

Круглов А.И. ГОС-202 ML4

$$\boxed{1} \quad X_1, \dots, X_n \sim N(0, \sigma^2), \quad Y_i = X_i^2 + \varepsilon_i, \quad \varepsilon_i \sim \text{N(0, } \sigma^2 \text{)}, \quad E\varepsilon_i = 0; \quad \hat{y}(x) = \hat{\theta}x$$

$$EY_i = EX_i^2 + E\varepsilon_i = EX_i^2 = \sigma^2$$

$$MSE = E((Y - \hat{y}(x))^2) = \text{Bias}^2 + \text{Variance} + \text{Irreducible}$$

$$\text{Bias} = E\hat{y}(x) - EY = E\hat{\theta} \cdot x - \sigma^2$$

$$\text{Variance} = E((\hat{y}(x) - E\hat{y}(x))^2) = D\hat{\theta} \cdot x^2$$

$$\text{Irreducible} = D\varepsilon_i$$

$$\text{Beware: } MSE = (E\hat{\theta} \cdot x - \sigma^2)^2 + (D\hat{\theta} \cdot x^2) + D\varepsilon_i$$