# $CNF \cdot SAT$

## Anatoly Weinstein

January 9, 2024

TODO A CNF-SAT is an equation ...

# 1 Equations

Set Notation · CNF 
$$\{\{a, \overline{b}\}, \{b, \overline{c}, \overline{d}\}, \{\overline{a}, \overline{c}\}\}$$
  
Arithmetic · CNF  $(a + \overline{b}) \cdot (b + \overline{c} + \overline{d}) \cdot (\overline{a} + \overline{c})$   
Negated · DNF  $\neg (ab + \overline{b}cd + ac)$ 

#### **Formulas**

**Zugzwang.** If a disjunction consists of one literal, the value of this literal is fixed.

$$\{X\} \in S \Rightarrow X = \top$$

**Absorption.** If a disjunction is a superset of another disjunction, it can be ignored.

$$\forall X \subseteq Y: X \cdot Y \equiv_{\mathrm{CNF}} X$$

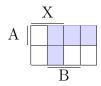
Proof:  $(X \Rightarrow Y) \vee \neg X$  makes Y irrelevant.



The Absorption Rule allows to freely create new disjunctions, as long it's consisting of an already existing disjunction.

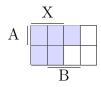
#### Common Contradiction.

$$(A+X)\cdot (B+\overline{X}) \equiv_{\mathrm{CNF}} (BX+A\overline{X})$$



#### Common Part.

$$(A+X)\cdot(B+X) \equiv_{\text{CNF}} (X+AB)$$



### Common and Contradicting Part

$$(A+B+C)\cdot(A+\neg B+D) \equiv_{\text{CNF}} (A+(B+C)(\neg B+D)) \equiv_{\text{CNF}} (A+BD+C\overline{B})$$

