From local to global... computationally.

SIAM CSE, Emerging Directions in Computational Topology,

1 March 2021

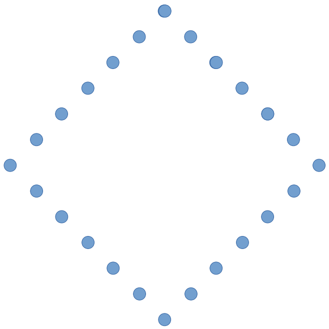
Paweł Dłotko

Dioscuri Centre in TDA, IMPAN.

Integration of data according to their proximity

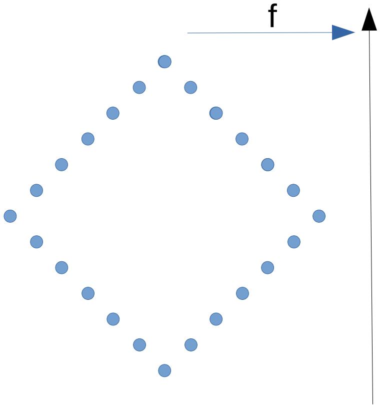
1. Mapper algorithm,
2. Ball Mapper algorithm,
3. their combinations,
4. and new descriptors.

Conventional Mapper algorithm



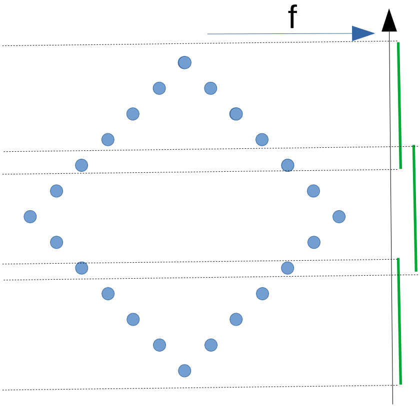
Point cloud X.

Conventional Mapper algorithm



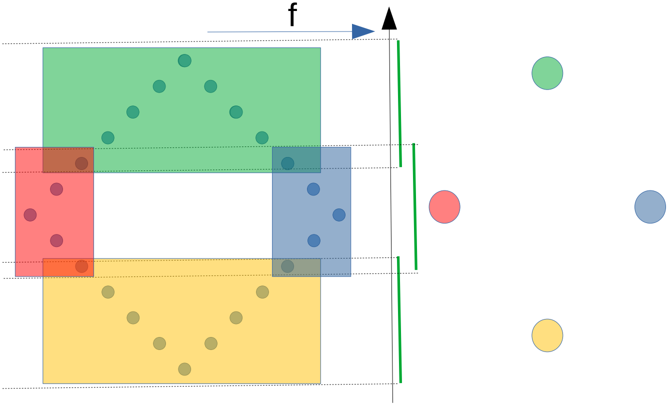
Point cloud X, $f : X \to \mathbb{R}^n$.

Conventional Mapper algorithm



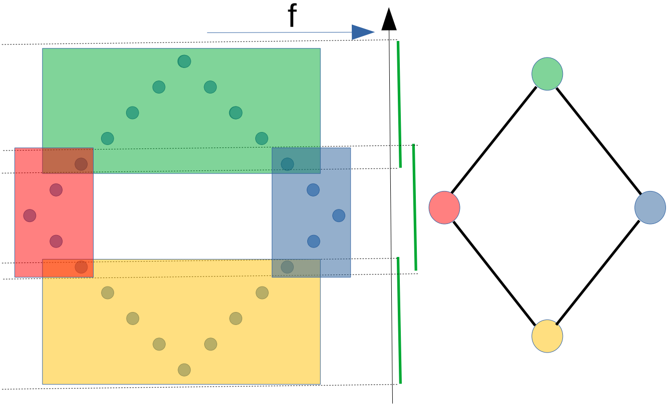
Point cloud X, $f : X \to \mathbb{R}^n$, I - cover of $f(X)$.

