Cal-II Finals

Name:
Abdul-Rehman

SECTION:
CS-D

ROLL NO:
19L-1135

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Q-1

$$\sum_{n=1}^{\infty} \frac{x^n}{2^n \ln(n)}$$

Sol Given series converges at

$$2 \leq x < 2$$

Applying Ratio test

$$a_n = \frac{x^n}{2^n \ln(n)} \qquad a_{n+1} = \frac{x^{n+1}}{2^{n+1} \ln(n+1)}$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x^{n+1}}{2^{n+1} \cdot \ln(n+1)} \cdot \frac{2^n \ln(n)}{x^n}\right|$$

$$= \frac{\cancel{x^n} \cdot x}{\cancel{2^n} \cdot 2} \cdot \frac{\cancel{2^n} \ln(n)}{\cancel{x^n}}$$

$$= \frac{\ln(n)}{\ln(n+1)} \cdot x/2$$

Applying n limit.

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{\ln(n)}{\ln(n+1)}\right| \cdot x/2$$

$$= \lim_{n\to\infty} \left|\frac{1/n}{1/n+1}\right| \cdot x/2$$

$$= x/2 \, (1)$$

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$$x/2 < 1 \Rightarrow -2 < x < 2$$

So it converges at radius 2

$$\frac{2^x}{2^n \ln(n)} = \int_2^\infty \frac{1}{\ln(n)}$$

$$\int_2^\infty \frac{1}{\ln(n)} = |e^\infty - e^2|$$

According to integral test, the series diverge at $x = 2$.

Put ~~x=2~~ $x = -2$ in eq

$$\lim_{n \to \infty} \frac{(-2)^n}{\ln(n)} = a_n = \frac{1}{\ln(n)}$$

Applying integral test

a_n is positive for all n

$$\frac{1}{\ln(n+1)} = a_{n+1} > a_n = \frac{1}{\ln(n)}$$

So, $\int_1^\infty \frac{1}{\ln(n)}$ diverges

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Q-2

(a)

Sol

$$\sum_{n=1}^{\infty} n e^{-n}$$

Sol

$a^n = n e^{-n}$, $a^{n+1} = (n+1) e^{-(n+1)}$

Using ratio test

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{(n+1) e^{-(n+1)}}{n e^{-n}}$$

$$= \lim_{n\to\infty} \frac{(n+1) \cdot \cancel{e^{-n}} \cdot e^{-1}}{n \cdot \cancel{e^{-n}}}$$

$$= \frac{1}{e} \lim_{n\to\infty} \frac{n+1}{n}$$

$$= \frac{1}{e} \lim_{n\to\infty} \frac{\cancel{n}(1+1/n)}{\cancel{n}}$$

$$= \frac{1}{e} \lim_{n\to\infty} (1+1/n)$$

$$= 1/e \, (1+1/\infty) \qquad (\because \text{As } 1/\infty = 0)$$

$$= 1/e \, (1+0)$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1/e$$

As $\frac{1}{e} = 0.369 < 1$, So, according to limit the series $\boxed{\sum_{n=1}^{\infty} n e^{-n} \text{ is convergent.}}$

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(b)

$$f(x) = \ln(1+x)$$

Sol

Let $x = 0$

$f(0) = \ln(1+0) = \ln(1) \Rightarrow 0$

Now,

$f(x) = \ln(1+x)$

$f'(x) = \frac{d}{dx} \ln(1+x)$

$= \frac{1}{1+x}$

$f(0) = \ln(1+0) = \ln(1) = 0$

$f'(0) = \frac{1}{1+0} = 1$

$f''(x) = \frac{d}{dx}\left(\frac{1}{1+x}\right) \Rightarrow \frac{d}{dx}(1+x)^{-1}$

