САНКТ-ПЕТЕРГБУРГСКИЙ НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ УНИВЕРСИТЕТ

ИНФОРМАЦИОННЫХ ТЕХНОЛОГИЙ, МЕХАНИКИ И ОПТИКИ

Курсовая работа по дискретной математике

*«Синтез комбинационных схем»*

Работу выполнил:

Анищенко Анатолий

P3112

Проверил:

Санкт-Петербург

2018 г.

|  |  |  |
| --- | --- | --- |
| **№ Вар.** | **Условие, при которых f = 1** | **Условие, при которых f = d** |
| **131** |  |  |

1. **Составление таблицы истинности**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Аыва** | **X1** | **X2** | **X3** | **X4** | **X5** | **X5 X3** | | **(X5 X3)10** | **X4 X2 X1** | | | **(X4 X2 X1)10** | **| - |** | **F** |
| **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0** | 0 | 0 | 0 | **0** | 0 | **0** |
| **1** | 0 | 0 | 0 | 0 | 1 | 1 | 0 | **2** | 0 | 0 | 0 | **0** | 2 | **1** |
| **2** | 0 | 0 | 0 | 1 | 0 | 0 | 0 | **0** | 1 | 0 | 0 | **4** | 4 | **1** |
| **3** | 0 | 0 | 0 | 1 | 1 | 1 | 0 | **2** | 1 | 0 | 0 | **4** | 2 | **1** |
| **4** | 0 | 0 | 1 | 0 | 0 | 0 | 1 | **1** | 0 | 0 | 0 | **0** | 1 | **d** |
| **5** | 0 | 0 | 1 | 0 | 1 | 1 | 1 | **3** | 0 | 0 | 0 | **00** | 3 | **1** |
| **6** | 0 | 0 | 1 | 1 | 0 | 0 | 1 | **1** | 1 | 0 | 0 | **4** | 3 | **1** |
| **7** | 0 | 0 | 1 | 1 | 1 | 1 | 1 | **3** | 1 | 0 | 0 | **4** | 1 | **d** |
| **8** | 0 | 1 | 0 | 0 | 0 | 0 | 0 | **0** | 0 | 1 | 0 | **2** | 2 | **1** |
| **9** | 0 | 1 | 0 | 0 | 1 | 1 | 0 | **2** | 0 | 1 | 0 | **2** | 0 | **0** |
| **10** | 0 | 1 | 0 | 1 | 0 | 0 | 0 | **0** | 1 | 1 | 0 | **6** | 6 | **0** |
| **11** | 0 | 1 | 0 | 1 | 1 | 1 | 0 | **2** | 1 | 1 | 0 | **6** | 4 | **1** |
| **12** | 0 | 1 | 1 | 0 | 0 | 0 | 1 | **1** | 0 | 1 | 0 | **2** | 1 | **d** |
| **13** | 0 | 1 | 1 | 0 | 1 | 1 | 1 | **3** | 0 | 1 | 0 | **2** | 1 | **d** |
| **14** | 0 | 1 | 1 | 1 | 0 | 0 | 1 | **1** | 1 | 1 | 0 | **6** | 5 | **0** |
| **15** | 0 | 1 | 1 | 1 | 1 | 1 | 1 | **3** | 1 | 1 | 0 | **6** | 3 | **1** |
| **16** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | **0** | 0 | 0 | 1 | **1** | 1 | **d** |
| **17** | 1 | 0 | 0 | 0 | 1 | 1 | 0 | **2** | 0 | 0 | 1 | **1** | 1 | **d** |
| **18** | 1 | 0 | 0 | 1 | 0 | 0 | 0 | **0** | 1 | 0 | 1 | **5** | 5 | **0** |
| **19** | 1 | 0 | 0 | 1 | 1 | 1 | 0 | **2** | 1 | 0 | 1 | **5** | 3 | **1** |
| **20** | 1 | 0 | 1 | 0 | 0 | 0 | 1 | **1** | 0 | 0 | 1 | **1** | 0 | **0** |
| **21** | 1 | 0 | 1 | 0 | 1 | 1 | 1 | **3** | 0 | 0 | 1 | **1** | 2 | **1** |
| **22** | 1 | 0 | 1 | 1 | 0 | 0 | 1 | **1** | 1 | 0 | 1 | **5** | 4 | **1** |
| **23** | 1 | 0 | 1 | 1 | 1 | 1 | 1 | **3** | 1 | 0 | 1 | **5** | 2 | **1** |
| **24** | 1 | 1 | 0 | 0 | 0 | 0 | 0 | **0** | 0 | 1 | 1 | **3** | 3 | **1** |
| **25** | 1 | 1 | 0 | 0 | 1 | 1 | 0 | **2** | 0 | 1 | 1 | **3** | 1 | **d** |
| **26** | 1 | 1 | 0 | 1 | 0 | 0 | 0 | **0** | 1 | 1 | 1 | **7** | 7 | **0** |
| **27** | 1 | 1 | 0 | 1 | 1 | 1 | 0 | **2** | 1 | 1 | 1 | **7** | 5 | **0** |
| **28** | 1 | 1 | 1 | 0 | 0 | 0 | 1 | **1** | 0 | 1 | 1 | **3** | 2 | **1** |
| **29** | 1 | 1 | 1 | 0 | 1 | 1 | 1 | **3** | 0 | 1 | 1 | **3** | 0 | **0** |
| **30** | 1 | 1 | 1 | 1 | 0 | 0 | 1 | **1** | 1 | 1 | 1 | **7** | 6 | **0** |
| **31** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | **3** | 1 | 1 | 1 | **7** | 4 | **1** |

