

## Heat Equation With Backward Euler (Fixed Point Iteration)

We will solve heat equation

$$u' = \alpha \Delta u \quad (1)$$

using backward Euler.

We use the discrete Laplacian operator for triangle meshes:

$$\Delta u_i = \frac{1}{2} \sum_j (\cot \alpha_{ij} + \cot \beta_{ij})(u_j - u_i) \quad (2)$$

Applying backward Euler to Eq(1), we get for each vertex  $i$ :

$$u_i^{k+1} = u_i^k + \tau \alpha \Delta u_i^{k+1} \quad (3)$$

Substituting Eq(2) into Eq(3) and using fixed point method, we have

$$u_i^{k+1} = u_i^k \quad \forall i \quad (4)$$

$$u_i^{k+1} = u_i^{k+1} + \frac{\tau \alpha}{2} \sum_j (\cot \alpha_{ij} + \cot \beta_{ij})(u_j^{k+1} - u_i^{k+1}) \quad (5)$$

For each call to backward Euler, after initialization by Eq(4), we perform several iterations of Eq(3) on the mesh.