USF Math 108 Final Exam w/Mitchell Schoenbrun Version 1

Write your Name Here: ______________________

This is a long exam, but you will have plenty of time. I suggest that you look through the problems and do the easier ones first. The numbers in brackets [1] tell you how many points a problem will count. Showing your work is encouraged.

Definition of a periodic function $f(x+p)=f(x)$	**Basic Identities** $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ $\sec\theta=\dfrac{1}{\cos\theta}$ $\csc\theta=\dfrac{1}{\sec\theta}$
Definition of an odd/even function $f(-x)=f(x)-Even$ $f(-x)=-f(x)-Odd$	**Period for Trig Functions** Sine, Cosine 2π or 360° Tangent π or 180°
Inverse Trig Function Domain: Ranges Sine - Domain [-1,1] Range $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ or $\left[-90^\circ,90^\circ\right]$ Cosine - Domain [-1,1] Range $[0,\pi]$ or $\left[0^\circ,180^\circ\right]$ Tangent - Domain $\mathbb{R}$ Range $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ or $\left[-90^\circ,90^\circ\right]$	**Product to Sum Identities** $\sin\alpha\sin\beta=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$ $\cos\alpha\cos\beta=\frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$ $\sin\alpha\cos\beta=\frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$ $\cos\alpha\sin\beta=\frac{1}{2}\left[\sin(\alpha+\beta)-\sin(\alpha-\beta)\right]$
Pythagorean Identities $\sin^2\theta+\cos^2\theta=1$ $1-\sin^2\theta=\cos^2\theta$ $\tan^2\theta+1=\sec^2\theta$ $1+\cot^2\theta=\csc^2\theta$	**Co-Function Identities** $\sin(90^\circ-\theta)=\cos(\theta)$ $\cos(90^\circ-\theta)=\cos(\theta)$
Even/Odd Identities $\sin(-\theta)=-\sin(\theta)$ $\cos(-\theta)=\cos(\theta)$ $\tan(-\theta)=-\tan(\theta)$	**Other Trig Functions** $\tan\theta=1/\cot\theta$ $\sec\theta=1/\cos\theta$ $\csc\theta=1/\sin\theta$
Sum and Difference Identities $\sin(\alpha\pm\beta)=\sin(\alpha)\cos(\beta)\pm\cos(\alpha)\sin(\beta)$ $\cos(\alpha\pm\beta)=\cos(\alpha)\cos(\beta)\mp\sin(\alpha)\sin(\beta)$	**Half Angle Identities** $\sin\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{2}}$ $\cos\frac{\theta}{2}=\pm\sqrt{\frac{1+\cos\theta}{2}}$
Law of Sines $\dfrac{\sin\angle A}{a}=\dfrac{\sin\angle B}{b}=\dfrac{\sin\angle C}{c}$	**Law of Cosines** $c^2=a^2+b^2-2ab\cos\angle C$
SOH CAH TOA $\sin=\frac{o}{h}$ $\cos=\frac{a}{h}$ $\tan=\frac{o}{a}$	**Exponential Functions & Logs** $y=B^x\rightarrow\log_B y=x$
Double Angle Identities $\sin 2\theta=2\sin\theta\cos\theta$ $\cos 2\theta=\cos^2\theta-\sin^2\theta=$ $2\cos^2\theta-1=$ $1-2\sin^2\theta$ $\tan 2\theta=2\tan\theta/1-\tan^2\theta$	**De Moivre's Formula** $(\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$ **Euler's Formula** $e^{i\theta}=\cos\theta+i\sin\theta$

