Sorbonne Université

Numerical Algorithms

Practical 2

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Janvier 2023

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1.

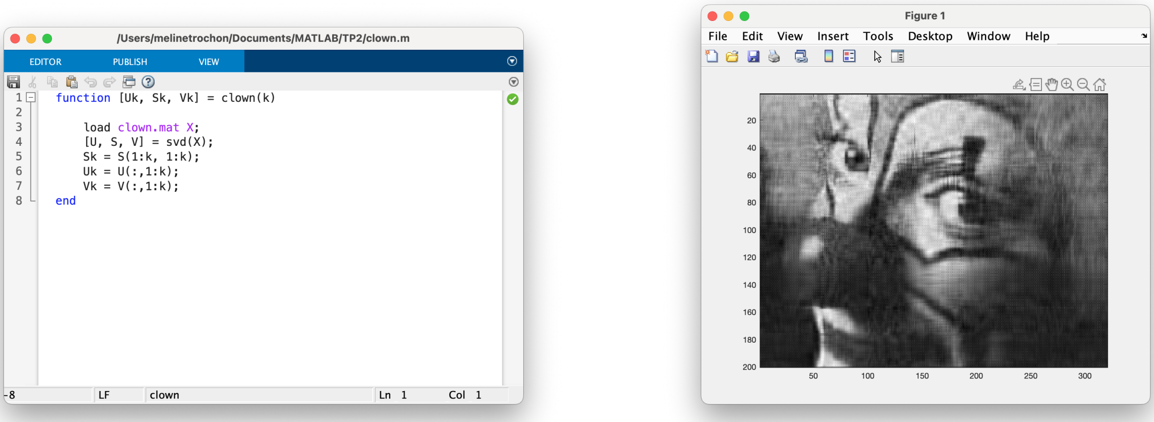


FIGURE 1 – The algorithm to compute the compression of the image the clown and the algorithm for k=20

2. A good candidate to be a measure of the quality of the compression would be $\frac{\sigma_k}{\sigma_1}$. Indeed, we want to trunks all the singular values that are to small in comparison to the bigger ones.

2 Exercise 7

1. We want to solve $\min_f \|g - Kf\|_2^2$ (1). We can use the SVD-decomposition which gives $\|g - Kf\|_2^2 = \|g - U\Sigma V^t f\|_2^2$

We complete U such that [U $\tilde{U}$] is square orthogonal. then,

$\|g - U\Sigma V^t f\|_2^2 = \| \begin{pmatrix} U^t \\ \tilde{U}^t \end{pmatrix} (g - U\Sigma V^t f)\|_2^2$

(the norm doesn't change)

$= \| \begin{pmatrix} U^t g - \Sigma V^t f \\ \tilde{U}^t g \end{pmatrix} \|_2^2$

$= \|U^t g - \Sigma V^t f\|_2^2 + \|\tilde{U}^t g\|_2^2$

The second term does not depend of f, so, we just have to solve the first one. That is to say that we are searching the solution of $U^t g - \Sigma V^t f = 0$ Which means : $f^* = \Sigma^{-1} V U^t g$.

Σ^{-1} is a diagonal matrix with $\sigma_i, i = 1, .., n$ as values. We can show by identification that $\Sigma^{-1} V U^t g = \sum_{i=1}^{p} \frac{u_i^t}{\sigma_j} v_i$

2.

