CNF · SAT

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TODO A CNF-SAT is an equation . . .

1   Equations

$$
\begin{array}{rc}
\text{Set Notation} \cdot \text{CNF} & \{\{a,\bar{b}\}, \{b,\bar{c},\bar{d}\}, \{\bar{a},\bar{c}\}\} \\
\text{Arithmetic} \cdot \text{CNF} & (a+\bar{b}) \cdot (b+\bar{c}+\bar{d}) \cdot (\bar{a}+\bar{c}) \\
\text{Negated} \cdot \text{DNF} & \neg(ab + \bar{b}cd + ac)
\end{array}
$$

Formulas

Zugzwang. If a disjunction consists of one literal, the value of this literal is fixed.

$$\{X\} \in S \Rightarrow X = \top$$

Absorption. If a disjunction is a superset of another disjunction, it can be ignored.

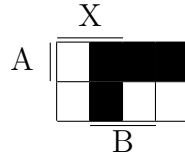$$\forall X \subseteq Y : X \cdot Y \equiv_{\text{CNF}} X$$

Proof: $(X \Rightarrow Y) \vee \neg X$ makes Y irrelevant.

The Absorption Rule allows to freely create new disjunctions, as long it's consisting of an already existing disjunction.
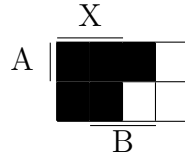
Common Contradiction.

$$(A + X) \cdot (B + \overline{X}) \equiv_{\mathrm{CNF}} (BX + A\overline{X})$$

X

A

B

Common Part.

$$(A + X) \cdot (B + X) \equiv_{\mathrm{CNF}} (X + AB)$$

X

A

B

Common and Contradicting Part

$$(A+B+C)\cdot(A+\neg B+D) \equiv_{\mathrm{CNF}} (A+(B+C)(\neg B+D)) \equiv_{\mathrm{CNF}} (A+BD+C\overline{B})$$

A

C

D

B