Solar Neutrino Survival Probability Scan

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Abstract

This note accompanies the `Python 3` code file `20190620MSWscanAllenergyNoSympy.py` provided at Github repository. For those who use the Jupyter notebook environment (or JupyterLab, our environment), we provide a notebook with identical code at Graph solar neutrino survival probability.

I. INTRODUCTION

This note is intended for the physics student with some exposure to the subject of neutrinos and flavor change who might appreciate a look at some simple Python computer code for correctly calculating and graphing transition probabilities of neutrinos produced in solar plasma where there is a matter potential (the Mikheyev, Smirnov, Wolfenstein or MSW effect [1] [2]). For those who would like additional information, in our article Standard Osc Equation Inappropriate we discuss why the standard two-flavor neutrino oscillation equation may not give correct results when the MSW potential is added, demonstrating with computer code and graph. In Derivation of MSW equations we present a detailed derivation of the two-flavor model equations for the mass splitting Δm^m_{12} and mixing angle θ^m_{12} with MSW potential. See [3] for an interesting history of the development of the MSW effect by one of the participants (A. Smirnov) with details of the physics or [4] for a comprehensive collection of lectures on neutrino physics, including the MSW equations.[1]

II. BACKGROUND

Solar neutrinos are produced in several branches of the solar fusion reaction chains [6] [7] (See Fig. 1) The *p*-*p* chain, accounting for $\sim 98.5\%$ of solar luminosity, is composed of three branches, the *pp*I, *pp*II and *pp*III branches (not labelled in Fig. 1, but comprise the vertical endpoints going left to right). Two ν_e (electron neutrino) are produced every time four protons p are burned to produce an alpha-particle α [8], e.g., two pp ν_e, or one pp ν_e and one $^7_4$Be ν_e [2], etc. d denotes deuterium, i.e., $^2_1$H. Neutrino species are labelled in red in the figure. The product of the radiative capture $^7_4\mathrm{Be} + p \to {}^8_5\mathrm{B} + \gamma$ (γ denotes photon) decays weakly as $^8_5\mathrm{B} \to {}^8_4\mathrm{Be}^* + e^+ + \nu_e$ (* denotes excited state), which in turn disintegrates into two alpha particles (that intermediate step not shown explicitly in the figure).

Only electron neutrinos ν_e are produced, the decay inevitably involving a fusion product with an unstable proton number (could say neutron deficient), i.e., the ν_e neutrinos are produced in β^+ decays of a proton to a neutron, e.g., in $p + p \to {}^2_1\mathrm{H} + e^+ + \nu_e$ and $^8_5\mathrm{B} \to {}^8_4\mathrm{Be}^* + e^+ + \nu_e$ (or similarly in an electron capture, e.g., $^7_4\mathrm{Be} + e^- \to {}^7_3\mathrm{Li} + \nu_e$). Conservation

[1] Also, the 2016 PDG Review of Particle Physics [5] (or its 2018 edition) has an excellent chapter (14) devoted to neutrino flavor physics, though this is more of a technical reference than a tutorial.

[2] $^7_4$Be denotes atomic mass (total protons and neutrons) of 7 and atomic number (protons) of 4.

