Rel. Design Quiz 3 Answers
Schema Normalization

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1 Introduction

In this document I'll provide the solutions for the Relational Design Database Schema Normalization quiz questions from the (infamous) dbclass MOOC kindly provided by Prof. Jennifer Widom.

Since markdown does not convey maths as it should, I created this document in order to preserve the intended notation as well as maintain cohesion; thus, since I got into all this trouble anyway solutions are a bit more detailed than the other (plain markdown) solutions in most chapters.

An **important** note is that the provided answers are the ones that were generated in my instance but the system is smart enough to generate *different* possible answers and mixes up the order as well, so your mileage may vary.

Finally you should use this documents (and my solutions for the dbcourse in general) as a *reference* and **not** just copy-paste; you're just hurting yourself if you do so. Now without further delays, off we go!

2 Question 1

Let's consider a relation $\mathcal{R}(A, B, C, D, E)$ and has the following multivalued functional dependencies:

- $A \twoheadrightarrow B$
- $B \twoheadrightarrow D$

Let's now suppose that we *decompose* the relation $\mathcal{R}$ into 4^{rth} Normal Form. Depending on the order in which we deal with $4NF$ violations, we can get different decompositions. Which of the following relation schemas could be in the final $4NF$ decomposition?

2.0.1 Q1 Options

In my instance the options to select the answer from, were the following:

a) $\mathcal{S}_a : ACDE$

b) $\mathcal{S}_b : ABCE$

c) $\mathcal{S}_c : BD$

d) $\mathcal{S}_d : CE$

2.0.2 Q1 Answer

Recall from lectures that during the decomposition into $4NF$ we need our relations to have a key inside them. In this case we only have multivalued functional dependencies but please remember that since $4NF$ is *stricter* than BCNF then iff $A \twoheadrightarrow B$, then we have also that $A \rightarrow B$. Now obviously, we have to decompose the relation in order to see which schemas are in the final result but before we do that, let's add in full our existing rules.

- $A \twoheadrightarrow B$
- $B \twoheadrightarrow D$
- $A \twoheadrightarrow D$ (implied, due to transitive rule)

Let's start by decomposing our relation using $A \twoheadrightarrow B$ initially, and we get the following results.

$$\mathcal{R}(A, B, C, D, E) \overset{A \twoheadrightarrow B}{=} \begin{cases} \mathcal{R}_1(A, B) \\ \\ \mathcal{R}_2(A, C, D, E) \end{cases}$$

Now we can obviously see that $\mathcal{R}_1$ is indeed in $4NF$ but due to the implied MVD, $A \twoheadrightarrow D$, $\mathcal{R}_2$ relation is not. We proceed then to break up the $\mathcal{R}_2$ relation as follows.

$$\mathcal{R}_2(A, C, D, E) \overset{A \twoheadrightarrow D}{=} \begin{cases} \mathcal{R}_3(A, D) \\ \\ \mathcal{R}_4(A, C, E) \end{cases}$$

Thankfully we do not need to do any more work as both $\mathcal{R}_1, \mathcal{R}_2$ and $\mathcal{R}_3$ are indeed in $4NF$. This was the quirkest decomposition that had to be done, the other two are really straightforward as you just follow the MVD rules; for completeness I'll show one of the two. The one starting with $B \twoheadrightarrow D$.

$$\mathcal{R}(A, B, C, D, E) \overset{B \twoheadrightarrow D}{=} \begin{cases} \mathcal{R}_1(B, D) \\ \\ \mathcal{R}_2(A, B, C, E) \end{cases}$$

Now we can obviously see that $\mathcal{R}_1$ is indeed in $4NF$ but due to the MVD, $A \twoheadrightarrow B$, $\mathcal{R}_2$ relation is not. We proceed then to break up the $\mathcal{R}_2$ relation as follows.

$$\mathcal{R}_2(A, B, C, E) \overset{A \twoheadrightarrow B}{=} \begin{cases} \mathcal{R}_3(A, B) \\ \\ \mathcal{R}_4(A, C, E) \end{cases}$$

Now the answer, given my instance options is obviously option **c**, $\mathcal{S}_c : BD$.

3 Question 2

Let's consider a relation $\mathcal{R}(A, B, C, D, E)$ which is a relation in Boyce-Codd Normal Form (BCNF). Support that ABC is the *only* key of $\mathcal{R}$. Which of the following functional dependencies is guaranteed to hold for our relation $\mathcal{R}$?

3.0.1 Q2 Options

In my instance the options to select the answer from, were the following:

a) $\mathcal{F}_a : ABCD \rightarrow E$

b) $\mathcal{F}_b : ABE \rightarrow D$

c) $\mathcal{F}_c : BCE \rightarrow A$

d) $\mathcal{F}_d : ACDE \rightarrow E$

3.0.2 Q2 Answer

Recall from lectures that in order for a relation to be in Boyce-Codd Normal Form in all of it's functional dependencies there has to be a key in the left hand side. To find if a left-hand side is a key you have to find its *closure*, which we will do now.