[6] 1) Simplify these expressions as much as possible ____________________

a) $\dfrac{6x^7y^3z^2}{\left(2xy^{-2}z^2\right)^2}$

b) $\dfrac{\sqrt{5}\cdot\sqrt{5^4}}{\sqrt{20}}$ ____________________

[8] 2) For the function $f\left(x\right)=\dfrac{\left(x-4\right)\left(x+1\right)}{\left(x-5\right)\left(x+3\right)}$

a) List any vertical asymptotes ____________________

b) List any horizontal asymptotes ____________________

c) List any zeros of the function ____________________

d) List the *y*-intercept of the function ____________________

3) Indicate which of these functions are odd, even, neither or both [6]

a) $f\left(x\right)=\sin\left(x\right)$ ____________________

b) $f\left(x\right)=x^3$ ____________________

c) $f\left(x\right)=x^2\cos\left(x\right)$ ____________________

[5] 4) What is the solution set of this inequality ______________________

$|5x-4|<9$

[5] 5) Find **all** θ such that $\theta=\cos^{-1}(1)$ ______________________

[5] 6) Find all solutions, real or complex to this equation. ______________________

Hint: you will need to use the **rational root** theorem at least once.

$x^4-2x^3-2x^2-2x-3=0$

[5] 7) Find all solutions to the equation $2\cos^2\theta - 7\cos\theta + 3 = 0$ ____________________

[6] 8) Verify the identity $\tan\theta + \cot\theta = \sec\theta\csc\theta$

[9] 9) Find the exact values (no calculators please)

a) $\sin\frac{\pi}{6}$ ______________________

b) $\cos 135^{\circ}$ ______________________

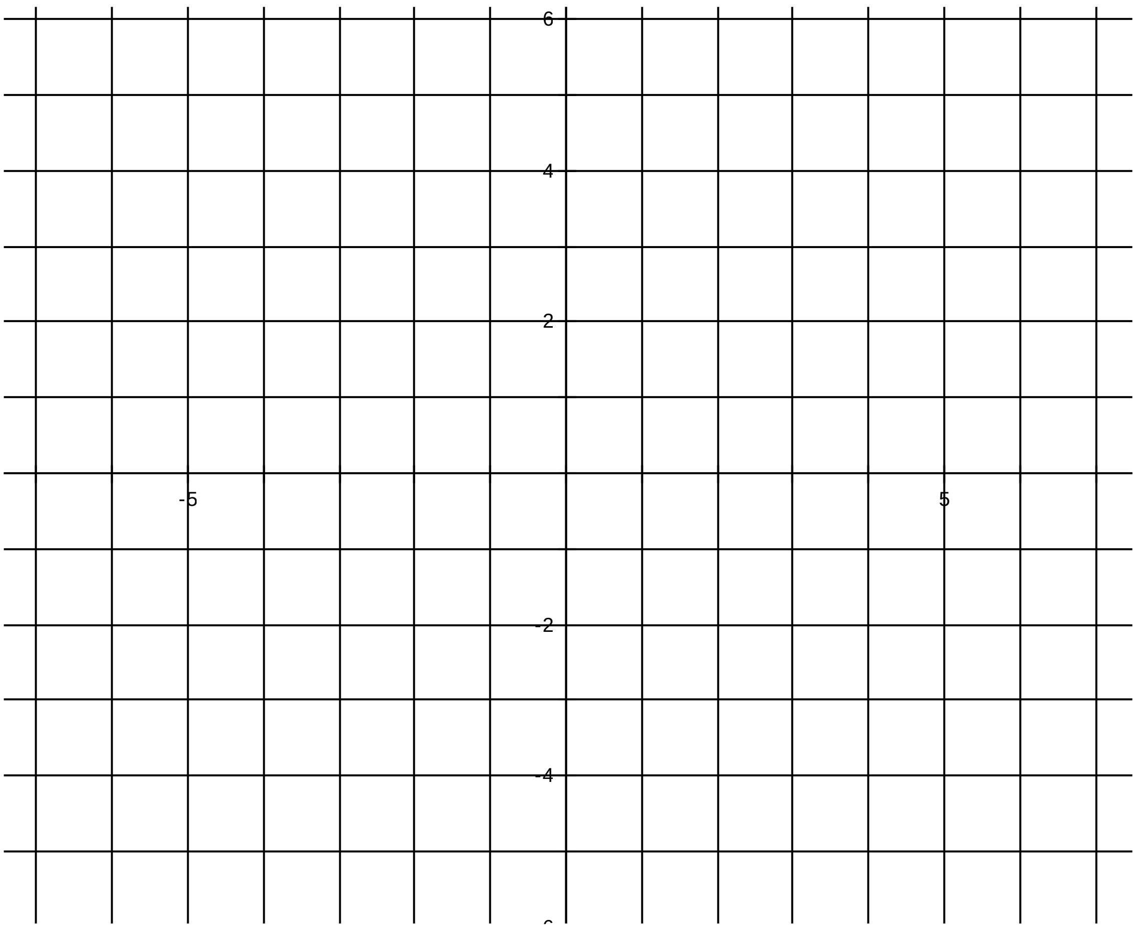
c) $\tan\frac{33\pi}{2}$ ______________________

