

## Wave Equation With Improved Euler

We will solve wave equation

$$u'' = \Delta u - \lambda u' \quad (1)$$

using improved Euler:

$$u^{k+1} = u^k + \frac{1}{2}\tau(u'^k + u'^{k+1}) \quad (2)$$

First we break Eq(1) into two equation:

$$u' = v \quad (3)$$

$$v' = \Delta u - \lambda v \quad (4)$$

We use the discrete Laplacian operator for triangle meshes:

$$\Delta u_i = \frac{1}{2} \sum_j (\cot \alpha_{ij} + \cot \beta_{ij})(u_j - u_i) \quad (5)$$

Applying improved Euler to Eq(3, 4) and using Eq(5), we get update rules for each vertex  $i$ :

$$v_i^{k+1} = v_i^k + \frac{\tau}{2} \sum_j (\cot \alpha_{ij} + \cot \beta_{ij})(u_j^k - u_i^k) \quad (6)$$

$$u_i^{k+1} = u_i^k + \frac{\tau}{2}(v_i^k + v_i^{k+1}) \quad (7)$$