Sat Solver Optimisations

Anatoly Weinstein

January 2, 2024

This document contains research into optimisations of CNF-SAT problems.

1 Definitions

Boolean Function. A boolean function $f : V \subseteq Variables \to \{0,1\}$ with truthy sets $P \subseteq \mathcal{P}(Variables)$ yields 1 if $V \in P$, else yields 0.

Boolean Equation. We define a boolean formula by the following context-free grammar with start variable S, a given variable set $Variable = \{x_i : i \in [n-1]\}$ of n variables and a set of binary operations $Operation = \{\cdot, +, \oplus, \dots\}$ and an assigned boolean function $f_{Operation} : \mathcal{P} \subseteq \{L, R\} \to \{0,1\}$, where L and R is the truthiness of left and right respectively.

$$
\begin{array}{ccc}
S & \to & (S) \\
S & \to & S\ Operation\ S \\
S & \to & \neg S \\
S & \to & Variable
\end{array}
$$

Conjunctive normal form. We define a boolean formula in conjunctive normal form by the following context-free grammar with start variable S and a given variable set $Variable = \{x_i : i \in [n-1]\}$ of n variables.

$$
\begin{array}{ccc}
S & \to & (Disjunction) \\
S & \to & S \cdot (Disjunction) \\
Disjunction & \to & Literal \\
Disjunction & \to & Disjunction + Literal \\
Literal & \to & Variable \\
Literal & \to & \neg Variable
\end{array}
$$

2 Transformation Techniques

UCNF-SAT. *Unmixed conjunctive normal form.*

TODO

3-CNF-SAT. A formula with only three literals per disjunction.

TODO

3 Optimisation Techniques

Contradiction. A formula with contradictional disjunctions is not satisfiable.

$$\forall A : X \cdot \neg X \cdot A(X) = \bot$$

Common Literals. A formula with common literals is not satisfiable.

$$\forall A : X \cdot \neg X \cdot A(X) = \bot$$