• Question # 6.2 (b)

→ Pumping Lemma

According to pumping Lemma II

Let

$w \in L$ and $|w| \geq n$

So, w can be divided into 5 parts

$w = UVXYZ$

So, Let

$U = \Lambda$, $X = \Lambda$, $z = a^{2^{n-k}}$

$V = a^{2^{k-m}}$, $Y = a^{2^m}$

So, $|VY| > 0$ and $|VXY| \leq P$

Now, by using pumping Lemma I I

UV^iXY^iZ

$= \Lambda\ (a^{2^{k-m}})^i\ \Lambda\ (a^{2^m})^i\ a^{2^{n-k}}$

Put $i = 10$

$= a^{2^{10k-10m}}\ a^{2^{10m}}\ a^{2^{n-k}}$

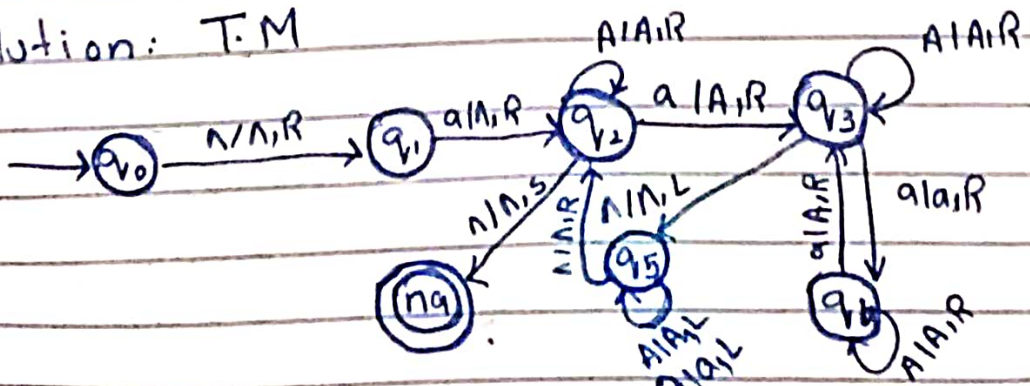
$= a^{2^{10k}}\ a^{2^{n-k}} \neq a^{2^{n+9k}} \notin L$

. So, this language is context free Language.

• Question 6.2 b

$L = \{a^{2^n} \mid n \geq 0\}$

Solution: T.M



Q5(a)

