Numerical Approximation of Non-Linear Equations Using Adams-Bashforth Method

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Introduction

- Non-linear equations are difficult to solve analytically.
- Numerical methods are often used to approximate the solutions.
- One such method is the Adams-Bashforth method.
- The Adams-Bashforth method is a predictor-corrector method that uses past values of the function to approximate future values.
- The predictor step uses previous function evaluations to predict the next value.
- The corrector step uses a weighted average of function evaluations to correct the prediction.
- Let's consider an example to illustrate the method.

Adams-Bashforth Method: Example

Consider the initial value problem

$$y'(t) = -2ty(t)^2,$$
$$y(0) = 1,$$

with solution $y(t) = \frac{1}{1+t^2}$.
Suppose we want to approximate y at $t = 1$ using the Adams-Bashforth method.

Adams-Bashforth Method: Example (cont'd)

- We start by computing the values of y and t for $t = 0$ and $t = 0.5$, using the fourth-order Runge-Kutta method or any other method.
- Let $w_0 = y(0)$ and $w_1 = y(0.5)$ be the initial values.
- The Adams-Bashforth method is a predictor-corrector method that uses the formula

$$w_{i+1} = w_i + \frac{h}{2}\left(3f(t_i, w_i) - f(t_{i-1}, w_{i-1})\right), \quad i = 1, 2, \ldots,$$

to compute the next approximations.

Adams-Bashforth Method: Example (cont'd)

Using $h = 0.5$, we have

- $w_0 = 1$, $w_1 = 0.8$ (using fourth-order Runge-Kutta)
- $w_2 = w_1 + \frac{h}{2}\left(3f(t_1, w_1) - f(t_0, w_0)\right)$
- $w_2 = 0.6396$
- The exact solution is $y(1) = \frac{1}{2}$.
- The error is $|w_2 - y(1)| \approx 0.1396$.

Non-Linear Population Equation

- A non-linear population equation is given by:

$$\frac{dy}{dt} = 0.2y - 0.01y^2$$

- This equation describes how a population y changes over time t.
- The equation is non-linear due to the quadratic term y^2.

Numerical Approximation

- We will use the Adams-Bashforth method to approximate the solution to the non-linear population equation.
- This method is a multi-step method that uses a predictor-corrector scheme.
- The Taylor method is used to compute the first step.

Python Code (Part 1)

```
# Python code for numerical approximation
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd

# Define the function f(y, t)
def f(y, t):
    return 0.2 * y - 0.01 * y**2

# Set the initial conditions
y0 = 6
t0 = 2000
tf = 2020
h = 1
```

Python Code (Part 2)

```
# Compute the number of steps
N = int((tf - t0) / h)

# Create arrays to store the solution
t = np.zeros(N+1)
y = np.zeros(N+1)
t[0] = t0
y[0] = y0

# Use the Taylor method to compute the first
    step
y[1] = y[0] + h * f(y[0], t[0]) + h**2 / 2 * (f
    (y[0], t[0]) - 0.2 * y[0])
t[1] = t[0] + h
```

Python Code (Part 3)

```
    # Use the Adams-Bashforth method to compute the
        remaining steps
for i in range(2, N+1):
    # Predictor step
    y_pred = y[i-1] + h * (3/2 * f(y[i-1], t[i-1])
        - 1/2 * f(y[i-2], t[i-2]))
    t_pred = t[i-1] + h

    # Corrector step
    y[i] = y[i-1] + h * (5/12 * f(y_pred, t_pred) +
        2/3 * f(y[i-1], t[i-1]) - 1/12 * f(y[i-2],
        t[i-2]))
    t[i] = t[i-1] + h
```

Python Code (Part 4)

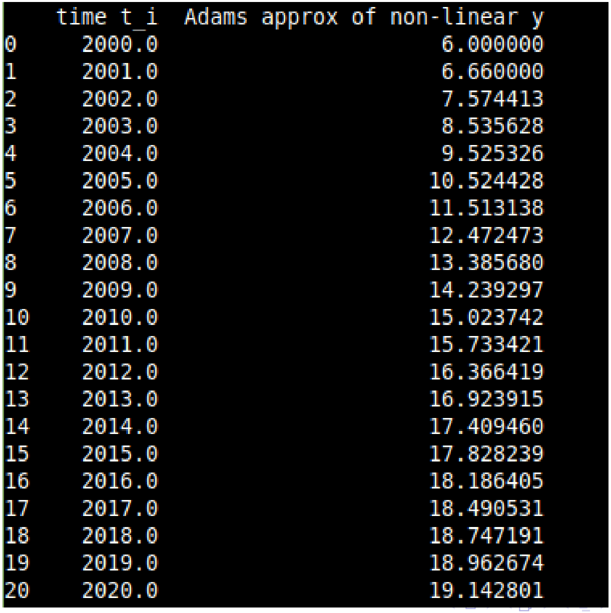
```
# Create a table of the numerical approximation
data = {'time␣t_i': t, 'Adams␣approx␣of␣non-
   linear␣y': y}
df = pd.DataFrame(data)
print(df)

# Plot the solution
plt.plot(t, y, label='Adams-Bashforth␣Method')
plt.title('Non␣Linear␣Population␣Equation')
plt.legend(loc='best')
plt.xlabel('time␣(yrs)')
plt.ylabel('Population␣in␣billions')
plt.show()
```

Results

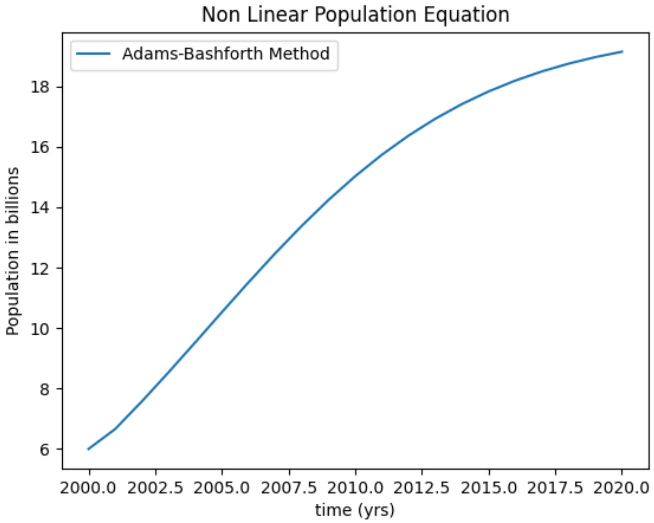
- The numerical approximation of the non-linear population equation is stored in arrays `t` and `y`.
- A table of the numerical approximation is created using the `pandas` library.
- A plot of the solution is created using the `matplotlib` library.

Table of Numerical Approximation



```
    time t_i  Adams approx of non-linear y
0     2000.0                      6.000000
1     2001.0                      6.660000
2     2002.0                      7.574413
3     2003.0                      8.535628
4     2004.0                      9.525326
5     2005.0                     10.524428
6     2006.0                     11.513138
7     2007.0                     12.472473
8     2008.0                     13.385680
9     2009.0                     14.239297
10    2010.0                     15.023742
11    2011.0                     15.733421
12    2012.0                     16.366419
13    2013.0                     16.923915
14    2014.0                     17.409460
15    2015.0                     17.828239
16    2016.0                     18.186405
17    2017.0                     18.490531
18    2018.0                     18.747191
19    2019.0                     18.962674
20    2020.0                     19.142801
```

Plot of The Solution


Non Linear Population Equation
Adams-Bashforth Method
Population in billions
time (yrs)