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Dynamics and Aeroelastic Analysis of Missiles

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An aeroelastic analysis for missiles, rockets, and projectiles has been developed that involves structural analysis, structural dynamics, aeroelasticity, and flight dynamics. The structural formulation is based on geometrically-exact mixed finite elements, and slender body theory is used for the aerodynamics. The analysis is outlined, and preliminary results are presented. These results indicate limit cycle oscillations develop near values of published critical thrust loads and that aerodynamic loads have a noticeable effect on the results. For increasing velocities, one finds aerodynamics playing the role of increasing the effective flexibility.

Introduction

In the design of missiles, higher speeds, more demanding maneuvers, and higher flexibility mean an increased role for structural deformation in preliminary design and simulation. Current missile preliminary design methodology does not facilitate multidisciplinary design optimization. Furthermore, even though the designs are driven by stiffness not by strength, no attempt has been made until now to take advantage of the elastic couplings afforded by use of composites. Aeroelastic phenomena can affect the originally planned missile trajectory, and in this case the extent of the coupling between flight mechanics and aeroelasticity should be understood. Furthermore, the insightful understanding of structural dynamics of flying missiles is an essential part in getting the structural design requirements which will lead to high performance. For example, the mechanical joints of missile greatly affect missile body flexibility[1] and the range of missile stiffness should be known at the preliminary design phase for optimum design in terms of maneuverability and stability. The Stability problems of missiles fall into various categories, including static aeroelastic instabilities,[2,3] flutter instabilities under thrust,[4–8] resonance-type response caused by coupling between the rocket spin and its bending natural modes,[9,10] etc. Our work has focused on being able to treat all these. The stability problem due to thrust is strictly a dynamic stability issue, but aeroelastic phenomena may influence it.[11] Unlike conventional flight vehicles, however, the static and dynamic aeroelastic instabilities may be coupled with flight dynamics modes. Finally, trajectory optimization[12] may place demands on the missile that create high loads. All these can be conducted within one framework, in which structural formulation is based on geometrically exact beam finite elements and aerodynamic theories which vary according to the flight regime and missile geometry. The motivations for specific aspects of the present work are well depicted in Fig. 1. The specifics of work done to date are described below.

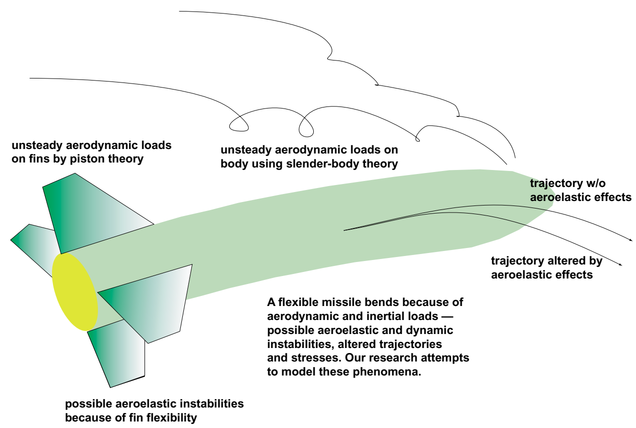

Fig. 1 Schematic of missile problem

Theoretical Basis

Structural Formulation

The structural part of the formulation comes from the mixed variational formulation based on the exact intrinsic equations for dynamics of moving beams.[13] The modifications to the original variational principle are the inclusion of the gravitational potential energy and appropriate energy variation for dealing with rigid-body dynamics, the analysis of which is needed for the missile time-marching scheme. The frames presented here are the undeformed beam cross-sectional frame (the b basis), the deformed cross-sectional frame (the B basis), and the inertial frame (the i basis). Here we follow the same rule for the variable notation as shown in Ref.,[13] except that the subscript o rep-