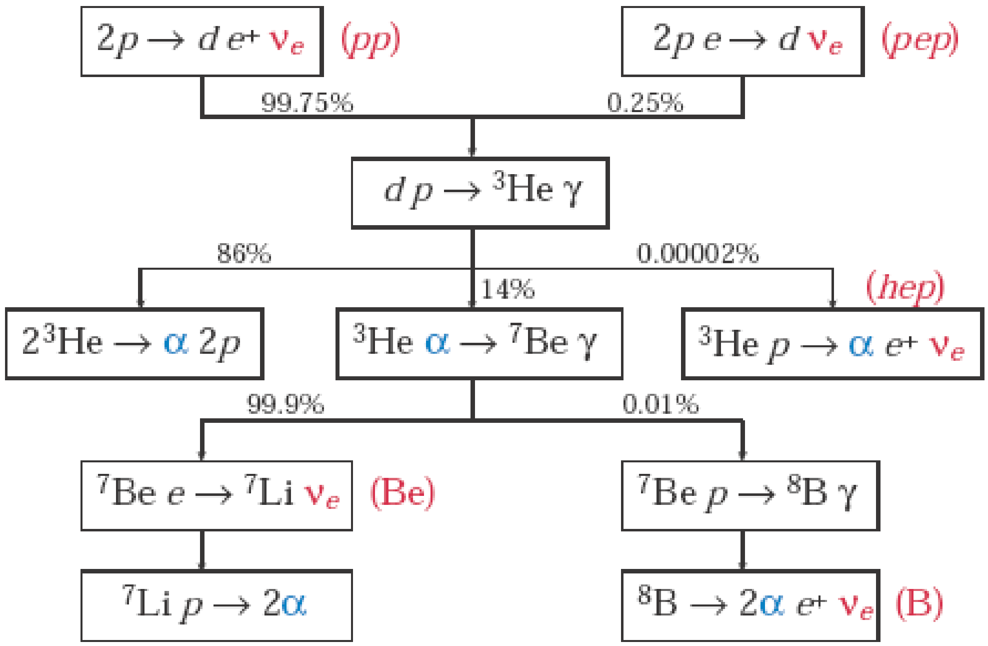

FIG. 1. The *p*-*p* chain. Figure from [7].

of electric charge requires that the transmutation of a proton to a neutron, $p^+ \to n^0$, be accompanied by the emission of a positive electric charge particle[3], a positron e^+ (or notation $\bar{e}$, this being the antiparticle of the electron). The baryon number is conserved, the neutron replacing the initial proton (both of baryon number 1). Electron lepton number must be conserved, so the created positron (an anti-electron with electron lepton number -1)[4] must be accompanied by a lepton with $+1$ electron lepton number and this lepton must not have an electric charge (since the lost positive electric charge of the proton is already restored by creation of a positron in β^+-decay, or zeroed by the e^- in electron capture): This means an electron neutrino ν_e must be emitted, it having electron lepton number $+1$ and zero electric charge. See [9], [8], [6].

III. TWO-FLAVOR MODEL RATIONALE

In section §14.8.2.2 *solar neutrino survival probability* in [5], it is explained that with $|\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,\mathrm{eV}^2 \gg |\Delta m^2_{21}|$ there is little MSW effect for the ν_1-ν_3 mixing at the

[3] If an electron is captured, it is not necessary to emit a positron.

[4] Or the $+1$ lepton number of the e^- consumed in the electron capture must be restored.

available solar electron densities. If the electron density required for MSW resonance is Eq. (14.58) in [5]:

$$N_e^{res} = \frac{\Delta m^2 \cos 2\theta}{2E\sqrt{2}G_F}\,, \tag{1}$$

then the required electron density for solar neutrinos produced with energies $E < 10\,\mathrm{MeV}$ corresponding to $|\Delta m^2_{31}|$ is a factor of 10 larger than the maximum N_e in the solar core. Let us check that assertion.

Working directly with the `B16-GS98` SSM (Standard Solar Model) data[5] we find that the ^{8}B neutrino flux maximum emission point is found at normalized solar radius $r_\odot = 0.0460$ ($r_\odot = 0$ is the center of the Sun, $r_\odot = 1$ the "surface"). The neutrino emissions from the *p*-*p* nuclear reaction chains follow an ordered sequence of shells (overlapping to some degree) radially out from high-density, high-temperature regions to relatively lower density and temperature, with ^{8}B emission nearest the inner core [11], so we will compare the result of our calculation of N_e^{res} required for a $10\,\mathrm{MeV}$ neutrino to the electron number density from our SSM data at $r_\odot = 0.0460$, which is $5.4670\,\times 10^{25}\,\mathrm{cm}^{-3}$.

We will use data from a 2016 global fit [12]: $m^2_{13} = 2.457\,\times 10^{-3}\,\mathrm{eV}^2$ and $\theta_{13} = 8.5°$. In Eq. (2b) below we converted θ_{13} to radians and multiplied by two in the argument of cosine. We included a lot of detail in Eq. (2) since the natural unit conversion back to an SI quantity (we want electron number density per cm^3 ultimately) can trip up the unwary. You can see that we are ending up with GeV^3 in Eq. (2c), so we insert in the denominator of Eq. (2d) a "custom" (we prefer inspecting the equation at hand and sculpting $\hbar c$ rather than following specific rules) $\hbar c$ quantity, cubed to cancel the GeV^3 and leave us the desired cm^{-3} in Eq. (2f) and Eq. (2g). Just find $\hbar$ in eV in `SciPy` constants or a reference like [5] (or convert from J s) and divide by 10^9 to give you GeV s. Take c in m s and multiply by 10^2 to obtain c in cm s. The product of $\hbar$ and c will then give you the $\hbar c$ in the desired units, i.e., $\hbar c = 1.9733\,\times 10^{-14}\,\mathrm{GeV\,cm}$. Convert the squared mass splitting in eV^2 to GeV^2 (divide by 10^{18}) and the neutrino production energy from MeV to GeV (divide by 10^3). The Fermi

