Homework # 12
Name: Zaeem Yousaf
Roll No: L19-1196
Section: BSCS-2E1
Submitted To: Abdul Hafeez Sheikh

Date: __/__/20__ Day: M T W T F S

Chapter # 12: Summary

partial differential evaluation and boundry value problems in Rectangular coordinates.

* **Separable partial differential evaluation.**

linear second order PDE. Homogeneous. non-homogeneous. Solution. Separable evaluation. Separation costant. superposition principle. classification of linear second order.

The general form of a linear second partial differential evaluation in two independent variables x and y is given as

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} + Fu = G.$$

* **Classical evaluation & boundry value problems**

for the remainder of this and the next chapter we shall be consemed principlly with finding product solations of the partial differential evaluations; $k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, $k > 0$ $\rightarrow$ (1)

$a^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$ $\rightarrow$ (2) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ $\rightarrow$ (3)

or slight variations of these evaluations these classical evaluations of mathematical physics are known.

- one dimentional evaluation
- Laplace's evaluation
- Heat evaluation

* **Heat evaluation:** consider a thin rod of length

with an initial evaluation temperature f(x) throughout and whose ends are held at temprature zero for all time t>0. If the rod shown in the Figure Satisfies all the conditions given in the previous formulas the temprature $u(x,t)$ in the rod is determined from the boundry value problem $k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ $0<x<L, t>0$ $u(0,t)=0$, $u(L,t)=0, t>0$ $u(x,0)=f(x)$ $0<x<L$. the homogeneous boundry conditions together with the homogeneous equation

* **Wave equation:** Solution of a boundry value problem by a Separation of variables. Standing waves. normal modes. First normal mode. fundamental frequency: we are now in a position to solve the boundry value. the vertical displacement of $u(x,t)$ of the vibrating String of length L. $a^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, $0<x<L$ $t>0$
$u(0,t)=0$, $u(L,t)=0$, $t>0$ →②
$u(x,0)=f(x)$, $\frac{\partial u}{\partial t}\Big|_{t=0} = g(x)$, $0<x<L$.

* **Laplace's Equation.** Suppose we wish to find the steady state temprature $u(x,y)$ in a rectangular plate with insolated boundries. when no heat escapes from the lateral faces of the plate, we solve laplace's equation. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
$\frac{\partial u}{\partial x}\Big|_{x=0} = 0$, $\frac{\partial u}{\partial x}\Big|_{x=a} = 0$ $u(x,0) = 0$

Date: __/__/20__ Day: M T W T F S

* **Non-homogeneous equations & boundry conditions:** The method of Separation of variables may not be applicable to a boundry value problem when the partial differential equation or boundry conditions are non-homogeneous, for example when the heat is generated at a conditions rate r within a rod of finite length. The form of heat equation. $k\frac{\partial^2 u}{\partial x^2} + r = \frac{\partial u}{\partial t}$, $u = v + \psi$

* **use of Generalized Fourier Series:** fourier series, Fourier Cosine Series, Fourier Sin Series are three ways of expanding a function in terms of an orthogonal matric functions, in terms of set of any functions. $\{\phi_n(x)\}$ that is orthogonal with respect to a weight function $[a, b]$ many of these called generalized Fourier series.

* **Boundry value problems involving fourier Series in two variables.** Here $\frac{\partial u}{\partial n}$ denotes the normal derivative of u the directional derivative in the perpendicular to the boundry. A boundry condition of first sight is called Drichelt condition. A boundry condition of second sight is called neuman condition. A boundry condition of third type is called Robin condition