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resents missile reference point for taking care of the rigid body motion. The variables with subscript b and o are measured in the b frame, except for u_o, the basis for which is the inertial frame. The contribution of all gravitational forces is handled by means of its potential energy, which is written as

$$G = \int_0^\ell mge_3^T(u_o + C_o^T(r_b + u_b + C^T\xi_B))dx_1 \quad (1)$$

where r_b is the position from the missile body reference point, u_o is the displacement of missile reference point in the i frame, u_b is the displacement of the points on missile reference line in the b frame, ξ_B is the mass offset from the missile reference line, m is mass per unit length, C_o is the rotation matrix from i frame to b frame, and C is the rotation matrix from b frame to B frame. The kinematic relationships and the expressions for the velocities and the generalized strains can be written as

$$v_o = C_o\dot{u}_o \quad (2)$$

$$\widetilde{\omega}_o = -\dot{C}_oC_o^T \quad (3)$$

$$V_B = C\left[v_o + \dot{u}_b + \widetilde{\omega}_o(r_b + u_b)\right] \quad (4)$$

$$\Omega_B = \left(\frac{\Delta - \frac{\widetilde{\theta}}{2}}{1 + \frac{\theta^T\theta}{4}}\right)\dot{\theta} + C\omega_o \quad (5)$$

$$\gamma = C(e_1 + u_b') - e_1 \quad (6)$$

$$\kappa = \left(\frac{\Delta - \frac{\widetilde{\theta}}{2}}{1 + \frac{\theta^T\theta}{4}}\right)\theta' \quad (7)$$

where the $\widetilde{(\ )}$ operator converts a 3×1 column matrix to its 3×3 antisymmetric dual matrix. The orientation of B frame with respect to b frame is represented using Rodrigues parameters, which have been applied to nonlinear beam problems with success. The rotation matrix relating B frame to the b frame is written as

$$C = \frac{\left(1 - \frac{1}{4}\theta^T\theta\right)\Delta - \widetilde{\theta} + \frac{1}{2}\theta\theta^T}{1 + \frac{1}{4}\theta^T\theta} \quad (8)$$

For the orientation of the missile body frame (i.e. the b frame), however, the regular use of the Rodrigues parameters is insufficient because of their well known singularity at a rotation angle of 180°. Thus, the direction cosines of b in i are used as rotational variables for the rigid-body motion of the missile. The strain and force measures, along with velocity and momentum measures, are related through the constitutive laws in the form

$$\begin{Bmatrix} P \\ H \end{Bmatrix} = \begin{bmatrix} m\Delta & -m\widetilde{\xi} \\ m\widetilde{\xi} & I \end{bmatrix} \begin{Bmatrix} V \\ \Omega \end{Bmatrix}$$
$$\begin{Bmatrix} F \\ M \end{Bmatrix} = [S] \begin{Bmatrix} \gamma \\ \kappa \end{Bmatrix} \quad (9)$$

The weak form then leads to

$$\begin{aligned}
&\int_{t_1}^{t_2}\int_0^\ell \Bigg\{ \left[\overline{\delta q}_B'^T - \overline{\delta q}_B^T\widetilde{\kappa} - \overline{\delta\psi}_B^T(\widetilde{e}_1 + \widetilde{\gamma})\right] F_B \\
&+ \left(\overline{\delta\psi}_B'^T - \overline{\delta\psi}_B^T\widetilde{\kappa}\right) M_B \\
&- \Big[\dot{\overline{\delta q}}_B^T - \overline{\delta q}_B^T\widetilde{\Omega}_B - \overline{\delta\psi}^T\widetilde{V}_B + \delta v_o^T C^T \\
&+ \delta\omega_o^T(\widetilde{r}_b + \widetilde{u}_b)C^T\Big] P_B \\
&- \left(\dot{\overline{\delta\psi}}_B^T - \overline{\delta\psi}\widetilde{\Omega}_B + \delta\omega_o^T C^T\right) H_B \\
&+ \overline{\delta F}^T\left[e_1 - C^T(e_1 + \gamma)\right] - \overline{\delta F}'^T u_b \\
&- \overline{\delta M}^T\left(\Delta + \frac{1}{2}\widetilde{\theta} + \frac{1}{4}\theta\theta^T\right)\kappa - \overline{\delta M}'^T\theta \\
&- \overline{\delta P}^T\left[v_o + \widetilde{\omega}_o(r_o + u_b) - C^T V_B\right] + \dot{\overline{\delta P}}^T u_b \qquad (10) \\
&- \overline{\delta H}^T\left(\Delta + \frac{1}{2}\widetilde{\theta} + \frac{1}{4}\theta\theta^T\right)(C\omega - \Omega_B) \\
&+ \dot{\overline{\delta H}}^T\theta - \overline{\delta q}_B^T f_B - \overline{\delta\psi}_B^T m_B\Bigg\} dx_1 dt \\
&+ \int_{t_1}^{t_2}\Big(\delta G - \delta {v_o^*}^T P_o - \delta {\omega_o^*}^T H_o - \delta u_o^T f_o \\
&- \delta\psi_o^T m_o\Big) dt = -\int_0^\ell \Big(\overline{\delta q}_B^T\hat{P}_B + \overline{\delta\psi}_B\hat{H}_B \\
&-\overline{\delta P}^T\hat{u}_b - \overline{\delta H}^T\hat{\theta}\Big)\Big|_{t_1}^{t_2} dx_1 + \int_{t_1}^{t_2}\Big(\overline{\delta q}^T\hat{F} \\
&+\overline{\delta\psi}^T\hat{M} - \overline{\delta F}^T\hat{u} - \overline{\delta M}^T\hat{\theta}\Big)\Big|_0^\ell dt
\end{aligned}$$

