CSDS 552 – Deep Learning

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Types of Learning

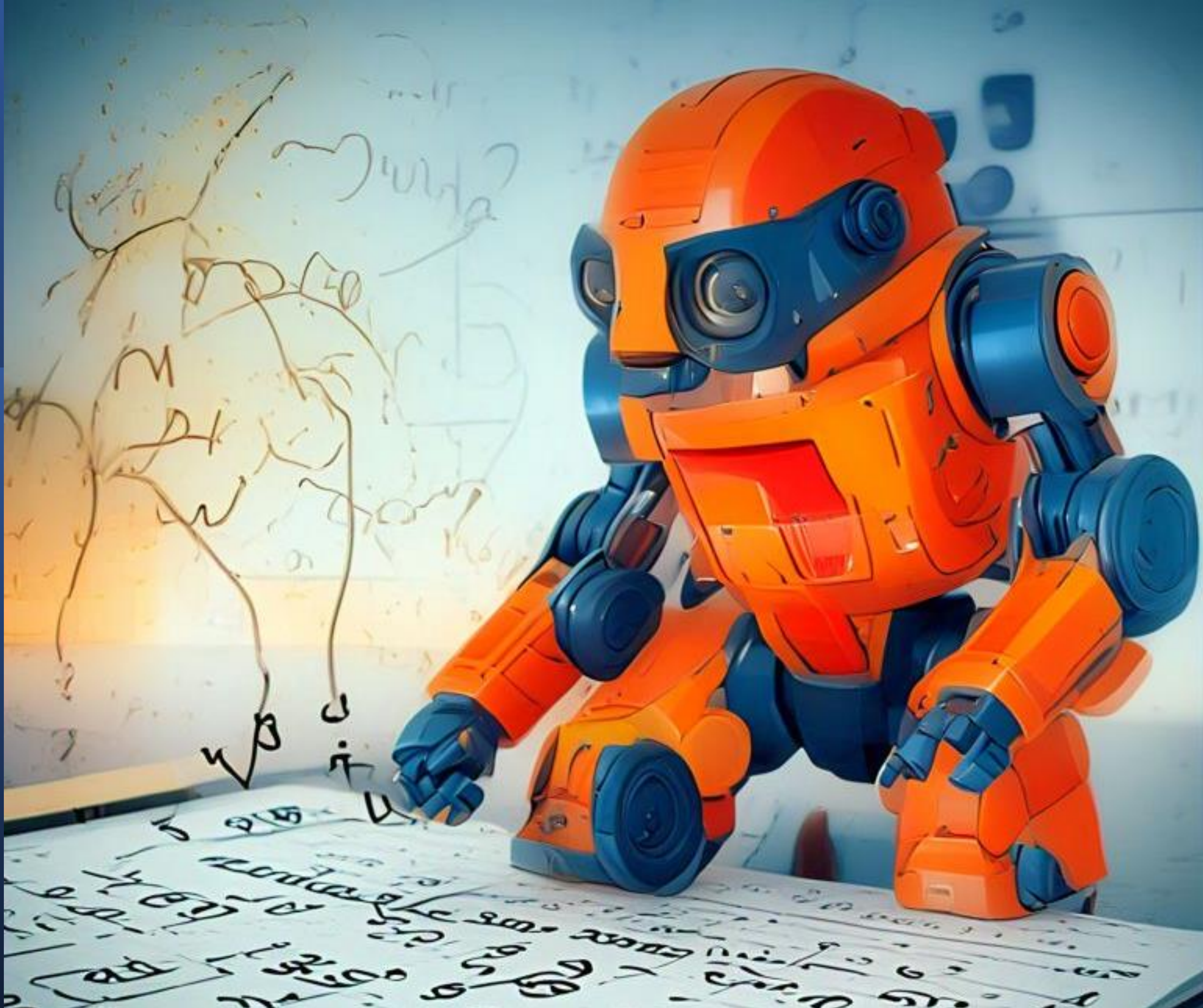
Supervised

The outcome is provided along with the data.

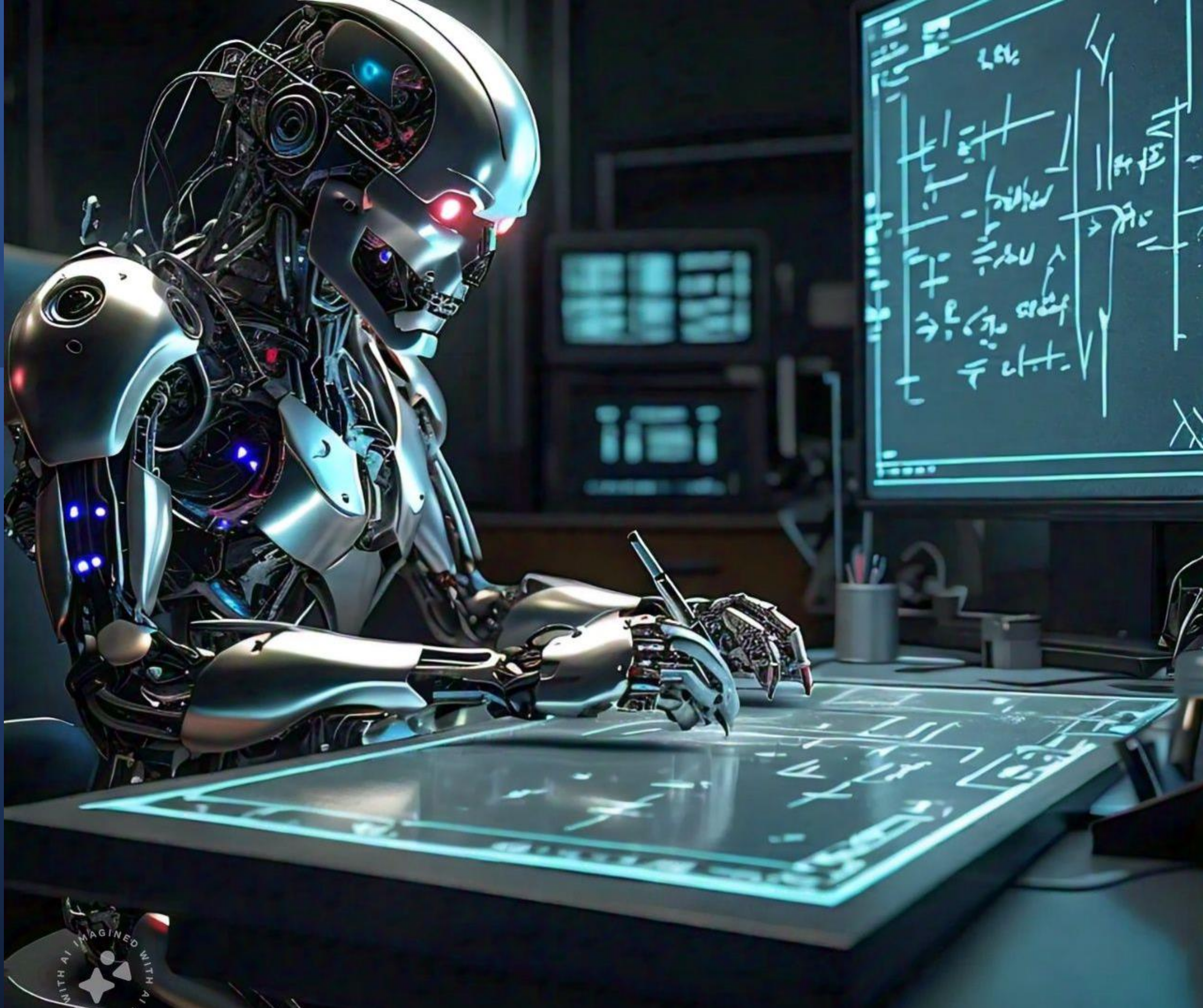
Unsupervised

The outcome is NOT provided along with the data.

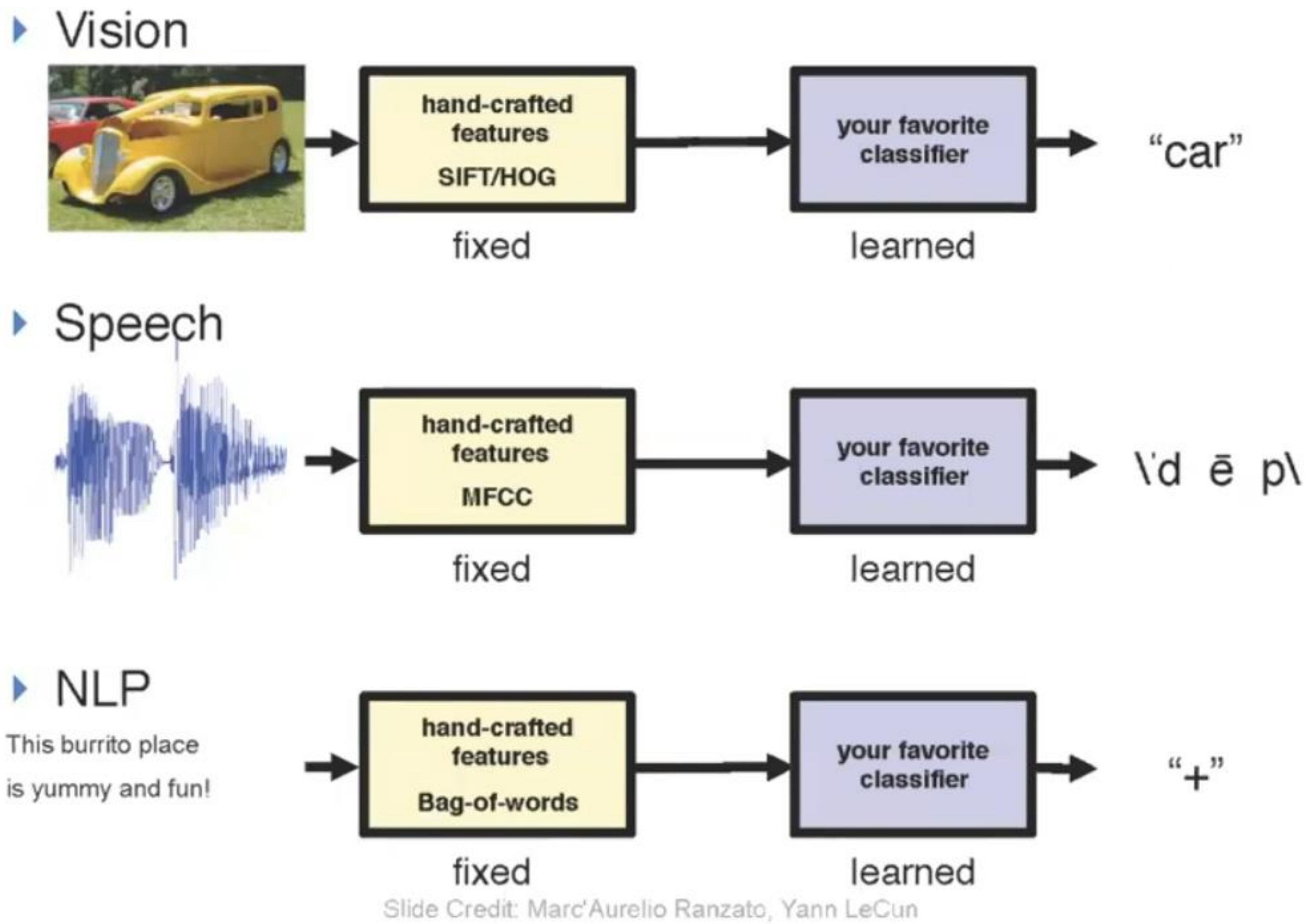
What is Machine Learning?



What is Deep (Machine) Learning?