Conventional Mapper algorithm



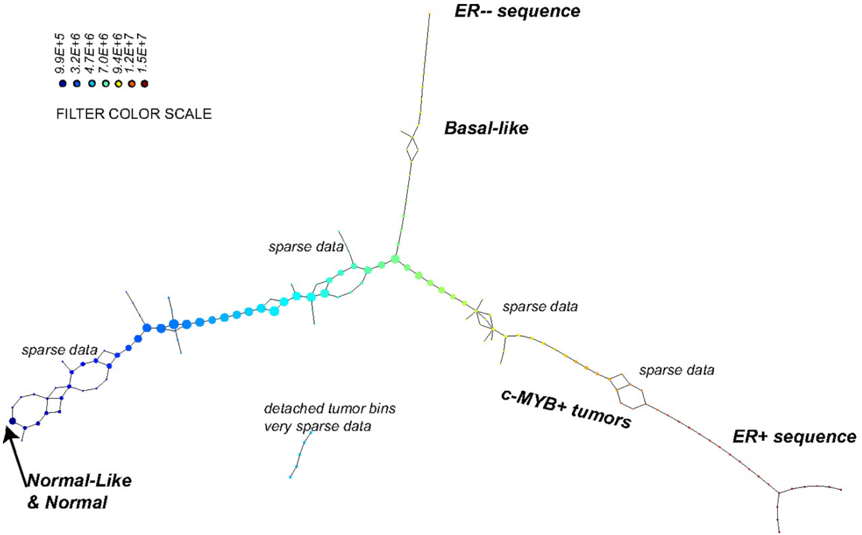
Point cloud X, $f : X \to \mathbb{R}^n$, I - cover of $f(X)$, clustering in $f^{-1}(i)$, for $i \in I$.

Conventional Mapper algorithm



Point cloud X, $f : X \to \mathbb{R}^n$, I - cover of $f(X)$, clustering in $f^{-1}(i)$, for $i \in I$, edges correspond to nonempty intersections of clusters.

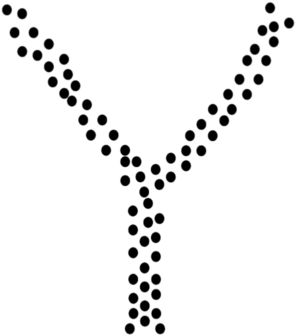
Mapper is the most well know tool of TDA.



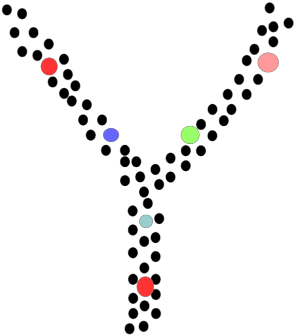
Conventional Mapper algorithm, knobs to adjust.

1. Cover of the line.
2. **Lens function.**
3. Clustering algorithm.
4. It is not easy to choose them all.
5. We may be tempted to "make up" lenses.
6. But, this may be dangerous as for every graph G there exist a point X cloud being a grid and a function $f : X \to \mathbb{R}$ giving a Mapper graph G plus one disconnected vertex.

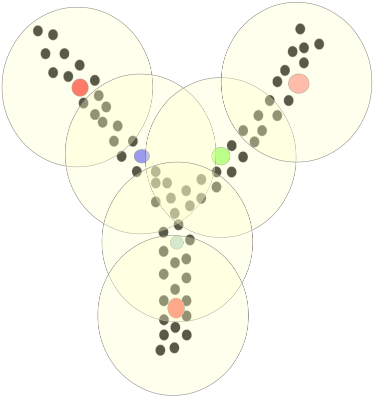
Ball Mapper algorithm.



