Final exam

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Of course, both $k1$ and $k2$and min $(k1, k2)$ are ¿ 0. To increase the flow from source to sink (while maintaining a feasible flow), decrease the flow in all of C's backward arcs by min (k1, k2) and increase the flow in all of C's forward arcs by min$(k1, k2)$. This will maintain conservation of flow and ensure that no arc capacity constraints are violated. Because the last arc in C is a forward arc leading into the sink, we have found a new feasible flow and have increased the total flow into the sink by min$(k1, k2)$. We now adjust the flow in the arc $a0$ to maintain conservation of flow. To illustrate Case 2, suppose we have found the feasible flow in Figure 12. For this flow, $(so, 1)R; (so, 2)I; (1, 3)I; (1, 2)I$ and $R; (2, si)R$; and $(3, si)I$.

We begin by labeling arc (so, 2) and node 2 (thus (so, 2) is a forward arc). Then we label arc (1, 2) and node 1. Arc (1, 2) is a backward arc, because node 1 was unlabeled before we labeled arc (1, 2), and arc (1, 2) is in R. Nodes so, 1, and 2 are labeled, so we can label arc (1, 3) and node 3. [Arc (1, 3) is a forward arc, because node 3 has not yet been labeled.] Finally we label arc (3, si) and node si. Arc (3, si) is a forward arc, because node si has not yet been labeled. We have now labeled the sink via the chain $C(so, 2)$ (1, 2) (1, 3) (3, si). With the exception of arc (1, 2), all arcs in the chain are forward arcs. Because $i(so, 2) = 3; i(1, 3) = 4; i(3, si) = 1$; and $r(1, 2) = 2$, we have

min $r(x, y) = 2$ and min $i(x, y) = 1$

Thus, we can increase the flow on all forward arcs in C by 1 and decrease the flow in all backward arcs by 1. The new result, pictured in Figure 13, has increased the flow from source to sink by 1 unit (from 2 to 3). We accomplish this by diverting 1 unit that was transported through the arc (1, 2) to the path 1–3–si. This enabled us to transport an extra unit from source to sink via the path so–2–si. Observe that the concept of a backward arc was needed to find this improved flow. If the sink cannot be labeled, then the current flow is optimal. The proof of this fact relies on the concept of a cut for a network.

Choose any set of nodes V' that contains the sink but does not contain the source. Then the set of arcs (i, j) with i not in V' and j a member of V' is a cut for the network.