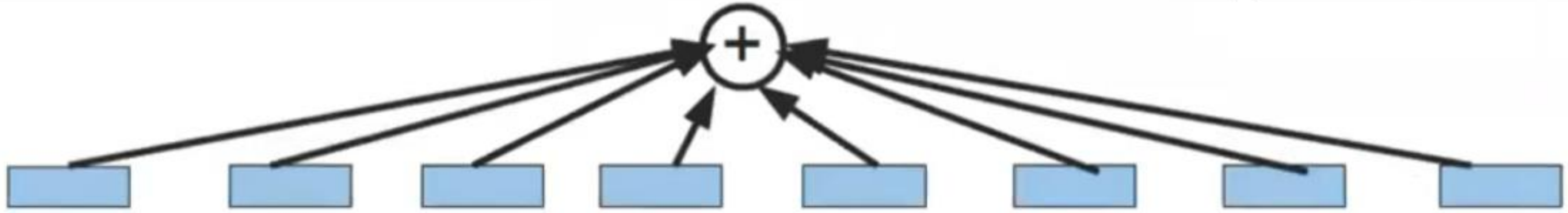


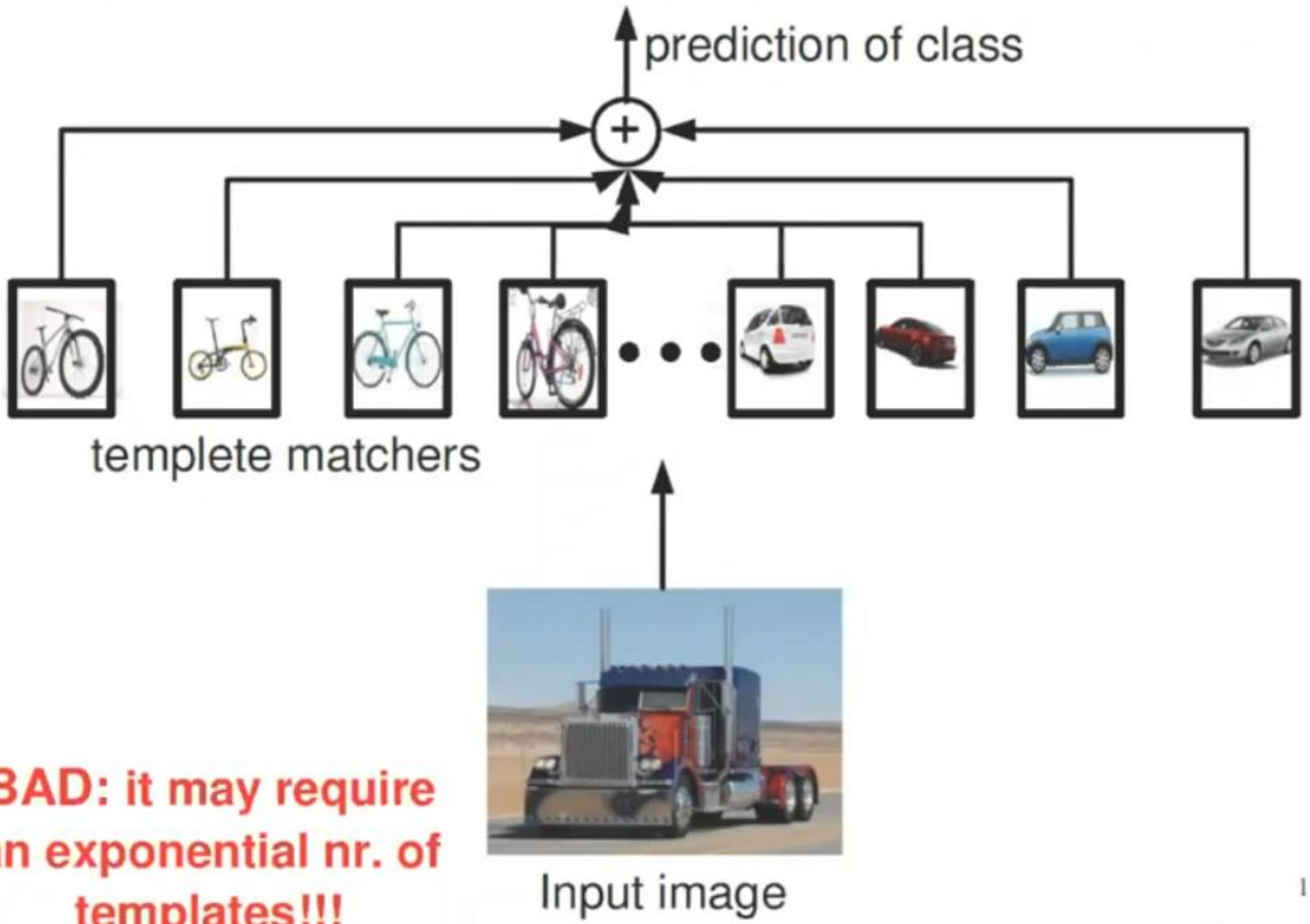
Feature Engineering



Linear Combination (shallow approach)

$$f(x) = \sum_i \alpha_i g_i(x)$$



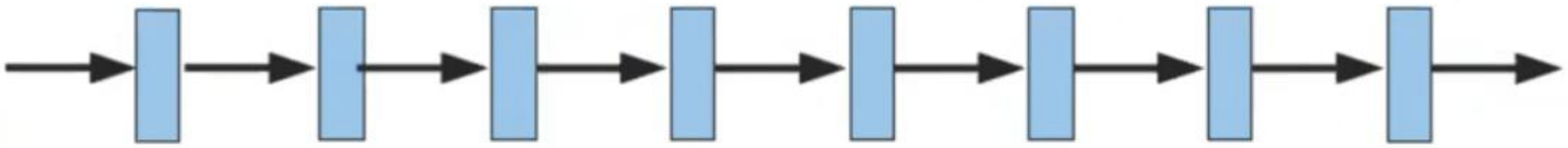


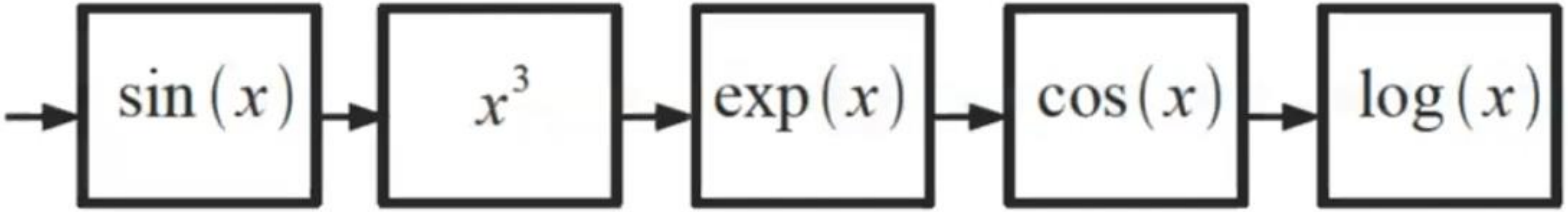
BAD: it may require an exponential nr. of templates!!!

Ranzato

Combination (deep approach)

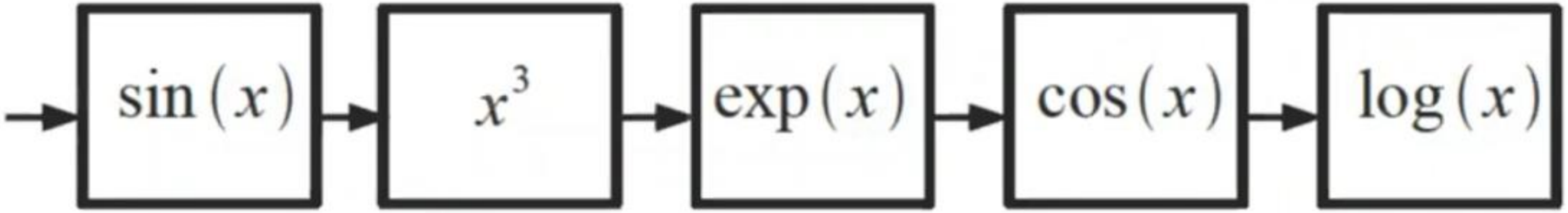
$$f(x) = g_1(g_2(\dots(g_n(x)\dots))$$

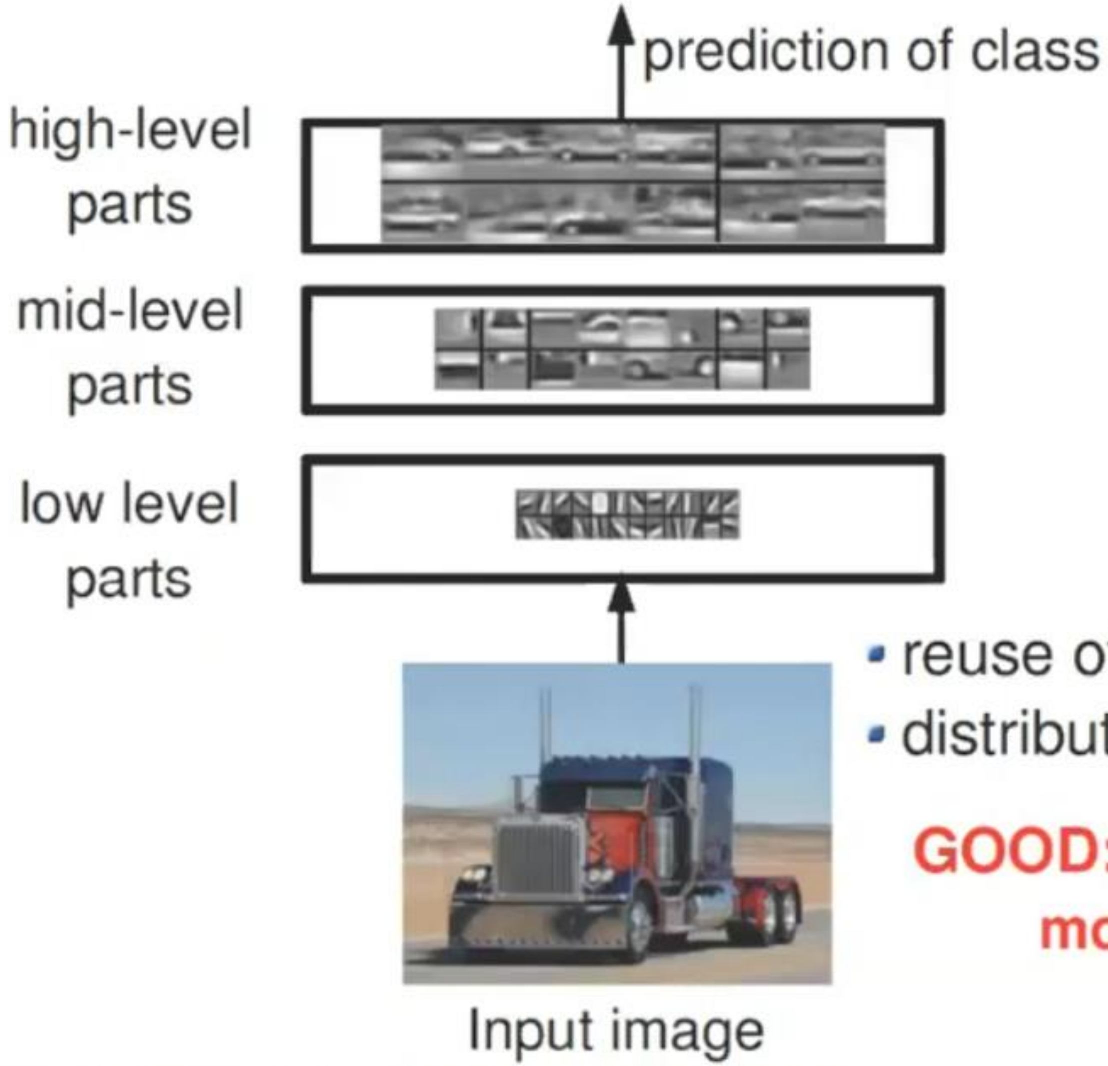




Combination (deep approach)

