Probability and Statistics

19L-1196 Assignment # 2

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Q # 1

	H	M	
F	0.2	0.3	0.5
SW	0.4	0.1	0.5
	0.6	0.4	1

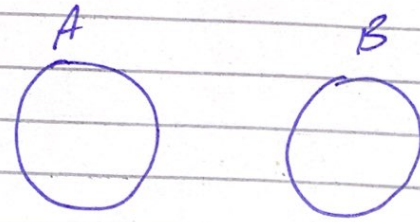
a) $p(W) = 1 - p(F)$
$= 1 - 0.5 = 0.5$

b) $p(MF) = p(F) - p(HF)$
$= 0.5 - 0.2 = 0.3$

c) $p(H) = p(HF) + p(HW)$
$= 0.2 + 0.4 = 0.6$

Q2

A and B are disjoint



given : $P(A \cup B) = 5/8$

$P(A) = 3/8$

$P(C \cap D) = 1/3$

$P(C) = 1/2$

a) $P(A \cap B) = 0$ Ans

(i) Since disjoint, overlap $= 0$

(ii) $P(B) = P(A \cup B) - P(A)$

$= 5/8 - \frac{3}{8} = \frac{1}{4}$ Ans

(iii)

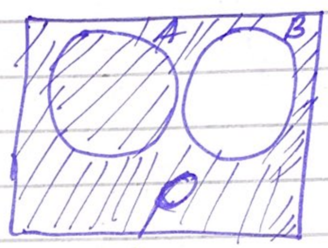
$P(A \cap B^c)$

$= P(A) - P(A \cap B)$

$= \frac{3}{8} - 0 = \frac{3}{8}$ Ans

(iv)

$P(A \cup B^c)$



$= P(A) + P(P)$

$= P(A) + (1 - P(A \cup B))$

$= \frac{3}{8} + \left(1 - \frac{5}{8}\right)$

$= \frac{3}{8} + \frac{3}{8} = \frac{6}{8} = \frac{3}{4}$ Ans

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b) Are A and B independent?

$P(A \cap B) = 0$

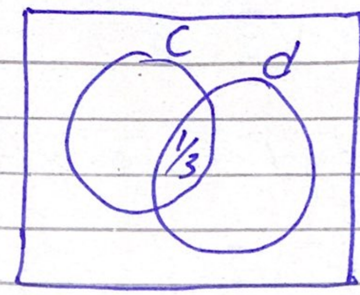
$P(A) \times P(B) = \frac{3}{8} \times \frac{1}{4} \neq 0$

$P(A) \times P(B) \neq P(A \cap B)$

Hence not independent

C)

c And d independent



Then $p(c) \times p(d) = p(c \cap d)$

(i) $p(d) = \dfrac{p(c \cap d)}{p(c)} = \dfrac{1/3}{1/2} = \dfrac{2}{3}$

(ii) $p(c \cap d^c)$

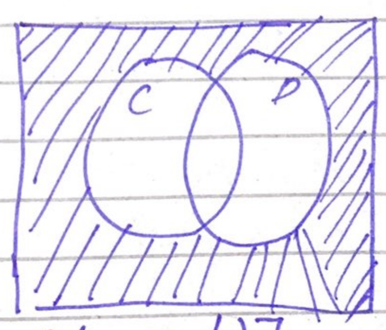
$= p(c) - p(c \cap d)$

$= \dfrac{1}{2} - \dfrac{1}{3} = \dfrac{1}{6}$ Ans

(iii) $p(c^c \cap d^c)$

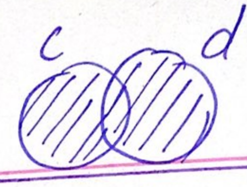
~~= p(~~

$= 1 - (P(c \cup d))$



$1 - [p(c) + p(d) - p(c \cap d)]$

$= 1 - \left[\dfrac{1}{2} + \dfrac{2}{3} - \dfrac{1}{3}\right] = \dfrac{1}{6}$ Ans.



iv) $p(C|D) = p(C \cup D)$

$= p(c) + p(d) - p(c \cap d)$

$= \frac{1}{2} + \frac{2}{3} - \frac{1}{3} = \frac{5}{6}$ Ans.