1. **Представление булевой функции в аналитическом виде**

**КДНФ:** f = v v v v v v v v v v v v v v

**ККНФ:** f =

1. **Минимизация булевой функции методом Квайна-Мак-Класки**

Нахождение простых импликант (максимальных кубов).

Получение кубов различной размерности кубического комплекса K(f) и выделение из них простых импликант:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Ko(ƒ) N(ƒ) | | | K1(ƒ) | | | | K2(ƒ) | | | | K3(ƒ) | Z(ƒ) | |
| 1 | 00001 | \* | 1 | 000X1 | 1-6 | \* | 1 | 00XX1 | 1-16 | \* | X0XX1 | 1 | 00X1X |
| 2 | 00010 | \* | 2 | 00X01 | 1-7 | \* | 2 | X00X1 | 1-23 | \* |  | 2 | 001XX |
| 3 | 00100 | \* | 3 | X0001 | 1-10 | \* | 3 | X0X01 | 2-24 | \* |  | 3 | 0X10X |
| 4 | 01000 | \* | 4 | 0001X | 2-6 | \* | 4 | 00X1X | 4-9 |  |  | 4 | X1X00 |
| 5 | 10000 | \* | 5 | 00X10 | 2-8 | \* | 5 | 001XX | 6-19 |  |  | 5 | 1X00X |
| 6 | 00011 | \* | 6 | 0010X | 3-7 | \* | 6 | 0X10X | 6-21 |  |  | 6 | 0XX11 |
| 7 | 00101 | \* | 7 | 001X0 | 3-8 | \* | 7 | X1X00 | 9-27 |  |  | 7 | 0X1X1 |
| 8 | 00110 | \* | 8 | 0X100 | 3-9 | \* | 8 | 1X00X | 11-26 |  |  | 8 | X011X |
| 9 | 00110 | \* | 9 | 01X00 | 4-9 | \* | 9 | 0XX11 | 13-30 |  |  | 9 | XX111 |
| 10 | 01100 | \* | 10 | X1000 | 4-11 | \* | 10 | X0X11 | 13-32 | \* |  | 10 | X0XX1 |
| 11 | 11000 | \* | 11 | 1000X | 5-10 | \* | 11 | X01X1 | 16-33 | \* |  |  |  |
| 12 | 00111 | \* | 12 | 1X000 | 5-11 | \* | 12 | 0X1X1 | 16-31 |  |  |  |  |
| 13 | 01011 | \* | 13 | 00X11 | 6-12 | \* | 13 | X011X | 19-34 |  |  |  |  |
| 14 | 01101 | \* | 14 | 0X011 | 6-13 | \* | 14 | 10XX1 | 23-33 | \* |  |  |  |
| 15 | 10011 | \* | 15 | X0011 | 6-15 | \* | 15 | XX111 | 28-36 |  |  |  |  |
| 16 | 10101 | \* | 16 | 001X1 | 7-12 | \* |  |  |  |  |  |  |  |
| 17 | 10110 | \* | 17 | 0X101 | 7-14 | \* |  |  |  |  |  |  |  |
| 18 | 11001 | \* | 18 | X0101 | 7-16 | \* |  |  |  |  |  |  |  |
| 19 | 11100 | \* | 19 | 0011X | 8-12 | \* |  |  |  |  |  |  |  |
| 20 | 01111 | \* | 20 | X0110 | 8-17 | \* |  |  |  |  |  |  |  |
| 21 | 10111 | \* | 21 | 0110X | 9-14 | \* |  |  |  |  |  |  |  |
| 22 | 11111 | \* | 22 | X1100 | 9-19 | \* |  |  |  |  |  |  |  |
|  |  |  | 23 | 100X1 | 10-15 | \* |  |  |  |  |  |  |  |
|  |  |  | 24 | 10X01 | 10-16 | \* |  |  |  |  |  |  |  |
|  |  |  | 25 | 1X001 | 10-18 | \* |  |  |  |  |  |  |  |
|  |  |  | 26 | 1100X | 11-18 | \* |  |  |  |  |  |  |  |
|  |  |  | 27 | 11X00 | 11-19 | \* |  |  |  |  |  |  |  |
|  |  |  | 28 | 0X111 | 12-20 | \* |  |  |  |  |  |  |  |
|  |  |  | 29 | X0111 | 12-21 | \* |  |  |  |  |  |  |  |
|  |  |  | 30 | 01X11 | 13-20 | \* |  |  |  |  |  |  |  |
|  |  |  | 31 | 011X1 | 14-20 | \* |  |  |  |  |  |  |  |
|  |  |  | 32 | 10X11 | 15-21 | \* |  |  |  |  |  |  |  |
|  |  |  | 33 | 101X1 | 16-21 | \* |  |  |  |  |  |  |  |
|  |  |  | 34 | 1011X | 17-21 | \* |  |  |  |  |  |  |  |
|  |  |  | 35 | X1111 | 20-22 | \* |  |  |  |  |  |  |  |
|  |  |  | 36 | 1X111 | 21-22 | \* |  |  |  |  |  |  |  |

