Heat Equation With Improved Euler

We will solve heat equation

$$u' = \alpha \Delta u \tag{1}$$

using improved Euler.

We use the discrete Laplacian operator for triangle meshes:

$$\Delta u_i = \frac{1}{2} \sum_j (\cot \alpha_{ij} + \cot \beta_{ij})(u_j - u_i) \tag{2}$$

Applying improved Euler to Eq(1), we get for each vertex i:

$$u_i^{k+1} = u_i^k + \frac{\tau}{2} \alpha (\Delta u_i^k + \Delta u_i^{k+1}) \tag{3}$$

The problem with this is in u_i^{k+1}, which is what we're trying to compute, appears on both sides of the equation.But we will replace the u_i^{k+1} on the right side by the Euler approximation for u_i^{k+1}:

$$u_i^{k+1} = u_i^k + \frac{\tau}{2} \alpha (\Delta u_i^k + \Delta (u_i^k + \tau * \Delta u_i^k)) \tag{4}$$

Then we update each vertex's temperature using the update rule in Eq 4.