Q1 a

int s, i, n ③
cin >> n ①
s = 0 ①
for (i = n ; i >= 1 , i --) — $i = n$ ①, $i >= 1$ $n+1$, $i--$ n
s++ n

$T(n) = 3n + 7$

$T(n) = O(n)$

$3n + 7 <= kn$

$3 + \frac{7}{n} <= k$

$n_0 = 1$ $k >= 10$

$T(n) <= 10n$

Hence $T(n) = O(n)$
$n \geq 1$

Q1/b

int Sum, i, j, n ④
sum = 0 ①
cin >> ①
for (i = 1 ; i < n ; i = i x 2 — $i = 1$ ①, $i < n$ $\log_2 n$, $i = i \times 2$ $\log_2 n$

$$\text{for}\,(j=\overset{1}{1};\ \overset{\log_2 n}{j<n},\ \overset{\log_2 n}{j=j\times 2})$$

$$\text{sum}++; \quad \log_2 n$$

$$T(n) = 7 + \log_2 n + \log_2 n + \log_2 n$$

$$+ 3\log_2 n \times \log_2 n$$

$$T(n) = 7 + \cancel{2}3\log_2 n + 3\log_2^2 n$$

$$T(n) < k \log_2^2 n$$

$$|T(n)| = 7\log_2^2 n + 3\log_2^2 n + 3\log_2^2 n$$

$$13 \log_2^2 n$$

$$n_0 > 1 \quad \text{and} \quad k >= 13$$

$$T(n) <= 13 \log_2^2 n$$

$$= O(\log_2^2 n)$$

Q1/C

```
int sum, i, j, k, n          (5)
  sum = 0                    (1)
  k = 0                      (1)
  cin >> n;                  (1)
     (1)          (n+1)        (n)
  for(i=0,      i<n,         ++i
     k = 0          (n)
     sum++;         (n)
                 (n)       (n(n/3)+1)   (n(n/3))
     for(j =     n;        j>0;         j=j-3)
        sum ++ ;  n(n/3)
        k=k x sum   n(n/3)
        cout<< k ;  n(n/3)
     }
     cout << k ;   (n)
```

$T(n) = 5n\left(\frac{n}{3}\right) + 6n + 9$

$T(n) <= kn^2$

$T(n) <= \left(\frac{5}{3} + 6 + 9\right)(n^2) = \left(\frac{50}{3} n^2\right)$

$n_0 >= 1$

$O(n^2)$ $\quad$ $n_0 >= 1$

Q1/d

int sum, i, j, k, n 5

sum = 0 1

cin >> n 1

$\log_2 n$

1 $\log_2 n$

for(i=1, i<n; i=i×2

$\log_2 n$ $n \times \log_2 n$ $n \times \log_2 n$

for(j=0, j<n; ++j)

$n \times \log_2 n$ $n \log_2^2 n$ $n \log_2^2 n$

for(k=1, k<=n, k=k×2

sum++; $n \log_2^2 n$

$$T(n) = 3n\log_2^2 n + 3\log_2 n + 3\log_2 n$$

$$+ 8$$

$$T(n) <= k n \log_2^2 n$$

$$(3+3+3+8)\, n \log_2^2 n$$

$$T(n) <= 17\, n \log_2^2 n$$

$$O(n \log_2^2 n)$$

$$n_0 > 1$$

Q1/e

```
int i, j, Sum, n                         A
 cin >> n                                1
          1        log2n+1     log2n
for(i=1, i<=n, i=i×2

     cout<<i ;               log2n
     Sum=0;       log2n
       log2n
   for(j=1; j<=i, j=j×2

      Sum++;
      cout <<i;
   }
   cout << sum            log2n
```

Q2

Insertion sort $4n^2$
merge sort $32n \lg n$

$4n^2 \leq 32n \lg n$

$n \leq 8 \lg n$

$\frac{n}{8} \leq \lg n$

$2^{\frac{n}{8}} = 2^{\lg n}$

$\left(2^{\frac{n}{8}}\right)^8 = (n)^8$

$2^n = n^8$

$2 = n^{\frac{8}{n}}$

$n \leq 43$

Q3 $100n^2 = 2^n$

By trial and error and with graph help,

$n = 15$

Q4 $\Theta(n^3)$

$T(n) = \frac{1}{8}n^3 - 5n^2$

$K_2 n^3 \le T(n) \le K_1 n^3$

$\frac{1}{9}n^3 \le T(n) \le (\frac{1}{8}+5)n^3$

$K_2 = \frac{1}{9}$ $K_1 = \frac{13}{8}$

$n_0 \geq 1$

Q5 Selection Sort best case and worst case is $O(n^2)$ hence average case is $\Theta(n^2)$

```
Void InsertionSort (array[ ], size){
    for (J=1 TO n-1)
        smallest = j
    for (i = J+1 TO n)
        if array[i] < array[smallest]
            then smallest = i
        swap (Array[J], array[smallest])
    }
}
```

last element is already become sorted if $n-1$ are sorted.