$f''(x) = -1(1+x)^{-2}$

$f''(0) = -1(1+0)^{-2}$

$f''(0) = -1$

$f'''(x) = -1 \cdot -2(1+x)^{-3}$

$f'''(x) = 2(1+x)^{-3}$

$f''''(x) = -6(1+x)^{-4}$

$f'''(0) = 2(1+0)^{-3}$

$f'''(0) = 2$

$f''''(0) = -6(1+0)^{-4}$

$f''''(0) = -6$

Maclaurin's series

$$f(x) = f(x) + f'(x)x + f''(x)\frac{x^2}{2} + f'''(x)\frac{x^3}{6} + f''''(x)\frac{x^4}{24}$$

$$\ln(1+x) = 0 + x - \frac{x^2}{2} + \frac{x^3}{3} - \left(\frac{6 \cdot x^4}{24}\right)$$

$$\boxed{\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}}$$

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Q-2
(b)

$$f(x) = \frac{\ln(1+x)}{x}$$

Sol

Let $x = 0$

$$f'(x) = x\frac{d}{dx}\ln(1+x) - \ln(1+x)\frac{d}{dx}(x)$$

$$f'(x) = x\frac{d}{dx}\ln(1+x) - \ln(1+x) \Rightarrow \boxed{\frac{x}{1+x} - \ln(1+x)}$$

$$f''(x) = \frac{d}{dx}\left(\left(\frac{x}{(1+x)}\right) - \ln(1+x)\right) \Rightarrow \frac{d}{dx}\left(x\cdot(1+x)^{-1} - \ln(1+x)\right)$$

$$f''(x) = -x\cdot(1+x)^{-2} + \cancel{(1+x)^{-1}} - \frac{1}{\cancel{1+x}} \Rightarrow \boxed{-x\cdot(1+x)^{-2}}$$

$$f'''(x) = \frac{d}{dx}\left(-x\cdot(1+x)^{-2}\right) \Rightarrow -x\cdot -2\cdot(1+x)^{-3} + (1+x)^{-2}\cdot(-1)$$

$$\Rightarrow \boxed{2x\cdot(1+x)^{-3} - (1+x)^{-2}}$$

$$f''''(x) = \frac{d}{dx}\left(2x\cdot(1+x)^{-3} - (1+x)^{-2}\right)$$

$$f''''(x) = 2x\cdot -3(1+x)^{-4} + (1+x)^{-3}(2) - (-2\cdot(1+x)^{-3})$$

$$f''''(x) = \boxed{-6x(1+x)^{-4} + 2(1+x^{-3}) + 2(1+x)^{-3}}$$

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From upper part, ~~when~~

$f'(x) = \frac{x}{1+x} - \ln(1+x)$, As $\ln(1+x) = 0$

① $\frac{1}{1+x} = 1$

$f'(x) = \left(\frac{1}{1+x}\right) \cdot x - \ln(1+x)$

② $-(1+x)^{-2} = -1$

③ $2(1+x)^{-3} = 2$

④ $-6(1+x)^{-4} = -6$

$f'(x) = (1)x - 0$

$f'(x) = x$

$\boxed{f'(0) = 0}$

$f''(x) = -x(1+x)^{-2}$

$f''(x) = x \cdot (-(1+x)^{-2})$

$f''(x) = x \cdot (-1)$

$f''(x) = -x$

$\boxed{f''(0) = 0}$

$f'''(x) = 2x \cdot (1+x)^{-3} - (1+x)^{-2}$

$f'''(x) = x(2(1+x)^{-3}) + (-1+x)^{-2})$

$f'''(x) = x(2) + (-1)$

$f'''(x) = 2x - 1$

$f'''(0) = 2(0) - 1$

$\boxed{f'''(0) = -1}$

$f''''(x) = -6x(1+x)^{-4} + 2(1+x)^{-3} + 2(1+x)^{-3}$

$f''''(x) = x(-6(1+x)^{-4}) + (2(1+x)^{-3}) + (2(1+x)^{-3})$

$f''''(x) = x(-6) + (2) + (2)$

$f''''(x) = -6x + 4$

$f''''(0) = -6(0) + 4$

$\boxed{f''''(0) = 4}$

In Maclaurin's series

$f(x) = f(x) + xf'(x) + f''(x)\frac{x^2}{2} + f'''(x)\frac{x^3}{6} + f''''(x)\frac{x^4}{24}$