[6] 10) Sketch a graph of $f(x) = 2\cos(x - \pi)$.

What is the amplitude and period of this function.

Amplitude = ____________

Period = ____________



[6] 11) Simplify and find the roots of this equation.

$4\left(x^2-3x+2\right)-3\left(x^2-2x+1\right)=0$ ________________________

[9] 12) Calculate and write the results in the form a+bi

a) $\left(5-3i\right)-\left(7-i\right)$ ________________________

b) $\left(2+i\right)\left(5-3i\right)$ ________________________

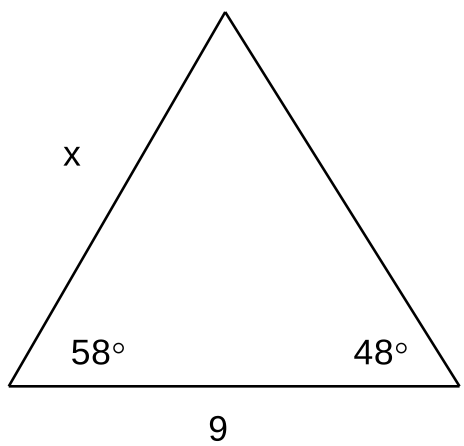
c) $\dfrac{5+2i}{1-i}$ ________________________

[6] 13) Simplify the following expressions as much as possible.

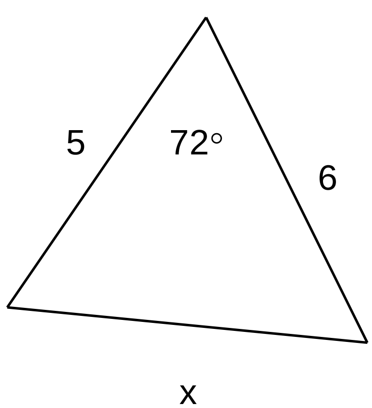
a) $\log_4 64=$ ________________________

b) $\log_3 30+\log_3 9-\log_3 10=$ ________________________

[6] 14) Find the length of the side labeled *x* in the following diagram.
You can leave answers in terms of sines and cosines.
Hint: Consider using the Law of Sines ___________________________



[6] 15) Find the length of the side labeled *x* in the following diagram
You can leave answers in terms of sines and cosines.
Hint: consider using the Law of cosines ___________________________



[6] 16) Use a sum, difference or half angle formula to find the exact values, not a calculator value. [6]

a) $\sin\left(15^{\circ}\right)$ ____________________

b) $\cos\left(105^{\circ}\right)$ ____________________

Optional Extra Credit
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[3] Extra Credit 2) What are the **three** cubed roots of *i*, eg. $\sqrt[3]{-1} =$ [3]

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[3] Extra Credit What is the solution to these three equations in three unknowns. Use any method, however, consider row reduction.

$x+y+z=6$

$2x-y+z=3$

$x+y-z=0$