where f_o and m_o are column matrices containing the measure numbers of force and moment vectors acting on the rigid body. The unknowns are F_B and M_B, the sectional force and moment measures in the B basis, respectively; P_B and H_B, the sectional linear and angular momentum measures in the B basis, respectively; γ and κ, the force and moment strains, respectively; V_B and Ω_B, the linear and angular velocity measures of the beam reference line in the B basis, respectively; and f_B and m_B, the external force and moment, respectively. Algebraic expressions defining certain variables in terms of displacement and rotation variables are denoted by $(\ )^*$.

Now space-time finite elements are used to obtain the time history of the missile motion, which is needed to investigate the nonlinear dynamics of the missile in flight. After integration by parts of the additional energy expression due to rigid-body motion, the unknowns are neither differentiated with respect time nor space from henceforth, so that constant shape functions may be used for them. Since the weak form is linear in the virtual quantities and they may be differentiated with respect to both space and time, and linear/bilinear shape functions are used for them.

Thus,

$$\begin{aligned}
&\overline{\delta q}_B = \overline{\delta q}_{i_s}(1-\xi)(1-\tau) + \overline{\delta q}_{i_f}(1-\xi)\tau \\
&\quad + \overline{\delta q}_{i+1_s}\xi(1-\tau) + \overline{\delta q}_{i+1_f}(1-\xi)\tau && u = u_i \\
&\overline{\delta\psi}_B = \overline{\delta\psi}_{i_s}(1-\xi)(1-\tau) + \overline{\delta\psi}_{i_f}(1-\xi)\tau \\
&\quad + \overline{\delta\psi}_{i+1_s}\xi(1-\tau) + \overline{\delta\psi}_{i+1_f}(1-\xi)\tau && \theta = \theta_i \\
&\overline{\delta F} = \overline{\delta F}_i(1-\xi) + \overline{\delta F}_{i+1}\xi && F = F_i \\
&\overline{\delta M} = \overline{\delta M}_i(1-\xi) + \overline{\delta M}_{i+1}\xi && M = M_i \\
&\overline{\delta P} = \overline{\delta P}_{i_s}(1-\tau) + \overline{\delta P}_{i_f}\tau && P = P_i \\
&\overline{\delta H} = \overline{\delta H}_{i_s}(1-\tau) + \overline{\delta H}_{i_f}\tau && H = H_i
\end{aligned}$$

where subscript s and f the variable values at the starting and final time. It should be noted that the rigid-body virtual quantities, not listed above, do not have spatial dependence. With these shape functions, if we just consider structural dynamics, the mixed variational formulation takes the form

$$F(X_s, X_f, X) = 0 \tag{11}$$

where X is a column matrix of all structural variables and X_s and X_f are its initial and final values. The Jacobian matrix of the above set of nonlinear equations can be obtained analytically and is found to be very sparse. So, if the initial conditions and boundary conditions are specified, the final values after one time step can be found very efficiently using the Newton-Raphson method, and time history is obtained by doing time marching iteration. The structural part of the above formulation has been well validated against the stability subject to thrust.