Q6 $T(n) = 2m + 3m + 4m = P-1$

a) $m(2+3+4 \cdots n)$

$m\left(\frac{P-1}{2} + 4 + (P-2)\right)$

$m\left(\frac{P-1}{2}\right)(P+2) = O(mp^2)$

B) it is possible when two subarrays are merged at each level. e.g there are A, B, C, & D arrays with each having 5 elements then $P = A+B$ and $Q = C+D$ will be performed and $P+Q$ will be added.

it will reduce the time complexity to $O(mp)$

(1) >

if $y = 1$
  return x (1)

otherwise
  $c := x \times x$ (1)
  ans := findpower[$y/2$]

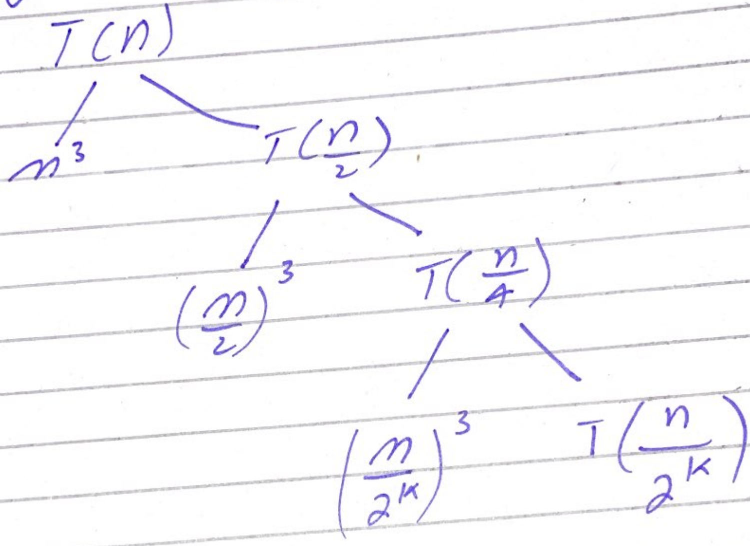
 if y is odd
   return $x \times$ ans (1)
 otherwise return ans

$$T(n) = \begin{cases} \cancel{\square}\ O(1) & y = 1 \\ T(\frac{n}{2}) + 2 & y > 1 \end{cases}$$

Q8/a

$$T(n) = T\left(\frac{n}{2}\right) + O(n^3)$$

Using Tree method



$$T(n) = n^3 + \frac{n^3}{8} + \frac{n^3}{64} \cdots \frac{n^3}{8^k}$$

$$T(1) = \frac{n^3}{8^k} \qquad n^3 = 8^k$$

$$k = \log_2 n$$

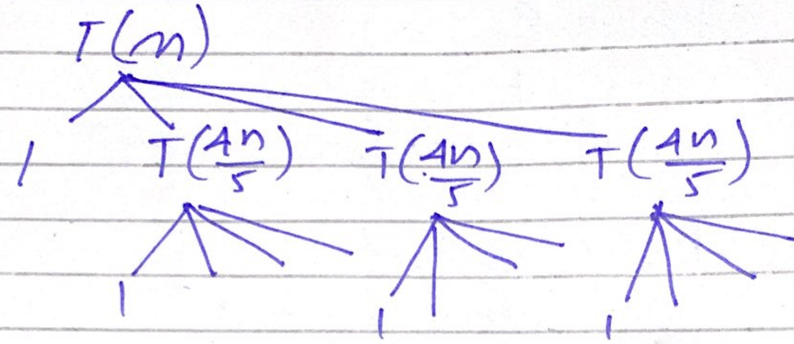
$$n^3 = 2^{3k} \Rightarrow n = 2^k$$

$$k = \log_2 n = \lg n$$

$$T(n) = n^3\left(1 + \frac{1}{8} + \frac{1}{64} \cdots \frac{1}{8^k}\right)$$

① 8/b

$$T(n) = 3T\left(\frac{4n}{5}\right) + O(1)$$



$$S = 3^0 + 3^1 + 3^2 \cdots 3^k$$

$$T\left(\frac{4^k n}{5^k}\right) = T(1)$$

$$4^k n = 5^k$$

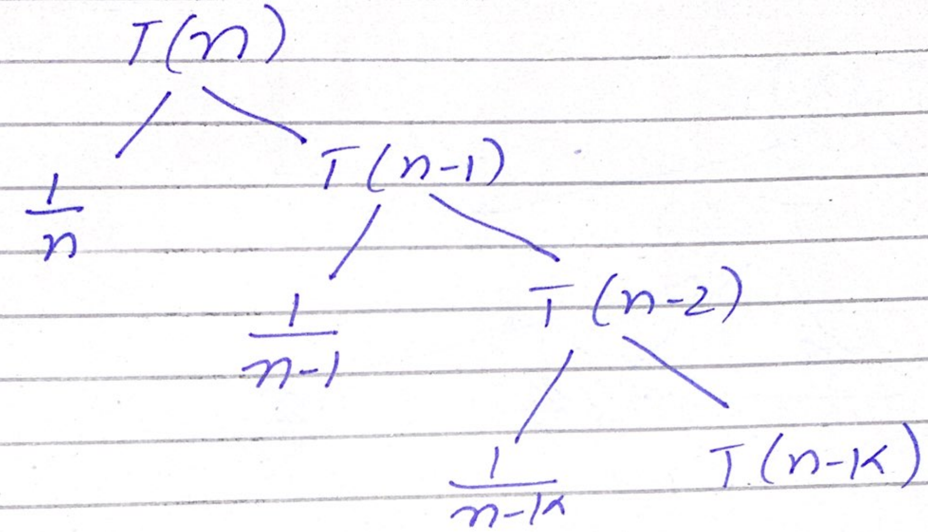
$$n = \left(\frac{5}{4}\right)^k \Rightarrow k = \log_{\frac{5}{4}} n$$