Pumping Lemma

$L = \{a^n b^{m+n} a\, b^n \mid m, n \geq 0$

By pumping Lemma 1

XY^iZ

$|XY| \leq n$

$|Y| > 0$

So Let

$Z = b\ a\ b$

$XY = a^n$

$X = a^{n-k}$

$Y = a^k$

So, by pumping Lemma I

$a^{n-k}\ (a^k)^i\ (b^m a^m b^m)$

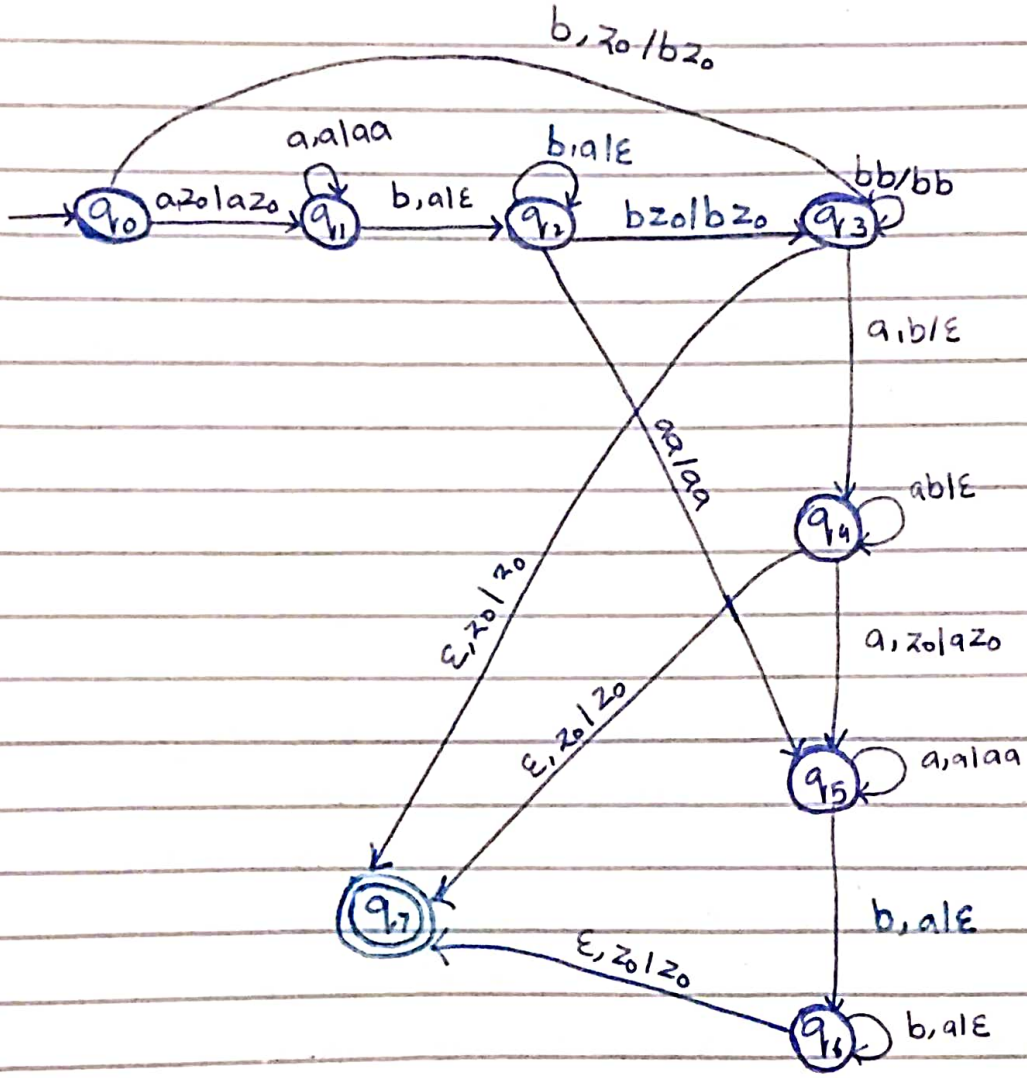
Putting $i = 2$

$a^{n-k}\ (a^{2k})\ (b^m a^m b^m)$

$a^{n+k}\ b^m\ a^m\ b^m$

Hence, $a^{n+k}\ b^m\ a^m\ b^m$ not a part of Language. So, its not a regular Language.

Push down automata :



. Question 6.5 (d)

$$L = \{xyx \mid x, y \in \{a,b\}^* \text{ and } |x| \geq 1\}$$

Puming Lemma:

Let $\quad \therefore X \in \{a,b\}^*$

$X = a^n b$

$Y = \wedge$

By Using pumping Lemma

Let

AB^iC

$AB = a^n$; $C = b$

So,

$|AB| \leq n$

Let

$A = a^{n-k}$ and $B = a^k$ $\therefore |B| > 0$

Putting in pumping Lemma eq,

$a^{n-k}(a^k)^i b$

Put $i = 3$

$a^{n-k} a^{3k} b = a^{n+2k} \notin L$

Hence 'L' is not a regular Language so it is non-regular Language

- Push Down Automatta

For the given Language we Can't construct PDA because

Let $X =$ aaa bbb abb

$Y = \wedge$

then the string will become

aaa bbb $a^b a^b$ a bbb abb

So, if stack is empty we need to push a or b and Let assume we are push a at inital points and if we

have a on input and a on the top of stack we need to push "a" but we don't know how many a's we need to pop with "b's"

