Homework # 3
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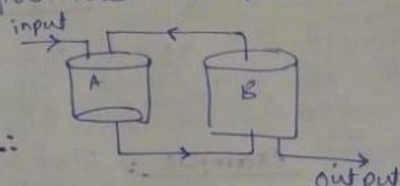
Chapter 3 **Summary** : **Linear Model** : $\frac{dx}{dt} = kx$

$x(t_0) = x_0$: **Non-linear Model** : $\frac{dP}{dt} = kP \quad k > 0$

Linear-non-linear System: Radio Active series

$\frac{dx}{dt} = -\lambda_1 x$, $\frac{dy}{dt} = \lambda_1 x - \lambda_2 y$. **Connected Mixing Tank** : $\frac{dx_1}{dt}$ = input rate of salt — output rate of salt.



Predatory model:

$\frac{dx}{dt} = -ax, \ a > 0$

predator preys the animal, animal gets reduced and predator increases. but when victims decreases predator starts dying and leave the places and move somewhere else.

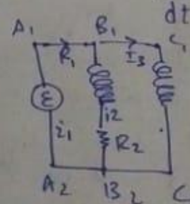
$\frac{dy}{dt} = dy - cxy = \frac{dy}{dt} = dy - cxy = y(d - cx)$

Competition Model: $\frac{dx}{dt} = ax$ and $\frac{dy}{dt} = ay$

$\frac{dx}{dt} = ax - by$, $\frac{dy}{dt} = cy - dx$, $\frac{dx}{dt} = ax - bxy$

$\frac{dy}{dt} = cy - dxy$, $\frac{dx}{dt} = a_1 x - b_1 x^2$, $\frac{dy}{dt} = a_2 y - b_2 y^2$

Kirchhoff's first law:



$i_1(t) = i_2(t) + i_3(t)$

$E(t) = i_1 R_1 + L_1 \frac{di_2}{dt} + i_2 R_2$

For loop $A_1 B_1 C_1 C_2 B_2 A_2 A_1$ we find $E(t)$

$= i_1 R_1 + L_2 \left(\frac{di_3}{dt}\right)$

$= i_2 R_2 + \frac{di_3}{dt}, L_2(i_1 + R_2)$, $i_2 = R_1 + \frac{di_2}{dt} + L_2\left(\frac{di_3}{dt}\right)$

half-life: $\frac{dA}{dt} = kA$, $A(0) = A_0$, its solution is $A_0 e^{kt} = A(t)$: **example**: plotonium decay: 0.043% A_0 Then 99.957% of the substance remains. find decay constant: $0.99957 A_0 = A(15) \Rightarrow 0.99957 A_0 = A_0 e^{15k}$, solving $k = \frac{\ln 2}{0.000002867} = 24180$-year.

Age of fossils:

A fossilized is found to contain one thousandth of C14 level found in living matter, estimate the age of fossil.

$A(t) = A_0 e^{kt}$. **example**: Find value of k (decay)

$\frac{1}{2} A_0 = A(5600) = \frac{1}{2} A_0 = A_0 e^{5600k} \Rightarrow t = \frac{\ln(1000)}{0.00012378}$ $= 55800$ year.

Newton's law of Cooling

working: $\frac{dT}{dt} = k(T - T_m)$, where k is constant of proportionality in the temprature. $t > 0$

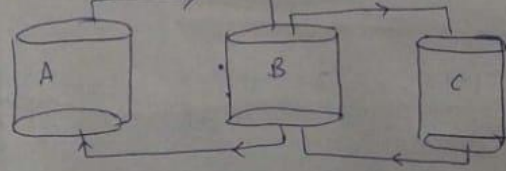
T_m is ambiant temprature. **example**: Cooling cake. Ambiant temp: 300° F. 3 minutes later its temprature is 200° F. how long will it take to reach 70° F. $\frac{dT}{dt} = k(T - 70) \Rightarrow T(0) = 300$

$\frac{dT}{T-70} = k\,dt$, $\ln(T-70) = t + C_1$, $T(t) = 70$

since, $\lim_{t\to\infty} T(t) = 70$

Mixture of two Salts

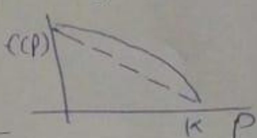
Two salts of different concentration. when one salt is dissolved into another. the the rate of charge in concentration can be calculated by differential method using input rate and its concentration and output and its concentration



Population growth: $\frac{dP}{dt}\cdot\frac{1}{P} \Rightarrow \frac{dP}{dt} = kP$

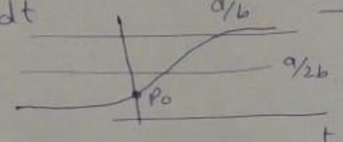
$\frac{dP}{dt}\cdot\frac{1}{P} = f(P)$ or $\frac{dP}{dt} = P f(P)$

$\frac{dP}{dt} = P(r - \frac{r}{k}P) \Rightarrow \frac{dP}{dt} = P(a-bP)$, human population can be predicted by logestic equation and logestic curve.



Solution of logestic Equation:

$\left(\frac{\frac{1}{a}}{P} + \frac{b/a}{a-bP}\right)dP = dt$



logestic growth:

for example student carrying flue virous returns to isolated college campus of 1000, student. it is assumed that rate at which the virus spreads is proportional not only to the number x of ineffected student but also the number of students not affected.

Example: Determine the number of students infected after 6 days if it is further observed that after 4 days $x(4) = 50$

$\frac{dx}{dt} = kx(1000 - x)$, $x(0) = 1$

$x(t) = \frac{1000k}{k + 999k e^{-1000kt}} = \frac{1000}{1 + 999 e^{-kt(1000)}}$

Now using $x(4) = 50$ we determine k from

$50 = \frac{1000}{1 + 999 e^{-1000kt}} \Rightarrow x(6) = \frac{1000}{1 + 999 e^{-5.9436}}$

$x(t) = \frac{1000}{1 + 999 e^{-0.99906t}}$