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[1] Вспомним ноутбук с заказа

1) diff-in-diff (набл.) подход: $\{X_{i,t}\}$, $\{Y_{i,t}\}$ – контроль и тест, i – номер позиции, t – время

Аддитивность эффектов: $EX_{i,t} = \alpha + \beta_t$, $EY_{i,t} = \alpha + \beta_t + \tau + \delta_t$; $\hat{\delta}_t = (Y_{0,t} - Y_{0,1}) - (X_{0,t} - X_{0,1})$

$E\hat{\delta}_t = EY_{0,t} - EY_{0,1} - EX_{0,t} + EX_{0,1} = E\frac{1}{n}\sum_{i=1}^{n} Y_{i,t} - E\frac{1}{n}\sum_{i=1}^{n} Y_{i,1} - E\frac{1}{n}\sum_{i=1}^{n} X_{i,t} + E\frac{1}{n}X_{i,1} = EY_{i,t} - EY_{i,1} - EX_{i,t} + EX_{i,1} =$

$= \alpha + \beta_t + \tau + \delta_t - \alpha - \beta_1 - \tau - \delta_1 - \alpha - \beta_t + \alpha + \beta_1 = \delta_t - \delta_1 = \delta_t$ $\Rightarrow$ $\hat{\delta}_t$ несмещенная

Известно: $DX_{i,t} = DY_{i,t} = \sigma^2$, $corr(X_{i,t}, X_{j,1}) = corr(Y_{i,t}, Y_{j,1}) = \rho$

$D\hat{\delta}_t = E(\hat{\delta}_t - E\hat{\delta}_t)^2 = E(\hat{\delta}_t - \delta_t)^2 = E\hat{\delta}_t^2 - 2E\hat{\delta}_t\delta_t + \delta_t^2 = D(Y_{0,t} - Y_{0,1}) + D(X_{0,t} - X_{0,1}) - 2 \cdot$

$\cdot cov(Y_{0,t} - Y_{0,1},\ X_{0,t} - X_{0,1}) = DY_{0,t} + DY_{0,1} + DX_{0,t} + DX_{0,1} - 2cov(Y_{0,t}, Y_{0,1}) - 2cov(X_{0,t}, X_{0,1}) = 4\frac{\sigma^2}{n} - 4\sigma^2\rho =$

$= \frac{8(1-\rho)\sigma^2}{n}$

2) cookie-cookie-day

$\hat{\delta}_t = Y_{0,t} - X_{0,t}$, $EX_{i,t} = \alpha + \beta_t + \tau$

$E\hat{\delta}_t = EY_{0,t} - EX_{0,t} = \alpha + \beta_t + \tau + \delta_t - \alpha - \beta_t - \tau = \delta_t$ $\Rightarrow$ $\hat{\delta}_t$ несмещенная

$D\hat{\delta}_t = DY_{0,t} + DX_{0,t} - 2cov(Y_{0,t}, X_{0,t}) = \frac{\sigma^2}{n} + \frac{\sigma^2}{n} + \frac{2T\sigma^2}{n} = \frac{2(T+1)\sigma^2}{n}$

3) $\frac{8(1-\rho)\sigma^2}{n} < \frac{2(T+1)\sigma^2}{n}$ при $4(1-\rho) < T+1$, т.е. $T > 3 - 4\rho$