

Unconstrained flight and stability analysis of a flexible rocket using a detailed finite-element based procedure

D. J. McTavish, D. R. Greatrix & K. Davidson

*Ryerson University, Department of Aerospace Engineering,
Toronto, Canada*

Abstract

This paper presents a flight and aeroelastic stability study of a small lightweight rocket being developed at Ryerson University for small payload delivery to high altitudes. The emphasis is on a relatively sophisticated approach to the structural modeling coupled to a coefficient-based aerodynamic analysis. The flexible rocket model is derived from a standard small-motion finite-element model and is fully unconstrained in three dimensions, supporting arbitrary translation and rotation. The centripetal and Coriolis terms required for large body angular rates are all included. The methodology is part of a more general effort to implement an unconstrained flexible vehicle modeling procedure that models structures *as they are* without significant restriction or fallback to degenerate structural forms or manually produced models. The finite-element method is used for its ability to produce accurate detailed geometrical models of structures and to exploit the commercial software that facilitates the production of such models. In this case study the reduced-order version of the high-detail model is chosen to address potential rocket fin flexibility issues. The aerodynamic model of the rocket treats the fuselage and fins as separate aerodynamic entities for which simple body-based theoretical and/or empirical theory can be applied. Aerodynamic coefficients were obtained from the Missile DATCOM software package, and built into interpolation tables to cover the speed range from subsonic to supersonic.

Keywords: rocket flight, aeroelastic stability, flexible body dynamics.



1 Introduction

1.1 The flexible flight vehicle problem

Although high-quality finite-element models arguably produce the best models of flexible structures of non-trivial geometry, as used for static or modal analysis (small motion problems), such models are typically abandoned when the leap to large-motion problems is made. We broadly classify aerostructural (aeroelastic) stability issues into two classes: those that involve detrimental motions of the vehicle as a whole interacting with control surfaces or possibly body flexibility (wing flutter and divergence, for example), and those that lead to localized detrimental structural action (panel flutter, etc.). The methodology described in this paper is finite-element based and thus able in principle to model any part of the vehicle to an arbitrary level of detail. It is therefore applicable to both classes of aerostructural problems. The case study of this paper, however, is concerned only with whole-vehicle stability.

1.2 Agenda of the present research

The study of the aerostructural behavior involves the two disciplines of unsteady aerodynamics and structural dynamics. In principle, the problem of modeling flexible structures, whether restricted to small motion or with large (unconstrained) motion allowed is an established analytical technology. The practice of flexible structure modeling is another matter. The agenda of this paper is to introduce and promote *a complete methodology for the structural modeling of flexible flight vehicles* that satisfies the following requirements:

- Fully three-dimensional.
- Supports large gross vehicle motion coupled with small relative deformation due to structural flexibility. (Large vehicle motion includes both translational and angular velocity and displacement.)
- Includes provision for large deformation rates.
- Allows arbitrary geometrical complexity of the structure, for which it is assumed that the finite-element method is used.
- Accepts finite-element models in a form that can be extracted from commercial finite-element tools. (Practically, this means assembled mass and stiffness matrices without explicit knowledge of the element types and associated element shape functions.)
- Provides for reduced-order modeling.
- Is consistent with available and future aerodynamic analysis techniques.

The *purpose of this methodology* is to support the following interdependent flight-vehicle issues...

- Investigation of aerostructural behaviour.
- Trajectory performance.
- Control-structure interaction.

This new methodology is described and applied to a specific flight vehicle for illustration – the SPHADS rocket of Ryerson University. While our objective is



the development of flexible vehicle structural modeling practice, the flexible rocket example includes a basic discrete-component based aerodynamic model for the illustration of its use in a complete aeroelastic stability analysis.