$$f(x) = \log(\cos(\exp(\sin^3(x))))$$



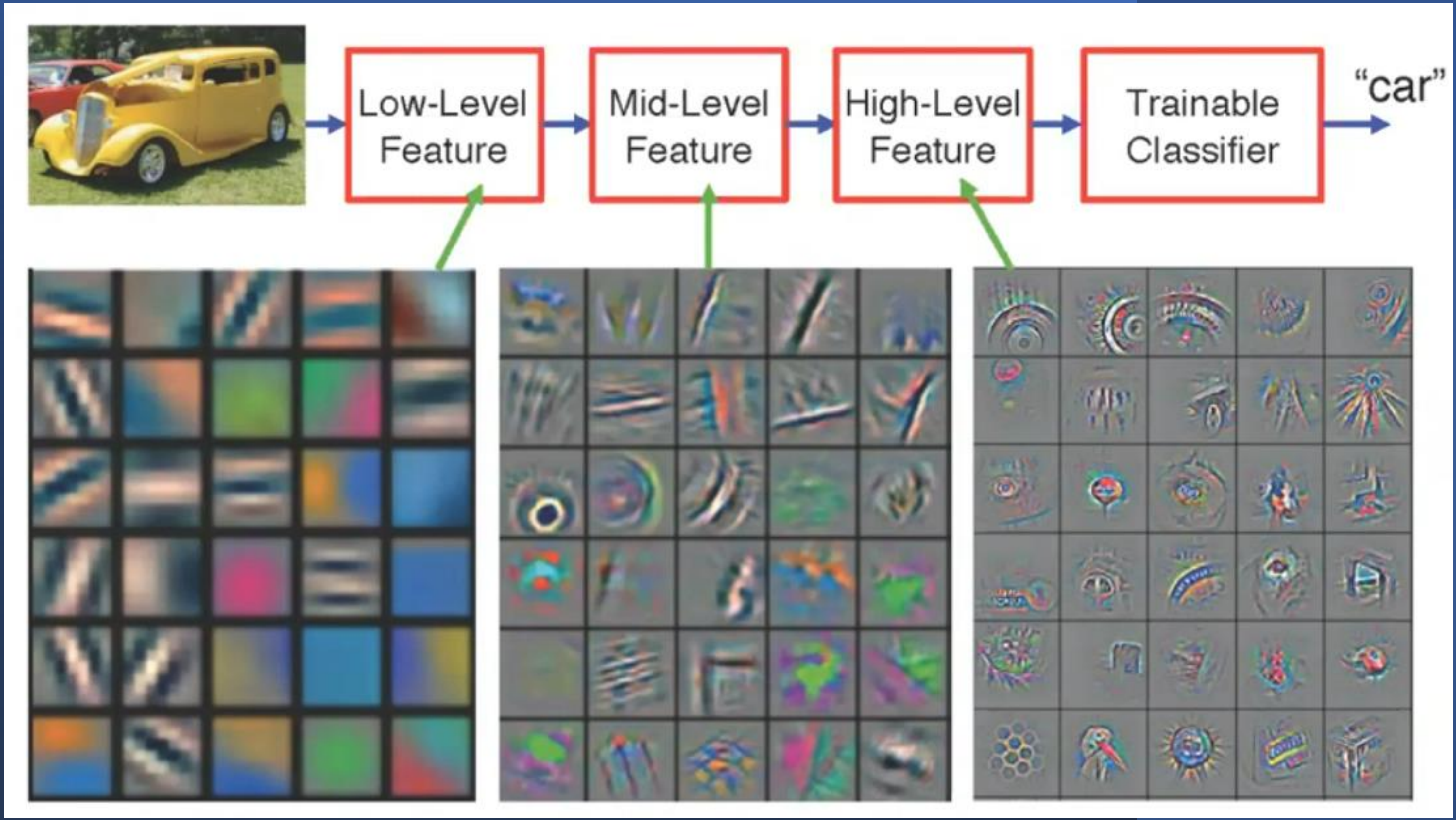


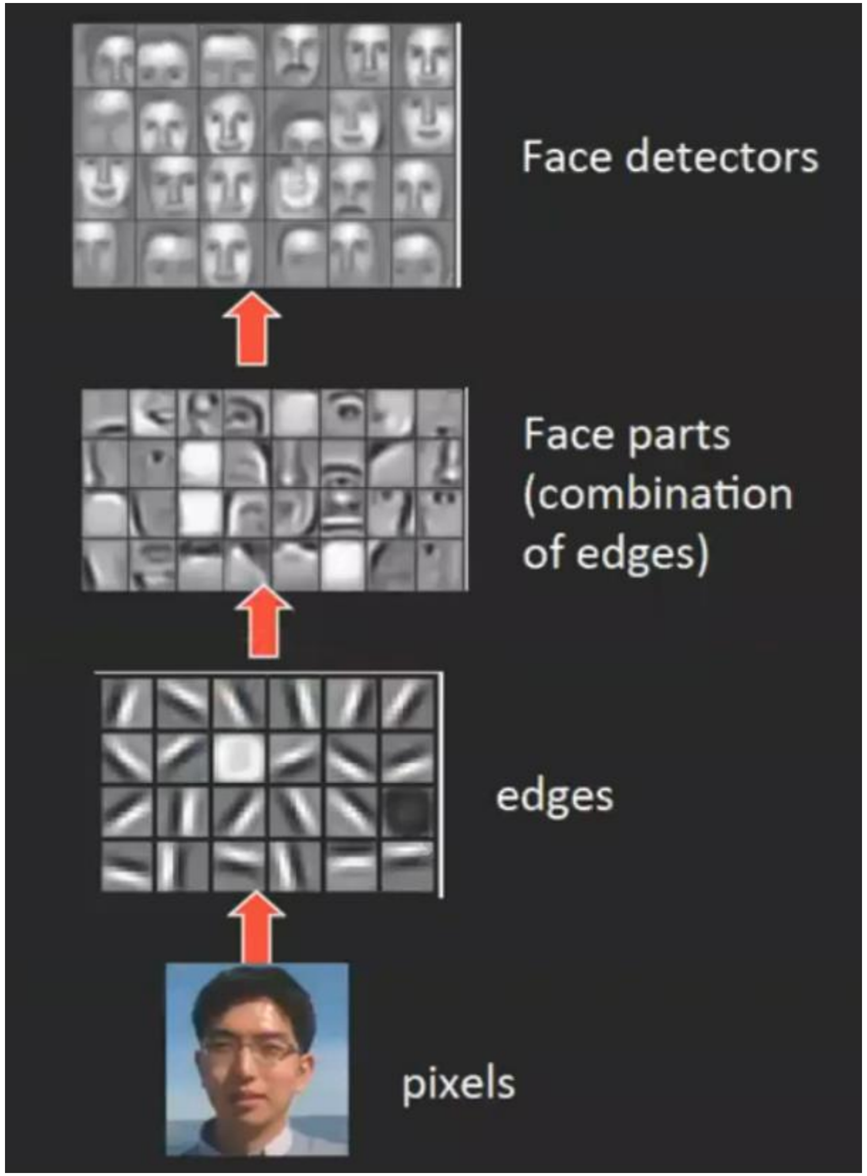
- reuse of intermediate parts
- distributed representations

GOOD: (exponentially) more efficient

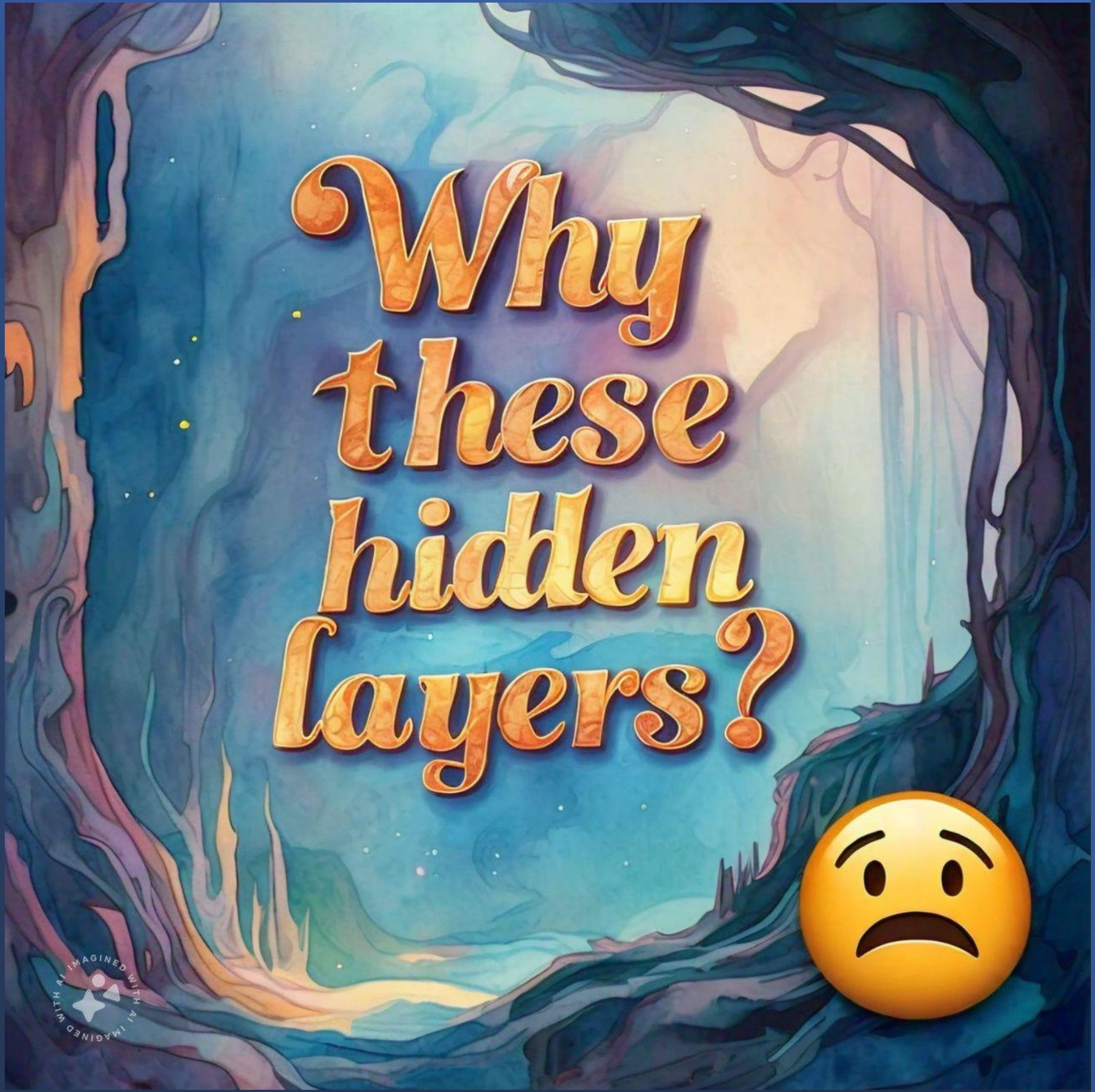
Lee et al. "Convolutional DBN's ..." ICML 2009

Ranzato

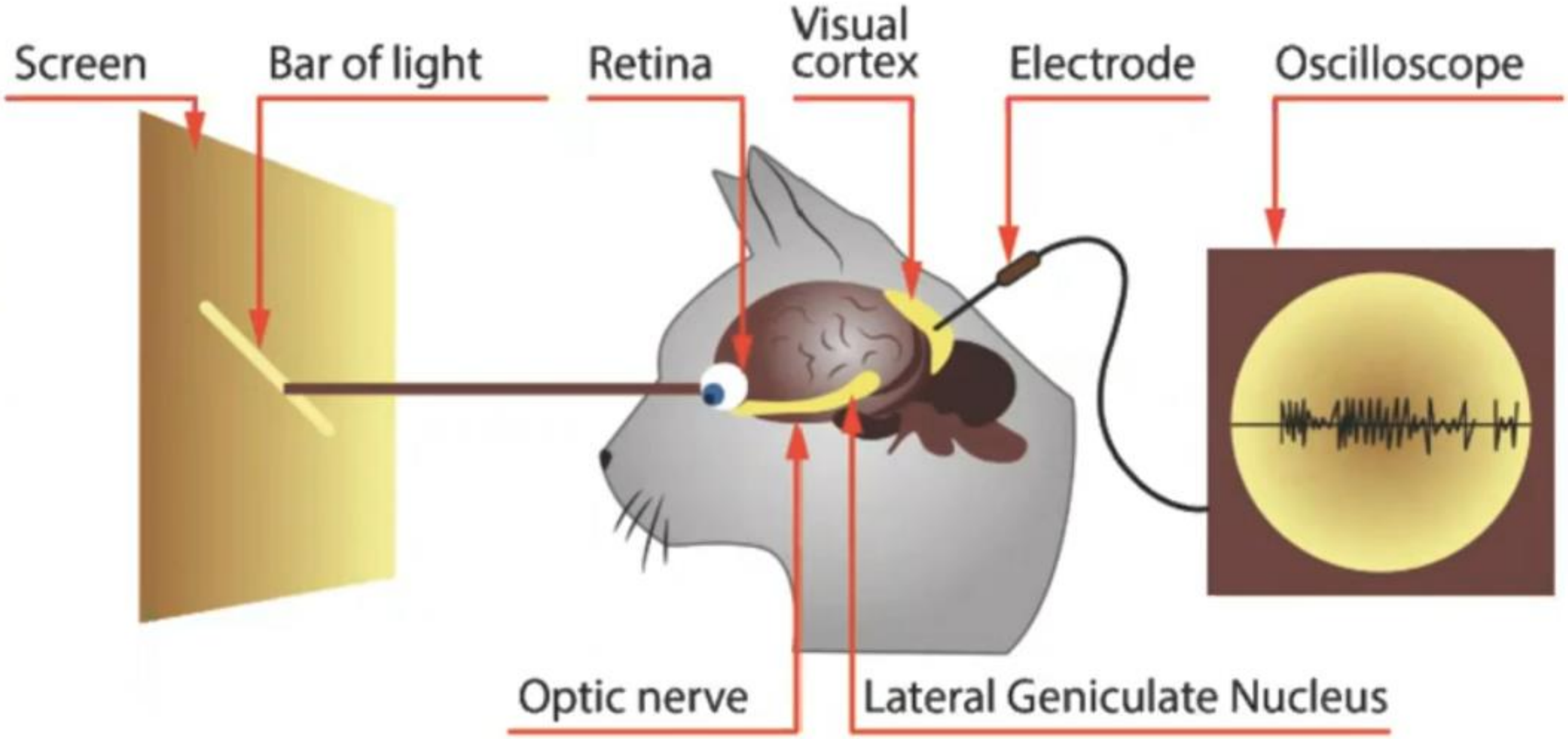

Low-Level
Feature
Mid-Level
Feature
High-Level
Feature
Trainable
Classifier
"car"

