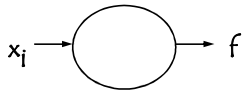
Outline

- Digital CMOS design
  - Boolean algebra
  - Basic digital CMOS gates
  - Combinational and sequential circuits
  - Coding - Representation of numbers

CMOS Circuits

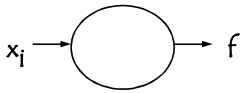
- Combinational logic



$f(x_1, \ldots, x_i, \ldots, x_n)$

CMOS Circuits

- Sequential logic

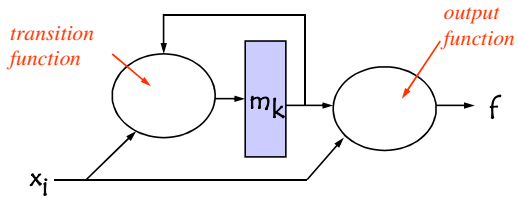


$f(x_1, \ldots, x_i, \ldots, x_n, m_1, \ldots, m_k, \ldots, m_p)$

$m_k(x_1, \ldots, x_i, \ldots, x_n, m_1, \ldots, m_k, \ldots, m_p)$

CMOS Circuits

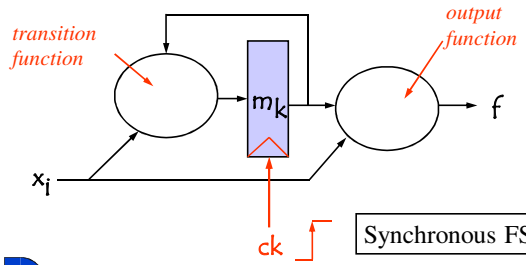
- Sequential logic



Finite State Machine (FSM)

CMOS Circuits

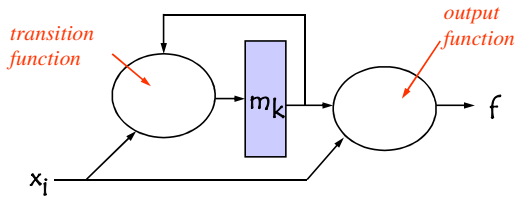
- Finite State Machine



Synchronous FSM

CMOS Circuits

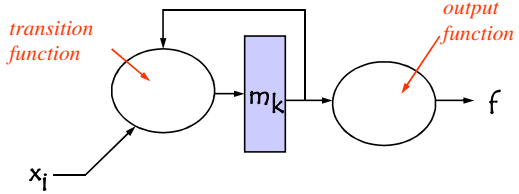
- Finite State Machine



Mealy FSM

CMOS Circuits

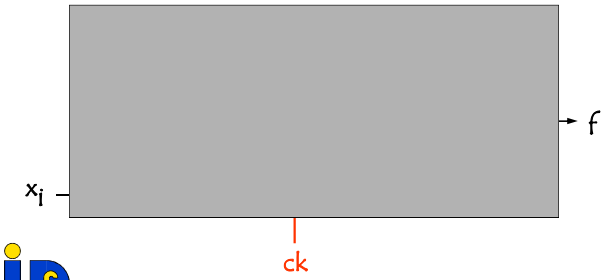
- Finite State Machine



Moore FSM

Sequential Circuits

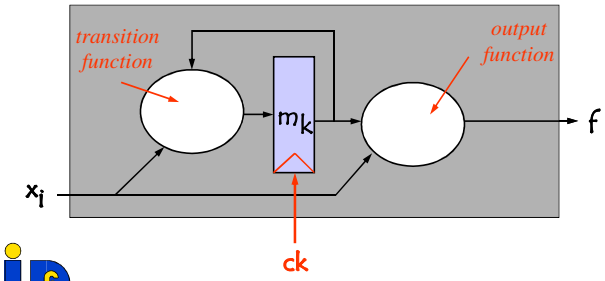
- Sequential Circuit



Sequential Circuits

How can I design a sequential circuit ?

I need a method to go from a black box to a white box



Sequential Circuits

How can I design a sequential circuit ?

The starting point (what I know)

What the circuit is supposed to do

Capture this knowledge into a representation that can be transposed into Boolean functions

The aim

How it will do it

number of required memory elements

Sequential Circuits

Representation of a sequential circuit ?

- State graph

$G = (X, Y, S, T, O)$

X : input variables
Y : output variables
S : states
T : transitions
O : output set conditions

Sequential Circuits

- $G = (X, Y, S, T, O)$

T : set of transitions

$t \in T, \quad t = (s, s', f)$

$s \in S$: source state

$s' \in S$: target state

$f \in B_n$: transition condition

Sequential Circuits

- $G = (X, Y, S, T, O)$

O : output set conditions

$o \in O, \quad o = (y, s, c)$

$y \in Y$: output variable

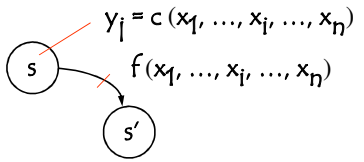
$s \in S$: state

$c \in B_n$: output set condition

Sequential Circuits

- Let consider a graph $G = (X, Y, S, T, O)$

Graphic representation of G



Sequential Circuits

Representation of a sequential circuit ?