$$S = \frac{1\left(3^{\log_{\frac{5}{4}} n} - 1\right)}{3-1}$$

$$= \frac{n^{\log_{\frac{5}{4}} 3} - 1}{2}$$

$$O(\log(n^4))$$

1 8/c $T(n) = T(n-1) + \frac{1}{n}$



$T(n) = \frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} + \dots + \frac{1}{n-k}$

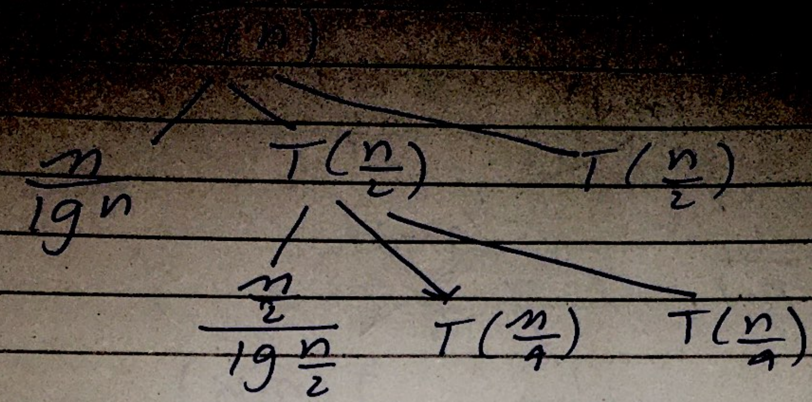
$n-k = 1$

$T(n) = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots \frac{1}{n}$

In reverse order.

$T(n) = \ln(n) + O(1)$

$= O(\ln(n))$



$$= 2^k \left(\frac{n}{2^k}\right) / \lg\left(\frac{n}{2^k}\right)$$

$$\frac{n}{\lg\left(\frac{n}{2^k}\right)} = 1$$

$$\cancel{n} 1 = \lg\left(\frac{n}{2^k}\right) = \lg n - \lg 2^k = 1$$

$$\lg n = \lg 2^k$$

$$k = \lg n$$

$$T(n) = \frac{n}{\lg n} + \frac{n}{\lg n - \lg 2} + \frac{n}{\lg n - \lg 4}$$

$$\frac{n}{\lg n - \lg 2^k}$$

$$T(n) = n\left(\frac{1}{\lg n} + \frac{1}{\lg n - \lg 2} \cdots \frac{1}{\lg n - 2^k}\right)$$

$$= n\left(\frac{1}{\lg n} + \cdots \frac{1}{2} + \frac{1}{1}\right)$$

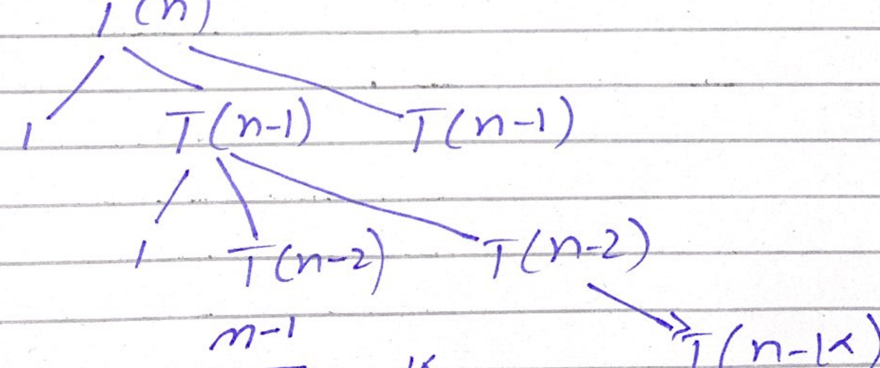
$$= n\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} \cdots \frac{1}{\lg n}\right)$$

$$= n \ln(\lg n)$$

$$O(n \ln(\lg(n))$$

or $O(n \lg^2 n)$

Q 8/e $T(n) = 2T(n-1) + \Theta(1)$



$$T(n) = \sum_{k=0}^{n-1} 2^k$$

$T(n) = 1 + 2 + 4 \cdots 2^k$

$T(n-k) = T(1)$

$n-k = 1$

$k = n-1$

$T(n) = 1 + 2 + 4 \cdots 2^{n-1}$

Geometric Serses

$r = 2 \qquad n \Rightarrow n-1$

$$S = \frac{1(2^{n-1} - 1)}{2-1}$$

$$= \frac{2^n - 1}{2} = O(2^n)$$