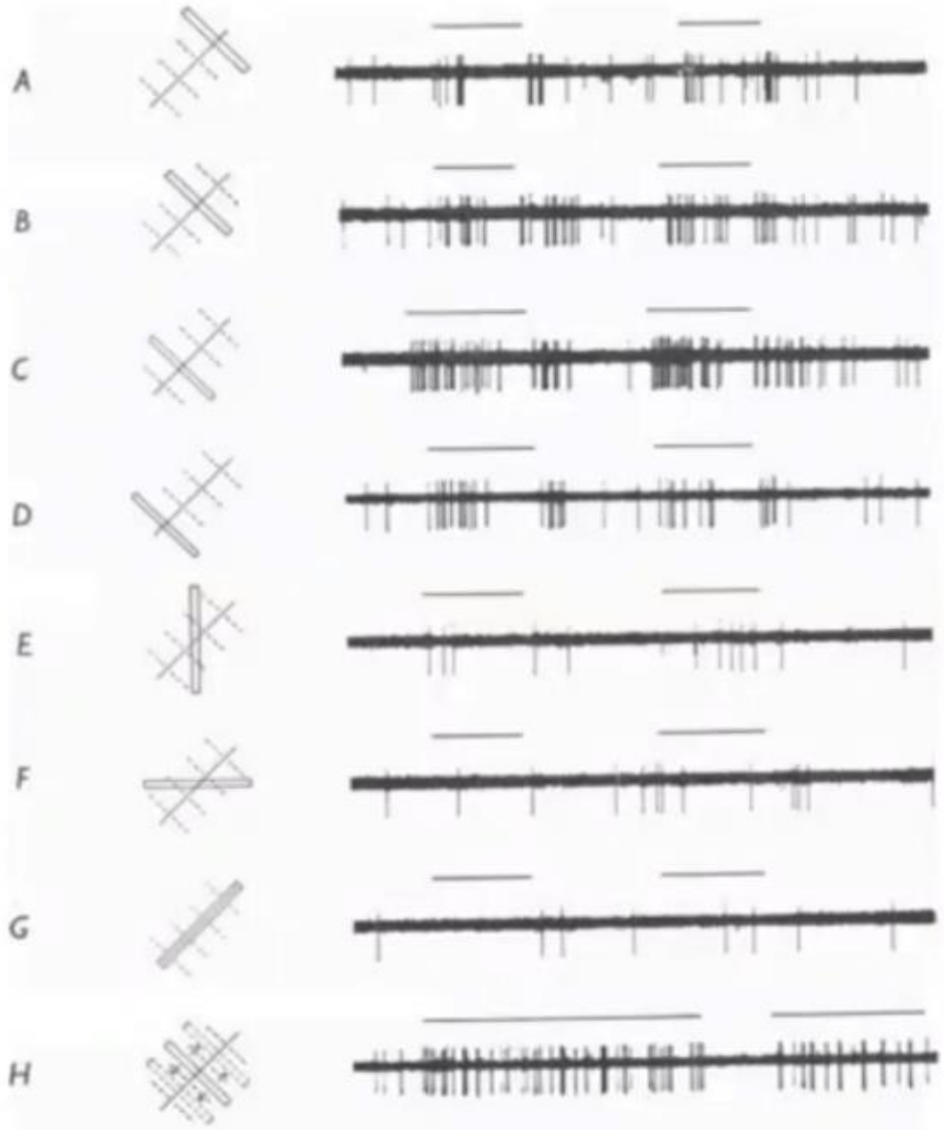


Face detectors
Face parts
(combination
of edges)
edges
pixels


Why
these
hidden
layers?

Visual Cortex of a Cat (HUBEL; WIESEL; 1959)





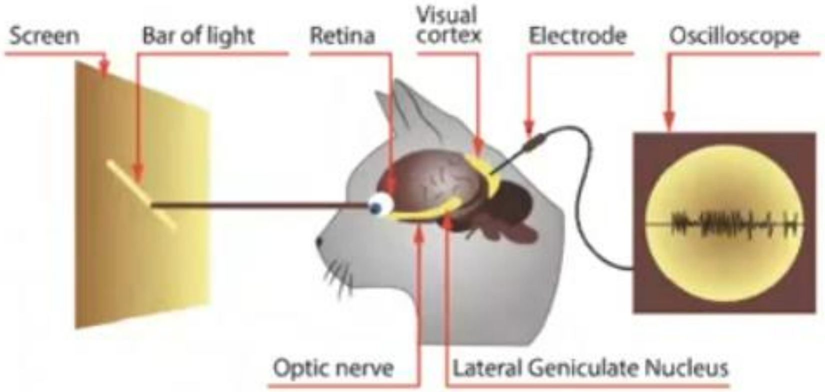
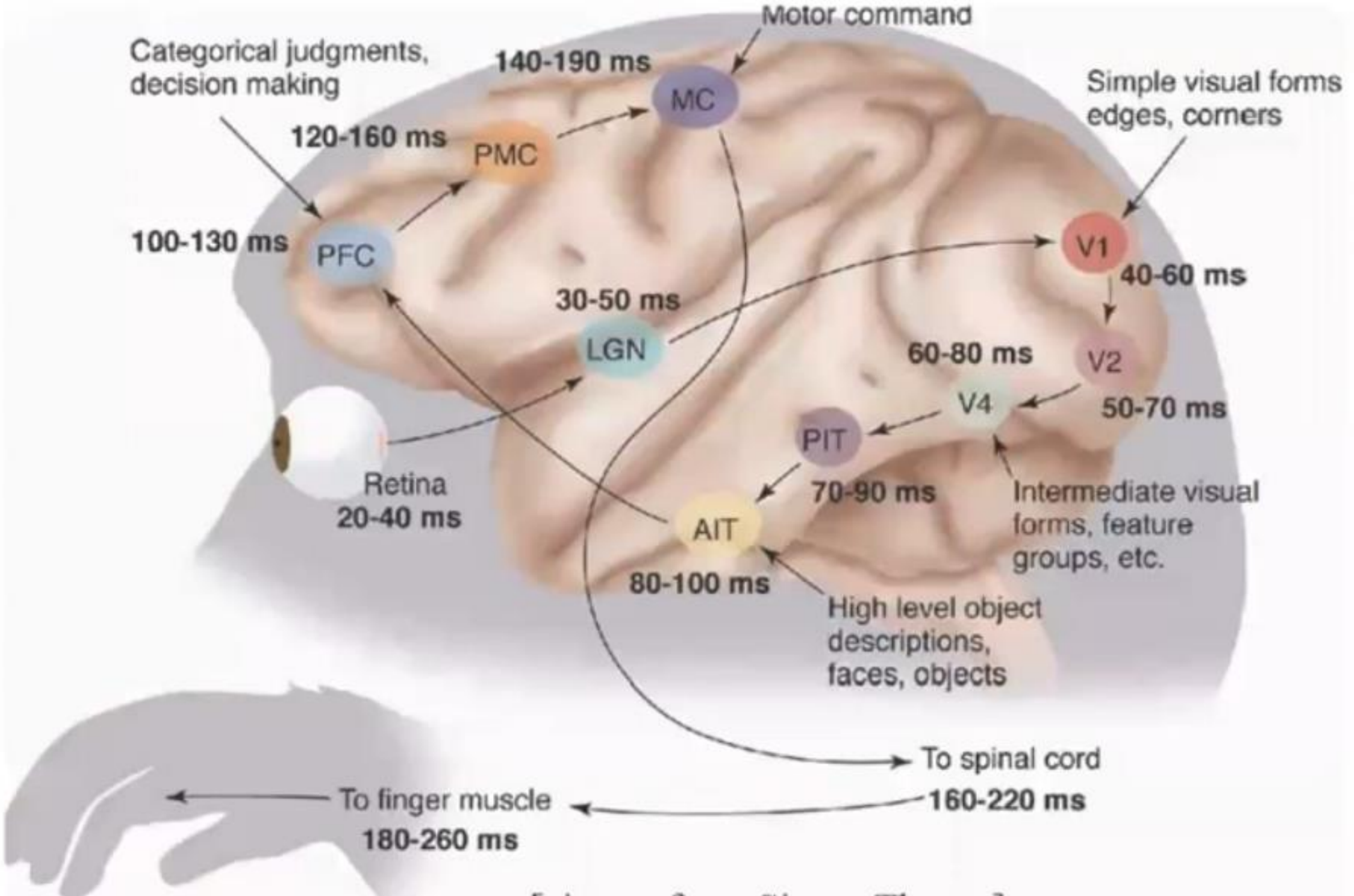


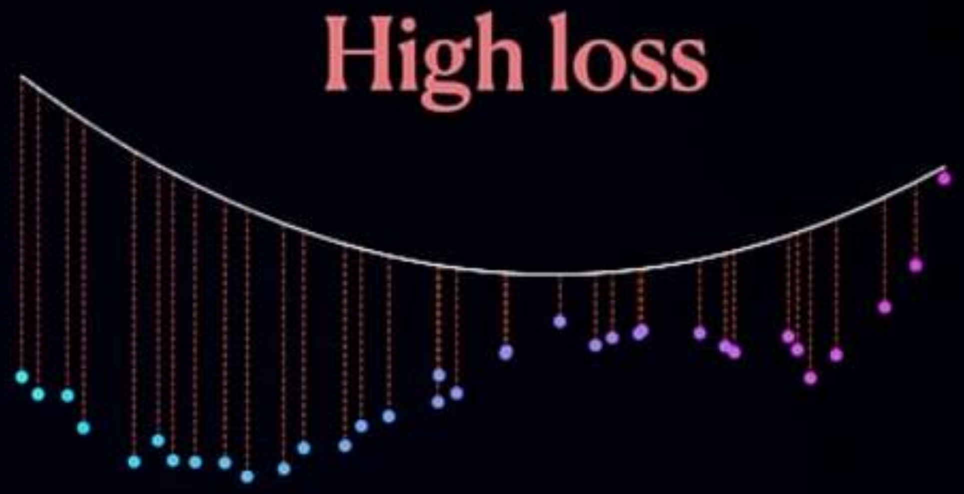
Image credits: https://distillery.com/

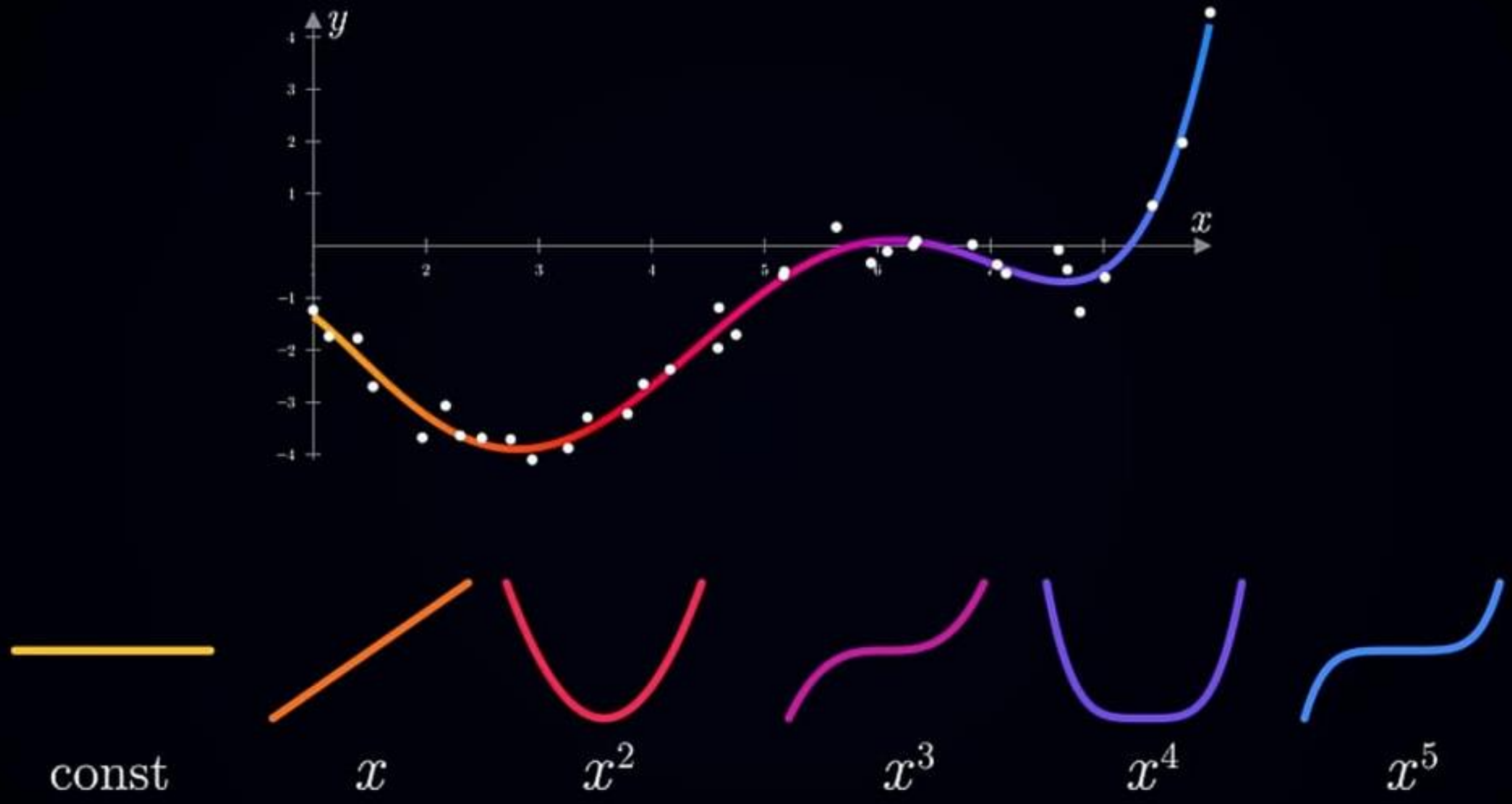
The ventral (recognition) pathway in the visual cortex