1.3 The SPHADS-1 rocket

A relatively low cost delivery of scientific payloads to the upper atmosphere for meteorological, environmental, ionospheric, and micro-gravity studies by universities and other research agencies is the principal objective of the SPHADS (Small-Payload, High-Altitude Delivery System) program underway at Ryerson University's Propulsion Research Facility (PRF). (See Greatrix and Karpynczyk [1].) Additional lower-atmosphere applications are also under consideration for SPHADS, such as high-g loading tests for payload packages, or aerodynamic heating evaluation of exposed test materials. Initial efforts in the design of potential prototype variants are being concentrated at the lower end of the atmospheric spectrum (< 100 km altitude), where a lower-cost delivery system would be of utility to researchers.

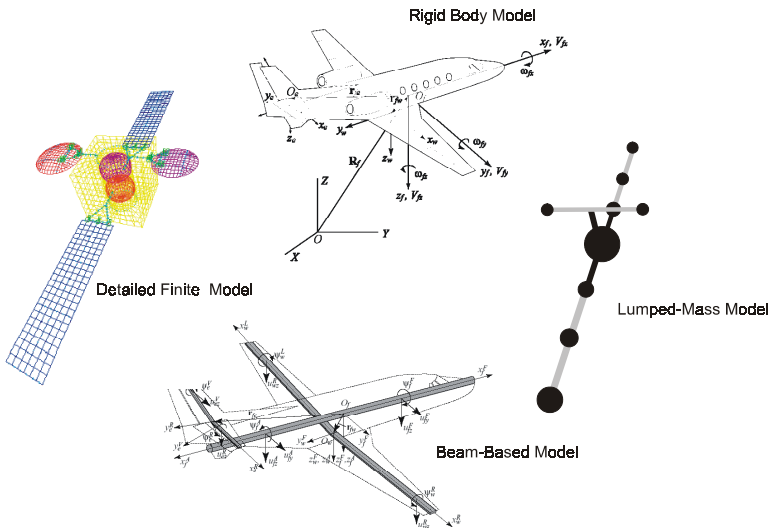


Figure 1: Survey of approaches to flexible vehicle modeling.

2 Structural modeling

Fig.1 provides a pictorial survey of the various approaches to flexible body modeling, including the degenerate case of ignoring flexibility completely. The remaining approaches include the use of a detailed finite-element model (our agenda), the substitution of simple “analytic” ideal structures (often uniform beams) for all or some of the flexible body substructures, and the somewhat

primitive approach using lumped-mass modeling for the system inertia combined with rigid offsets and interconnecting distributed flexibility.

In this study we prefer to take the detailed finite-element approach, modeling the vehicle *as is*, in full geometrical detail. This is not normal practice due to the practical difficulty of producing a number of terms in the unrestrained motion equations.

2.1 The equations of unrestrained flexible body motion

Eqn.(1) presents the form of the motion equations used in this study. All quantities are expressed in a body-fixed reference frame located arbitrarily in the body. The equations are non-linear in the motion variables, and the coefficient matrices marked “total” generally \mathbf{q} -dependent.

$$\begin{bmatrix} m\mathbf{I} & -\mathbf{c}_{\text{total}}^{\times} & \mathbf{P} \\ \mathbf{c}_{\text{total}}^{\times} & \mathbf{J}_{\text{total}} & \mathbf{H}_{\text{total}} \\ \mathbf{P}^T & \mathbf{H}_{\text{total}}^T & \mathbf{M} \end{bmatrix} \begin{bmatrix} \mathbf{v}_B \\ \boldsymbol{\omega}_B \\ \dot{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \mathbf{G} \\ \mathbf{f} \end{bmatrix} - \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{f}_{\sigma} \end{bmatrix} - \begin{bmatrix} m\boldsymbol{\omega}_B^{\times} & -\boldsymbol{\omega}_B^{\times}\mathbf{c}_{\text{total}}^{\times} & 2\mathbf{P}_{\omega}(\boldsymbol{\omega}_B) \\ \mathbf{c}_{\text{total}}^{\times}\boldsymbol{\omega}_B^{\times} & \boldsymbol{\omega}_B^{\times}\mathbf{J}_{\text{total}} & 2\mathbf{H}_{\omega,\text{total}}(\boldsymbol{\omega}_B) \\ -\mathbf{P}_{\omega}^T(\boldsymbol{\omega}_B) & -\mathbf{H}_{\omega,\text{total}}^T(\boldsymbol{\omega}_B) & -2\mathbf{M}_{\omega}(\boldsymbol{\omega}_B) \end{bmatrix} \begin{bmatrix} \mathbf{v}_B \\ \boldsymbol{\omega}_B \\ \dot{\mathbf{q}} \end{bmatrix} \quad (1)$$