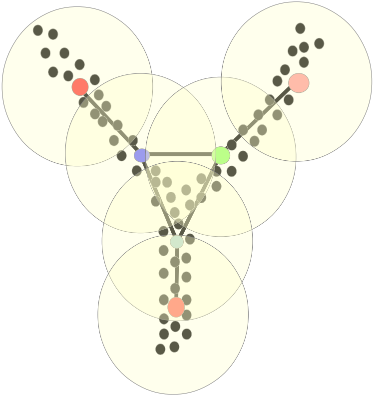
Ball Mapper algorithm.



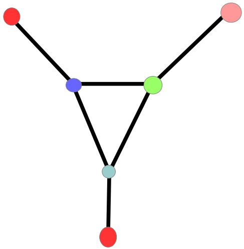
Ball Mapper algorithm.



Ball Mapper algorithm.



Ball Mapper algorithm.



Ball Mapper implementation.

BallMapper: The Ball Mapper Algorithm

The core algorithm is described in "Ball mapper: a shape summary for topological data analysis" by Pawel Dlotko, (2019) <arXiv:1901.07410>. Please consult the following youtube video <https://www.youtube.com/watch?v=M9Dm1nl_zSQfor> the idea of functionality. Ball Mapper provide a topologically accurate summary of a data in a form of an abstract graph. To create it, please provide the coordinates of points (in the points array), values of a function of interest at those points (can be initialized randomly if you do not have it) and the value epsilon which is the radius of the ball in the Ball Mapper construction. It can be understood as the minimal resolution on which we use to create the model of the data.

Version:	0.2.0
Imports:	igraph, scales, networkD3, testthat, fields, methods, stringr
Published:	2019-08-20
Author:	Pawel Dlotko [aut, cre]
Maintainer:	Pawel Dlotko <pdlotko at gmail.com>
License:	MIT + file LICENCE
NeedsCompilation:	no
CRAN checks:	BallMapper results

Downloads:

Reference manual:	BallMapper.pdf
Package source:	BallMapper_0.2.0.tar.gz
Windows binaries:	r-devel: BallMapper_0.2.0.zip, r-release: BallMapper_0.2.0.zip, r-oldrel: BallMapper_0.2.0.zip
macOS binaries:	r-release: BallMapper_0.2.0.tgz, r-oldrel: BallMapper_0.2.0.tgz
Old sources:	BallMapper archive

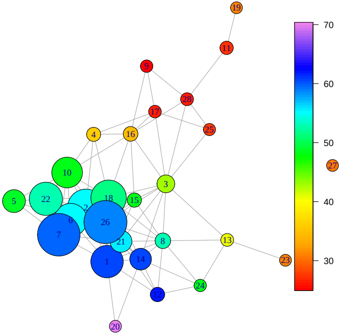
Linking:

Please use the canonical form `https://CRAN.R-project.org/package=BallMapper` to link to this page.

`https://cran.r-project.org/web/packages/BallMapper`

`https://github.com/dgurnari/pyBallMapper`

Support for Brexit in 2016 referendum.

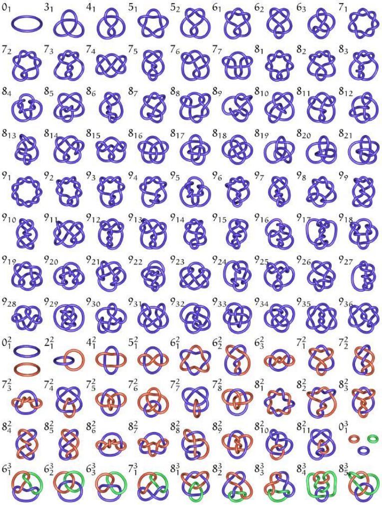


https://arxiv.org/abs/1909.03490
https://ukandeu.ac.uk/constituency-voting-patterns-illustrate-a-close-election-ahead/

Where shall we go next?

1. Ball Mapper as simpler to use Mapper (a few used-cases are joint work with Simon and Wanling Rudkin).
2. Dimension and curvature–based coloring functions.
3. Conventional Mapper on Ball Mapper.
4. The whole story told in the space of knots.