[picture from Simon Thorpe]

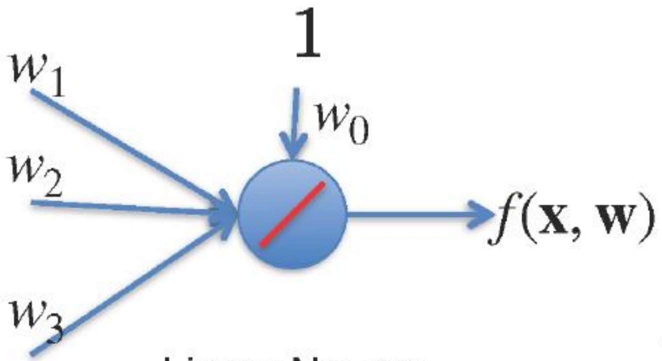

Low loss


High loss

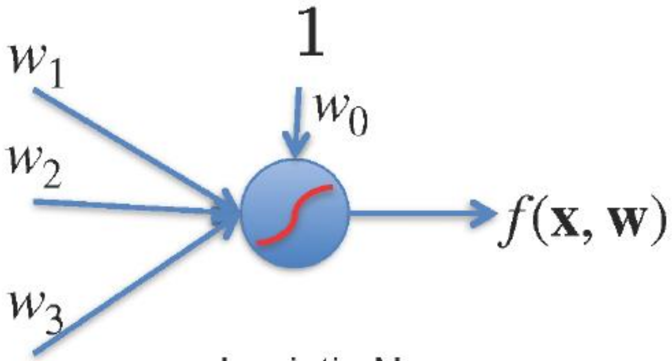


y
x
4
3
2
1
−1
−2
−3
−4
2
3
4
5
6
const
x
x^2
x^3
x^4
x^5

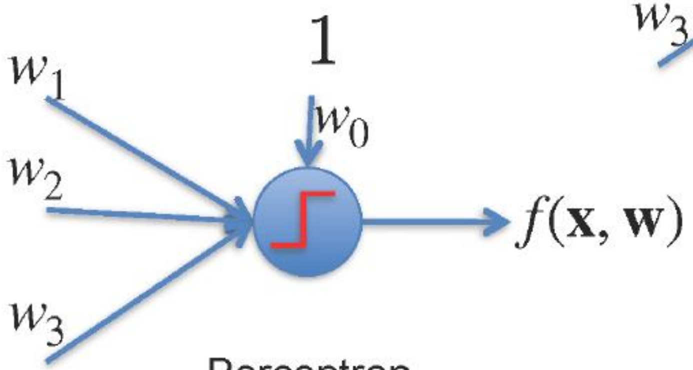
Types of Neurons



Linear Neuron



Logistic Neuron



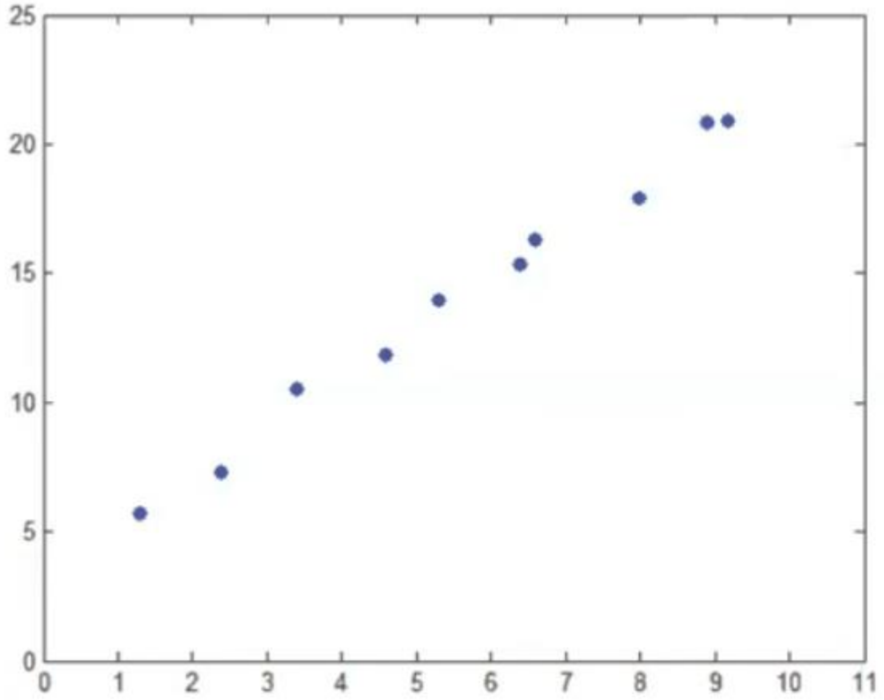
Perceptron

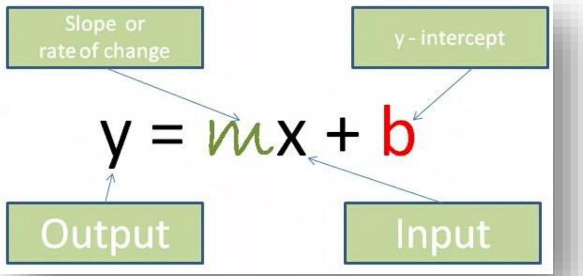
Potentially more. Require a convex loss function for gradient descent training.

Linear Regression

Linear Regression

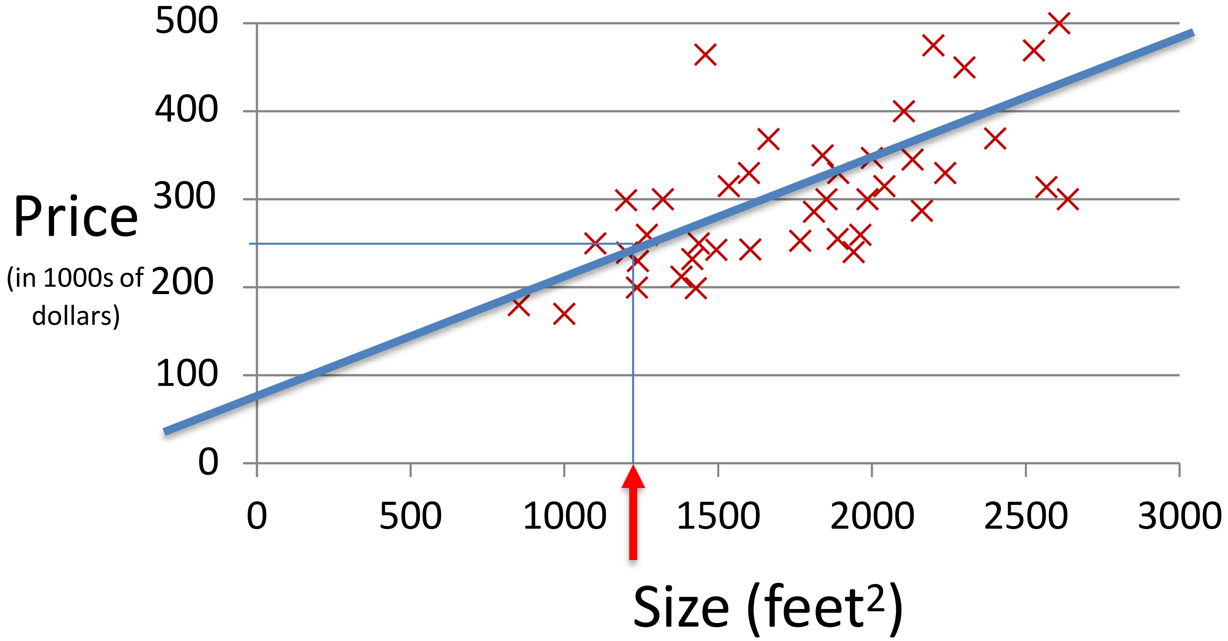
x	t
1.3	5.7
2.4	7.3
3.4	10.5
4.6	11.8
5.3	13.9
6.6	16.3
6.4	15.3
8.0	17.9
8.9	20.8
9.2	20.9





Linear Regression with one variable

Size ($feet^2$)	Price $(\times 1000)$
1500	190
2250	285
2740	420
2318	300
2500	350
1250	180
…	…



Notation:

$\boldsymbol{m}$ = Number of training samples

$\boldsymbol{n}$ = Number of features

$\boldsymbol{x_j}$ = jth feature

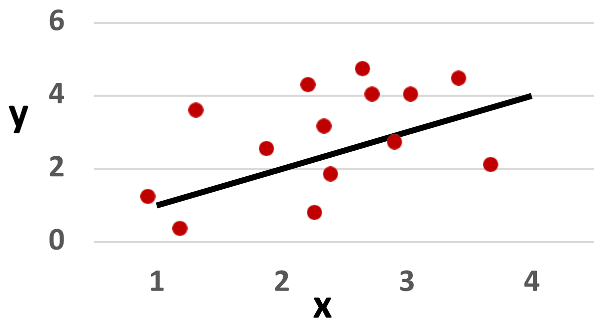
$\boldsymbol{y}$ = Label

$(\boldsymbol{x^i}, \boldsymbol{y^i})$: the ith sample in the dataset

Linear Regression with one variable

$$\boldsymbol{h_\theta(x) = \theta_0 + \theta_1 x}$$

Linear regression with one variable, univariate linear regression, simple linear regression



Choose θ_0, θ_1 such that $h_\theta(x) \approx y$ for our training examples (x, y).

Parameters

$$\theta_0, \theta_1$$

Cost function

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

Goal:

$$Minimize_{\theta_0,\theta_1} J(\theta_0, \theta_1)$$

A simplified case

Hypothesis

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Parameters

$$\theta_0, \theta_1$$

Cost function

$$J(\theta_0, \theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

Goal:

$$Minimize_{\theta_0,\theta_1} J(\theta_0, \theta_1)$$

Assume $\boldsymbol{\theta_0 = 0}$

Hypothesis

$$h_\theta(x) = \theta_1 x$$

Parameters

$$\theta_1$$

Cost function

$$J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

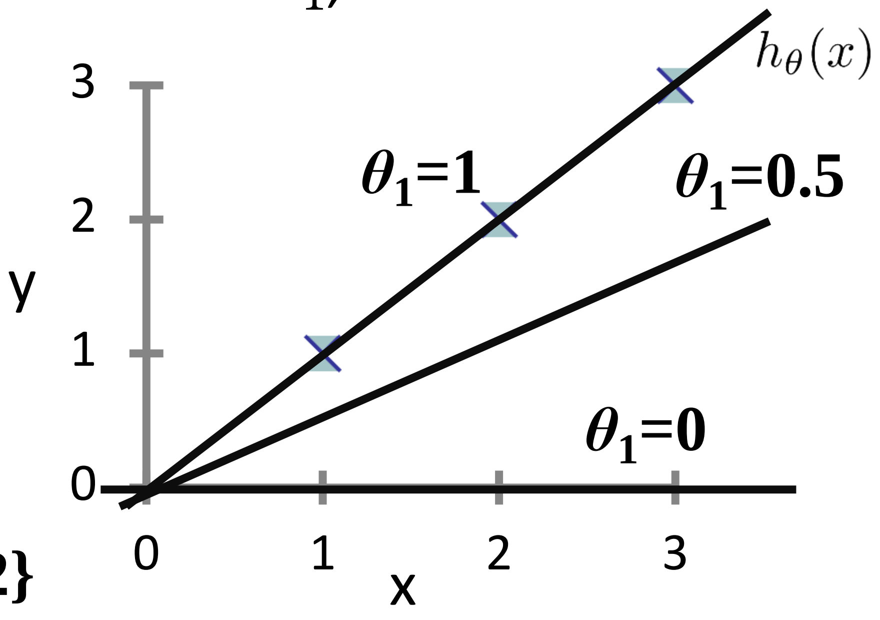
Goal:

$$Minimize_{\theta_1} J(\theta_1)$$

$h_\theta(x) = \theta_1 x$

(for a fixed θ_1, this is a function of x)

x	y
1	1
2	2
3	3



$\theta_1 = \{0,1,2\}$

$$J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(\theta_1 x^{(i)} - y^{(i)}\right)^2$$