In these equations, the motion variables are...

$\mathbf{v}_B, \boldsymbol{\omega}_B$ Body inertial velocity and angular velocity
 \mathbf{q} Flexible motion coordinates

the external forces are...

\mathbf{F}, \mathbf{G} Net external force and moment about the body-frame origin
 \mathbf{f} Generalized forces associated with the flexible coordinates

For simple linear elasticity the material resistance forces take the form: $\mathbf{f}_{\sigma} = \mathbf{K}\mathbf{q}$. The definitions of the various embedded coefficient matrices are omitted for brevity and may be found in McTavish [2].

The full three-dimensional simulation model was produced according to the process illustrated in fig.2 and described following.

2.1.1 Detailed finite-element model

ANSYS was used to produce a standard undamped dynamic model

$$\mathbf{M}\ddot{\mathbf{q}}^a + \mathbf{K}\mathbf{q}^a = \mathbf{f}^a \quad (2)$$

corresponding to fig.3. The “a” superscript denotes the nature of the FEM coordinates being small absolute displacements. The base model consists of 390

nodes with 376 elements of various types and well over 2,000 coordinates (degrees of freedom).

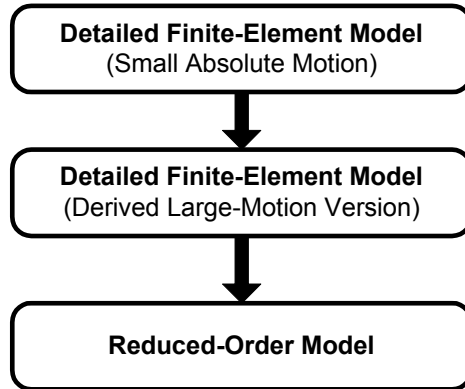


Figure 2: Simulation model creation.

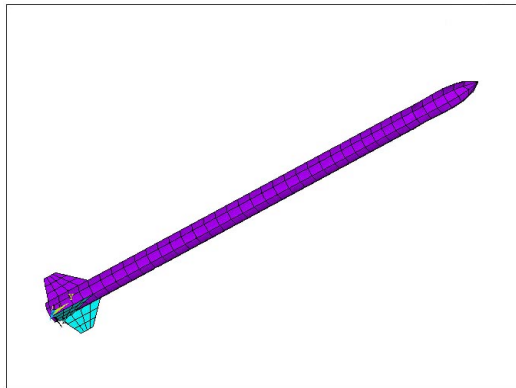


Figure 3: Finite-element model of the SPHADS rocket.

2.1.2 Detailed FE-based unrestrained motion model

Using the detailed procedure defined in McTavish [2], an unrestrained model was produced from the standard finite-element model. The motion equation produced is in the form of eqn.(1). While space in this short paper format is too limited to reproduce the procedure, we highlight its main features:

- Generated exclusively from the mass and stiffness matrices of the small-motion FE-model – no knowledge of low-level shape functions required.
- Based on consistent mass principles, no “lumped mass” approximations.
- Uses the full geometrical description of the vehicle as embodied in the small-motion FE model – no reduction to degenerate structural forms (ideal beams, etc.).

- All terms present in the equation of unrestrained motion are produced, either exactly or by quality automatic approximation.

