SORBONNE UNIVERSITÉ

ADVANCED NUMERICAL ALGORITHMS

Practical 1 : - Monte Carlo Computations

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I detailed the Monte Carlo method just once in the first question

1 Exercise 6

1. Let consider the unity square C with an area of 1. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space such that $\Omega = C = [0,1] \times [0,1], \mathcal{F} = \mathcal{B}(\Omega)$ and $\mathbb{P}$ is uniform probability law. We want an estimation of $\frac{\pi}{4}$, knowing that we have a random variable $X : \Omega \to \{0,1\}$ and $A \in C$ such that $X(\omega) = 1$ if $\omega \in A$ and 0 otherwise

So we just have to choose A $\subset$ C such that $\mathbb{P}(X = 1) = \frac{\pi}{4} \to \int_A 1dx = \frac{\pi}{4}$.For that, we can just pick A = $\{x \in C, x^2 + y^2 < 1\}$

Then, we want to repeat the random experiment X many times, let us denote $(X_n)_{n\in\mathbb{N}}$ the sequence of random variables. X_n are independents, have the same probability law and $\mathbb{E}(X_1) = \frac{\pi}{4} < \infty$

So, thanks to the strong law of large numbers we have :

$$\frac{1}{n}\sum_1^k X_k \to \mathbb{E}(X_1)$$

Because $\mathbb{E}(X_1) = \frac{\pi}{4}$, we just have to compute $4 \times \frac{1}{n}\sum_1^k X_k$ to have an estimation of π

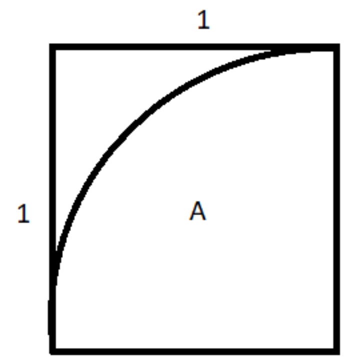

FIGURE 1 – illustration of the idea

```
function p = pi_estimation(ni)
    N =  1;
    n = 0;
    while (n<ni)
        x = rand(1);
        y = rand(1);
        if (x*x + y*y <= 1)
            N = N + 1;
        end
    n = n + 1;
    end
    p = 4*N/n;
end
```

I found $\pi = 3.1418$ with n = 100 000 000

2 Exercise 7

Having an estimation of the volume of $\mathbb{B}_3$ and $\mathbb{B}_4$ is very similar of what we did in the previous exercise.

We just have to be careful of the dimension to compute the volume. In 2D, $A_2 = \frac{1}{4}\mathbb{B}_2 = \frac{1}{2^2}\mathbb{B}_2$ We deduce that $A_3 = \frac{1}{8}\mathbb{B}_3$ and $A_4 = \frac{1}{16}\mathbb{B}_4$

So we have the matlab code above :

```
function v = vs3_estimation(ni)
    N =  1;
    n = 0;
    while (n<ni)
        x = rand(1);
        y = rand(1);
        z = rand(1);
        if (x*x + y*y + z*z <= 1)
            N = N + 1;
        end
    n = n + 1;
    end
    v = 8*N/n;
end
```

```
function v = vs4_estimation(ni)
    N =  1;
    n = 0;
    while (n<ni)
        x = rand(1);
        y = rand(1);
        z = rand(1);
        t = rand(1);
        if (x*x + y*y + z*z + t*t<= 1)
            N = N + 1;
        end
    n = n + 1;
    end
    v = 16*N/n;
end
```

Which give us for n = 100 000 000 :
$V(\mathbb{B}_3) = 4.1892$ $V(\mathbb{B}_4) = 4.9351$

3 Exercise 8

1. To compute C and P,the same method used in exercise 1 and 2 can be used. So we have :

```
function e = Ec(ni)
    n = 0;
    a = 0;
    s = 0;
    while(n<ni)
        x = randn(1, 1);
        a = exp(x) - 1;
        if (a<0)
            a = 0;
        end
        s = s + a;
        n = n + 1;
    end
    e = s/ni;
end
```

```
function e = Ep(ni)
    n = 0;
    a = 0;
    s = 0;
    while(n<ni)
        x = randn(1, 1);
        a = 1 - exp(x);
        if (a<0)
            a = 0;
        end
        s = s + a;
        n = n + 1;
    end
    e = s/ni;
end
```

we have for n = 100 000 000 : C = 0.8870 P = 0.2384

2. We want to express $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt$ in function of erfc(x) = $\frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-t^2} dt$

For that, we just have to do a change of variable $y = -t\sqrt{2}$ to invert the boundaries of the integral and vanish the over two.

So far, we get :

$$\Phi(x) = \frac{1}{2} erfc(-\frac{x}{\sqrt{2}})$$

Then, we can check the values of C and P ith mathlab thanks to the formula given

I obtained C = 0.8871 P = 0.2384

These results are very similar with what we get with the Monte-Carlo method which show the accuracy of this method. However, we have to do a lot of iteration to have this accuracy