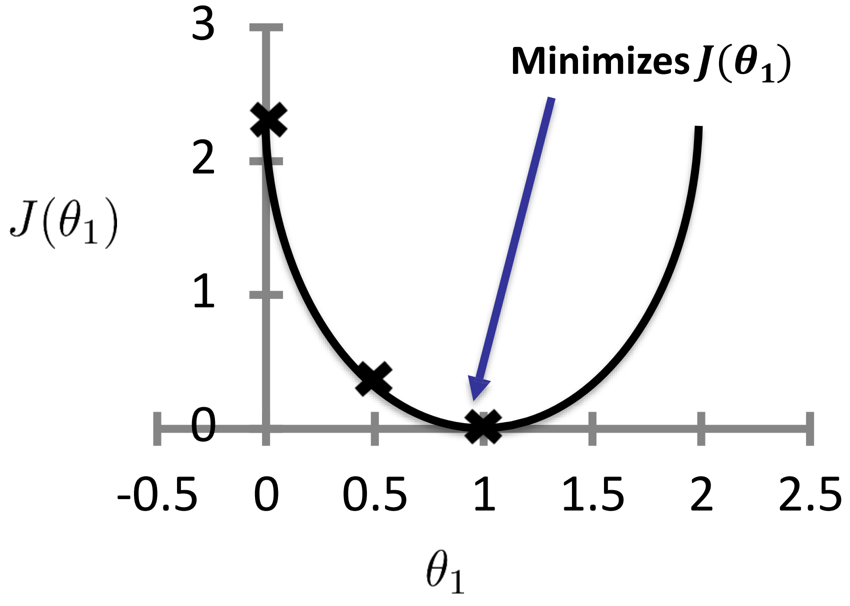
$J(\theta_1)$

(function of the parameter θ_1)

$$J(1) = \frac{1}{2(3)}\left((1-1)^2 + (2-2)^2 + (3-3)^2\right) = 0$$

$$J(0.5) = \frac{1}{6}\left((0.5-1)^2 + (1-2)^2 + (1.5-3)^2\right) = 0.58$$

$$J(0) = \frac{1}{6}\left((1)^2 + (2)^2 + (3)^2\right) = 2.3$$



Using both of the "knobs"

Hypothesis

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Parameters

$$\theta_0, \theta_1$$

Cost function

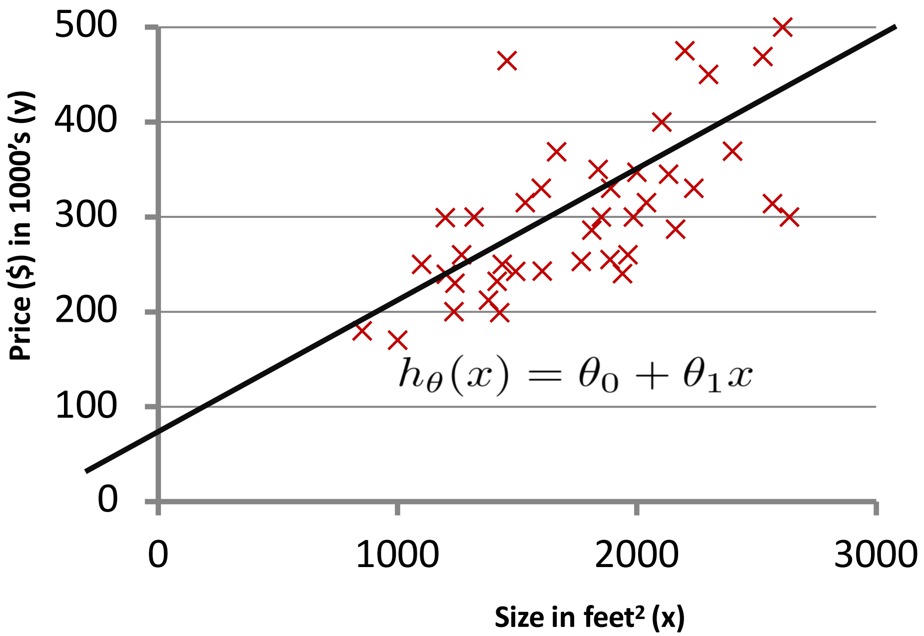
$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

Goal:

$$Minimize_{\theta_0, \theta_1} J(\theta_0, \theta_1)$$

$$h_\theta(x) = \theta_0 + \theta_1 x$$

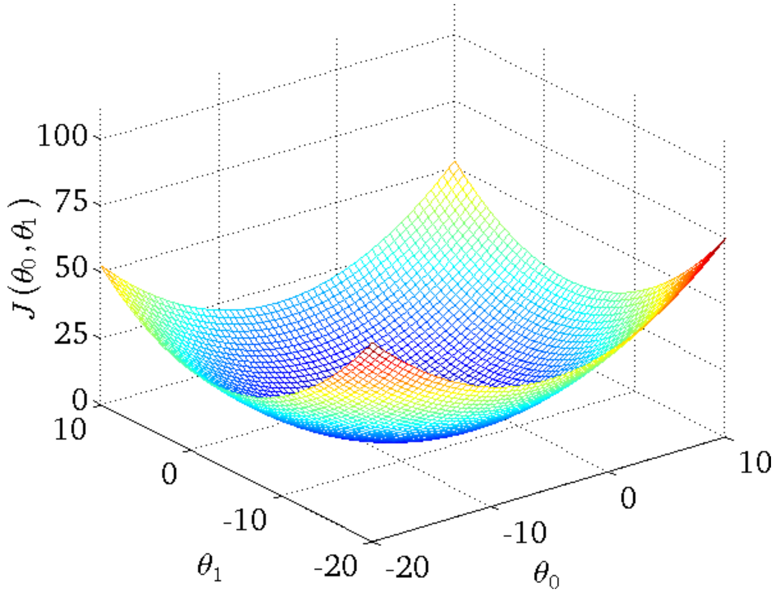
(for fixed θ_0, θ_1, this is a function of x)



$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$



Gradient Descent Algorithm

The Gradient Descent Algorithm

Goal: Minimize $J(\theta_0, \theta_1)$

Outline:

- Start with some (θ_0, θ_1) - For example (0,0)
- Keep updating (θ_0, θ_1) to reduce $J(\theta_0, \theta_1)$
  - Until we **<u>hopefully</u>** reach **<u>a</u>** minimum

A simplified version of gradient descent

Assume again that we set $\theta_0 = 0$ and our hypothesis and cost function practically have only one coefficient,θ_1.

$$h_\theta(x) = \theta_1 x$$

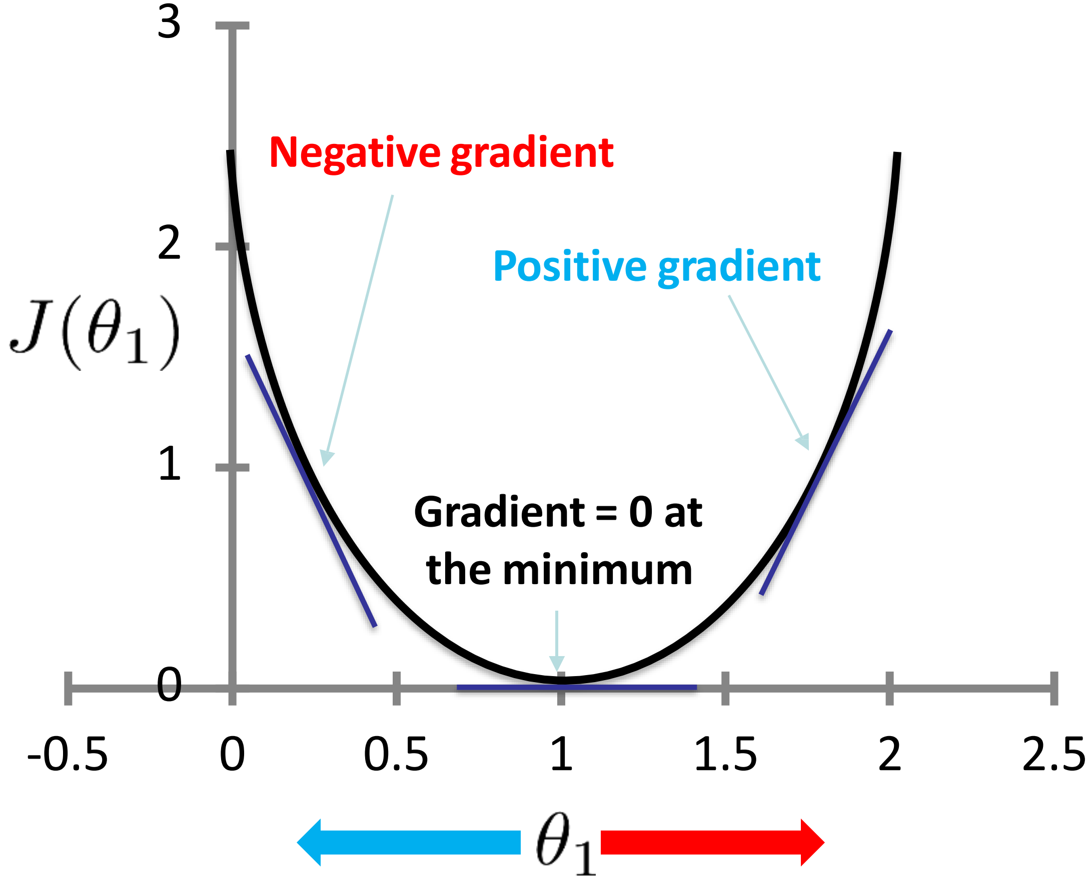
$repeat\ until\ convergence\{$

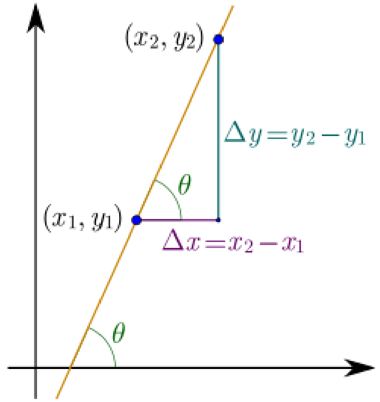
$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1}(J(\theta_1))$$

$\}$

$$J(\theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(\theta_1 x^{(i)} - y^{(i)}\right)^2$$

