Rel. Algebra Quiz Answers

From Stanford's DBClass

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Introduction

In this document I'll provide the solutions for the Relational Algebra quiz questions from the (infamous) dbclass MOOC kindly provided by Prof. Jenifer Widom.

Since markdown does not convey maths as it should, I created this document in order to preserve the intended notation as well as maintain cohesion; thus, since I got into all this trouble anyway solutions are a bit more detailed than the other chapters.

An **important** note is that the provided answers are the ones that were generated in my instance but the system generates different possible answers and mixes up the order as well so your mileage may vary.

Finally you should use this document (and my solutions for the dbcourse in general) as a *reference* and not just copy-pasta; you're just hurting yourself if you do so.

Question 1

Suppose relations $\mathcal{R}(A, C)$ and $\mathcal{S}(B, C, D)$ have the following tuples shown in Tables 1 and 2 respectively.

Table 1

$\mathcal{R}$	
A	**C**
3	3
6	4
2	3
3	5
7	1

Table 2

$\mathcal{S}$		
B	**C**	**D**
5	1	6
1	5	8
4	3	9

Compute the *natural join* of $\mathcal{R}$ and $\mathcal{S}$. Which of the following tuples is in the result? Assume each tuple $\mathcal{T}$ has schema $\mathcal{T}$:(A, B, C, D).

Q1 Options

In my instance the options to select the answer from, were the following:

a) $\mathcal{T}_a : (3, 3, 5, 8)$

b) $\mathcal{T}_b : (5, 1, 6, 4)$

c) $\mathcal{T}_c : (3, 4, 3, 9)$

d) $\mathcal{T}_d : (6, 4, 3, 9)$

Q1 Answer

In order to be certain which of the four (4) tuples provided is the correct one, we should calculate the *natural join* of relations $\mathcal{R}$ and $\mathcal{S}$ first. Remember that the definition of *natural join* is to join the relations on the columns that have the *same* name in both relations. Observing relations $\mathcal{R}$, $\mathcal{S}$ we see that they have only one (1) commonly named column, which is: C. With that information we are now able to calculate the *natural join* of the provided relations.

Table 3: natural join of $\mathcal{R}$,$\mathcal{S}$

$\mathcal{R}$

A	**C**
3	3
6	4
2	3
3	5
7	1

$\bowtie$

$\mathcal{S}$

B	**C**	**D**
5	1	6
1	5	8
4	3	9

$=$

$\mathcal{R} \bowtie \mathcal{S}$

A	**B**	**C**	**D**
3	4	3	9
2	4	3	9
3	1	5	8
7	5	1	6

We can easily see from the result and highlighted row that the tuple from our given options that is part of the result is $\mathcal{T}_c : (3, 4, 3, 9)$.

Question 2

Suppose relations $\mathcal{R}(A, C)$ and $\mathcal{S}(B, C, D)$ have the following tuples shown in Tables 4 and 5 respectively.

Table 4

$\mathcal{R}$

A	**C**
3	a
6	t
2	g
3	c
7	t

Table 5

$\mathcal{S}$

B	**C**	**D**
c	1	6
a	5	8
t	3	9

Compute the *theta-join* of $\mathcal{R}$ and $\mathcal{S}$ with the condition $\mathcal{R}.B = \mathcal{S}.B$ AND $\mathcal{R}.A < \mathcal{S}.C$ Which of the following tuples is in the result? Assume each tuple has schema $\mathcal{T} : (A, \mathcal{R}.B, \mathcal{S}.B, C, D)$.

Q2 Options

In my instance the options to select the answer from, were the following:

a) $\mathcal{T}_a : (4, c, c, 7, 8)$

b) $\mathcal{T}_b : (9, t, t, 8, 9)$

c) $\mathcal{T}_c : (7, t, t, 8, 9)$

d) $\mathcal{T}_d : (1, a, a, 8, 9)$

Q2 Answer

As you might remember in *theta join* we join the relations based on a given condition, which usually contains one or more *predicates* that need to be satisfied to join the respective tuples. Our condition which we join contains two different branches, one being p_1: $\mathcal{R}.B = \mathcal{S}.B$ and the other p_2: $\mathcal{R}.A < \mathcal{S}.C$; since these two conditions are join using an AND operator both have to hold at the same time.