- State graph

Transpose into a graphic representation the expected behavior of a sequential system

$G = (X, Y, S, T, O)$

Sequential Circuits

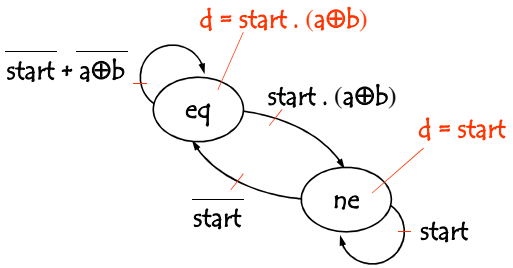
- Example

Two signals a, b each transmitting a series of bits (1 bit at a cycle)

Design a system that sets a flag d if the value transmitted by a is different than b

Sequential Circuits

- Example



Sequential Circuits

Representation of a sequential circuit ?

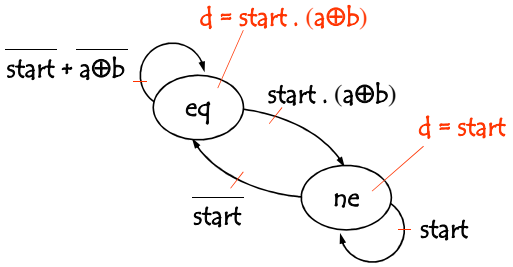
- State graph

$G = (X, Y, S, T, O)$

define the number of memory elements required to represent $S : M$

Sequential Circuits

- Example



2 states ⟶ 1 memory element

Sequential Circuits

Representation of a sequential circuit ?

- State graph

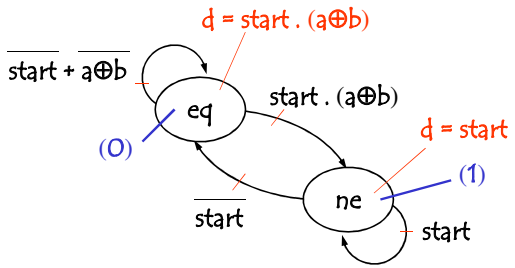
$G = (X, Y, S, T, O)$

define the number of memory elements required to represent $S : M$

represent each state s by a vector of M

Sequential Circuits

- Example



2 states ⟶ 1 memory element

Sequential Circuits

Representation of a sequential circuit ?

- State graph

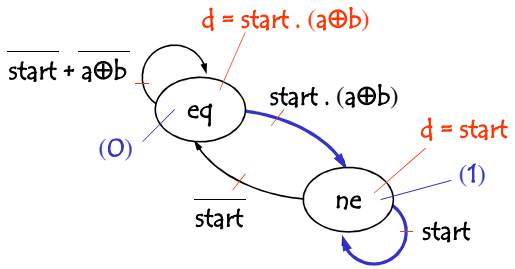
$G = (X, Y, S, T, O)$

define the *transition function* : m_k

m_k = sum of the Boolean function of the transitions that have as target a state where m_k=1

Sequential Circuits

- Example



$m_1 = \overline{m_1}.start.(a \oplus b) + m_1.start$

Sequential Circuits

Representation of a sequential circuit ?

- State graph

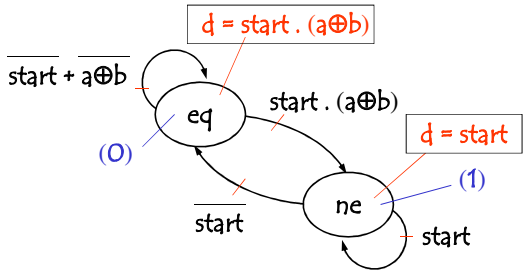
$G = (X, Y, S, T, O)$

define the *output function* : y_i

y_i = sum of the output conditions concerning y_i

Sequential Circuits

- Example



$d = \overline{m_1}.start.(a \oplus b) + m_1.start$

Sequential Circuits

Representation of a sequential circuit ?

- State graph

$G = (X, Y, S, T, O)$

implement the Boolean functions : m_k , y_i

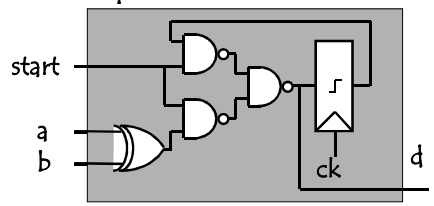
Sequential Circuits

- Example

$m_1 = \overline{m_1}.start.(a \oplus b) + m_1.start$

$d = \overline{m_1}.start.(a \oplus b) + m_1.start$

$m_1 = start .(a \oplus b) + start.m_1$



Sequential Circuits

How can I design a sequential circuit (summary) ?

- $G = (X, Y, S, T, O)$
  - graphic representation of the behavior
  - define the number of memory elements required to represent S : M
  - represent each state by a vector of M
  - define the *transition function*
  - define the *output function*
  - implement the Boolean functions

Sequential Circuits

How can I design a sequential circuit (summary) ?

- G = (X, Y, S, T, O)
  - check the completeness
    for each state s : $\sum_{t_i=(s, s'_i, f_i)} f_i = 1$
  - check the exclusivity
    for each state s : $\sum_{\substack{t_i=(s, s'_i, f_i) \\ t_j=(s, s'_j, f_j) \\ i \neq j}} f_i \cdot f_j = 0$