Direction of step





Gradient = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

The limiting case is when x_2 approaches x_1

$= \frac{\Delta y}{\Delta x} as \Delta x \rightarrow 0 = \frac{dy}{dx}$

$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1}(J(\theta_1))$$

$\theta_1 := \theta_1 - \alpha(negative)$: Increases θ_1

$\theta_1 := \theta_1 - \alpha(positive)$: Decreases θ_1

Step size

Gradient = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

The limiting case is when x_2 approaches x_1

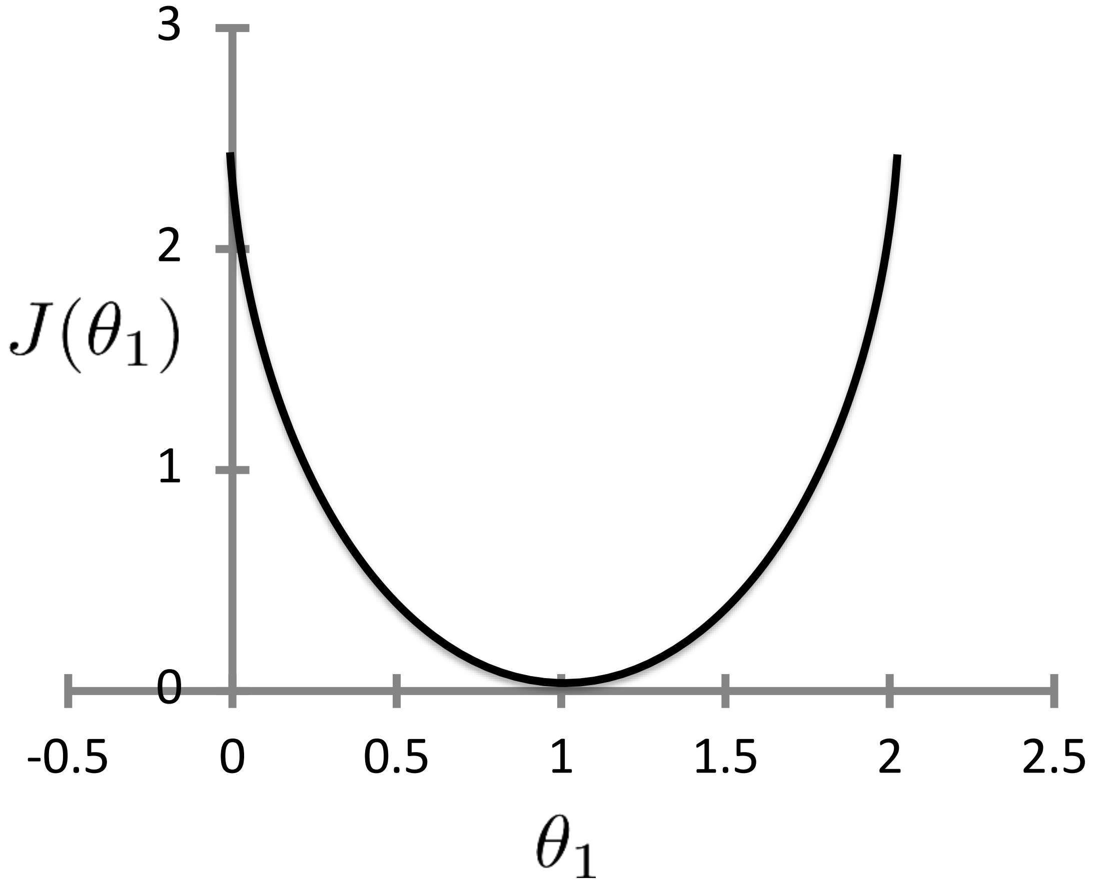
$= \frac{\Delta y}{\Delta x} as \Delta x \rightarrow 0 = \frac{dy}{dx}$

$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1}\big(J(\theta_1)\big)$$

$\theta_1 := \theta_1 - \alpha\,(negative)$: Increases θ_1

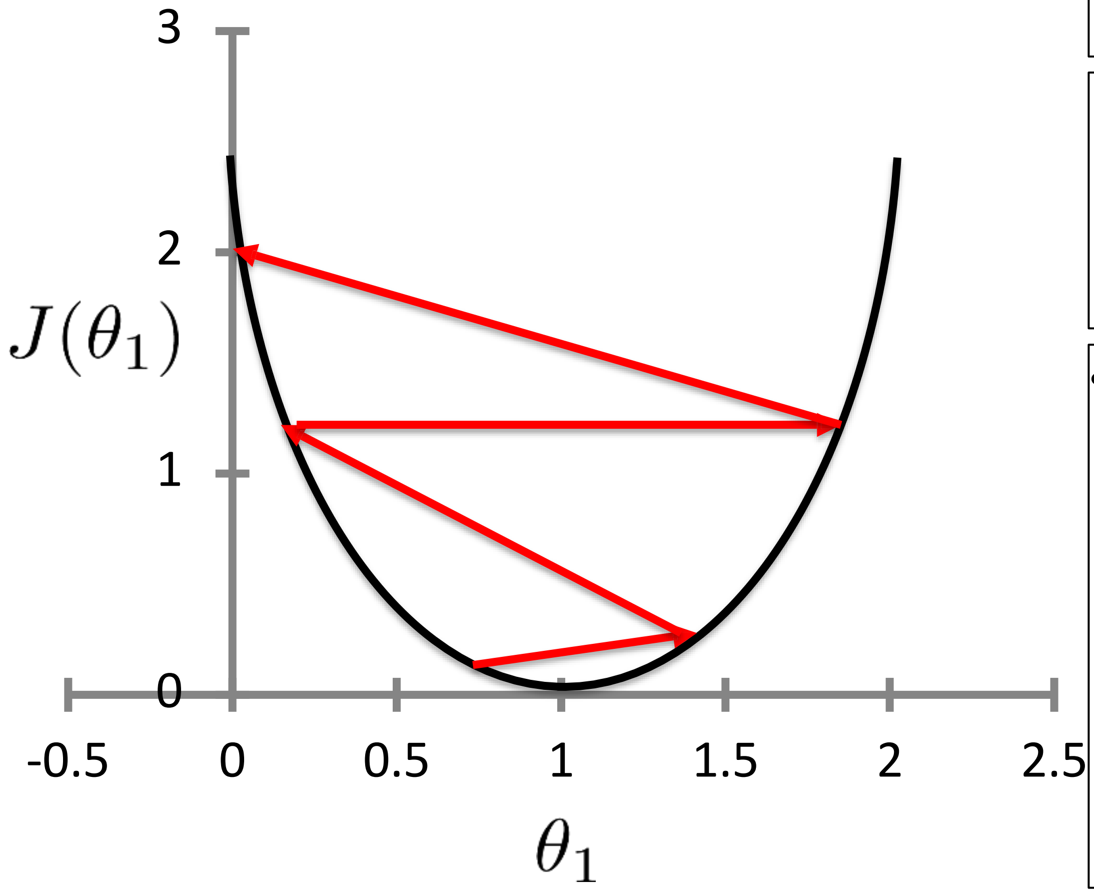
$\theta_1 := \theta_1 - \alpha\,(positive)$: Decreases θ_1

- In addition to α, the steepness of the gradient also control the step size.
- The step sizes become smaller as we get closer to the minimum, even with a fixed α.
- With α too small, takes a long time to reach the minimum



$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

Step size



$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

Gradient = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

The limiting case is when x_2 approaches x_1

$= \frac{\Delta y}{\Delta x} as \Delta x \rightarrow 0 = \frac{dy}{dx}$

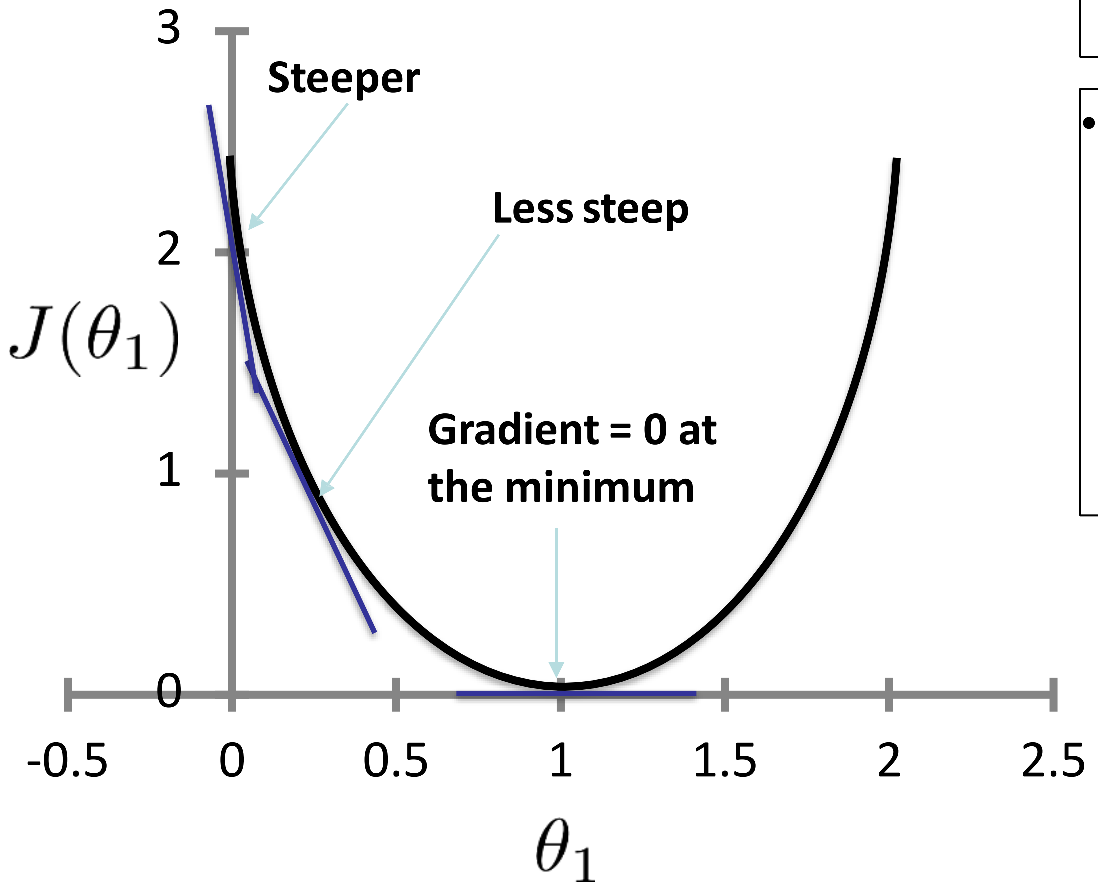
$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1}\left(J(\theta_1)\right)$$

$\theta_1 := \theta_1 - \alpha(negative)$: Increases θ_1

$\theta_1 := \theta_1 - \alpha(positive)$: Decreases θ_1

- In addition to α, the steepness of the gradient also control the step size.
- The step sizes become smaller as we get closer to the minimum, even with a fixed α.
- With α too small, takes a long time to reach the minimum
- With α too big, we can miss the minimum, and may fail to converge

Step size



$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

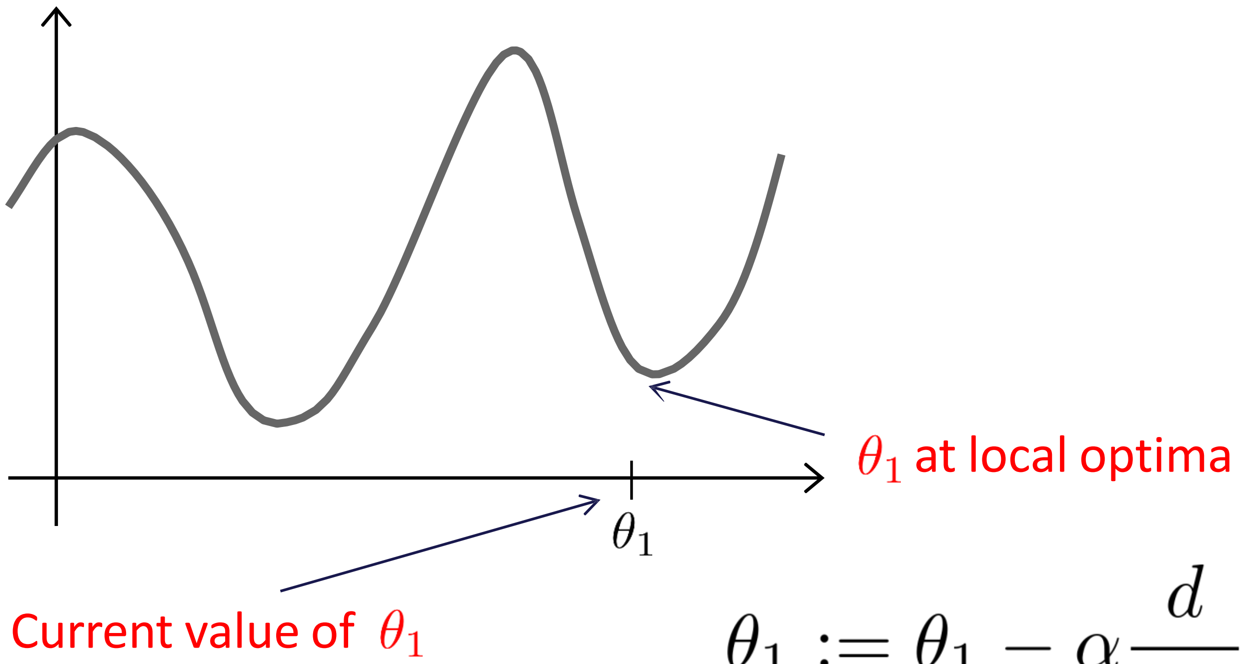
Gradient = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

The limiting case is when x_2 approaches x_1

$= \frac{\Delta y}{\Delta x} as \Delta x \rightarrow 0 = \frac{dy}{dx}$

- In addition to α, the steepness of the gradient also control the step size.
- The step sizes become smaller as we get closer to the minimum, even with a fixed α.
- No need to decrease α with time or number of steps

The problem of local optima



$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$

Gradient Descent by Hand

X	Y
1	2
2	4
3	6

$$\theta_1 = 0.1$$
$$\alpha = 0.1$$

$$repeat\ until\ convergence\{$$
$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1}(J(\theta_1))$$
$$\}$$

$$J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(\theta_1 x^{(i)} - y^{(i)}\right)^2$$

Gradient Descent by Hand

x	Y
1	2
2	4
3	6

$$\theta_1 = 0.1$$

$$\alpha = 0.5$$

$$repeat\ until\ convergence\{$$

$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1}(J(\theta_1))$$

$$\}$$

$$J(\theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(\theta_1 x^{(i)} - y^{(i)}\right)^2$$

Gradient Descent by Hand

X	Y
1	2
2	4
3	6

$$\theta_1 = 0.1$$
$$\alpha = 0.9$$

$$repeat\ until\ convergence\{$$
$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1}(J(\theta_1))$$
$$\}$$

$$J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(\theta_1 x^{(i)} - y^{(i)}\right)^2$$

Gradient Descent by Hand

x	Y
2	4
4	8
6	12

$$\theta_1 = 0.5$$
$$\alpha = 0.1$$

$$repeat\ until\ convergence\{$$
$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1}(J(\theta_1))$$
$$\}$$

$$J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(\theta_1 x^{(i)} - y^{(i)}\right)^2$$

Gradient descent algorithm

$$\text{repeat until convergence } \{$$
$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \qquad (\text{for } j = 0 \text{ and } j = 1)$$
$$\}$$

(Where α is the learning rate. E.g., 1, 0.1, 0.01, 0.001,… etc.)