[5] See [10]. In another of our articles, available at Bounds on Neutrino Packet Lengths, we incidentally include an appendix demonstrating how to read in and work with the SSM data.

coupling constant G_F is usually given as $1.1663787 \times 10^{-5}\,\mathrm{GeV}^{-2}$ so it may be used as is:

$$N_e^{res} = \frac{\Delta m_{13}^2 \cos 2\theta_{13}}{2E\sqrt{2}G_F} \tag{2a}$$

$$= \frac{\left[2.457\ \times 10^{-21}\,\mathrm{GeV}^2\right] \cos(0.2967)}{(2)\,(10\ \times 10^{-3}\,\mathrm{GeV})\,(1.4142)\,\left(1.1663787 \times 10^{-5}\mathrm{GeV}^{-2}\right)} \tag{2b}$$

$$= \frac{2.3496\ \times 10^{-21}\,\mathrm{GeV}^2}{3.2990\ \times 10^{-7}\,\mathrm{GeV}^{-1}} \tag{2c}$$

$$= \frac{2.3496\ \times 10^{-21}\,\mathrm{GeV}^2}{\left(3.2990\ \times 10^{-7}\,\mathrm{GeV}^{-1}\right)(\hbar c\,\mathrm{GeV\,cm})^3} \tag{2d}$$

$$= \frac{2.3496\ \times 10^{-21}\,\mathrm{GeV}^2}{\left(3.2990\ \times 10^{-7}\,\mathrm{GeV}^{-1}\right)(7.6835\ \times 10^{-42}\,\mathrm{GeV}^3\,\mathrm{cm}^3)} \tag{2e}$$

$$= \frac{2.3496\ \times 10^{-21}\,\mathrm{GeV}^2}{2.5348\ \times 10^{-48}\,\mathrm{GeV}^2\,\mathrm{cm}^3} \tag{2f}$$

$$= 9.2695\ \times 10^{26}\,\mathrm{cm}^{-3} \tag{2g}$$

The MSW contribution of ν_1-ν_3 mixing at the available solar electron densities is indeed insignificant, given that about 10 times more N_e ($\mathcal{O}(10^{26}\,\mathrm{cm}^{-3})$) than available N_e ($\mathcal{O}(10^{25}\,\mathrm{cm}^{-3})$) is required to reach MSW resonance even for the upper end of the solar neutrino energies at 10 MeV. We will therefore use a two-state model, omitting the Δm_{13}^2 mass split and θ_{13}.[6]

IV. TWO-FLAVOR SURVIVAL PROBABILITY

Our computer code uses an equation assuming adiabatic conversion (more on that shortly) and provides an averaged 2ν survival probability for solar electron neutrinos:

$$P^{2v}(\Delta m_{21}^2, \theta_{12}; N_e) = \frac{1}{2}\,(1 + \cos 2\theta_m \cos 2\theta) \tag{3}$$

This equation is identical to equation (3) in [14] and, with removal of a so-called "jump" probability term [7], equation (14.84) in [5] or equation (22) in [14]. By *jump probability*, we refer to a non-adiabatic correction for propagation of a neutrino in the Sun, the probability

[6] [13] estimates the Δm_{13}^2 and θ_{13} correction at $\leq 12\%$ and provides equations to incorporate those parameters, as does [5].

[7] The P_c term in Eq. 4 below.

of a transition $\nu_{2m} \rightarrow \nu_{1m}$ on the path from production in the core on out to the vacuum (so "adiabatic" in this context means that jump does not happen).