$\frac{\ln(1+x)}{x} = 0 + 0 + 0 + (-1)\frac{x^3}{6} + \frac{x^4}{24}$

$\boxed{\frac{\ln(1+x)}{x} = -\frac{x^3}{6} + \frac{x^4}{6}}$

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Q-3

(A) Amount of water = 60,000 gallons

(x_0) Amount of dye = 5 Kg

(rate) Cleaning rate = 200 gal/min

$Q(t)$, t

1 Kg = 0.264172053 gallon
5 Kg = 1.320860265 gallon

Sol

$Q(t)$ is the amount of dye in water at time "t".

So, By initial-value Problem formula:

$$\frac{dx}{dt} = Kx, \quad x(t_0) = x_0$$

where, K is constant of proportionality.

We can say that,

As rate is taking water, so

$$\frac{dQ}{dt} = \frac{Q(t)\ (\text{in Kg})}{\text{Amount of water}} \times \text{rate} \times -1$$

$$\frac{dQ}{dt} = \frac{Q(t)}{60000\,\text{gal}} \times 200\,\text{gal/min}$$

$$\text{''} = -\frac{Q(t)}{300\ \text{min}}$$

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$$\boxed{\frac{dQ}{dt} = \frac{-Q(t)}{300 \text{ min}}}$$, According Initial value problem

$\dot{x}(t_0) = 5$

where t is in min and Q is in Kg

So,

$Q(0) = 5$ —(ii)

$$\int \frac{1}{-Q(t)} dQ = \int \frac{1}{300} (dt)$$

$$-1 \ln(Q(t)) = \frac{1}{300} \cdot t + c$$

$$\ln(Q(t)^{-1}) = \frac{1}{300} \cdot t + c$$

$$e^{\ln(Q(t)^{-1})} = e^{\frac{t}{300} + c}$$

$$Q(t)^{-1} = e^{t/300 + c}$$

$$\boxed{e^{-t/300 + c} = Q(t)}$$

$$e^{-t/300} \cdot e^{c} = Q(t)$$

$$e^{-t/300} \cdot c = Q(t)$$

Let $t = 0$, so From (ii)

$$e^{-0/300} \cdot c = Q(0)$$

$$e^{0} \cdot c = Q(0)$$

$$1 \cdot c = 5$$

$$c = 5$$

Thus,

$$Q(t) = 5 \cdot e^{-t/300}$$

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Q-4

$$y' + ay = be^{-\lambda x}$$

Sol

As we know,

$$y' + P(x)y = q(x)$$

So, $P(x) = a$, $q(x) = be^{-\lambda x}$

Now,

$$I(x) = e^{\int P(x)dx}$$

$$I(x) = e^{\int a\,dx}$$

$$I(x) = e^{a\int 1\,dx}$$

$$\boxed{I(x) = e^{ax}}$$

To find general solution,

$$y(x) = \frac{1}{I(x)}\left(\int I(x)\,q(x)\,dx + c\right)$$

$$y(x) = \frac{1}{e^{ax}}\left(\int e^{ax}\cdot be^{-\lambda x}\,dx + c\right)$$

$$y(x) = \frac{1}{e^{ax}}\left(\int e^{ax-\lambda x}\cdot b\,dx + c\right)$$

$$y(x) = \frac{1}{e^{ax}}\left(b\int e^{ax-\lambda x}\,dx + c\right)$$

$$y(x) = \frac{b}{e^{ax}}\int e^{ax-\lambda x}\,dx + \frac{c}{e^{ax}}$$

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$$y(x) = \frac{b}{e^{ax}} \int e^{(a-\lambda)x} dx + \frac{c}{e^{ax}}$$

$$y(x) = \frac{b}{\cancel{e^{ax}}} \frac{\cancel{e^{ax}} \cdot e^{-\lambda x}}{a-\lambda} + c e^{-ax}$$