An easy way to go about it (and in order to see the whole process) is to apply one *predicate* at a time and apply the other predicates to the resulting relation. This not an efficient way of performing that operation but in my opinion illustrates the "inner-guts" of the operation and that'll do just fine for our intents and purposes.

Let's start by applying the first *predicate*, p_1: $\mathcal{R}.B = \mathcal{S}.B$ as is shown below.

Table 6: theta join of $\mathcal{R}$,$\mathcal{S}$ using predicate p_1

$\mathcal{R}$	
A	**C**
3	a
6	t
2	g
3	c
7	t

$\bowtie_{p_1}$

$\mathcal{S}$		
B	**C**	**D**
c	1	6
a	5	8
t	3	9

$\rightarrow$

$\mathcal{R}1_{\mathcal{R}\bowtie_{p_1}\mathcal{S}} =$

$\mathcal{R} \bowtie_{p_1} \mathcal{S}$				
A	$\mathcal{R}.B$	$\mathcal{S}.B$	**C**	**D**
1	a	a	7	8
4	c	c	5	6
7	t	t	8	9
9	t	t	8	9

Now let's apply the second *predicate*, p_2: $\mathcal{R}.A < \mathcal{S}.C$ to the relation; the gist of it is that since we already have done the join using p_1, we use the select operator (σ) in order to filter out the invalid tuples based on p_2. For our purposes we assume that in $\mathcal{R}_1$, $\mathcal{R}.A$ column is $\mathcal{R}_1.A$ and $\mathcal{S}.C$ is $\mathcal{R}_1.C$. So without further delay Table 7.
We can easily see from the result and highlighted row that the tuple from our given options that is part of the result is $\mathcal{T}_c : (7, t, t, 8, 9)$.

Table 7: select from $\mathcal{R}_1$ using p_2 predicate

$\sigma_{p_2}(\mathcal{R}_1) =$

$\mathcal{R} \bowtie_{p_1 \text{ AND } p_2} \mathcal{S}$				
A	$\mathcal{R}.B$	$\mathcal{S}.B$	**C**	**D**
1	a	a	7	8
4	c	c	5	6
7	t	t	8	9

Question 3

Consider a relation $\mathcal{R}(A, C)$ with r tuples, all unique within $\mathcal{R}$, and a relation $\mathcal{S}(B, C)$ with s tuples, all unique within $\mathcal{S}$. Let t represent the number of tuples in $\mathcal{R} \bowtie$ [1] $\mathcal{S}$. Which of the following triples of values (r, s, t) is possible?

Q3 Options

In my instance the options to select the answer from, were the following:

- **a)** $\mathcal{T}_a : (2, 3, 9)$
- **b)** $\mathcal{T}_b : (5, 5, 50)$
- **c)** $\mathcal{T}_c : (5, 3, 1)$
- **d)** $\mathcal{T}_d : (3, 3, 27)$

Q3 Answer

This is tricky to answer, you have to know that the upper bound of the possible number of tuples in a natural join result set *cannot* be more than the product of the number of tuples in each of the relations. More formally the following invariant holds $t \leq rs$. In our case the product of the number of tuples r and s which satisfies this rule is only tuple $\mathcal{T}_c : (5, 3, 1)$ and thus is the correct answer.

Question 4

Consider a relation $\mathcal{R}(A, C)$ with r tuples, all unique within $\mathcal{R}$, and a relation $\mathcal{S}(B, C)$ with s tuples, all unique within $\mathcal{S}$. Let t represent the number of tuples in $\mathcal{R} -$ [2] $\mathcal{S}$. Which of the following triples of values (r, s, t) is possible?