Knots and their properties

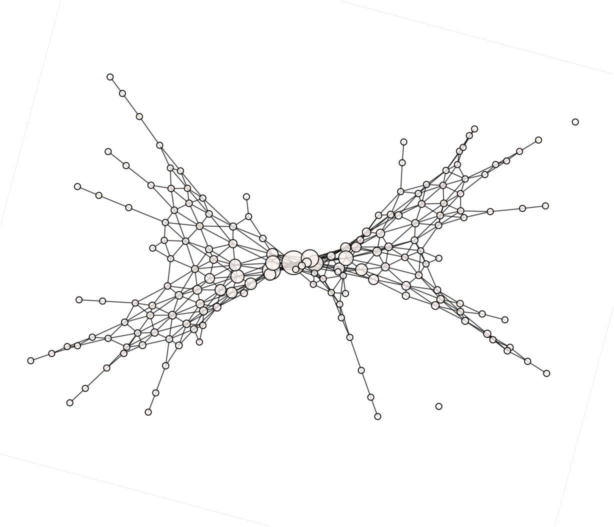


With Davide Gurnari and Radmila Sazdanovic

Knots and their properties

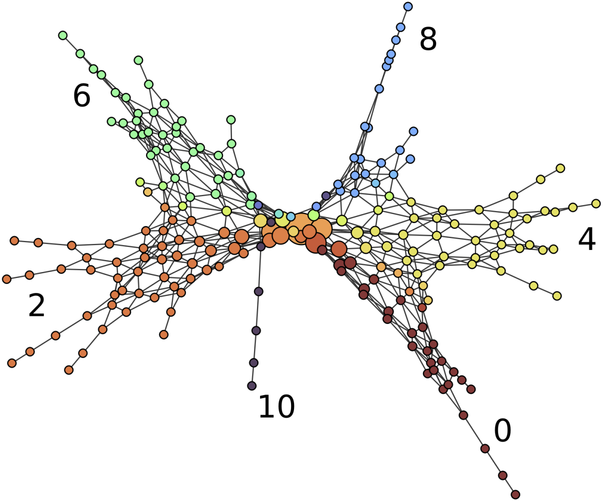
1. A knot is an embedding of S^1 to $\mathbb{R}^3$ up to continuous deformations (isotopies).
2. A number of so called knot polynomials (Alexander, Jones, HOMPFLY-PT) have been introduced to describe knots.
3. As well as Khovanov Homology theory.

Ball Mapper on Jones data



Space of Jones Polynomials for knots up to 15 crossings.

Ball Mapper on Jones data: signature

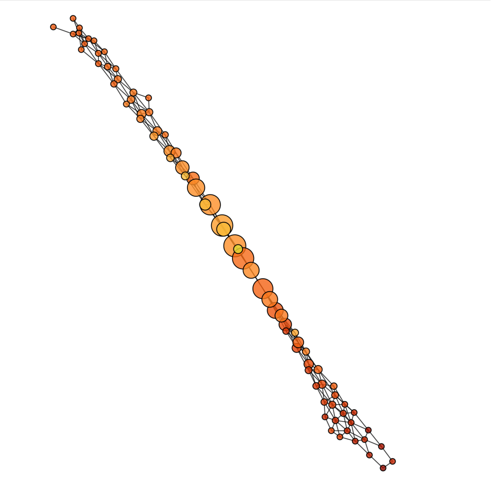


Space of Jones Polynomials for knots up to 17 crossings colored by signature.

Ball Mapper on Alexander data

Space of Alexander Polynomials for knots up to 15 crossings.

Alexander data: signature

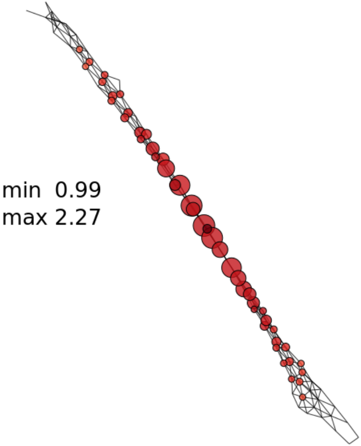


Space of Alexander Polynomials for knots up to 15 crossings colored by signature.

