Probability & Statistics

19L-1196 TO: Sir Mubashir Qayyum

Section: 4A

Characteristics function.

$$\phi_x(t) = E(e^{itx})$$

$$= \int_a^b \frac{1}{b-a} e^{itx}\, dx$$

$$= \frac{1}{b-a} \int_a^b e^{itx}\, dx = \frac{1}{b-a}\left[\frac{e^{itx}}{it}\right]_a^b$$

$$= \frac{1}{(b-a)it}\left(e^{itb} - e^{ita}\right)$$

$$\therefore e^{itx} = 1 + \frac{itx}{1!} + \frac{i^2t^2x^2}{2!} + \cdots$$

Substituting e^{itb} and e^{ita} expansion

$$\frac{1}{(b-a)it}\left(1 + \frac{itb}{1} + \frac{i^2t^2b^2}{2} \cdots - 1 - \frac{itta}{1} \cdots\right)$$

$$E(e^{itx}) = \left(1 + \frac{it(b+a)}{2} + \frac{i^2t^2(b+\cdots)}{3!}\right)$$

$$\phi_x'(t) = 0 + \frac{i(b+a)}{2} + i^2 t \cdots$$

value of ϕ_x' at '0'

$$\phi_x'(0) = 0 + \frac{i(a+b)}{2} + 0 + \cdots$$

$$\frac{\phi_x'(0)}{i} = \frac{a+b}{2} = \mu_1'$$

Now Driving Variance

$$Var(x) = E(x^2) - (Ex)^2$$

$$E(x) = \frac{1}{b-a}\int_a^b x\,dx = \cancel{\frac{\cancel{(b-a)}}{\cancel{(b-a)}}}\frac{a+b}{2} \quad *$$

$$(Ex)^2 = \frac{1}{b-a}\int_a^b x^2\,dx = \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b$$

$$= \frac{1}{b-a}\left(\frac{b^3}{3} - \frac{a^3}{3}\right) = \frac{1}{b-a}\left(\frac{b^3-a^3}{3}\right)$$

$$Var(x) = E(x^2) - (Ex)^2$$

$$= \frac{b^3-a^3}{3(b-a)} - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{a^2+ab+b^2}{3} - \frac{a^2+2ab+b^2}{4}$$

$$= \frac{a^2-2ab+b^2}{12} = \frac{(b-a)^2}{12}$$