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HomeWork #6

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Chapter #3

Summary

Ex 3.1

Growth and decay:

$$\frac{dx}{dt} = kx, \quad x(t_0) = x_0$$

where k is a constant of proportionality which occurs when involving growth or decay. The rate of growth of certain populations is proportional to the population present at time t. e.g. Knowing population at time "t_0", we can predict in future when $t > t_0$. The constant of proportionality k can be determined from initial value problem using a measure of x at time $t_1 > t_0$

Half Life:

In Physics, half life is a measure of stability of radioactive substances. It is the time it takes for one-half of atom to disintegrate.

$$A(t) = A_0 e^{kt}$$

If 0.043 % atoms have disintegrated then 99.957% remains. For k, we use

$$0.9995\,A_0 = A(15)$$

$$0.99957\,A_0 = A_0 e^{kt}$$

$$A(t) = A_0 e^{-0.00028[illegible]t}$$

$$A(t) = \frac{1}{2} A_0$$

Solving for t gives $\frac{1}{2}A_0 = A_0 e^{-0.0000285\,t}$

$$t = \frac{\ln 2}{0.0000285} = 24.180 \text{ years}$$

Carbon Dating:

It is based on the fact that by comparing amount of C-14 present in a fossil with constant ratio in atmosphere we can obtain age of fossil. e.g

$$\frac{1}{2}A_0 = A(5600) \Rightarrow \frac{1}{2}A_0 = A_0 e^{5600K}$$

$$K = \frac{-\ln 2}{5600} = -0.00012378 \Rightarrow A(t) = \frac{1}{1000}A_0 = A_0 e^{-0.00012378\,t}$$

$$t = \frac{\ln 1000}{0.00012378} = 558000$$

Newton's law of ~~...~~ cooling / warming:

It is: $\frac{dT}{dt} = K(T - T_m)$, where K is constant, T is temperature of medium.

E.g. $\frac{dT}{dt} = K(T-70)$, $T(0) = 300$

$$\frac{dT}{t-70} = Kdt \Rightarrow \ln(t-70) = Kt + C_1$$

$T = 70 + C_2 e^{kt}$, where $t=0$, $T=300$, So $300 = 70 + C_2$

$C_2 = 230$, $T = 70 + 230e^{kT}$, $T(3) = 200$ leads to $e^{3K} = \frac{13}{23}$

$$K = \frac{1}{3}\ln\frac{13}{23} = 0.190$$

$$T = 70 + 230e^{-0.190t}.$$

Mixture:

Mixing of two fluids sometimes give rises to first order differential equation.

Shot on Y91
Vivo AI camera

$\frac{dA}{dt}$ = (Input rate of salt) − (output rate of salt)
$= R_{in} - R_{out}$

$\frac{dA}{dt} + \frac{1}{100} A = 6$, $A(0) = 50$

$\frac{d}{dt}\left[e^{t/100} A\right] = 6e^{t/100} \Rightarrow A(t) = 600 + c\,e^{-t/100}$

∴ when $t = 0$, $A = 50$, $c = -550$

$A(t) = 600 - 550\,e^{-t/100}$

$r_{out} = 2$, $r_{in} - r_{out} = 3 - 2 = 1$ gal/min

so, tank will contain $300 + t$

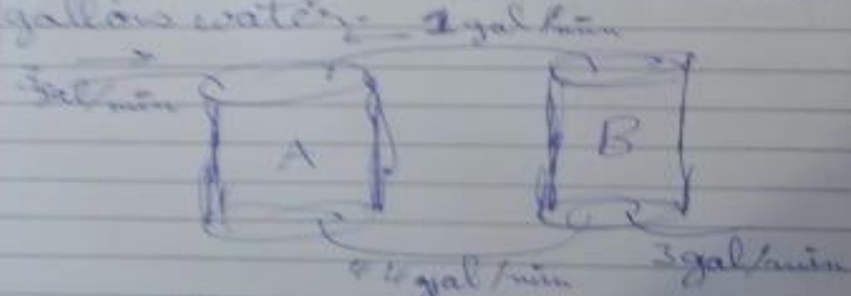
$c(t) = A/300+t$

$R_{out} = \left(\frac{A}{300+t}\right) \cdot (2) = \frac{2A}{300+t} \Rightarrow \frac{dA}{dt} = 6 - \frac{2A}{300+t}$

Ex 3.3

Mixtures:

Consider tank A contains 50 gallons in which is 25 pounds of salt and tank B contain 50 gallons water:



$\frac{dx_1}{dt} = (3)\cdot(0) + 1\left(\frac{x_2}{50}\right) - 4\left(\frac{x_1}{50}\right)$

$= -\frac{2}{25} x_1 + \frac{1}{50} x_2$

$$\frac{dx_2}{dt} = \frac{4 \cdot x_1}{50} - \frac{3 \cdot x_2}{50} - \frac{1 \cdot x_2}{50}$$

$$= \frac{2}{25}x_1 - \frac{2}{25}x_2$$

Predator-Prey Model:

* Let $x(t)$ and $y(t)$ denote fox and rabbit population.

If there were no rabbits.

$$\frac{dx}{dt} = -ax, \quad a>0$$

when rabbits are present

$$\frac{dx}{dt} = -ax + by$$

If there are no foxes.

$$\frac{dy}{dt} = dy, \quad d>0$$

when foxes are present

$$\frac{dy}{dt} = dy - cxy$$