$$y(x) = \frac{b}{a-\lambda} \cdot e^{-\lambda x} + c e^{-ax} \quad -(1)$$

Assume, $a \neq \lambda$

So, $y \to 0$ as $x \to \infty$, verify

From (1)

$$\lim_{x\to\infty} y(x) = \lim_{x\to\infty} \left(\frac{b}{a-\lambda} \cdot e^{-\lambda x} + c e^{-ax} \right)$$

$$= b \lim_{x\to\infty} \left(\frac{e^{-\lambda x}}{a-\lambda} \right) + c \lim_{x\to\infty} \left(e^{-ax} \right)$$

$$\lim_{x\to\infty} y(x) = \frac{b}{a-\lambda} \lim_{x\to\infty} \left(e^{-\lambda x} \right) + c \lim_{x\to\infty} \left(e^{-ax} \right)$$

$$= \frac{b}{a-\lambda} \lim_{x\to\infty} -\lambda e^{-\lambda x} + c \lim_{x\to\infty} e^{-ax}$$

By applying limit rules

$$\lim_{x\to\infty} y(x) = \frac{b \cdot -\lambda}{a-\lambda} \lim_{x\to\infty} e^{-\lambda x} + c \lim_{x\to\infty} e^{-ax}$$

$$= \frac{b \cdot (-\lambda)}{a-\lambda} (0) + c(0)$$

$$\boxed{\lim_{x\to\infty} y(x) = 0}$$

Hence, Proved

$y \to 0$, when $x \to \infty$

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Q-5

$$(3xy+y^2)+(x^2+xy)\frac{dy}{dx}=0$$

Sol

~~As be~~ $(x^2+xy)dy = -(3xy+y^2)dx$

$$\underbrace{(3xy+y^2)dx}_{M} + \underbrace{(x^2+xy)dy}_{N} = 0$$

$M = (3xy+y^2)$ | $N = x^2+xy$

$\frac{d}{dy}M = 3x+2y$ | $\frac{dN}{dx} = 2x+y$

As $M(y) \neq N(x)$, so the given differential eqn is $\boxed{\text{not exact.}}$

Given:

$\Rightarrow \mu(x,y) = \frac{1}{xy(2x+y)}$

Converting given equation into exact differential eqn

$$\frac{1}{xy(2x+y)}(x^2+xy)dy + \frac{1}{xy(2x+y)}(3xy+y^2)dx = 0$$

$$\frac{1}{y(2x+y)}(x+y)dy + \frac{1}{x(2x+y)}(3x+y)dx = 0$$

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$$\underbrace{\frac{3x+y}{x(2x+y)}}_{m} dx + \underbrace{\frac{(x+y)}{y(2x+y)}}_{n} dy = 0$$

$$\frac{dm}{dy} = \cancel{3x+y}\ x(2x+y)(1) + (3x+y)(x)$$
$$= 2x^2 + xy \cancel{+3x^2} + 3x^2 + xy$$
$$= 5x^2 + 2xy$$

$$\frac{dn}{dx} = y(2x+y)(1) + (x+y)(2y)$$

$$\frac{dn}{dx} = \cancel{2xy + y^2 + 2xy + 2y^2}\ 2x^2 + xy + 3x^2 + xy$$
$$\cancel{= 3y^2 + 4xy} \quad = 5x^2 + 2xy$$

So, $\frac{dm}{dy} = \frac{dn}{dx}$

$$\phi(x,y) = c = \int m\,dx + g(y)$$
$$= \int (5x^2 + 2xy)\,dx + g(y)$$
$$\phi(x,y) = \int ($$

$$\phi(x,y) = \frac{1}{2}x^2y^2 +$$

$$y(x) = \frac{-\sqrt{c_1 + x^4}}{x} - x$$

$$\cancel{\sqrt{c_1+x^4}} \quad \cancel{\frac{-\sqrt{c_1+x^4}}{x}} \quad \text{or } y(x) = \frac{\sqrt{c_1+x^4}}{x} - x$$