Q4 Options

In my instance the options to select the answer from, were the following:

- **a)** $\mathcal{T}_a : (5, 10, 10)$
- **b)** $\mathcal{T}_b : (5, 2, 0)$
- **c)** $\mathcal{T}_c : (10, 5, 15)$

[1] natural-join
[2] minus operation

d) $\mathcal{T}_d : (5, 3, 2)$

Q4 Answer

The correct answer is $\mathcal{T}_d$, as since all tuples are unique to $\mathcal{R}$ and $\mathcal{S}$ respectively, $\mathcal{R} - \mathcal{S}$ will contain those tuples which *are* in $\mathcal{R}$ but *not* in $\mathcal{S}$. $\mathcal{T}_a$ and $\mathcal{T}_c$ options are immediately excluded due to the result being larger than the number of initial tuples in $\mathcal{R}$ while the other interesting option that would worth exploring is that of $\mathcal{T}_b$. In this case we have a value which is to have zero tuples in the result, but that can't happen as $\mathcal{R}$ in this particular option has 5 tuples while $\mathcal{S}$ has 2 thus the minimum result being 3. In fact should none tuples of $\mathcal{S}$ match those in $\mathcal{R}$ (and hence subtracted) we would get just $\mathcal{R}$ which would equal to 5 tuples for t.

Question 5

Suppose relations $\mathcal{R}(A, C)$ and $\mathcal{S}(B, C, D)$ have the following tuples shown in Tables 8 and 9 respectively.

Table 8

$\mathcal{R}$	
A	**C**
1	2
3	4
5	6

Table 9

$\mathcal{S}$		
B	**C**	**D**
2	4	6
4	6	8
4	7	9

Compute the *natural join* of $\mathcal{R}$ and $\mathcal{S}$. Which of the following tuples is in the result? Assume each tuple $\mathcal{T}$ has schema $\mathcal{T}$:(A, B, C, D).

Q5 Options

In my instance the options to select the answer from, were the following:

a) $\mathcal{T}_a : (3, 4, 2, 6)$

b) $\mathcal{T}_b : (1, 2, 4, 8)$

c) $\mathcal{T}_c : (1, 2, 4, 6)$

d) $\mathcal{T}_d : (5, 6, 7, 8)$

Q5 Answer

As I have previously explained *natural join*, I won't repeat myself here; thus let's head straight for the solution, which is shown in Table 10.

Table 10: natural join of $\mathcal{R}$,$\mathcal{S}$

$\mathcal{R}$	
A	**C**
1	2
3	4
5	6

$\bowtie$

$\mathcal{S}$		
B	**C**	**D**
2	4	6
4	6	8
4	7	9

$=$

$\mathcal{R} \bowtie \mathcal{S}$			
A	**B**	**C**	**D**
1	2	4	6
3	4	6	8
3	4	7	9

We can easily see from the result and highlighted row that the tuple from our given options that is part of the result is $\mathcal{T}_a : (1, 2, 4, 6)$.

Question 6

Suppose relations $\mathcal{R}(A, C)$ and $\mathcal{S}(B, C, D)$ have the following tuples shown in Tables 11 and 12 respectively.

Table 11

$\mathcal{R}$	
A	**C**
1	2
3	4
5	6

Table 12

$\mathcal{S}$		
B	**C**	**D**
2	4	6
4	6	8
4	7	9

Compute the *theta-join* of $\mathcal{R}$ and $\mathcal{S}$ with the condition $\mathcal{R}.A < \mathcal{S}.C$ AND $\mathcal{R}.B < \mathcal{S}.D$ Which of the following tuples is in the result? Assume each tuple has schema $\mathcal{T} : (A, \mathcal{R}.B, \mathcal{S}.B, C, D)$.

Q6 Options

In my instance the options to select the answer from, were the following:

a) $\mathcal{T}_a : (1, 2, 4, 7, 9)$

b) $\mathcal{T}_b : (1, 2, 2, 6, 8)$

c) $\mathcal{T}_c : (3, 4, 4, 7, 8)$

d) $\mathcal{T}_d : (5, 6, 4, 6, 9)$

Q6 Answer

As I previously explained the *theta join* and it's "inner-guts" I won't repeat myself here; thus let's head directly for the solution which is shown in Table .

