Sat Solver Optimisations

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This document contains research into optimisations of CNF-SAT problems.

1 Definitions

Boolean Function. A boolean function $f : V \subseteq Variables \rightarrow \{0,1\}$ with truthy sets $P \subseteq \mathcal{P}(Variables)$ yields 1 if $V \in P$, else yields 0.

SAT *Boolean Satisfyability Equation.*

We define a boolean formula by the following context-free grammar with start variable S, a given variable set $Variable = \{x_i : i \in [n-1]\}$ of n variables and a set of binary operations $Operation = \{\cdot, +, \oplus, \dots\}$ and an assigned boolean function $f_{Operation} : \mathcal{P} \subseteq \{L, R\} \rightarrow \{0,1\}$, where L and R is the truthiness of left and right respectively.

$$\begin{array}{ccc} S & \rightarrow & (S) \\ S & \rightarrow & S\ Operation\ S \\ S & \rightarrow & \neg S \\ S & \rightarrow & Variable \end{array}$$

CNF *Unmixed conjunctive normal form.*

We define a boolean formula in conjunctive normal form by the following context-free grammar with start variable S and a given variable set $Variable = \{x_i : i \in [n-1]\}$ of n variables.

$$\begin{array}{ccc} S & \rightarrow & (Disjunction) \\ S & \rightarrow & S \cdot (Disjunction) \\ Disjunction & \rightarrow & Literal \\ Disjunction & \rightarrow & Disjunction + Literal \\ Literal & \rightarrow & Variable \\ Literal & \rightarrow & \neg Variable \end{array}$$

2 Transformation Techniques

U-SAT *Unmixed conjunctive normal form.* Given a *CNF-SAT* formula, we call it *unmixed*, if every disjunction consists of either positive or negated literals.

3-CNF-SAT *Unmixed conjunctive normal form.* Given a *CNF-SAT* formula, we call it *unmixed*, if every disjunction consists of either positive or negated literals.

TODO

U3-SAT A formula that is *unmixed* and in

TODO

3 Optimisation Techniques

Contradiction. A formula with contradictional disjunctions is not satisfiable.

$$\forall A : X \cdot \neg X \cdot A(X) = \bot$$

Common Literals. A formula with common literals is not satisfiable.

$$\forall A : X \cdot \neg X \cdot A(X) = \bot$$