Sat Solver Optimisations

Anatoly Weinstein

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This document contains research into optimisations of 3-KNF-SAT problems.

1 Definitions

Boolean Function. A boolean function $f : V \subseteq Variables \to \{0, 1\}$ with truthy sets $P \subseteq \mathcal{P}(Variables)$ yields 1 if $V \in P$, else yields 0.

Boolean Equation. We define a boolean formula by the following context-free grammar with start variable S, a given variable set $Variable = \{x_i : i \in [n-1]\}$ of n variables and a set of binary operations $Operation = \{\cdot, +, \oplus, \dots\}$ and an assigned boolean function $f_{Operation} : \mathcal{P} \subseteq \{L, R\} \to \{0, 1\}$, where L and R is the truthiness of left and right respectively.

$$\begin{aligned} S &\to (S) \\ S &\to S\ Operation\ S \\ S &\to \neg S \\ S &\to Variable \end{aligned}$$

Conjunctive normal form. We define a boolean formula in conjunctive normal form by the following context-free grammar with start variable S and a given variable set $Variable = \{x_i : i \in [n-1]\}$ of n variables.

$$\begin{aligned} S &\to (Disjunction) \\ S &\to S \cdot (Disjunction) \\ Disjunction &\to Literal \\ Disjunction &\to Disjunction + Literal \\ Literal &\to Variable \\ Literal &\to \neg Variable \end{aligned}$$

2 Optimisation Techniques

Contradiction A formula with contradictional disjunctions is not satisfiable.

$$\forall A : X \cdot \neg X \cdot A(X) = \bot$$

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