When the energy gap between this propagating states ν_{2m} and ν_{1m} is at minimum (which it is at MSW resonance), there is a chance ν_{2m} may "jump" across that energy gap to ν_{1m} if the electron density $N_e(r)$ does not evolve smoothly enough (section §14.8.2.1 in [5]). Can we quantify this smoothness requirement? Yes, but we forewarned that most sources do not use the correct equations. In the solar context LMA (large mixing angle) solution the Landau-Zener formula is inappropriate, as is the double exponential formula [14].[8]

With jump probability term P_c, the survival probability equation Eq. 3 is modified as:

$$P_{ee} = \frac{1}{2}\left[1 + (1 - 2P_c)\cos 2\theta_m^\circ \cos 2\theta\right] \tag{4}$$

The superscript $^\circ$ on the matter mix angle indicates it is calculated at the production point. One expression for the jump probability P_c is given by equation (25) [14]:

$$P_c = \frac{\gamma^2(x_0)}{4} \tag{5}$$

x_0 here denotes the point of origin in the Sun. γ, typically called the "adiabaticity parameter," can be defined by equation (8) in [14] or equation (4.31) in [16]:

$$\gamma \equiv \frac{4E\left|\dot{\theta}_m\right|}{\Delta m_m^2} \tag{6}$$

Δ_m^2 is the mass split with MSW potential (we calculated the MSW potential in detail in our other articles cited). $\dot{\theta}_m = d\theta_m(x)/dx$ is the derivative of the mixing angle in matter with respect to radial location in the plasma. Remember that the mixing angle in matter[9] is a function of the matter potential, which is a function of neutrino energy and electron number density, $N_e(x)$. The absolute value is used because the rate of change could be negative. $\Delta m_m^2/(4E)$ is the energy eigenvalue of the neutrino mass states in matter (as we dealt with in detail in the cited article describing calculation of those eigenvalues). [14] places this in context by suggesting that an arbitrary neutrino state could be described in terms of the instantaneous eigenstates of the Hamiltonian in matter, ν_{1m} and ν_{2m}, as

$$|\nu\rangle = \psi_{1m}\,|\nu_{1m}\rangle + \psi_{2m}\,|\nu_{2m}\rangle \tag{7}$$

[8] We cannot give a full account of the subject here. See [15] for Clarence Zener's original 1932 presentation of energy level crossing and [14] for the proper use of the concept in the solar context. [16] is useful also.

[9] See Eq. 11 for $\cos 2\theta_m$ below.

The evolution equation in the base (ν_{1m}, ν_{2m}) would then be:

$$i\frac{d}{dx}\begin{pmatrix}\psi_{1m}\\ \psi_{2m}\end{pmatrix} = \begin{pmatrix}-\frac{\Delta m_m^2(x)}{4E} & -i\dot{\theta}_m(x)\\ i\dot{\theta}_m(x) & \frac{\Delta m_m^2(x)}{4E}\end{pmatrix}\begin{pmatrix}\psi_{1m}\\ \psi_{2m}\end{pmatrix} \tag{8}$$

This should make it clear that the adiabatic approximation corresponds to the situation where the ratio of the change in mixing angle (off-diagonal element in Eq. 8) to the energy eigenvalues (diagonal elements in Eq. 8):

$$\gamma \equiv \frac{4E\left|\dot{\theta}_m\right|}{\Delta m_m^2} \ll 1 \tag{9}$$

With $\gamma \ll 1$, the off-diagonal terms (often called the transition matrix elements in a general two-state system[10]) in the Hamiltonian Eq. 8 could be neglected, there will be no transitions between the eigenstates and therefore the neutrino mass states propagate independently on out of the Sun.

Following [14], we used equation 9 and SSM data we discussed earlier to calculate γ for neutrino production energy $E = 10\,\mathrm{MeV}$ at radial location $r_\odot = 0.0460$. Working with the SSM data we calculated the instantaneous rate of change of the matter potential was $-4.5919 \times 10^{-26}\,\mathrm{GeV/km}$ at that location, with corresponding $\dot{\theta}_m = -1.0877 \times 10^{-4}\,\mathrm{deg\,km^{-1}}$.