Table 13: theta join of $\mathcal{R}$,$\mathcal{S}$ using predicates p_1, p_2

$\mathcal{R}$	
A	**C**
1	2
3	4
5	6

$\bowtie_{p_1 \text{ AND } p_2}$

$\mathcal{S}$		
B	**C**	**D**
2	4	6
4	6	8
4	7	9

$\rightarrow$

$\mathcal{R}_{1\,\mathcal{R} \bowtie_{p_1 \text{ AND } p_2} \mathcal{S}} =$

$\mathcal{R}_1$				
A	$\mathcal{R}.B$	$\mathcal{S}.B$	**C**	**D**
1	2	2	4	6
1	2	4	6	8
1	2	4	7	9
3	4	2	4	6
3	4	4	6	8
3	4	4	7	9
5	6	4	6	8
5	6	4	6	8

We can easily see from the result and highlighted row that the tuple from our given options that is part of the result is $\mathcal{T}_a : (1, 2, 4, 7, 9)$.

Question 7

Suppose relation $\mathcal{R}(A, B, C)$ has the following tuples which are shown in Table 14.

Table 14

$\mathcal{R}$		
A	**B**	**C**
1	2	3
4	2	3
4	5	6
2	5	3
1	2	6

Compute the projection $\Pi_{C,B}(\mathcal{R})$. Which of the following tuples is in the result?

Q7 Options

In my instance the options to select the answer from, were the following:

a) $\mathcal{T}_a : (5, 6)$

b) $\mathcal{T}_b : (1, 2)$

c) $\mathcal{T}_c : (6, 4)$

d) $\mathcal{T}_d : (6, 2)$

Q7 Answer

Now the *projection* operator does something really simple, essentially selects a number of columns and returns them into a new relation; so the requested $\Pi_{C,B}(\mathcal{R})$ is the one shown in Table 15.

Table 15

$\Pi_{C,B}(\mathcal{R})$	
C	**B**
3	2
6	5
3	5
6	2

We can easily see from the result and highlighted row that the tuple from our given options that is part of the result is $\mathcal{T}_d : (6, 2)$.

Question 8

Suppose relations $\mathcal{R}(A, B, C)$ and $\mathcal{S}(A, B, C)$ have the following tuples shown in Tables 16 and 17 respectively.

Table 16

$\mathcal{R}$		
A	**B**	**C**
1	2	3
4	2	3
4	5	6
2	5	3
1	2	6

Table 17

$\mathcal{S}$		
A	**B**	**C**
2	5	3
2	5	4
4	5	6
1	2	3

Compute the *union* of $\mathcal{R}$ and $\mathcal{S}$. Which of the following tuples **DOES NOT** appear in the result?

Q8 Options

In my instance the options to select the answer from, were the following:

a) $\mathcal{T}_a : (1, 2, 3)$

b) $\mathcal{T}_b : (4, 5, 3)$

c) $\mathcal{T}_c : (4, 2, 3)$

d) $\mathcal{T}_d : (1, 2, 6)$

Q8 Answer

The *union* operator ($\cup$) all tuples which are present in *all* relations and returns them as the resulting set eliminating duplicate tuples (if any). The result of the $\mathcal{R} \cup \mathcal{S}$ is shown in Table 18.

Table 18: intersection of $\mathcal{R}$ with $\mathcal{S}$

$\mathcal{R}$

A	B	C
1	2	3
4	2	3
4	5	6
2	5	3
1	2	6

$\cup$

$\mathcal{S}$

A	B	C
2	5	3
2	5	4
4	5	6
1	2	3

$=$

$\mathcal{R}_1$

A	B	C
1	2	3
1	2	6
4	2	3
4	5	6
2	5	3
2	5	4

We can easily see from the result the tuple from our given options that **IS NOT** part of the result is $\mathcal{T}_b : (4, 5, 3)$.

Question 9

Suppose relations $\mathcal{R}(A, B, C)$ and $\mathcal{S}(A, B, C)$ have the following tuples shown in Tables 19 and 20 respectively.

Table 19

$\mathcal{R}$

A	B	C
1	2	3
4	2	3
4	5	6
2	5	3
1	2	6

Table 20

$\mathcal{S}$

A	B	C
2	5	3
2	5	4
4	5	6
1	2	3

Compute the *intersection* of the relations $\mathcal{R}$ and $\mathcal{S}$. Which of the following tuples is in the result?