2.1.3 Reduced-order model

The unrestrained model just described remains a high-order model involving all of the original FE model coordinates plus the added gross body-motion degrees of freedom. Model-order reduction is applied to bring the equations of motion down to a more manageable size and to be consistent with our approach to aerodynamic modeling. The reduced-order model is an ad-hoc specification of flexible-deformation shapes deemed to be relevant for the scope of the present study, being the interaction of fin flexibility with overall vehicle attitude stability. Typically, we would consider shape functions such as the following...

- Whole body modes taken from a free-free normal modes analysis of the entire vehicle
 - 1st bending modes (two directions)
 - 1st torsion bending mode
- Fin-only modes taken from a root-constrained normal modes analysis of a single fin
 - 1st bending (flapping) mode
 - 1st torsion mode

Samples of these shapes are shown in fig.4.

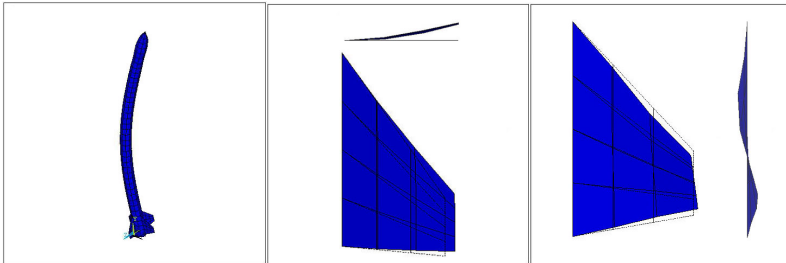


Figure 4: Sample shapes used in the reduced-order model.

3 Aerodynamic modeling

In this section, we review our approach to the aerodynamic modeling used for our flexible rocket case study. The basic approach is to use a component model of the vehicle consisting of discrete bodies for which the aerodynamic actions are computed separately; then appropriately applied to the structural response model. Our structural model and overall simulation contains the information required, in principal, to produce a complete deformed-surface model of the vehicle at any point in time. This presumes that an aerodynamic model is available that can produce a consistent and comprehensive description of the pressure distribution over the vehicle. The latter capability, for a time-varying surface description of an accelerating vehicle, lies at the extreme leading edge of modern CFD technology and is beyond the scope of this study.

Many references have been consulted to develop empirical-based formula to compute values of the aerodynamic coefficients across a range of flight conditions. Generally the available covers the static case of $\dot{\alpha} = \dot{\beta} = 0$ and does not address the non-static situation. We use interpolation-based functions for the coefficients using available “Missile DATCOM” numerical data (Blake [3]) that does include the angular rates.

3.1.1 Fuselage-body aerodynamics

The aerodynamic coefficients pertaining to the symmetrical rocket body (without fins) can be represented as

$$\begin{aligned} C_N &= C_{N_\alpha} \alpha + C_{N_{\dot{\alpha}}} (\dot{\alpha} + \dot{\beta}) \\ C_A &= C_{A_0} + C_{A_\alpha} \alpha + C_{A_{\dot{\alpha}}} (\dot{\alpha} + \dot{\beta}) \\ C_M &= C_{M_0} + C_{M_\alpha} \alpha + C_{M_{\dot{\alpha}}} \dot{\alpha} + C_{M_{\dot{\beta}}} \dot{\beta} \end{aligned} \quad (3)$$

where C_N , C_A , C_M are the normal force, axial force and moment coefficients respectively, and the angle of attack defined in the plane of the local air-relative vehicle velocity and the vehicle longitudinal axis (thus the sideslip angle β is zero by construction).