Составление импликантной таблицы:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 00001 | 00010 | 00011 | 00101 | 00110 | 01000 | 01011 | 01111 | 10011 | 10101 | 10110 | 10111 | 11000 | 11100 | 11111 |
| 00X1X |  | (\*) | \* |  | \* |  |  |  |  |  |  |  |  |  |  |
| 001XX |  |  |  | \* | \* |  |  |  |  |  |  |  |  |  |  |
| 0X10X |  |  |  | \* |  |  |  |  |  |  |  |  |  |  |  |
| X1X00 |  |  |  |  |  | (\*) |  |  |  |  |  |  | \* | (\*) |  |
| 1X00X |  |  |  |  |  |  |  |  |  |  |  |  | \* |  |  |
| 0XX11 |  |  | \* |  |  |  | (\*) | \* |  |  |  |  |  |  |  |
| 0X1X1 |  |  |  | \* |  |  |  | \* |  |  |  |  |  |  |  |
| X011X |  |  |  |  | \* |  |  |  |  |  | (\*) | \* |  |  |  |
| XX111 |  |  |  |  |  |  |  | \* |  |  |  | \* |  |  | (\*) |
| X0XX1 | (\*) |  | \* | \* |  |  |  |  | (\*) | (\*) |  | \* |  |  |  |

Множество существенных импликант:

Импликанты 1, 4, 6, 8, 9 и 10 - существенные, так как они покрывают вершины 1, 2, 6, 7, 9, 10, 11, 14 и 15, не покрытые другими импликантами. Вычеркнем из таблицы строки, соответствующие этим импликантам, а также столбцы, соответствующие вершинам, покрываемым существенными импликантами. Это все вершины.

Множество существенных импликант (максимальных кубов) образует ядро покрытия как его обязательную часть:

F =

Число букв в МДНФ совпадает с ценой покрытия Sa, а суммарное число букв и число термов совпадает с ценой покрытия Sb.

1. **Минимизация булевой функции на картах Карно**

**4.1 Определение МДНФ**

Для минимизации булевой функции от пяти переменных используем две четырехмерные карты Карно, различающиеся по переменной X1: (единичные покрытия)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | X4X5 | | | | | | X2X3 |  | **00** | **01** | **11** | **10** | | **00** |  | 1 | 1 | 1 | | **01** | d | 1 | d | 1 | | **11** | d | d | 1 |  | | **10** | 1 |  | 1 |  | | X1 = 0 | | | | |  |  | | --- | |  | | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | X4X5 | | | | | | X2X3 |  | **00** | **01** | **11** | **10** | | **00** | d | d | 1 |  | | **01** |  | 1 | 1 | 1 | | **11** | 1 |  | 1 |  | | **10** | 1 | d |  |  | | X1 = 1 | | | | | |

МДНФ имеет следующий вид:

F =

**4.2 Определение МКНФ**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | X4X5 | | | | | | X2X3 |  | **00** | **01** | **11** | **10** | | **00** | 0 |  |  |  | | **01** | d |  | d |  | | **11** | d | d |  | 0 | | **10** |  | 0 |  | 0 | | X1 = 0 | | | | | | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | X4X5 | | | | | | X2X3 |  | **00** | **01** | **11** | **10** | | **00** | d | d |  | 0 | | **01** | 0 |  |  |  | | **11** |  | 0 |  | 0 | | **10** |  | d | 0 | 0 | | X1 = 1 | | | | | |

|  |
| --- |
| *Получение МКНФ производится по нулевому покрытию булевой функции.*  *Минимальное нулевое покрытие определяется по тем же принципам, что и единичное, но обозначается .* |

МKНФ имеет следующий вид:

F =

1. **Преобразование минимальных форм булевой функции**
   1. **Факторное преобразование для МДНФ**

F = **(Sq=23)**

F = **(Sq=19)**

F = **(Sq=16)**

F = **(Sq=16)**

**5.2 Факторное преобразование для МКНФ**

F =  **(Sq=22)**

F =  **(Sq=20)**

F = **(Sq=18)**

1. **Синтез комбинационных схем в булевом базисе**