We found the mixing angle at birth ($r_\odot = 0.0460$) for this neutrino was $\theta_m = 73.86\,\mathrm{deg}$ and $\Delta m_m^2 = 1.2920 \times 10^{-4}\,\mathrm{eV}^2$ (compare to vacuum split $\Delta m_m^2 = 7.5 \times 10^{-5}\,\mathrm{eV}^2$). Though it was not required for the γ calculation, we made a rough estimate of the cumulative decrease of θ_m on the path towards the surface from production, multiplying $\dot{\theta}_m$ times the $3.1585 \times 10^5\,\mathrm{km}$ distance to $r_\odot = 0.5$. We estimated $\theta_m = 39.5\,\mathrm{deg}$ by $r_\odot = 0.5$, clearly headed for the vacuum value of $\theta_m = 33.48\,\mathrm{deg}$ on exit from the Sun. The path having been adiabatic, the ν_{2m} reverts to ν_2 with the vacuum mix angle restored and arrives at Earth as such (explaining the observed survival probability at SNO [18]).

We arrived at an adiabatic parameter value of $\gamma = 1.1181 \times 10^{-4} \ll 1$ and corresponding jump probability $P_c = 3.3627 \times 10^{-9}$ using Eq. 5 above, comparing well with [14] equation (27) approximating P_c in range $10^{-9} - 10^{-7}$ for a $10\,\mathrm{MeV}$ solar neutrino. As we expected, there is almost no jump probability, i.e., the transitions are adiabatic, for the range of interest for solar neutrinos with MSW effects, $E = (2 - 15)\,\mathrm{MeV}$ [14].

[10] See Richard Feynman's lectures on the ammonia maser and other two-state systems Chapters 9-11 [17].

V. GRAPHING SURVIVAL PROBABILITY

Using a two-flavor equation (Eq. 3), we graphed the *pp* and ^{8}B average solar neutrino survival probability (see Fig. 2) at Earth over neutrino production energies from 150 keV to 100 MeV. This range permits comparison of the asymptotic behavior of the curves, though *pp* neutrinos do not have energies above $\sim$ 420 keV, nor do ^{8}B neutrino energies extend above $\sim$ 16 MeV. Averaging over the region of neutrino production in the Sun (and the length of the path to Earth) renders negligible all interference terms, so only an average survival probability can be observed (no oscillations) [5]. Additionaly, at production energies above $\sim$ 5 MeV, the MSW effect results in a single ν_2 arriving at Earth as the ^{8}B electron neutrino [19], i.e., a single neutrino mass state cannot oscillate [20]. For comparison with our graph

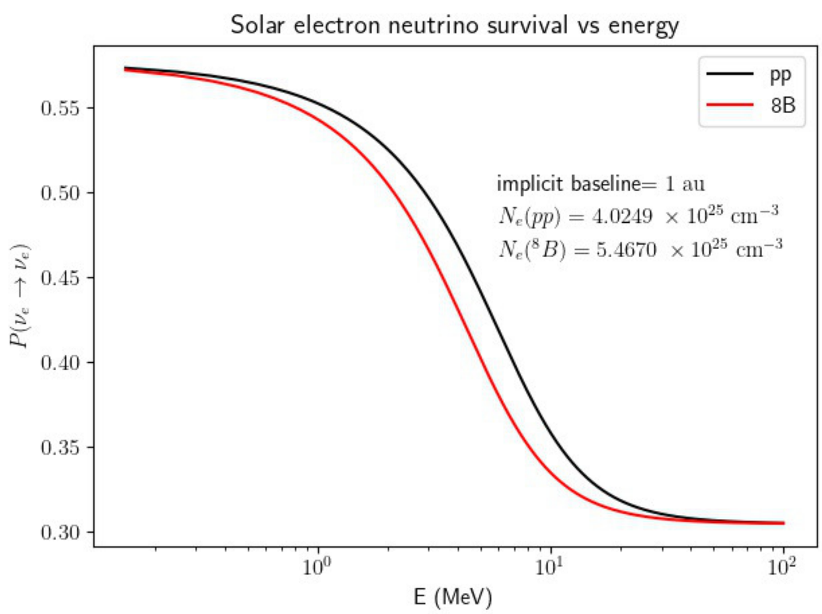

FIG. 2. Solar neutrino survival probability.

Fig. 2, the Borexino experiment 2017 published result [21] for the survival probability of the relatively low energy *pp* neutrinos detected at Earth was $P(\nu_e \to \nu_e) = 0.57 \pm 0.09$. over the approximate range $E = 320 - 420$ keV, i.e., the left side of our graph. Looking at the red curve for the ^{8}B neutrino, at 10 MeV the survival probability is near the published survival probability of 0.317 for 10 MeV ^{8}B solar electron neutrinos (observation by SNO, Sudbury Neutrino Observatory, 2016 summary [18], final joint fit to all of their data).