Aerodynamics

Available aerodynamics tools have been evaluated for computation of loads on missiles. Missile loads are very dependent on flight conditions and missile geometry. Several technical methods are extensively described in Refs. 14 and 15. The validity of slender-body theory, which is based on potential flow, has been well established by comparison with experimental data[16] for a wide range of Mach numbers. An extended slender-body theory is discussed by Ref. 17. An unsteady version of slender-body theory for aeroelasticity was presented in Ref. 18. For our purposes, the aerodynamic loads on a missile body can be calculated with sufficient accuracy for the sort of interdisciplinary tradeoff studies we anticipate doing by using slender-body theory augmented by a viscous cross-flow theory.[19] There are parts of the missile for which these methods are not suitable, and for these other methods will be used. For example, the loads on the missile fins and tail will be calculated by thin-airfoil theory in low-speed flight and by piston theory[20] in hypersonic flight. With the combination of the viscous cross-flow theory of Ref. 19 and the potential flow slender-body theory in Ref. 18, we can take into account the bending deformation and unsteadiness of the flow. The resulting equation then reduces to

$$\begin{aligned}
\frac{dN}{dx} =& -\rho_\infty \frac{dS}{dx}\left(U^2\frac{\partial\lambda}{\partial x} + U\frac{\partial\lambda}{\partial t}\right) + \eta c_d d\frac{\rho_\infty U^2}{2}\alpha^2 \\
& -\rho_\infty S\left(U^2\frac{\partial^2\lambda}{\partial x^2} + 2U\frac{\partial^2\lambda}{\partial x\partial t} + \frac{\partial^2\lambda}{\partial t^2}\right)
\end{aligned} \tag{12}$$

where $\lambda = -u_b - \alpha[x - x_o]$; N is the normal force column matrix; U and ρ_∞ are the freestream velocity and air density, respectively; α is the angle of attack and sideslip angle column matrix at the reference point; x_o is the location of the reference point; η is the ratio of the drag coefficient of a circular cylinder of finite length to that of a circular cylinder of infinite length; c_d is the drag coefficient of a circular cylinder and d is the missile diameter. In accordance with the lowest order of differentiation for the variables in the aerodynamic force expression, kinematic expressions are used to reduce the order. Here, the angle of attack and sideslip angle quantities can be determined by other kinematic quantities. Results according to this equation are in good agreement with existing experimental data. Fig. 2 represents the comparison between slender body theory and experiments[21] for steady flow when the angle of attack is 10°. The average normal force and pitching moment are in excellent agreement and the distributed force shows sufficiently good agreement for the purposes of our current research. Drag is very dependent on the configuration and flight conditions. Body, wings and tails have all contributions to the drag and body drag is dominant especially in supersonic flight regime. For the purposes of the current research, methods that are approximate, closed-form solutions or that at least require less computational effort, such as the modified Newtonian method and tangent cone method, have been employed. Also, in case of spin stabilized missiles, additional lift should be considered due to the effect of spin, which is called the Magnus effect.[22] All the available methods for missile aerodynamics are well documented in Ref. 23.

Stability Analysis without Aerodynamics

Based on the methodology set forth here, a computer code for investigation of the nonlinear dynamics of a missile has been developed. The various stability problems due to thrust which appear in the literature can be examined in terms of their time history. Considering the case without directional control considered by Beal in Ref. 4, when a small perturbation of the transverse deflection is imposed at the initial time and the thrust level is below Beal's critical value, the deflection indeed dies out in time. However, as expected, when the thrust level is a little larger than the critical value, the deflections grow until they reach an

