# CNF · SAT

Anatoly Weinstein

January 9, 2024

TODO A CNF-SAT is an equation ...

## 1 Equations

$$\begin{array}{cc}
\text{Set Notation} \cdot \text{CNF} & \{\{a, \overline{b}\}, \{b, \overline{c}, \overline{d}\}, \{\overline{a}, \overline{c}\}\} \\
\text{Arithmetic} \cdot \text{CNF} & (a + \overline{b}) \cdot (b + \overline{c} + \overline{d}) \cdot (\overline{a} + \overline{c}) \\
\text{Negated} \cdot \text{DNF} & \neg(ab + \overline{b}cd + ac)
\end{array}$$

### Formulas

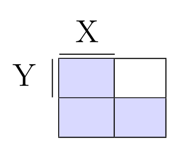
**Zugzwang.** If a disjunction consists of one literal, the value of this literal is fixed.

$$\{X\} \in S \Rightarrow X = \top$$

**Absorption.** If a disjunction is a superset of another disjunction, it can be ignored.

$$\forall X \subseteq Y : X \cdot Y \equiv_{\text{CNF}} X$$

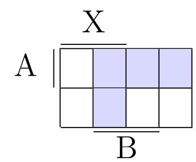
Proof: $(X \Rightarrow Y) \vee \neg X$ makes $Y$ irrelevant.



The Absorption Rule allows to freely create new disjunctions, as long it's consisting of an already existing disjunction.

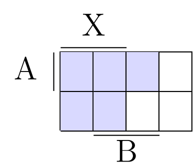
**Common Contradiction.**

$$(A + X) \cdot (B + \overline{X}) \equiv_{\text{CNF}} (BX + A\overline{X})$$



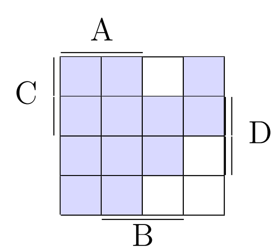

**Common Part.**

$$(A + X) \cdot (B + X) \equiv_{\text{CNF}} (X + AB)$$




**Common and Contradicting Part**

$$(A + B + C) \cdot (A + \neg B + D) \equiv_{\text{CNF}} (A + BD + C\overline{B})$$




Proof:

$$\begin{aligned} &(A + B + C) \cdot (A + \neg B + D) \\ \equiv_{\text{CNF}}\ &(A + (B + C)(\neg B + D)) \\ \equiv_{\text{CNF}}\ &(A + BD + C\overline{B}) \end{aligned}$$