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Q-6

$$\frac{dy}{dx} = 1 + x^2 - 2xy + y^2$$

Sol

$$\frac{dy}{dx} = 1 + x^2 - 2xy_p(x) + y_p^2(x)$$

$$\int \frac{dy/dx}{1 + x^2 - 2xy_p(x) + y_p^2(x)} = \int 1\, dx$$

Substitute $v = y_p(x)$

$$= \int \frac{dx}{1 + x^2 - 2xv + v^2}$$

$$= \ln|1 + x^2 - 2x(v) + v^2| + c$$

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Q-7

$y_1(x) = x^{-1}$

$2x^2y'' + 3xy' - y = 0, \; x > 0$

Sol

Here, $2x^2y'' + 3xy' - y = 0$

$$y'' + \frac{3}{2x}y' + g(x)y = f(x)$$

$$\Rightarrow 2x^2y'' + 3xy' - y = 0$$

$$y'' + \frac{3}{2x}y' - \frac{y}{2x^2} = 0$$

$g(x) = -1/2x^2$

$P(x) = 3/2x$, $f(x) = 0$

This Function is continuous on the interval except $u = 0$, So

$I = (0, \infty)$, So, P, g, f are continuous Function on $(0, \infty)$.

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Now,

$$2x^3 \frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0$$

$$y = x^{-1}$$

$$y = \frac{1}{x} \Rightarrow \frac{dy}{dx} = y' = -1/x^3$$

$$\frac{d^2y}{dx^2} = y'' = \frac{2}{x^3}$$

~~Put in~~

So, $2x^2\left(\frac{2}{x^3}\right) + 3\left(-1/x^2\right) - y = 0$

$$\boxed{y = \frac{4}{x} - 3/x^2 + c}$$

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Q-10

(a)

$$y'' + 3y' = 2x^4 + x^2 e^{-3x} + \sin 3x$$

$$m^2 + 3m = 0$$

$$m(m+3) = 0$$

$$\Rightarrow m_1 = 0, \; m_2 = -3$$

$$y_p = c_1 + c_2 e^{-3x}$$

$$2x^4 + x^2 e^{-3x} + \sin 3x$$

$$\therefore 2x^4 = D^5$$

$$x^2 e^{-3x} = (D-2)^{-2} + (D+3)^3$$

$$= [D+9]$$

$$\Rightarrow y_p^2 = m^5 (m+3)^3 (m+9)(m^2+3m) = 0$$

So,

$$\Rightarrow m_1 = m_2 = m_3 = m_4 = m_5 = 0$$

$$\Rightarrow m_6 = m_7 = m_8 = -3$$

$$\Rightarrow m_9 = -9$$

$$\Rightarrow m_{10} = 0$$

$$\Rightarrow m_{11} = -3$$

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$$y_p = Ax + Bx^2 + Cx^3 + Dx^4 + Ex^5 + Fe^{-3x} Ge^{-3x}x^2 + He^{-3x}x^3 + Ie^{-a}$$

$$y'' + 2y' + 2y = 3e^{-x} + 2e^{-x}\cos x + 4e^{-x}x^2 \sin x$$

By using ~~sy~~ superpostion,

$$y_{p_1} = Ae^{-x}$$

$$y_{p2} = Be^{-x}\cos x + Ce^{-x}\sin x$$

$$y_{p3} = (Dx^x + Ex + F)e^{-x}\cos x + (Gx^2 + Hx + I)e^{-x}\sin x$$

$$y = Ae^{-x} + Be^{-x}\cos x + (e^{-x}\sin x + Dx^2e^{-x}\cos x + Exe^{-x}\cos x + Fe^{-x}\cos x^{-1}e^{-x}\sin x + Hx\, e^{-x}\sin x + Ie^{-x}$$

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Q-11

$$ax^3y''' + bx^2y'' + cxy' + dy = g(x)$$

Sol

Let $y = x^m$

$y' = m(x^{m-1})$, $y'' = m(m-1)x^{m-2}$

$y''' = m(m-1)(m-2)x^{m-3}$

Charactreqn:

$a x^3 m(m-1)(m-2)x^{m-3} + b x^2 m(m-1)x^{m-2}$
$+ cxm(x^{m-1}) + dx^m = 0$

$a=1$, $b=4$, $c=1$, $d=-1$

$x^3 m(m-1)(m-2)x^{m-3} + 4x^2 m(m-1)x^{m-2} + xmx^{m-1}$
$+ dx^m = 0$

$m(m-1)(m-2) + 4m(m-1) + m - 1 = 0$

$m[(m-1)(m-2) + 4(m-1) + 1] = 1$

$m_1 = 1$, $(m-1)(m-2) + 4m - 4 + 1 = 1$

$m^2 + m = 2$

$m_2 = 2$, $m_3 = 1$

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$y_c = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{-x}$

when $g(x) = x^2$, assume $Ax^2 + Bx + C$

$y' = 2Ax + B$

$y'' = 2A$

$y''' = 0$

then

$Ax^2 + 2Ax^2 + Bx - Ax^2 + Bx + C = x^2$

$9Ax^2 + 2Bx + C = x^2$

$9A\cancel{x^2} = 1\cancel{x^2}$, $2Bx = 0$, $C = 0$

$A = 1/9$, $B = 0$, $C = 0$

$y_p = \frac{1}{9}x^2 + 0 + 0 \Rightarrow 1/9\, x^2$

$y = y_c + y_p$

$y = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{-x} + 1/9\, x^2$

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Q-12

$$f(x) = \begin{cases} -\pi & \text{if } -2\pi < x < \pi \\ x & \text{if } -\pi \le x < \pi \\ \pi & \text{if } \pi \le x < 2\pi \end{cases}$$

Sol As the given $f(x)$ is odd.
So, Fourier Series of an odd series is

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{P} x \qquad (-2\pi, 2\pi)$$

where

$$b_n = \frac{2}{P} \int_0^P f(x) \sin \frac{n\pi}{P} x \, dx$$

So, by applying values

$$b_n = \frac{2}{2\pi} \int_0^{2\pi} f(x) \sin \frac{n\pi}{2\pi} x \, dx$$, by re-arranging

it becomes

$$b_n = \frac{1}{\pi} \int_0^{\pi} x \sin \frac{n}{2} x \, dx + \frac{1}{\pi} \int_{\pi}^{2\pi} \pi \sin \frac{nx}{2} \, dx$$

let,

$u = x$

$du = dx$

And, $dv = \sin \frac{nx}{2} dx$

$v = \frac{-2}{n} \cos \frac{n}{2} x$

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So, by applying values

$$b_n = \frac{1}{\pi}\left(\left[-\frac{2}{n}x\cos\frac{n}{2}x\right]_{x=0}^{x=\pi} + \frac{2}{n}\int_0^{\pi}\cos\frac{n}{2}x\,dx\right)$$

$$+\int_{\pi}^{2\pi}\sin\frac{nx}{2}\,dx$$

$$b_n = \frac{1}{\pi}\left[\frac{-2\pi\cos(n\pi/2)}{n}\right]^{x=0} + \frac{2}{n\pi}\left[\frac{2\sin(n\pi/2)}{n}\right]_{x=0}^{x=\pi}$$

$$+\left[\frac{-2\cos(n\pi/2)}{n}\right]_{x=\pi}^{x=2\pi}$$

$$= \frac{-2\pi\cos(n\pi/2)}{n} + \frac{4\sin(n\pi/2)}{n^2\pi} + \frac{-2(-1)^n + 2\cos(n\pi)}{n}$$

$$= \frac{4\sin(n\pi/2) - 2n\pi(-1)^n}{n^2\pi}$$

Thus,

$$\boxed{f(x) = \sum_{n=1}^{\infty}\frac{4\sin(n\pi/2) - 2n\pi(-1)^n}{n^2\pi}\sin\frac{n}{2}x}$$

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