Correct: **Simultaneous update**

$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$
$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$
$$\theta_0 := \text{temp0}$$
$$\theta_1 := \text{temp1}$$

Incorrect:

$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$
$$\theta_0 := \text{temp0}$$
$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$
$$\theta_1 := \text{temp1}$$

Gradient descent algorithm

repeat until convergence $\{$

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \qquad \text{(simultaneously update } j = 0 \text{ and } j = 1\text{)}$$

$\}$

Gradient descent algorithm

$$
\begin{aligned}
&\text{repeat until convergence } \{ \\
&\quad \theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \\
&\qquad \text{(for } j = 1 \text{ and } j = 0) \\
&\}
\end{aligned}
$$

Linear Regression Model

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

$$\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) = \frac{\partial}{\partial \theta_j} \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

$$\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) = \frac{\partial}{\partial \theta_j} \frac{1}{2m} \sum_{i=1}^{m} \left(\theta_0 + \theta_1 x^{(i)} - y^{(i)} \right)^2$$

$$j = 0 : \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)$$

$$j = 0 : \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^{m} \left(\theta_0 + \theta_1 x^{(i)} - y^{(i)} \right)$$

$$j = 1 : \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \quad x^{(i)}$$

$$j = 1 : \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^{m} \left(\theta_0 + \theta_1 x^{(i)} - y^{(i)} \right) \quad x^{(i)}$$

Gradient descent algorithm

repeat until convergence {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \cdot x^{(i)}$$

}

update θ_0 and θ_1 simultaneously

$$w_j \leftarrow w_j - \eta \frac{\partial E}{\partial w_j}$$

$$w_j \leftarrow w_j + \Delta w_j$$

$$\Delta w_j = -\eta \frac{\partial E}{\partial w_j}$$

$$\Delta w_j = \eta (t - y) x_j^{(i)}$$

“Batch” Gradient Descent

“Batch”: Each step of gradient descent uses all the training examples.

$$
\begin{aligned}
&\text{repeat until convergence } \{ \\
&\quad \theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \\
&\quad \theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \cdot x^{(i)} \\
&\}
\end{aligned}
$$

Types of Gradient Descent

Batch gradient descent: Computes the gradient of the cost function with respect to to the parameters θ for the entire training dataset (m instances).

Stochastic gradient descent: SGD performs a parameter update for *each* training example $x^{(i)}$ and label $y^{(i)}$

Mini-batch gradient descent: Mini-batch gradient descent takes the best of both worlds and performs an update for random mini-batches of k training examples, where $k < m$.

Read: https://ruder.io/optimizing-gradient-descent/

Multivariate

Single feature (variable)

Size (feet^2) x	**Price ($1000)** y
2104	460
1416	232
1534	315
852	178
…	…

$$h_\theta(x) = \theta_0 + \theta_1 x$$

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Multiple features (variables)

Size (feet^2)	Number of bedrooms	Number of floors	Age of home (years)	Price ($1000)
2104	5	1	45	460
1416	3	2	40	232
1534	3	2	30	315
852	2	1	36	178
…	…	…	…	…

$$h_\theta(X) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \cdots + \theta_n x_n$$

Generally speaking, weights are indicating the importance of each feature.

Notation:

m = Number of training samples

n = Number of features

x = Feature

y = Label

($x_j^{(i)}$, $y_j^{(i)}$): the jth feature of the ith sample in the dataset

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Hypothesis

Previously:

$$\boldsymbol{h_\theta(x) = \theta_0 + \theta_1 x}$$

Now:

$$\boldsymbol{h_\theta(X) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \cdots + \theta_n x_n}$$

where, $X = [x_1\ x_2\ \ldots\ x_n]$ is the n-dimensional feature vector, and $\Theta = [\theta_0\ \theta_1\ \ldots\ \theta_n]$ is an (n+1)-dimensional vector of weights.

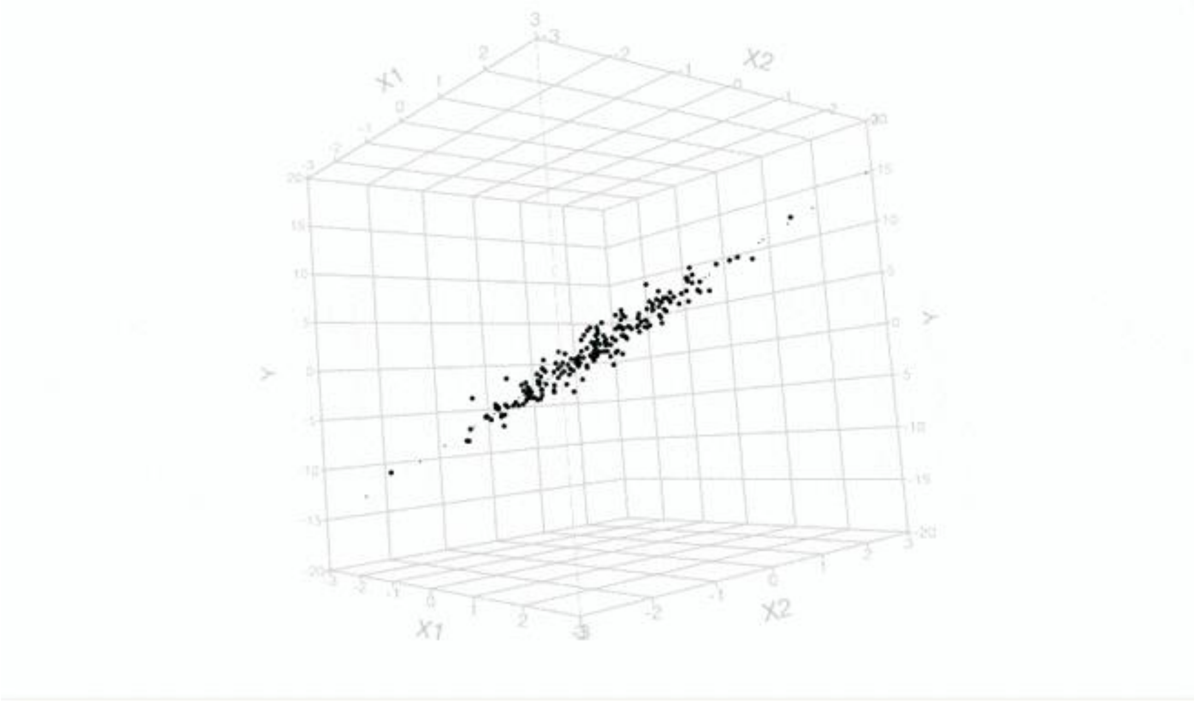
$$\boldsymbol{X} \in \mathbb{R}^{\boldsymbol{n}}$$
$$\boldsymbol{\Theta} \in \mathbb{R}^{\boldsymbol{n+1}}$$

Geometric Interpretation

$$\boldsymbol{h_\theta(X) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \cdots + \theta_n x_n}$$

where, $X = [x_1\ x_2\ \dots\ x_n]$ is the n-dimensional feature vector, and $\Theta = [\theta_0\ \theta_1\ \dots\ \theta_n]$ is an (n+1)-dimensional vector of weights.

$\boldsymbol{h_\theta(X) = \theta_0 + \theta_1 x_1 + \theta_2 x_2}$ (a 2-D hyperplane in 3-D)



https://www.jmp.com/en_us/statistics-knowledge-portal/what-is-multiple-regression/fitting-multiple-regression-model.html

Hypothesis

$$\boldsymbol{h_{\theta}(X) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \cdots + \theta_n x_n}$$

where, $X = [x_1\ x_2\ \ldots\ x_n]$ is the n-dimensional feature vector, and $\Theta = [\theta_0\ \theta_1\ \ldots\ \theta_n]$ is an (n+1)-dimensional vector of weights.

$$\boldsymbol{X} \in \mathbb{R}^{\boldsymbol{n}}, \boldsymbol{\Theta} \in \mathbb{R}^{\boldsymbol{n+1}}$$

To make this more uniform, assume $x_0 = 1$ to get:

$$\boldsymbol{h_{\theta}(x) = \theta_0 x_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \cdots + \theta_n x_n}$$

where, $X = [x_0\ x_1\ x_2\ \ldots\ x_n]$ is the (n+1)-dimensional feature vector, and

$\Theta = [\theta_0\ \theta_1\ \ldots\ \theta_n]$ is an (n+1)-dimensional vector of weights.

$$\boldsymbol{X} \in \mathbb{R}^{\boldsymbol{n+1}}, \boldsymbol{\Theta} \in \mathbb{R}^{\boldsymbol{n+1}}$$

Vectorizing the notation

$$\boldsymbol{h_\theta(x) = \theta_0 x_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \cdots + \theta_n x_n}$$

Now we can redefine our hypothesis as:

$\Theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_n \end{bmatrix}, X = \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix}$ and where $\Theta \in \mathbb{R}^{n+1}$ and $X \in \mathbb{R}^{n+1}$

and,

$$\boldsymbol{h_\Theta(X) = \Theta^T X}$$

Multivariate linear regression

For m-training instances

$$\boldsymbol{h_\theta(x) = \theta_0 x_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \cdots + \theta_n x_n}$$

$$\Theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_n \end{bmatrix}, X = \begin{bmatrix} x_0^{(1)} & x_0^{(2)} & x_0^{(3)} & & x_0^{(m)} \\ x_1^{(1)} & x_1^{(2)} & x_1^{(3)} & \dots & x_1^{(m)} \\ \dots & \dots & \dots & & \dots \\ x_n^{(1)} & x_n^{(2)} & x_n^{(3)} & & x_n^{(m)} \end{bmatrix}$$

$n \times 1$ $\qquad$ $n \times m$

$$h_\Theta(X) = \Theta^T X = [\theta_0\ \theta_1\ \dots \theta_n] \begin{bmatrix} x_0^{(1)} & x_0^{(2)} & x_0^{(3)} & & x_0^{(m)} \\ x_1^{(1)} & x_1^{(2)} & x_1^{(3)} & \dots & x_1^{(m)} \\ \dots & \dots & \dots & & \dots \\ x_n^{(1)} & x_n^{(2)} & x_n^{(3)} & & x_n^{(m)} \end{bmatrix} = [h_\theta(x^{(0)})\ \ h_\theta(x^{(1)})\ \dots\ h_\theta(x^{(m)})]$$

$1 \times n$ $\qquad$ $n \times m$ $\qquad = \qquad$ $1 \times m$

Multivariate Gradient Descent

Multivariate Gradient Descent

Hypothesis: $h_\theta\ (x) = h_\Theta(X) = \Theta^T X = \theta_0 x_0 + \theta_1 x_1 + \cdots + \theta_n x_n$

Parameter vector: $\Theta = \theta_0, \theta_1 \ldots \theta_n$

Feature vector: $\boldsymbol{X} = x_0 x_1 \cdots x_n$

Cost Function: $J(\theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$

Gradient descent:

repeat{

$\theta_j \coloneqq \theta_j - \alpha \frac{\partial}{\partial \theta_j} \left(J(\Theta) \right)$ //simultaneously update for all j = 0 … n

}

Gradient Descent

Previously (n=1):

Repeat $\{$

$$\theta_0 := \theta_0 - \alpha \underbrace{\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})}_{\frac{\partial}{\partial\theta_0}J(\theta)}$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x^{(i)}$$

(simultaneously update θ_0, θ_1)

$\}$

New algorithm $(n \geq 1)$:

Repeat $\{$

$$\theta_j := \theta_j - \alpha \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}$$

(simultaneously update θ_j for $j = 0, \ldots, n$)

$\}$

$$\theta_0 := \theta_0 - \alpha \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_0^{(i)}$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_1^{(i)}$$

$$\theta_2 := \theta_2 - \alpha \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_2^{(i)}$$

$\ldots$

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Sources

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