Homework # 11
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Date: ___/___/20__ Summary 11 Day: M T W T F S

Orthogonal function: The inner product of two functions f_1 and f_2 on interval $[a,b]$ is the number $(f_1, f_2) = \int_a^b f_1(x) f_2(x)\, dx = 0$. e.g $f_1(x) = x^2$, $f_2(x) = x^3$ are orthogonal on iterval $[-1,1]$ because $(f_1, f_2) = \int_{-1}^{1} f_1(x) f_2(x)\, dx = \int_{-1}^{1} x^2 x^3\, dx = 0$

Fruier Series: Suppose $\{\phi_n(x)\}$ is an infinite orthogonal set of function on interval $[a,b]$ we also if $y = f(x)$ is a function such as $f(x) = C_0\phi_0(x) + C_1\phi_1(x) + \cdots C_n\phi_n(x) \ldots$

$\{1, \cos\frac{\pi}{p}x, \cos\frac{2\pi}{p}x, \ldots\}$ $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos\frac{n\pi}{p}x\, dx \ldots) = \int_{-p}^{p} \cos\frac{n\pi}{p}x \sin\frac{n\pi}{p}x\, dx) = 0$

FOURIER Cosine and Sine series.

- Even and odd function
- properties of even and odd functions. The product of even.
- Fourier cosine and sine series. seq sequence of partial sums.
- Gibbs phenomenon
- if f is odd then $\int_{-a}^{a} f(x)\, dx = 0$

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Even and odd Functions

You may recall that a function f is said to be: **Properities of even & odd.**
even if $f(-x) = f(x)$ and
odd if $f(-x) = -f(x)$

Sturm - liouville problem

- Eigenvalues and eigen function
- self - adjoin form of a linear second, **Eigenvalues notition solutions only by vectors.**

$y'' + \frac{9\pi^2}{L^2} y = 0$ $\quad y = \sin \frac{3\pi x}{L}$;
$y(0) = 0, \; y(L) = 0$ solution to value

$y'' - 2y = 0$ sol is $y = 0$
$y(0) = 0, y(L) = 0$ solution to $y = 0$ Tairal solution,

Regular strurm Liouville problem

Theorm: let y_m and y_n be eigenvalues corresponding to eigen values λ_m and λ_n

then: $\frac{d}{dx}[r(x) y'_m] + (q(x) + \lambda_m p(x)) y_m = 0$

$(\lambda_m - \lambda_n) p(x) y_m y_n = y_m \frac{d}{dx}(r(x) y'_m$

Self - Adjoint Form: $\frac{1}{a(x)} e^{\int b(x) \ln(x) dx}$

$e^{\int (b/a) dx} y'' + \frac{b(x)}{a(x)} e^{\int b(a) dx} y' + \left(\frac{c(x)}{a(x)} e^{\int (b/a) d} + \lambda f(b/a) \right)$

$\frac{d}{dx}\left[e^{\int (b/a) dx} y' \right] + \left(\frac{c(x)}{a(x)} e^{\int (b/a) dx} + \lambda \frac{d(x)}{a(x)} e^{\int (b/a) dx} \right) y = 0$

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Parametric Bessel equation.

$\frac{d}{dx}[xy'] + (\lambda^2 x - \frac{n^2}{x})y = 0$ where we $r(x) = q(x) = -\frac{n^2}{x}$ and $P(x) = x$ and λ is $Y_n(\lambda x)$ only $J_n(\lambda x)$ is bounded at $x = 0$

$\int_0^b x J_n(\lambda_i x) J_n(\lambda_j x)\,dx = 0$, $\lambda_i \neq \lambda_j$

Legendre's Equation: let the variable

$\frac{d}{dx}[(1-x^2)y'] + n(n+1)y = 0$ we read off $r(x) = 1 - x^2$ $\frac{d}{dx}$ by $P(x) = 1$, and $\lambda = n(n+1)$,

$\{P_n(x)\}$, on $[-1,1]$. The orthogonality relation is $\int_{-1}^{1} P_m(x) P_n(x)\,dx = 0$, $m \neq n$

Fourier-Bessel Series: when the eigenvalues λ_i are defined by means of a boundary condition. $\alpha_2 J_n(\lambda b) + \beta_2 \lambda J_n'(\lambda b) = 0$

$f(x) = \sum_{i=1}^{\infty} c_i J_n(\lambda_i x)$. — (2) and

$c_i = \frac{\int_0^b x J_n(\lambda_i x) f(x)\,dx}{\int_0^b x J_n^2(\lambda_i x)\,dx}$

Differential Recurrance Relation:

$\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$; $\frac{d}{dx}[x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$

Square Norm: if $y = J_n(\lambda x)$, then we

$\frac{d}{dx}[xy'] + (\lambda^2 x - \frac{n^2}{x})y = 0$; $\frac{d}{dx}[xy']^2 + (\lambda^2 x^2 - n^2)\frac{d}{dx}(y)^2 = 0$

$2\lambda^2 \int_0^b x y^2\,dx = ([xy']^2 + (\lambda^2 x^2 - n^2)y^2]_0^b$

Case 1: if we choose $\alpha_2 = 1$ and $\beta_2 = 0$

$J_n(\lambda b) = 0$, There are an infinite number of positive roots x_i of eigenvalues are positive $\lambda_i = x_i/b$