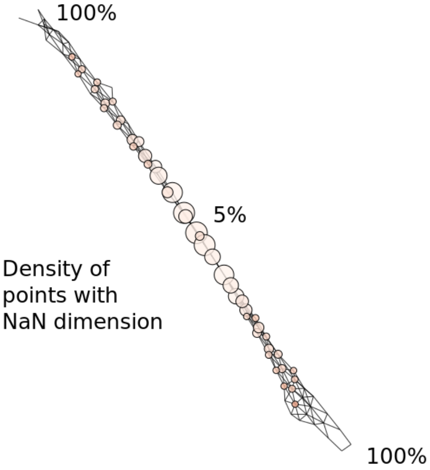
Non standard coloration

1. Local dimension:
   1.1 Distances to k-n-n and their ratios,
   1.2 Local PCA,
   1.3 Angles between nearest neighbors.
2. Curvature
3. Joint work with John Harvey.

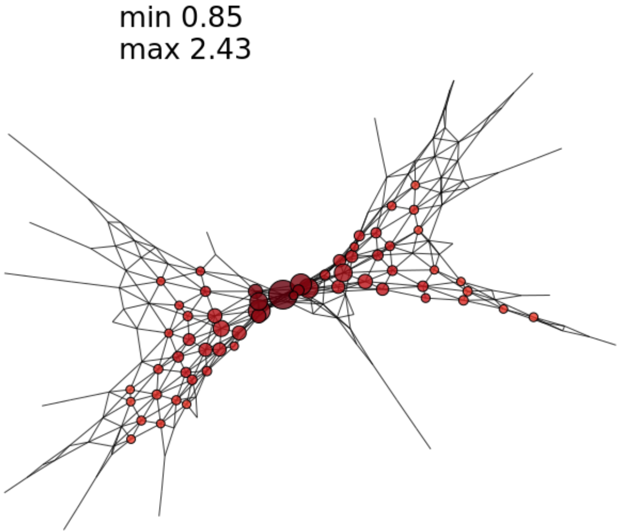
Alexander 15 crossings local dimension.



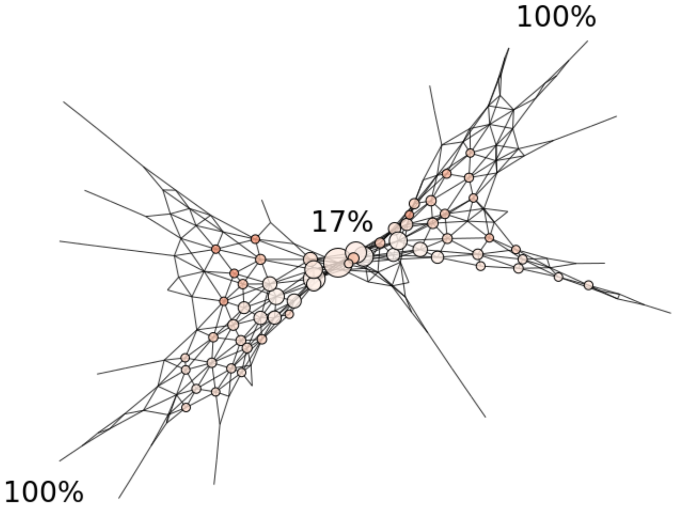
Alexander 15 crossings local dimension, failure percentage.



Jones 15 crossings local dimension.