- Question 6.9 (b)

$$B = \{ a^i b^j c^k \mid i \neq j \text{ or } i \neq k \}$$

Consider that

$$B' = \{ a^i b^i c^k \mid i = j \text{ or } i = k \}$$

Let B' is CFL

$$A = a^n b^n c^n$$

$$|A| \geq 3n$$

By pumping Lemma

$$A = vwxyz$$

So, $z = b^n c^n$, $v = n$, $x = \Lambda$

$$|wxy| \leq n$$

$$|wy| > 0$$

So,

$$w = a^{n-k}, \quad y = a^k$$

Putting in pumping Lemma equation

$$vw^i xy^i z$$

$$(\Lambda)\ (a^{n-k})^i\ (\Lambda)\ (a^k)^i\ (b^n c^n)$$

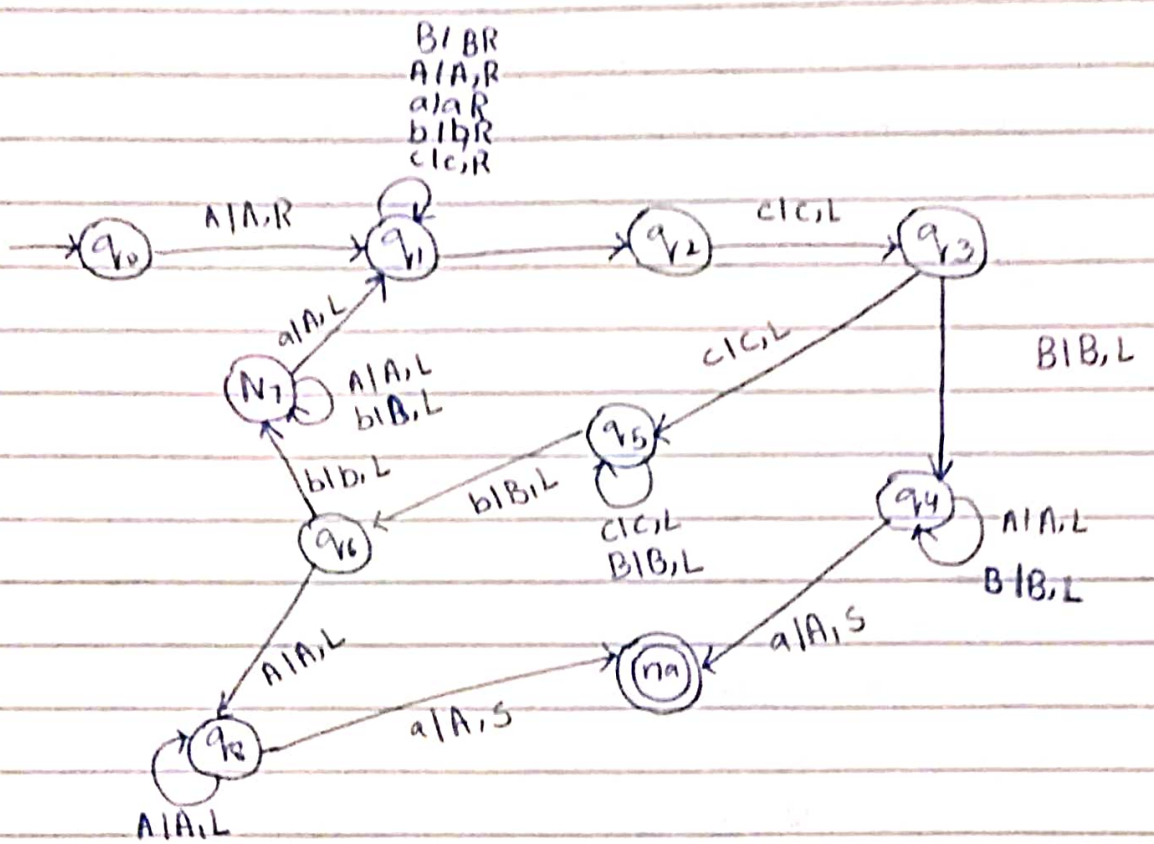
Put $n = 10$

$$a^{10n-10k}\ a^{10k}\ b^n c^n$$

$$a^{10n} b^n c^n \notin L$$

Hence B' is non CFL So B is also non CFL

(a) T.M


B/B,R
A/A,R
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