Q9 Options

In my instance the options to select the answer from, were the following:

a) $\mathcal{T}_a : (1, 2, 4)$

b) $\mathcal{T}_b : (2, 2, 6)$

c) $\mathcal{T}_c : (2, 5, 3)$

d) $\mathcal{T}_d : (2, 5, 4)$

Q9 Answer

The *intersection* operator ($\cap$) takes matching tuples which are present in *all* relations and returns them as the resulting set. The result of the $\mathcal{R} \cap \mathcal{S}$ is shown in Table 21.

Table 21: intersection of $\mathcal{R}$ with $\mathcal{S}$

$\mathcal{R}$		
A	**B**	**C**
1	2	3
4	2	3
4	5	6
2	5	3
1	2	6

$\cap$

$\mathcal{S}$		
A	**B**	**C**
2	5	3
2	5	4
4	5	6
1	2	3

$=$

$\mathcal{R}_1$		
A	**B**	**C**
1	2	3
2	5	3
4	5	6

We can easily see from the result and highlighted row that the tuple from our given options that is part of the result is $\mathcal{T}_c : (2, 5, 3)$.

Question 10

Suppose relations $\mathcal{R}(A, B, C)$ and $\mathcal{S}(A, B, C)$ have the following tuples shown in Tables 22 and 23 respectively.

Table 22

$\mathcal{R}$		
A	**B**	**C**
1	2	3
4	2	3
4	5	6
2	5	3
1	2	6

Table 23

$\mathcal{S}$		
A	**B**	**C**
2	5	3
2	5	4
4	5	6
1	2	3

Compute $(\mathcal{R} - \mathcal{S})$ union $(\mathcal{S} - \mathcal{R})$, often called the *symmetric difference* of $\mathcal{R}$ and $\mathcal{S}$. Which of the following tuples is in the result?

Q10 Options

In my instance the options to select the answer from, were the following:

a) $\mathcal{T}_a : (4, 5, 3)$

b) $\mathcal{T}_b : (2, 5, 4)$

c) $\mathcal{T}_c : (4, 5, 6)$

d) $\mathcal{T}_d : (1, 2, 3)$

Q10 Answer

This is the most complicated and tedious relation to produce, so we are going to do it in three (3) intermediate steps. We will first calculate each of the *minus* operations, namely $\mathcal{R}_1$: $\mathcal{R} - \mathcal{S}$ and $\mathcal{R}_2$: $\mathcal{S} - \mathcal{R}$. The resulting relations $\mathcal{R}_1$, $\mathcal{R}_2$ are shown into Tables 24 and 25 respectively. Finally we calculate the union of the resulting relations $\mathcal{R}_1$, $\mathcal{R}_2$ to get the final relation $\mathcal{R}_3$, which is shown in Table 26.

Table 24: $\mathcal{R}$ minus $\mathcal{S}$

$\mathcal{R}$		
A	**B**	**C**
1	2	3
4	2	3
4	5	6
2	5	3
1	2	6

$-$

$\mathcal{S}$		
A	**B**	**C**
2	5	3
2	5	4
4	5	6
1	2	3

$=$

$\mathcal{R}_1$		
A	**B**	**C**
1	2	6
4	2	3

Table 25: $\mathcal{S}$ minus $\mathcal{R}$

$\mathcal{S}$		
A	**B**	**C**
2	5	3
2	5	4
4	5	6
1	2	3

$-$

$\mathcal{R}$		
A	**B**	**C**
1	2	3
4	2	3
4	5	6
2	5	3
1	2	6

$=$

$\mathcal{R}_2$		
A	**B**	**C**
2	5	4

Table 26: $\mathcal{R}_1$ union $\mathcal{R}_2$

$\mathcal{R}_1$		
A	**B**	**C**
1	2	6
4	2	3

$\cup$

$\mathcal{R}_2$		
A	**B**	**C**
2	5	4

$=$

$\mathcal{R}_3$		
A	**B**	**C**
1	2	6
4	2	3
2	5	4

We can easily see from the result and highlighted row from $\mathcal{R}_3$ that the tuple from our given options that is part of the result is $\mathcal{T}_b : (2, 5, 4)$.