Design and Analysis of Algorithms

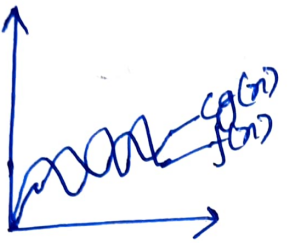
Tutorial-1

Name: Anchit Sinha
CST-SPL1
Roll no.: 55

Q- Asymptotic Notation

→ They help you find the complexity of an algorithm when input is very large.

1)- Big O



$f(n) = O(g(n))$

iff $f(n) \leq C \cdot g(n)$

$\forall\ n \geq n_0$

for some ~~weight~~ constant $C > 0$

$\Rightarrow g(n)$ is ~~tight~~ upper bound of $f(n)$.

2)- Big Omega (Ω)

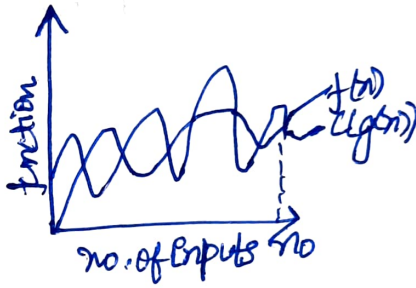
$f(n) = \Omega(g(n))$

$g(n)$ is tight lower bound of $f(n)$

$f(n) = \Omega(g(n))$

iff $f(n) \geq cg(n)$

$\forall\ n \geq n_0$ for some constant $C > 0$.



3)- Theta (Θ)

$f(n) = \Theta(g(n))$

$g(n)$ is both tight, upper and lower bound for $f(n)$

$f(n) = \Theta(g(n))$

iff

$C_1 g(n) \leq f(n) \leq C_2 \cdot g(n)$

$\forall\ n \geq (\max(n_1, n_2)$

for some constant $C_1 > 0$ and $C_2 > 0$

4)- Small o (o)

$f(n) = o(g(n))$

$g(n)$ is upper bound of fn $f(n)$.

$f(n) = o(g(n))$

when $f(n) < C\,g(n)$ $\forall\ n > n_0$.

and $\forall\ C > 0$

5)- Small Omega(ω)

$f(n) = \omega(g(n))$

$g(n)$ is lower bound of $f(n)$

$f(n) = \omega(g(n))$

when $f(n) > cg(n)$ $\forall n > n_0$ and $\forall c > 0$.

Q2- for(i=1 to n){i=i*2;}

```
for(i=1 to n)   // i=1,2,4,8 ---n.
{i=i*2;}        //O(1)
```

$$\Rightarrow \sum_{i=1}^{n} 1+2+4+8+\cdots n$$

GP k^{th} value $\Rightarrow T_k = a r^{k-1}$

$\Rightarrow 1 \times 2^{k-1}$

$\Rightarrow n = 2^k$

$\Rightarrow 2n = 2^k$

$\Rightarrow \log 2n = k \log 2$

$\Rightarrow \log_2 + \log n = k \log 2$

$\log n + 1 = k$

$O(k) = O(1+\log n)$

$= O(\log n)$

Q3- $T(n) = \{3T(n-1)$ if $n>0$, otherwise $1\}$

$T(n) = 3T(n-1)$ —①

put $n = n-1$

$T(n-1) = 3T(n-2)$ —②

from 1 and 2,

$\Rightarrow T(n) = 3(3T(n-2))$

$= 9T(n-2)$ —③

putting $n = n-2$ in ①,

$T(n) = 3(T(n-3))$ —④

$\Rightarrow T(n) = 27(T(n-3))$

$\Rightarrow T(n) = 3^k (T(n-k))$

putting n-K=0

$\Rightarrow n=k$.