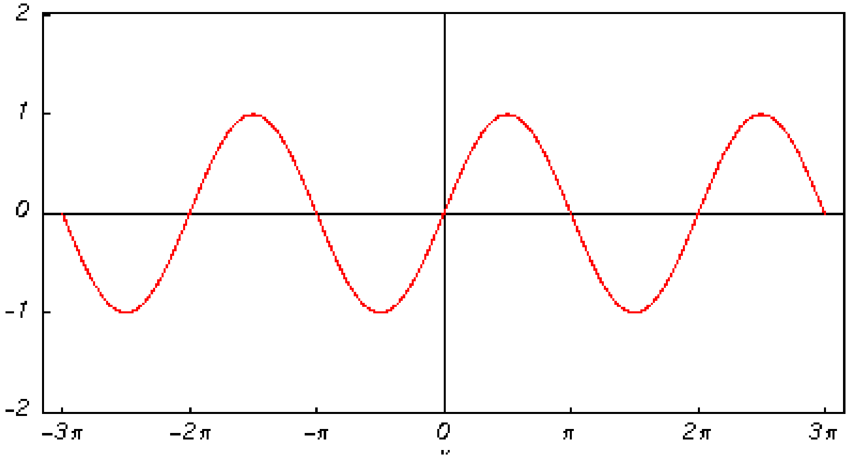
Jones 15 crossings local dimension, failure percentage.



Functorial Ball Mapper

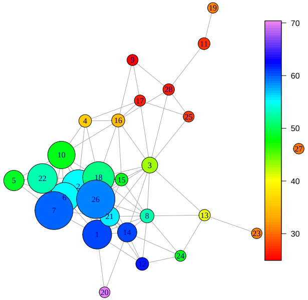
1. Suppose we have different descriptors for the fixed data.
2. How to compare the space of descriptors?
3. Where are they similar, and where are they different?
4. All boils down to the question how to visualize a function $f : \mathbb{R}^n \to \mathbb{R}^m$?

