19L-1196

probability & Statistics

Assignment # 5

Q # 1

Density function of x is given by

$$f(x) = \begin{cases} a + bx^2 & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

if $E(x) = \frac{3}{5}$ find a, b.

Properties: area under $a+bx^2 = 1$

$$\int_0^1 (a+bx^2)dx$$

$$\left[ax + \frac{bx^3}{3}\right]_0^1 = a(1) + \frac{b}{3} - (0+0)$$

$$= a + \frac{b}{3} = 1 \quad (1)$$

$$E(X) = \int_0^1 x \cdot f(x)\,dx$$

$$= \int_0^1 ax + bx^3\,dx$$

$$= \left[\frac{ax^2}{2} + \frac{bx^4}{4}\right]_0^1$$

$$\frac{a(1)}{2} + \frac{b(1)}{4} - 0 = \frac{a}{2} + \frac{b}{4} = \frac{3}{5}$$

(II)

Solving simultaneous equation

$$a + \frac{b}{3} = 1 \quad (i)$$

$$\frac{a}{2} + \frac{b}{4} = \frac{3}{5} \quad (ii)$$

$$3a + b = 3 \quad (i)$$

$$4a + 2b = \frac{24}{5} \quad (ii)$$

$$2a + b = \frac{12}{5} \quad (ii)$$

(i) − (ii)

$$3a - 2a + b - b = 3 - \frac{12}{5}$$

$$a = \frac{3}{5}$$ Ans

substitution $a = \frac{3}{5}$ into (i)

$$b = 3\left(\frac{3}{5}\right) + b = 3$$

$$b = 3 - \frac{9}{5} = \frac{6}{5}$$ Ans.

① #2

$$f_X(x) = \frac{\lambda}{2} e^{-\lambda|x|}$$

$$\int_{-3}^{-2} \frac{\lambda}{2} e^{-\lambda|x|} dx + \int_{0}^{3} \frac{\lambda}{2} e^{-\lambda|x|} dx$$

$$\frac{\lambda}{2}\left[\frac{e^{-\lambda|x|}}{-\lambda}\right]_{-3}^{-2} + \frac{\lambda}{2}\left[\frac{e^{-\lambda|x|}}{-\lambda}\right]_{0}^{3}$$

$$\frac{\lambda}{2}\left(\frac{e^{-3\lambda}}{-\lambda} - \frac{e^{-2\lambda}}{-\lambda}\right) + \frac{\lambda}{2}\left(\frac{e^{-3\lambda}}{-\lambda} - \frac{1}{-\lambda}\right)$$

$$= \frac{1}{2} - \frac{1}{2} e^{-2\lambda}$$

Talking lambda $= 0.5$

$$\frac{1}{2} - \frac{1}{2e} = 0.31$$ Ans.