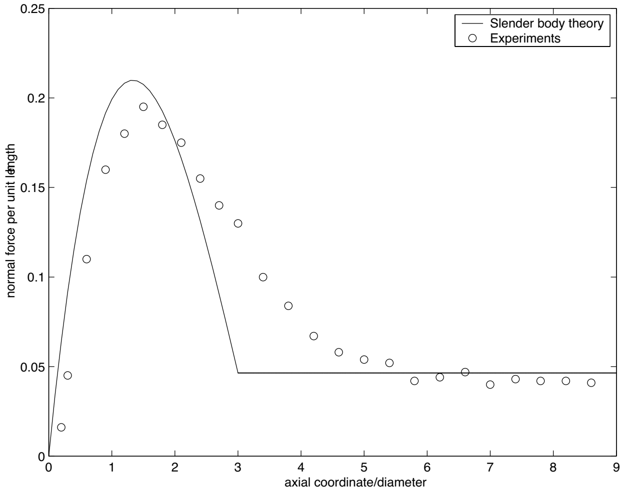

Fig. 2 Comparison of Slender body theory with Experiments

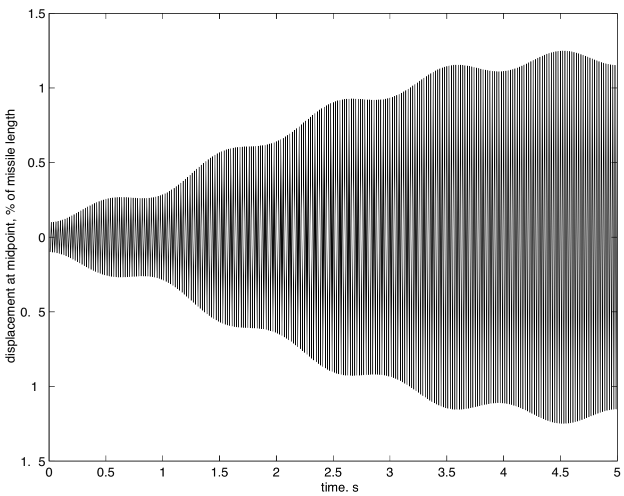

Fig. 3 Time history above critical thrust

oscillatory motion with bounded amplitude, suggesting a limit cycle. A typical result is shown in Fig. 3. Preliminary results for several cases show that limit cycles can develop from disturbances with thrust values that are either just below the critical value suggested by Beal[4] (Fig. 4) or just above it. However, the motion is divergent well above the critical value as shown in Fig. 5. To see if there are any aeroelastic effects on the stability as a function of thrust or on accelerated flight, Fig. 6 is used as a baseline missile configuration.[24] Both movable wings and fixed tail wings with a cruciform pattern have two sets of wedge-shaped panels. Around the nose, the large missile body drag is applied. The wing and tail are both under the influence of wave drag and skin friction drag. At the nozzle base, drag and thrust are both applied. The total drag force is distributed along the body, and the thrust force is balanced with the total drag. Results for cases with Mach numbers 2 and 2.5, where are presented in Figs. 7 and Fig. 8. The velocity increase

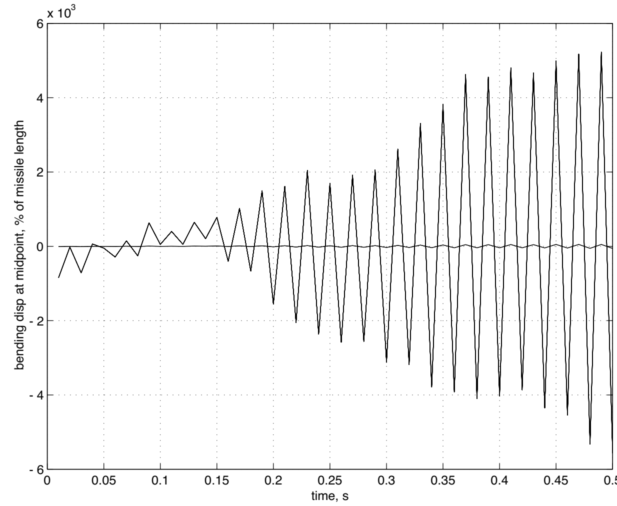

Fig. 4 Time history below critical thrust

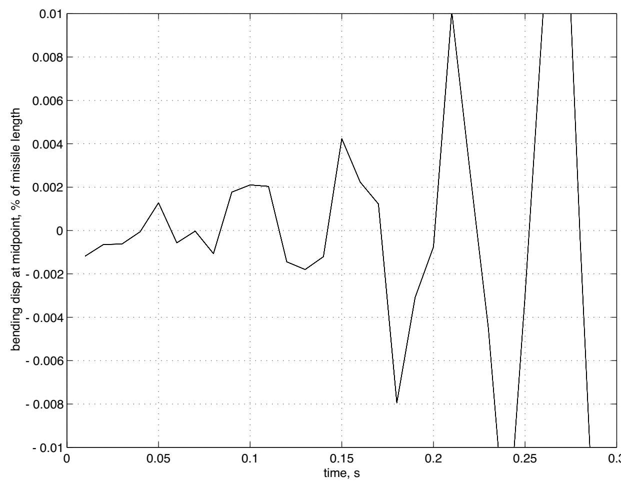

Fig. 5 Time history well above critical thrust

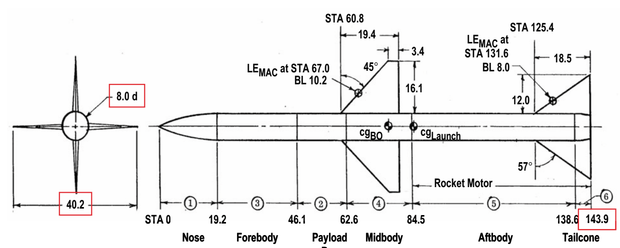

Fig. 6 Baseline missile configuration

noticeably affects the amplitudes of the response after small lateral disturbances are given. The flexural stiffnesses are relatively large, but the distributed drag forces appear to play the role of reducing the effective stiffness.

