Week HW 3, Statics analysis

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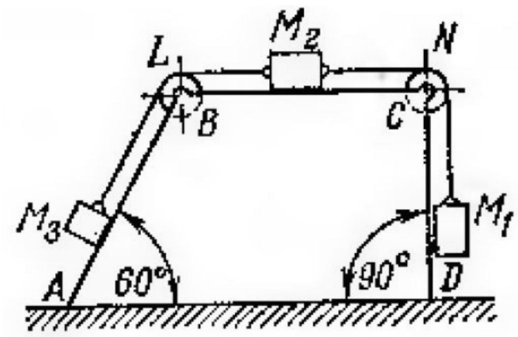
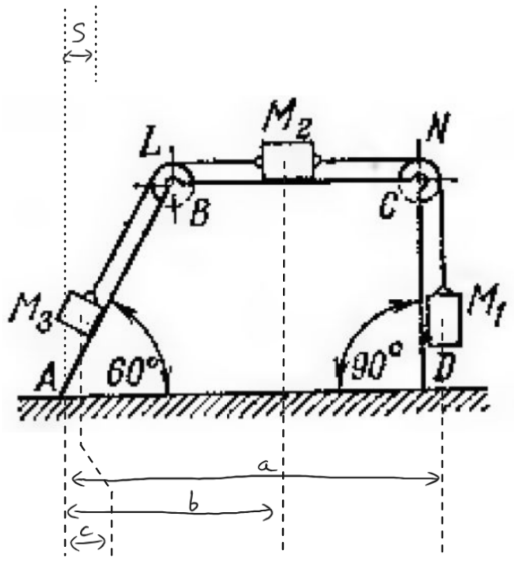
Task 1:

There are 3 weights with masses $M_1 = 20$ kg, $M_2 = 15$ kg, $M_3 = 10$ kg. They are connected by ideal string. This string goes through two pulleys L, N. When the M_1 weight goes down on 1 meter, the body $ABCD$ shifts on some distance S. The task is to find the distance of this movement according to the ground. Neglect the friction between the floor and $ABCD$.



Solution:



Since the center of mass remains constant, we equate its initial and final coordinates after displacement S of the body ABCD whose mass is neglected:

$$x_{C1} = x_{C2} \tag{1}$$

$$x_{C1} = \frac{M_1 a + M_2 b + M_3 c}{M_1 + M_2 + M_3} \tag{2}$$

$$x_{C2} = \frac{M_1(a - S) + M_2(b + h - S) + M_3(c + h\cos 60^\circ - S)}{M_1 + M_2 + M_3} \tag{3}$$

Since $x_{C1} = x_{C2}$

$$M_1 a + M_2 b + M_3 c = M_1(a - S) + M_2(b + h - S) + M_3(c + h\cos 60^\circ - S) \tag{4}$$

Expanding and simplifying:

$$M_1 a + M_2 b + M_3 c = M_1 a - M_1 S + M_2 b + M_2 h - M_2 S + M_3 c + M_3 h\cos 60^\circ - M_3 S \tag{5}$$

Canceling common terms:

$$M_1 S + M_2 S + M_3 S = h(M_2 + M_3\cos 60^\circ) \tag{6}$$

$$S(M_1 + M_2 + M_3) = h(M_2 + M_3 \cos 60^\circ) \tag{7}$$

Solving for S

$$S = \frac{h(M_2 + M_3 \cos 60^\circ)}{M_1 + M_2 + M_3} \tag{8}$$

Given values:

$$h = 1 \text{ m}, \quad M_1 = 20, \quad M_2 = 15, \quad M_3 = 10, \quad \cos 60^\circ = \frac{1}{2} \tag{9}$$

Substituting:

$$S = \frac{1 \times (15 + 10 \times \frac{1}{2})}{20 + 15 + 10} \tag{10}$$

$$S = \frac{1 \times (15 + 5)}{45} = \frac{1 \times 20}{45} = \frac{20}{45} = 0.44 \text{ cm} \tag{11}$$

Thus, the displacement is **0.44 cm to the left**.