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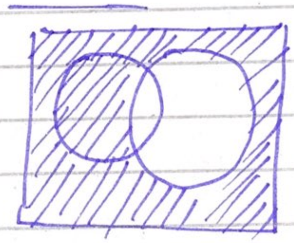
d) (i) $p(c \cup d) = p(c|d)$

$= p(c) + p(d) - p(c \cap d)$

$= \frac{1}{2} + \frac{2}{3} - \frac{1}{3} = \frac{5}{6}$ Ans.

(ii) $p(c \cup d^c)$



$= 1 - p(d)$

$= 1 - \frac{2}{3} = \frac{1}{3}$ Ans.

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e) Are c & d^c independent?

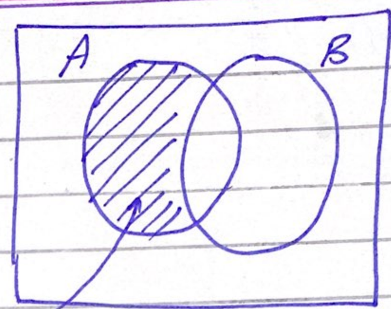
$p(c \cap d^c) = \frac{1}{6}$ (ii) from

$p(c) \times p(d^c) = \frac{1}{2} \times (1 - \frac{2}{3}) = \frac{1}{6}$

$p(c \cap d^c) = p(c) \times p(d^c)$

Hence independent.

1 3



Given: $p(A \cap B)$
$= p(A) \times p(B)$

a) $p(A \cap B^c)$

$= p(A) \times p(B^c)$

$= p(A) \times p(1 - p(B))$

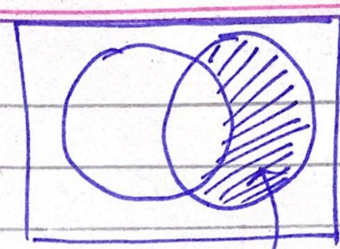
$= p(A) \times \left[1 - \dfrac{p(A \cap B)}{p(A)} \right]$

$= \cancel{p(A)} \times \dfrac{p(A) - p(A \cap B)}{\cancel{p(A)}}$

$= p(A) - p(A \cap B) = p(A \cap B^c)$

Shown $p(A \cap B^c) = p(A) \times p(B^c)$

Q3/b



$A^c \cap B$

$$p(A^c \cap B) = p(A^c) \times p(B)$$

$$= [1 - p(A)] \times p(B)$$

$$= \left[1 - \frac{P(A \cap B)}{P(B)}\right] \times P(B)$$

$$= \frac{P(B) - P(A \cap B)}{P(B)} \times P(B)$$

$$= P(B) - P(A \cap B)$$

$$= A^c \cap B$$

Shown

Q3/c

$$P(A^c \cap B^c) = P(A^c) \times P(B^c)$$

$$= [\cancel{\times} 1 - P(A)] \times [1 - P(B)]$$

$$= \left(1 - \frac{P(A \cap B)}{P(B)}\right) \times \left(1 - P(B)\right)$$

$$= \frac{P(B) - P(A \cap B)}{P(B)} \left(1 - P(B)\right)$$

$$\frac{P(B) - P(B)^2 - P(A \cap B) + P(B) \times P(A \cap B)}{P(B)}$$

$$= 1 - P(B) - \frac{P(A \cap B)}{P(B)} + P(A \cap B)$$

$$= 1 - [P(B) + P(A) - P(A \cap B)]$$

$$= 1 - (P(A) \cup P(B))$$

$$= P(A^c \cap B^c) \text{ shown}$$