Mappers to understand functions



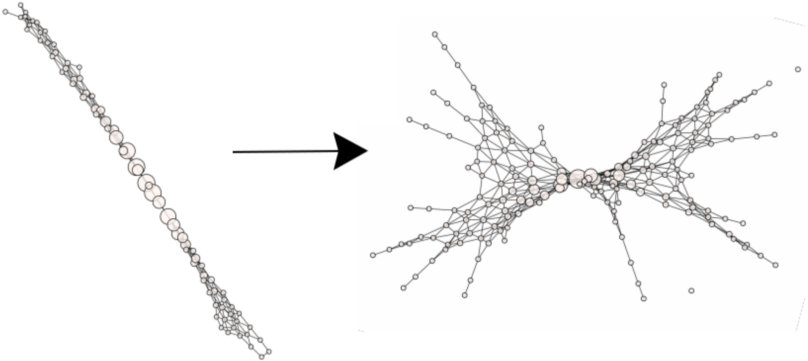
$f : \mathbb{R} \to \mathbb{R}$

Mappers to understand functions



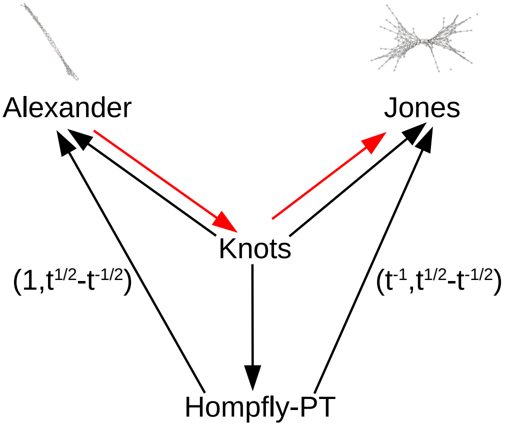
$$f : \mathbb{R}^n \to \mathbb{R}$$

Mappers to understand functions

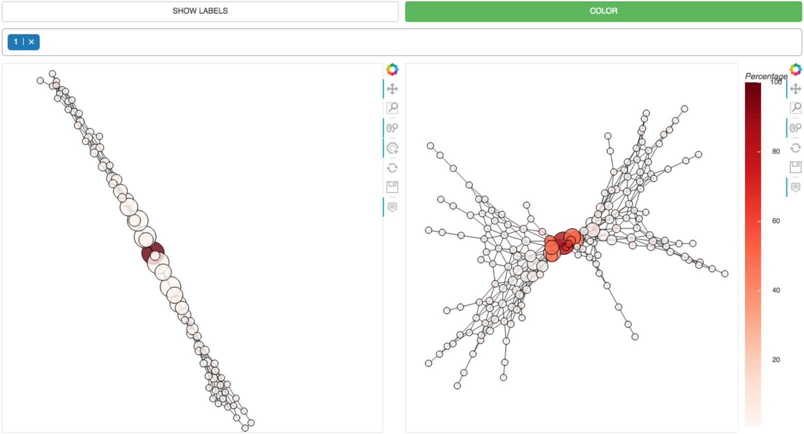


$$f : \mathbb{R}^n \to \mathbb{R}^m$$

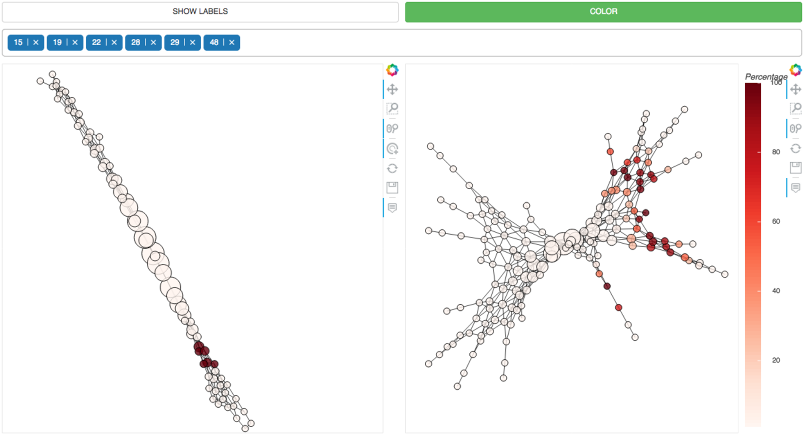
Mappers to understand functions



Ball Mapper comparison of data sets: Alexander vs Jones



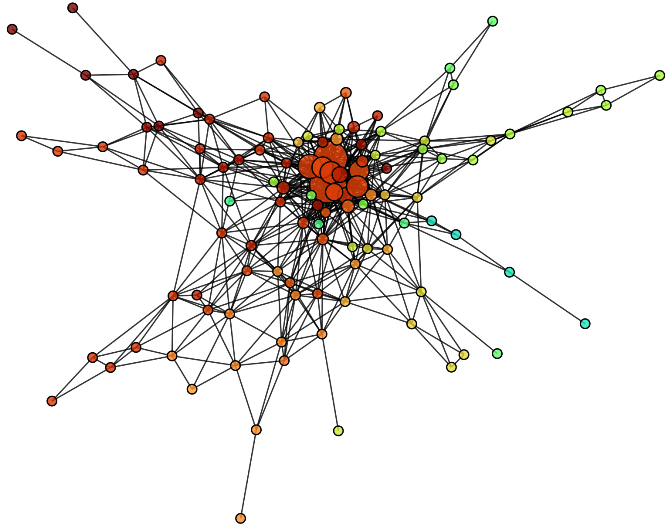
Ball Mapper comparison of data sets: Alexander vs Jones