Conclusion

A current trend in the development of missiles is in the direction of more flexibility, higher maneuverability, and higher speeds, all of which require a higher level of fidelity for calculations for stability, loads, control, and guidance. To address these and other issues,

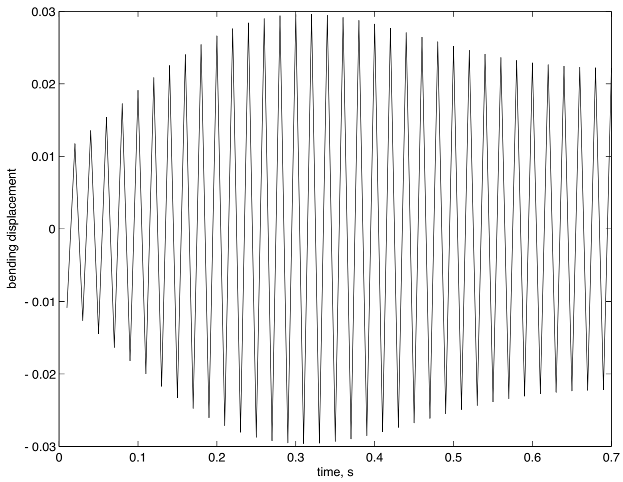

Fig. 7 Time history at Mach 2

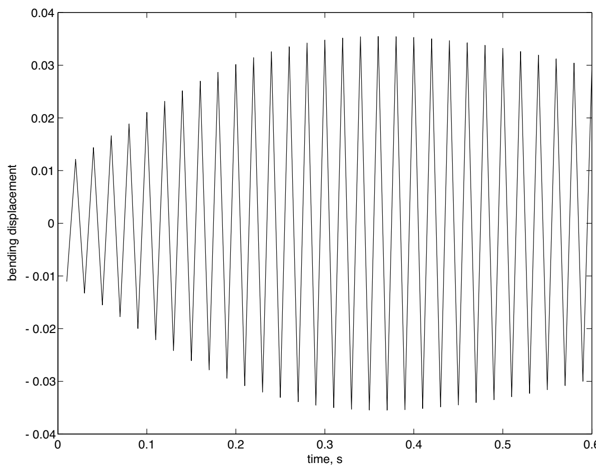

Fig. 8 Time history at Mach 2.5

an aeroelastic analysis for missiles has been developed and is outlined herein. Missile design requires a multidisciplinary effort, but in preliminary design, aeroelasticity is generally not taken into account. In the present effort, the missile body is modeled in terms of geometrically-exact, nonlinear, beam finite elements. Aerodynamics loads are based on slender body theory, and details of the present implementation will be in future papers. Preliminary results are presented for instabilities induced by thrust with and without aerodynamic effects. These results indicate that the aerodynamics serve to decrease the effective stiffness of the missile. As with general aircraft, velocity is an important aerodynamic element related with aeroelastic instability. The present code has been validated against several cases, especially the critical load under thrust without directional control. It is not surprising that the response is divergent far above the critical value given in several literature. But near the critical value whether the thrust is high or low, limit cycles were observed.

Much work needs to be done to investigate inflight dynamic loads and stability with optimum trajectories. The authors are not aware of published attempts to include the effects of aeroelasticity on the trajectory. With the present analysis a whole host of phenomena can now be investigated, including the dynamics of accelerating flight with thrust, varying flexural stiffness, free play and mechanical joints. It would be very conducive to design efforts to know of the effects of coupling between flight mechanics (i.e., trajectory optimization, constraints, etc.) and aeroelasticity (including internal loads and stability). A variety of dynamic and aeroelastic stability and loads problems over a wide range of steady-flight conditions, including spin and thrust will be analyzed in future papers.

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