3.1.2 Fin aerodynamics

Unlike the fuselage-body, we regard only the local “normal” force generated by the fins to be of significance for the present case study. For each fin we have, simply,

$$C_{N_i} = C_{N_\alpha} \bar{\alpha}_i \quad (4)$$

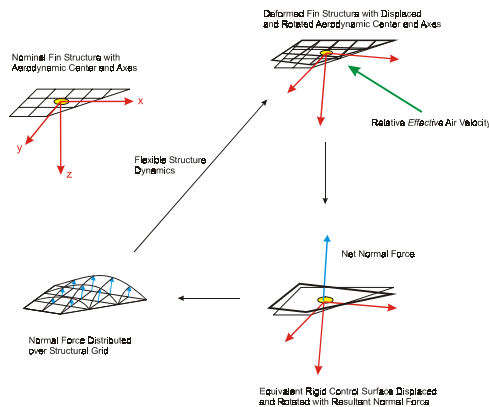


Figure 5: Discrete aerodynamics applied to the continuous fin structure.

3.1.3 Aerostructural interface

Aerodynamic “key-nodes” on the structural model provide the link from the structural response to the aerodynamic “state” of each aerodynamic component of the vehicle model. From this state, the aerodynamic actions are generated and then, with appropriate interpretation, are impressed back onto the structural model as input. Both gross motion and local deformation kinematics are used in evaluating the motion of the key-nodes relative to the surrounding air. The details of this procedure are discussed in [4].

Figure 5 illustrates the approach applied to the fin substructures.

A complete description of the aerodynamic modeling is provided in [4, 5].

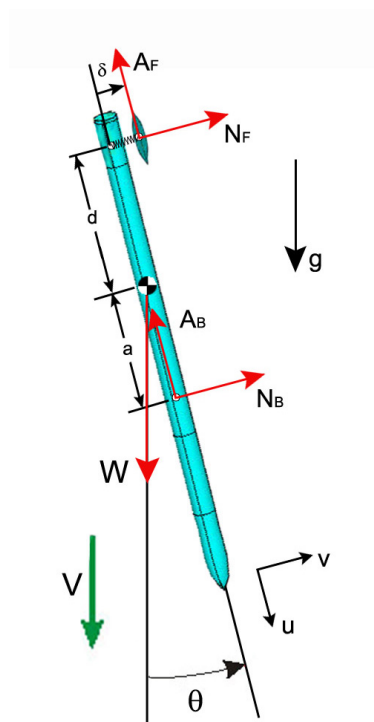


Figure 6: Planar analysis configuration.

4 Aeroelastic stability analysis

The stability study presented in this paper focus on the aeroelastic stability of the vehicle in unpowered steady flight. This restricted analysis allows a simplified analysis to be used as a validation of the unrestrained flight model. Details of the

analysis may be found in Davidson et al. [5], highlights of the results are presented here.

4.1 Planar aeroelastic stability analyses

Figure 6 shows the unrestrained version of the planar stability model. An equilibrium situation of steady velocity is achieved by study the free-fall case at terminal velocity, hence the downward vertical orientation of the nominal attitude.