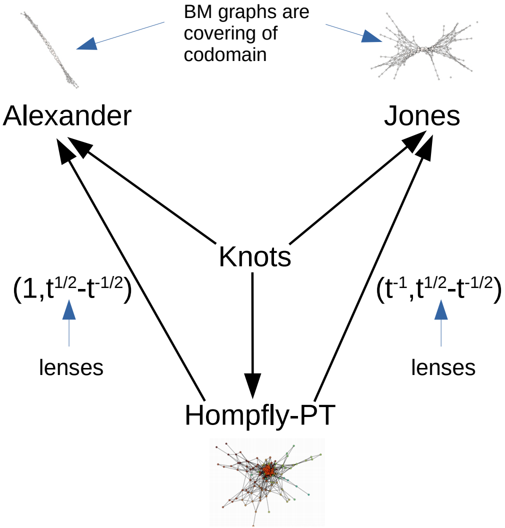
https://github.com/dgurnari/mapper_GUI

Two more exotic knot descriptors

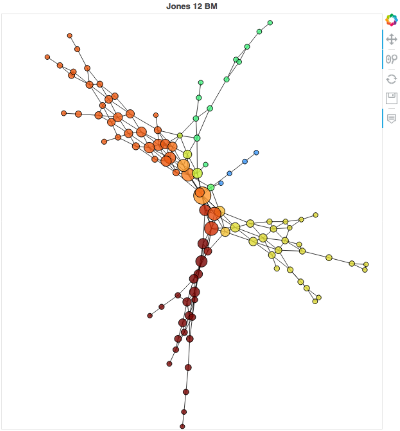
HOMFLY-PT: 15 crossings knots

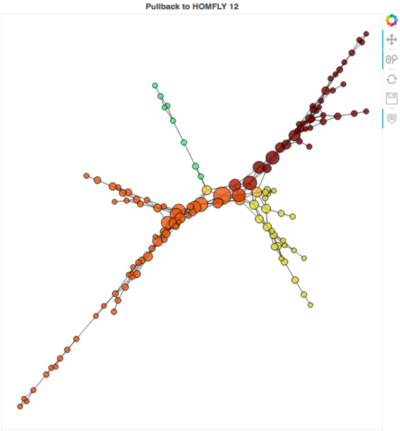


Conventional mapper on Ball Mapper

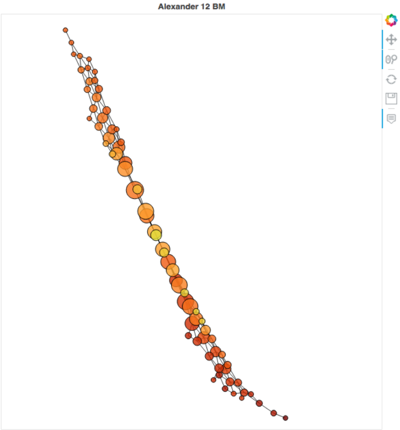


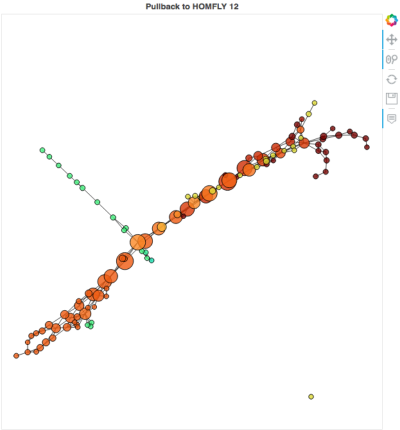
Mapper on Ball Mapper; Hompfly-PT $\longrightarrow$ Jones





Mapper on Ball Mapper; Hompfly-PT $\to$ Alexander





Mapper on Ball Mapper, general

$$\text{Very high dimensional data} \xrightarrow{\text{projection}} \text{High dimensional data}$$

$$\text{Mapper} \xrightarrow{\text{lens}} \text{Ball Mapper}$$

Take home

1. Careful with all the knobs in mapper!
2. Simpler methods like Ball Mapper first?
3. New coloration functions; local dimension and curvature.
4. Visualization of functions $f : \mathbb{R}^n \to \mathbb{R}^m$.
5. Mapper on Ball Mapper cascade - for high dimensional lenses.

Thank you for your time.

Dioscuri Centre in Topological Data Analysis
@Facebook

WE ARE HIRING POSTDOCS AND PHD STUDENTS!!

Paweł Dłotko
pdlotko @ impan.pl
pdlotko @ gmail
pawel_dlotko @ skype