In the computer code, we calculate the cosine of the mixing angle in matter (code function

PDGCST2Mvec) used in Eq. 3) directly with Eq. (14.57) from [5]:

$$\cos 2\theta_m = \frac{1 - \frac{N_e}{N_e^{res}}}{\sqrt{\left(1 - \frac{N_e}{N_e^{res}}\right)^2 + \tan^2 2\theta}} \tag{10}$$

In that equation, N_e^{res} is given by Eq. 1 above, although in the computer code (function NRESFvec) we made use of a convenient variant of that equation also given by PDG in Eq. (14.58) [5]:

$$N_e^{res} = 6.56 \times 10^6 \frac{\Delta m^2[eV^2]}{E[MeV]} \cos 2\theta \, \mathrm{cm}^{-3} N_A$$

The result is electron number per cm^3. The N_A term is Avogadro's number. We held constant the electron number density $N_e = 5.4670 \times 10^{25}$ cm^{-3} for ^{8}B and electron number density $N_e = 4.0249 \times 10^{25}$ cm^{-3} for *pp* and scanned over the energy range given earlier.

Aside from calculating the cosine rather than the typical sine of the angle, Eq. 10, may appear somewhat unfamiliar as most sources write the relevant equations using the ratio of the matter potential ΔV to the vacuum mass split Δm^2 rather than the ratio of neutrino production density $N_e(r)$ to resonant density N_e^{res}. For example, Eq. 10 could be rephrased as:

$$\cos 2\theta_m = \frac{\left(\cos 2\theta - \frac{\Delta V}{\Delta m^2}\right)}{\sqrt{\left(\cos 2\theta - \frac{\Delta V}{\Delta m^2}\right)^2 + \sin^2 2\theta}} \tag{11}$$

