\,x - b \quad (15)$$

$c2 := denom(op(1, op(2, exp8)))$

$$x\sqrt{2} - x^2 - 1 \quad (16)$$

$c3 := numer(op(2, op(2, exp8)))$

$$c\,x + d \quad (17)$$

$c4 := denom(op(2, op(2, exp8)))$

$$x^2 + x\sqrt{2} + 1 \quad (18)$$

$exp82 := numer(op(1, exp8)) = c1\cdot c4 + c3\cdot c2$

$$1 = (-a\,x - b)\left(x^2 + x\sqrt{2} + 1\right) + \left(x\sqrt{2} - x^2 - 1\right)(c\,x + d) \quad (19)$$

$exp83 := sort(simplify(exp82))$

$$1 = -a\,x^3 - c\,x^3 - \sqrt{2}\,a\,x^2 - b\,x^2 + \sqrt{2}\,c\,x^2 - d\,x^2 - a\,x - \sqrt{2}\,b\,x - c\,x + \sqrt{2}\,d\,x - b - d \quad (20)$$

$cc := lhs(exp83)$

$$1 \quad (21)$$

$cc2 := coeffs(rhs(exp83), x)$

$$-b - d, -c + \sqrt{2}\,d - a - \sqrt{2}\,b, -a - c, \sqrt{2}\,c - d - \sqrt{2}\,a - b \quad (22)$$

$exp810 := cc = cc2[1]$

$$1 = -b - d$$

$exp811 := coeff(op(1, exp83), x) = coeff(rhs(exp83), x)$

$$0 = -c + \sqrt{2}\, d - a - \sqrt{2}\, b \qquad \textbf{(24)}$$

$exp812 := coeff\left(op(1, exp83), x^2\right) = coeff\left(rhs(exp83), x^2\right)$

$$0 = \sqrt{2}\, c - d - \sqrt{2}\, a - b \qquad \textbf{(25)}$$

$exp813 := coeff\left(op(1, exp83), x^3\right) = coeff\left(rhs(exp83), x^3\right)$

$$0 = -a - c \qquad \textbf{(26)}$$

$exp\_final := solve(\{exp810, exp811, exp812, exp813\}, \{a, b, c, d\})$

$$\left\{a = \frac{1}{4}\sqrt{2}, b = -\frac{1}{2}, c = -\frac{1}{4}\sqrt{2}, d = -\frac{1}{2}\right\} \qquad \textbf{(27)}$$

Task 11

$rec11 := f(n+1) = f(n) + n + 1$

$$f(n+1) = f(n) + n + 1 \qquad \textbf{(28)}$$

$cond := f(0) = 1$

$$f(0) = 1 \qquad \textbf{(29)}$$

With function *rsolve:*

$ans11 := rsolve(\{rec11, cond\}, f)$

$$-n + (n+1)\left(\frac{1}{2}\, n + 1\right) \qquad \textbf{(30)}$$

$sort(simplify(ans11), n)$

$$\frac{1}{2}\, n^2 + \frac{1}{2}\, n + 1 \qquad \textbf{(31)}$$

Manually:

$op(1, op(2, rec11))$

$$f(n) \qquad \textbf{(32)}$$

$rec111 := op(1, rec11) - op(1, op(2, rec11)) = op(2, op(2, rec11)) + op(3, op(2, rec11))$

$$f(n+1) - f(n) = n + 1 \qquad \textbf{(33)}$$

$rec112 := subs\left(\{op(1, rec11) = \lambda, op(1, op(2, rec11)) = 1, op(2, rec111) = 0\}, rec111\right)$

$$\lambda - 1 = 0 \qquad \textbf{(34)}$$

$char := solve(rec112, \lambda)$

$$1 \qquad \textbf{(35)}$$

$rec113 := A \cdot (char)^n$

$$A \qquad \textbf{(36)}$$

Since 1 is the root of the characteristic equation, so we will look **for a particular solution** for (n+1) as

$partans := C\, n^2 + \mathrm{D}\, n$

$$C\, n^2 + \mathrm{D}\, n \qquad \textbf{(37)}$$

Then:

$rec1111 := C\,(n+1)^2 + \mathrm{D}\,(n+1) - C\, n^2 - \mathrm{D}\, n = rhs(rec111)$

$$C\,(n+1)^2 + \mathrm{D}\,(n+1) - C\, n^2 - \mathrm{D}\, n = n + 1 \qquad \textbf{(38)}$$

$rec1112 := subs(n = 0, rec1111)$

$$C + \mathrm{D} = 1 \qquad \textbf{(39)}$$

$rec1113 := subs(n = 1, rec1111)$

$$3\, C + \mathrm{D} = 2 \qquad \textbf{(40)}$$

$srec1111 := solve(\{rec1112, rec1113\}, \{C, \mathrm{D}\})$

$$\left\{C = \frac{1}{2}, \mathrm{D} = \frac{1}{2}\right\} \tag{41}$$

$rec114 := rec113 + \frac{1}{2}n^2 + \frac{1}{2}n$

$$A + \frac{1}{2}\,n^2 + \frac{1}{2}\,n \tag{42}$$

$rec115 := subs(n = 0, rec114) = 1$

$$A = 1$$

$A1 := solve(rec115, A)$

$$1 \tag{44}$$

$ans11 := sort(simplify(subs(A = A1, rec114)))$

$$\frac{1}{2}\,n^2 + \frac{1}{2}\,n + 1 \tag{45}$$

Let's check the answer:

$simplify\left(\frac{1}{2}(n+1)^2 + \frac{1}{2}\,(n+1) + 1 - ans11\right)$

$$n + 1 \tag{46}$$

$subs(n = 0, ans11)$

$$1 \tag{47}$$

Task 12

$rec12 := f(n+2) = -2\,f(n+1) + 3\,f(n)$

$$f(n+2) = -2\,f(n+1) + 3\,f(n) \tag{48}$$

$rsolve(\{rec12,\ f(0) = -4,\ f(1) = 5\},\ f)$

$$-\frac{9}{4}\,(-3)^n - \frac{7}{4} \tag{49}$$