$T(n) = 3^n [T(n-n)]$

$T(n) = 3^n T(0)$

$T(n) = 3^n \times 1$ $[T(0)=1]$

$T(n) = O(3^n)$

4- $T(n) = \{2T(n-1)-1$ if $n>0$, otherwise $1\}$

$T(n) = 2T(n-1)$

$T(n-1) = 2T(n-2)$

$T(n-2) = 2T(n-3)$

⋮

$T(1) = 2T(0)$

$T(0) = 1$

} n levels

Substituting value of $T(n-1)$ then $T(n-2)$ ---till $T(1)$ in eqn $T(n)$.

we get,

$T(n) = 2^n \times T(0)$

$T(n) = 2^n \times 1$

$= O(2^n)$

5-

```
int i=1,s=1;
while (s<=n)
{
  i++;
  s=s+i;
  printf("#");
}
```

i = 1 2 3 4 5 6 ---

s = 1+3+6+10+15+

Sum of $S = 1+3+6+10+ \dots +n$ — ①

also $s = 1+3+6+10+ \dots T_{n-1}+n$ — ②

from ① - ②,

$0 = 1+2+3+4+ \dots n - T_n$.

$\Rightarrow T_k = 1+2+3+4+\cdots+k$

$T_k = \frac{1}{2}k(k+1)$

$\Rightarrow$ For k iterations,

$1+2+3+\cdots+k < n$

$\Rightarrow \frac{k(k+1)}{2} < n$

$\Rightarrow \frac{k^2+k}{2} < n$

$\Rightarrow O(k^2) < n$

$\Rightarrow k = O(\sqrt{n})$

$\Rightarrow T(n) = O(\sqrt{n})$

6-
```
void function(int n){
    int i, count = 0;
    for(i=1; i*i<n; i++)
        count++;        //O(1)
}
```

as $i^2 < n$

$\Rightarrow i < \sqrt{n}$

$i = 1, 2, 3, 4, \cdots, \sqrt{n}$

$\sum_{i=1}^{n} 1+2+3+4+\cdots+\sqrt{n}$

$\Rightarrow T(n) = \frac{\sqrt{n} \times (\sqrt{n}+1)}{2}$

$T(n) = \frac{n \times \sqrt{n}}{2}$

$T(n) = O(n)$

7-
```
void function(int n)
{
    int i, j, k, count = 0;
    for(i=n/2; i<n; i++)
      for(j=1; j<n; j=j*2)
        for(k=1; k<n; k=k*2)
          count++
}
```

for k = k*2

k = 1, 2, 4, 8, ---- n

⇒ GP → a = 1, r = 2

$= \frac{a(r^n - 1)}{r - 1}$

$= \frac{1(2^k - 1)}{1}$

$n = 2^k$

$\Rightarrow \log n = k$

i	j	k
1	logn	logn * logn
2	logn	logn * logn
⋮		
n	logn	logn * logn

⇒ O(n * log n * logn)

⇒ $O(n \log^2 n)$

8- function(int n)

```
{
  if(n==1) return;          // O(1)
    for(i=1 to n) {         // i=1,2,3,4 ---n => O(n)
      for(j=1 to n) {       // j=1,2,3,4 --- n^n => O(n^2)
        printf("*");
      }
    }
  function(n-3);            T(n/3)
}
```

⇒ $T(n) = T(n/3) + n^2$

⇒ a = 1, b = 3, $f(n) = n^2$

$c = \log_3^1 = 0$

⇒ $n^0 = 1 < f(n) = n^2$

⇒ $T(n) = \theta(n^2)$

9- Void function(int n){
for(i=1 to n){ // O(n)
for(j=1; j<=n; j=j+i)
printf("*"); // O(1)
}
}

for i=1 ⇒ j=1, 2, 3, 4 ---n = n
for i=2 ⇒ j= 1, 3, 5, ---n = n/2
for i=3 ⇒ j = 1, 4, 7, ---n = n/3
⋮ ⋮
for i=n ⇒ j=1 --- 1

$$\Rightarrow \sum_{j=n}^{1} n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \cdots + 1$$

$$\Rightarrow \sum_{j=n}^{1} n\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \frac{1}{n}\right]$$

$$\sum_{j=n} n[\log n]$$

$$\Rightarrow T(n) = [n \log n]$$

$$T(n) = O(n \log n)$$

10-

Relation b/w n^k and c^n is

$n^k = O(c^n)$

as $n^k \leq ac^n \ \forall\ n \geq n_0$ and some constant $a > 0$.

for $n_0 = 1$

$c = 2$

$\Rightarrow 1^k \leq a 2^1$

$n_0 = 1$ and $c = 2$.