VI. COMPUTER CODE

The following is the Python code that produces the $P(\nu_e \to \nu_e)$ Fig. 2 graph above using Eq. 3. Use assumes you have SciPy, NumPy, Matplotlib, and Python 3 and know how to make code adjustments if you have different versions.

```
# coding: utf-8


from scipy import constants
# we want to be forced to prepend math. to all Python math functions
import math


import numpy as np
```

```
import matplotlib.pyplot as plt

import matplotlib as mpl

# Fermi coupling constant in GeV^-2
G_f = constants.value('Fermi coupling constant')
# speed of light in m/s
c = constants.value('speed of light in vacuum')
hbar = constants.value('Planck constant over 2 pi in eV s')
hbar_GeVs = hbar / 1e9
c_cm_s = c * 1e2
hbarcGeVcm = hbar_GeVs * c_cm_s
root2 = math.sqrt(2)
# Avogadro's number 6.022140857e+23
N_0 = constants.value('Avogadro constant')

# mass split 2-1, eV^2
delta_m_sqr = 7.5e-5
# mass split 2-1, GeV^2
delta_m_sqr_GeV = 7.5e-5 / 1e18
# vacuum mix angle theta 1-2
theta_12 = math.radians(33.48)

sinsqr2theta12 = math.sin(2*theta_12)**2
sin2theta = math.sin(2*theta_12)
cos2theta = math.cos(2*theta_12)

tan2theta = math.tan(2*theta_12)
tanSqrd2theta = tan2theta**2

# number of points/sample steps in the specified interval on x axis x_ax
n_E_steps = 1000
```

```
# per PDG 2016 eq (14.58): gen array of MSW N_e densities in e/cm^3
# for each neutrino production energy scanned, GeV in, convert to MeV locally
# equation expects mass split in eV^2
def NRESF (energy_sequence):
    "scan neutrino energy array GeV->MeV, gen array of N_res MSW e/cm^3"
    return ( 6.56e6 * delta_m_sqr * cos2theta * N_0 / (energy_sequence * 1e3) )
NRESFvec = np.vectorize(NRESF)

# per PDG 2016 eq (14.57): create array of cos2theta_m from res density array
# and N_e electron density for the target neutrino species in Sun
def PDGCST2M (MSWresMtrx,N_e):
    "scan N_res MSW e/cm^3 array, gen cos2theta matter angle for each"
    return ( ((1-(N_e / MSWresMtrx))/math.sqrt((1 - (N_e / MSWresMtrx))**2 +\
tanSqrd2theta)) )
PDGCST2Mvec = np.vectorize(PDGCST2M)

# energy scan range, GeV
E_min = 0.150e-3
E_max = 100e-3

# x_ax contains the energy scan steps, GeV
x_ax = np.linspace(E_min,E_max, n_E_steps, endpoint=True)

# scan neutrino energies and calculate the MSW resonance density e/cm^3for each
NeRes_matrix = NRESFvec(x_ax)

# generate cos2theta_m values using PDG 2016 tangent and density ratio equation
# only target density n_e changes on each species
N_e_pp = 4.0249e25 # pp neutrino max flux at 0.0990 radisu, e density in cm^-3
# provide index for this used to produce graph array below
n_e_pp_index = 0
```

```
N_e_8B = 5.4670e25 # 8B neutrino max flux at 0.0460 radisu, e density in cm^-3
# provide index for this used to produce graph array below
n_e_8B_index = 1

n_e = np.array( [N_e_pp, N_e_8B])
# want to loop over the number of different neutrino species densities avail
work_index = n_e.shape[0]
# create zeroed array of arrays to accept an array for each cos 2theta_m
# array computed for each density (all over same energy range)
PDGcos2thetamMtrx = np.zeros( (work_index,n_E_steps) )

# loop over the available neutrino species densities creating cos 2theta_m
# for each
for indx in range(work_index):
    PDGcos2thetamMtrx[indx] = PDGCST2Mvec(NeRes_matrix,n_e[indx])

# using PDG 2016 eq (14.84) with PDG matter angle cos2theta
# is assumed SNO detector at Earth, so you have no explicit baseline
def PVEVES13 (pdgcos2thmtrx):
    "return list of survival probabilities per 2013 solar eq as array"
    return(  ( cos2theta * pdgcos2thmtrx * 0.5) + 0.5 )
PVEVES13vec = np.vectorize(PVEVES13)

# graph surv prob with 2013 solar equation, using PDG cos2theta matter angle
# created with tangent and density ratios, passing in the cos2theta_m array
# for each neutrino species.
y_ax_pp = PVEVES13vec(PDGcos2thetamMtrx[n_e_pp_index])
y_ax_8B = PVEVES13vec(PDGcos2thetamMtrx[n_e_8B_index])

# ditch previous runs or end up with multiple plots or pieces of plots
plt.clf()
# tell Matplotlib to use LaTex fonts throughout figures
```

```
# (other than label, legend etc which we have to specify); expect wait!
mpl.rcParams.update({'font.size': 12, 'text.usetex': True})
# create a matplotlib figure, accept default aspect ration, dpi
fg1 = plt.figure()
# create an Axes object on it explicitly; graphs are drawn on it
ax1 = fg1.add_subplot(111)

ax1.set(xlabel='E (MeV)', ylabel=r'$P (\nu_e \rightarrow \nu_{e})$')

# want to plot MeV rather than GeV so adjust
# by multiplying the original GeV array by 1e3 element-wise to new array
x_ax_MeV = np.multiply(1e3, x_ax)

# plot first neutrino species data, pp
ax1.plot(x_ax_MeV,y_ax_pp, color='black',label='pp')
# plot next neutrino species data, 8B
ax1.plot(x_ax_MeV,y_ax_8B, color='red',label='8B')

ax1.set_xscale('log')
ax1.legend()

# add annotation giving baseline used; data coordinates for position
ax1.annotate( r'implicit baseline$=1\; \mathrm{au}$',xy=(6.0, 0.50), xycoords='data')
# add annotation giving electron density used; data coordinates for position
ax1.annotate( r'$N_e(pp) = 4.0249 \; \times 10^{25} \; \
\mathrm{cm}^{-3}$',xy=(6.0, 0.48), xycoords='data')
ax1.annotate( r'$N_e(^8B) = 5.4670 \; \times 10^{25} \; \
\mathrm{cm}^{-3}$', xy=(6.0, 0.46), xycoords='data')

ax1.set_title(r'Solar electron neutrino survival vs energy')
fg1.tight_layout()
plt.show()
```

```
plt.close()
```

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