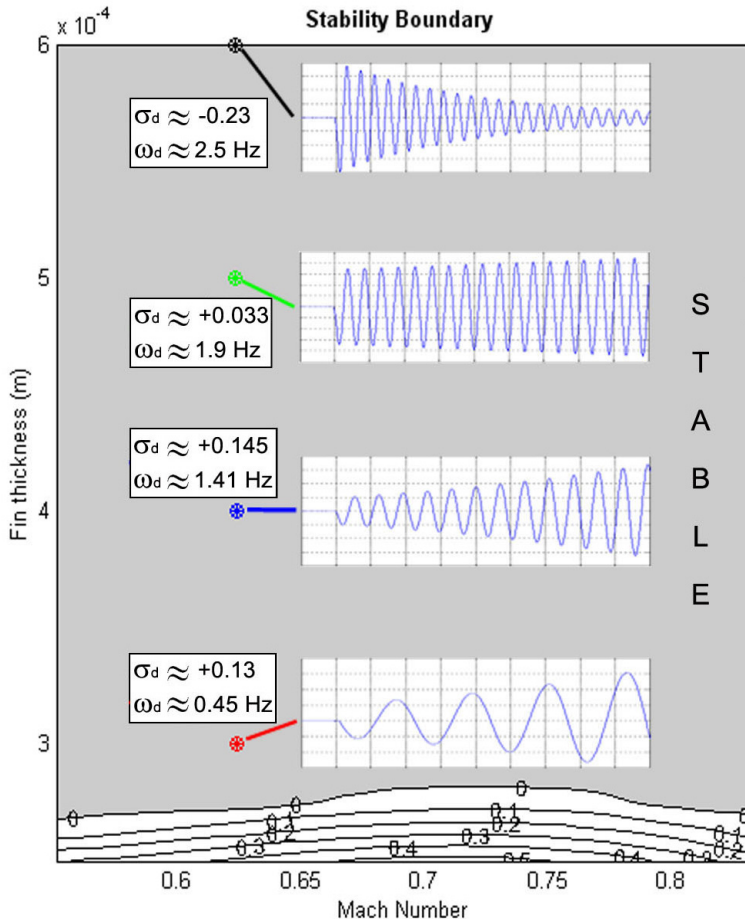


Figure 7: Planar stability diagram (AoA profiles inset for high-detail model).

The small motion of the vehicle relative to a reference frame falling at the terminal velocity is described by the relative pitch angle θ , the body-frame translational position of the body x and y such that $u = \dot{x}$ and $v = \dot{y}$, and the

flexible fin deflection δ . The equations of perturbed motion were set-up in the form

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{0} : \quad (\lambda^2 \mathbf{M} + \lambda \mathbf{C} + \mathbf{K}) \mathbf{x} = \mathbf{0} \quad (5)$$

and solved for the characteristic values and eigenvectors, at different values of fin thickness and local atmospheric conditions.

4.2 Comparison with fully unrestrained stability analysis

The three dimensional high-detail model was exercised by “dropping” the vehicle and allowing it to approach terminal velocity. A small side-gust wind disturbance was injected into the system to stimulate natural instability if present. The post-disturbance motion was monitored via angle-of-attack looking for growth or decay of the response. The plot in fig.7 shows contours of $\sigma_d = \text{Re}\{\lambda_i\}$ for the largest relevant eigenvalue found. The contour at $\sigma_d = 0$, then, represents neutral stability. The abscissas for the plot are fin thickness and speed expressed as Mach number. The results for the high-detail model suggest that the fin must be at least twice as thick to maintain aeroelastic stability at a particular speed than predicted by the simple low-detail model.

5 Concluding remarks

This paper has presented a aeroelastic stability study for a flexible rocket, combining a new sophisticated structural modeling approach with a basic discrete aerodynamic model. For the low aspect ratio fins under consideration, the analysis suggests that the fins can be made very thin. The high-detail model of the vehicle produces a higher value of fin thickness than a simple discrete model. Flight stability may not be the limiting factor in their design however.

Acknowledgements

This work was supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada.

References

- [1] Greatrix, D. R. & Karpynczyk, J., Rocket Vehicle Design for High-Altitude Payload Delivery, *CSME Forum SCGM*, Vol.5, 1998.
- [2] McTavish, D. J. ,Full and Reduced-Order Models for Flexible Body Motion: A Comprehensive Consistent-Mass Finite-Element Based Procedure, (submitted to the *AIAA Journal of Guidance, Control and Dynamics*, October 2004).



- [3] Blake, W. B., "Missile DATCOM, User's Manual – 1997 Fortran 90 Revision", Air Force Research Laboratory, Wright-Patterson Air Force Base, Ohio, 1998. AFRL-BA-WP-TR-1998-3009.
- [4] McTavish, D. J., Davidson, K. & Greatrix, D. R., Modeling of an Unconstrained Flexible Flight Vehicle, *CSME Forum*, June 2004.
- [5] Davidson, K., McTavish, D. J. & Greatrix, D. R., High-Detail Modeling of an Unconstrained Flexible Rocket Vehicle for Flight and Stability Analysis (submitted to CASI Journal, January 2005).