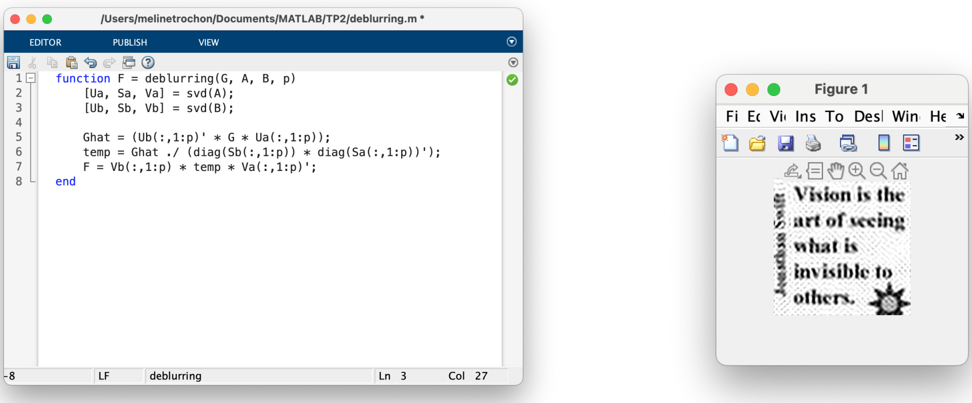


FIGURE 2 – The deblurring algorithm then the best image we found with p=52

3 Exercise 8

1. A^t is a stochastic Matrix if and only if

$$\sum_{j=0}^{n} A^t_{i,j} = 1,\ i = 1, ..., n \quad (1)$$

If $c_j = 0$, $A^t_{i,j} = 1/n$ and (1) is verified.
Let consider $c_j \neq 0$, we have

$$\begin{aligned} &\sum_{i=0}^{n} \frac{p g_{i,j}}{c_j} + \delta \\ &= \frac{p}{c_j} \sum g_{i,j} + \frac{n(1-p)}{n} \\ &= \frac{p c_{i,j}}{c_{i,j}} + \frac{n(1-p)}{n} = 1 \end{aligned}$$

Therefore, A is a stochastic matrix.

In addition, if we consider the vector $v \in (R)^n$ which is composed of 1, we can remark that, thanks to the properties of a stochastic matrix, we have Av = 1*v. So 1 is an eigenvalue of A.

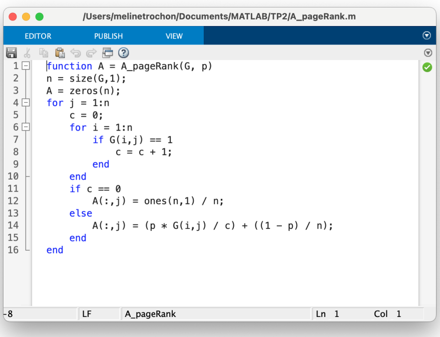
Then, we can prove that all the eigenvalues of A are less than 1. Indeed, we know by the Gershgorin circle theorem that :

$$\lambda_k \in \bigcup_{j=1,..,n} D(A_{j,j}, r_j), k = 1, .., n$$

where $r_j = \sum_{i \neq j} A_{i,j}$ and we know as showed above that (1), and $A_{i,j} \geq 0$, $i, j = 1, .., n$ So, we can conclude that $\lambda_k \in$[0,1], $k = 1, .., n$

2. A is non negative, so, by the Perron-Frobenius theorem we have the result that the spectral radius $\rho(A)$ is a unique eigenvalue of A. Then, because $\rho(A) = 1$ (showed previously) there exists an eigenvector associated to the eigenvalue 1 (non negative, by the theorem) . If we take x = $(\frac{1}{n}, .., \frac{1}{n})^t$ this verifies the equality $\sum_{i=1}^{n} x_i = 1$

3.



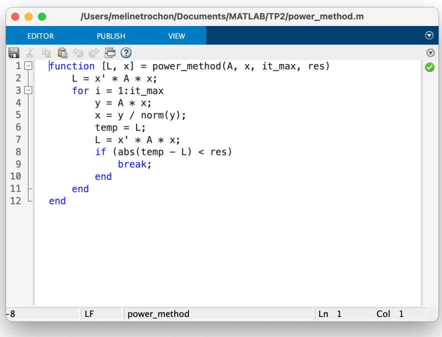


FIGURE 3 – Algorithms to create the matrix A and compute Λ and x

And we can provide a ranking according to their page_rank by sorting the vector x.