Homework # 4
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Summary Chp # 04.

→ Preliminary Theory - Linear equations

Theorem 4.1.1
existence of a Unique solution

let $a_n(x)$, $a_{n-1}(x)$ ... $a_1(x)$, $a_0(x)$ and $g(x)$ be continuous on an interval I and let $a_n(x) \neq 0$ for every x in this interval. If $x = x_0$ is any point in this interval then a solution $y(x)$ of initial value problem (1) exists on interval and is Unique.

Theorem 4.1.2
superposition principle - Homogenous Equations.

Let $y_1, y_2 ... y_k$ be solutions of homogenous n^{th} order differential equation (6) on interval I. then the combination

$$y = c_1y_1(x) + c_2y_2(x) + \dots + c_ky_k(x)$$

where c_i, $i = 1, 2, \dots, k$
are arbitrary constants,
is also a solution on
the interval.

Corollaries to theorem 4.1.2

(A) A constant multiple
$y = c_1y_1(x)$ of a solution $y_1(x)$ of
a homogenous linear equation.
(b) A homogenous linear
equation always posses
the solution $y = 0$

Definition 4.1.1

Linear Dependance/Independence

A set of function $f_1(x)$, $f_2(x)$,
$\dots$ $f_n(x)$ is said to be

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linearly dependent on an interval I if there exist constants $c_1, c_2 \dots c_n$ not all zero, such that

$$c_1 f_1(x) + c_2 f_2(x) + \dots + c_n f_n(x) = 0$$

for every x in the interval. If the set of functions is not linearly dependent on interval. It is said to be linearly independent.

Definition 4.1.2

Wronksian

Suppose each of functions $f_1(x), f_2(x) \dots f_n(x)$ possess at least $n-1$ derivatives. The determinant

$$W(f_1, f_2 \dots f_n) = \begin{vmatrix} f_1 & f_2 & \dots & f_n \\ f_1' & f_2' & & f_n' \\ \vdots & \vdots & & \vdots \\ f_1^{(n-1)} & f_2^{(n-1)} & & f_n^{(n-1)} \end{vmatrix}$$

where the primes denote derivatives, is called the wronksin of the functions.

Theorem 4.1.3
Criterion for linearly Independent solutions

Let $y_1, y_2 \dots y_n$ be n solutions of the homogenous linear n^{th}-order differential equation on an interval I. Then the set of solutions is linearly independent on I if and only if $W(y_1, y_2 \dots y_n) \neq 0$ for every x in the interval.

Definition 4.1.3
fundamental set of solutions

Any set $y_1, y_2 \dots y_n$ of n linearly indepent solutions of the homogenous linear n^{th} order differential equation on an interval I is said to be fundamental set of solutions on the interval.

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Theorem 4.1.4 Existence of a fundamental set:

There exists a fundamental set of solutions for homogenous linear nth order differential equation on interval I.

Theorem 4.1.5 General solution— Homogenous Equations

Let $y_1, y_2 \dots y_n$ be a fundamental set of solutions of the Homogenous linear nth order differential equation on an interval I. Then general solution of the equation on interval is

$$y = c_1 y_1(x) + c_2 y_2(x) + \dots + c_n y_n(x)$$

where $c_i, i = 1, 2 \dots n$ are arbitrary constants

Theorem 4.1.7 Superposition principle non-homogenous Equations

Let $y_{p_1}, y_{p_2} \dots y_{p_k}$ be k particular solutions of the nonhomogenous linear nth order differential equation on an interval

(12) $$a_n(x)y^{n} + a_{n-1}(x)y^{(n-1)} + \dots + a_1(x)y' + a_0(x)y = g_i(x)$$

where $i = 1, 2, \dots k$, then

(13) $$y_p = y_{p_1}(x) + y_{p_2}(x) + \dots + y_{p_k}(x)$$

is a particular solution of

$$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \dots + a_1(x)y' + a_0(x)y = g_1(x) + g_2(x) + \dots + g_k(x) \quad (14)$$