- $(ABCD)^{+} \stackrel{ABCD \rightarrow E}{=} ABCDE = \mathcal{R}$
- $(ABE)^{+} \stackrel{ABE \rightarrow D}{=} ABED$
- $(BCE)^{+} \stackrel{BCE \rightarrow A}{=} BCEA$
- $(ACDE)^{+} \stackrel{ACDE \rightarrow B}{=} ABCDE = \mathcal{R}$

Now we have *two* (2) promising options, $\mathcal{F}_a$ and $\mathcal{F}_d$ but if you again recall from lectures a part of the key **cannot** be on the right-hand side. From definition we know that ABC *is* a key, hence $\mathcal{F}_d$ is rejected. Thus the correct answer is option: **a**, $\mathcal{F}_a : ABCD$.

4 Question 3

Let's consider a relation $\mathcal{R}(A, B, C, D)$ for which one of the given sets of functional dependencies is $\mathcal{R}$ in Boyce-Codd Normal Form (BCNF)?

4.0.1 Q3 Options

In my instance the options to select the answer from, were the following:

a) $\mathcal{F}_a : BC \rightarrow A, AD \rightarrow C, CD \rightarrow B, BD \rightarrow C$

b) $\mathcal{F}_b : C \rightarrow B, BC \rightarrow A, A \rightarrow C, BD \rightarrow A$

c) $\mathcal{F}_c : AC \rightarrow D, D \rightarrow A, D \rightarrow C, D \rightarrow B$

d) $\mathcal{F}_d : A \rightarrow C, B \rightarrow A, A \rightarrow D, AD \rightarrow C$

4.0.2 Q3 Answer

Given the above sets, the only thing that we have to do in order to check if the given relation $\mathcal{R}$ is indeed in BCNF is actually do compute the *closures* of each set's functional dependencies and check if **all** are a key of $\mathcal{R}$. We will do this now for each set; if a violating functional dependency is found the rest of the set will be *skipped*.
Let's start with $\mathcal{F}_a$

- $(BC)^+ \overset{BC \rightarrow A}{=} BCA \neq \mathcal{R}$

This set does not fit the bill, let's try $\mathcal{F}_b$ next.

- $(C)^+ \overset{C \rightarrow B}{=} CB \neq \mathcal{R}$

Likewise, this set again seems that's no good, let's try $\mathcal{F}_c$ next.

- $(AC)^+ \overset{AC \rightarrow D}{=} (ACD)^+ \overset{D \rightarrow B}{=} ABCD = \mathcal{R}$
- $(D)^+ \overset{D \rightarrow A}{=} (DA)^+ \overset{D \rightarrow C}{=} (ACD)^+ \overset{D \rightarrow B}{=} ABCD = \mathcal{R}$

This set, looks like its the correct one – and indeed it is; thus the correct answer is option: **c**, $\mathcal{F}_c : AC \rightarrow D, D \rightarrow A, D \rightarrow C, D \rightarrow B$.

5 Question 4

Let's consider a relation $\mathcal{R}(A, B, C, D)$ which has following functional dependencies:

- $A \to B$
- $C \to D$
- $AD \to C$
- $BC \to A$

Suppose we decompose $\mathcal{R}$ into Boyce-Codd Normal Form (BCNF); which of the following schemas could **not** be in the result of the decomposition?

5.0.1 Q4 Options

In my instance the options to select the answer from, were the following:

a) $\mathcal{S}_a : AB$

b) $\mathcal{S}_b : CD$

c) $\mathcal{S}_c : AC$

d) $\mathcal{S}_d : ABC$

5.0.2 Q4 Answer

Again in order to find which one of the following does not belong to the decomposition of $\mathcal{R}$, we have to compute it as previously.

$$\mathcal{R}(A, B, C, D) \overset{A \to B}{\equiv} \begin{cases} \mathcal{R}_1(A, B) \\ \\ \mathcal{R}_2(A, C, D) \end{cases}$$

Now we can obviously see that $\mathcal{R}_1$ is indeed in BCNF but due to the FD, $C \to D$, $\mathcal{R}_2$ relation is not. We proceed then to break up the $\mathcal{R}_2$ relation as follows.

$$\mathcal{R}_2(A, C, D) \overset{C \to D}{\equiv} \begin{cases} \mathcal{R}_3(C, D) \\ \\ \mathcal{R}_4(A, C) \end{cases}$$

We can now see that $\mathcal{R}_1$, $\mathcal{R}_3$ and $\mathcal{R}_4$ are in BCNF; so the correct answer based on my instance options is option: **d**, which is $\mathcal{S}_d : ABC$.

6 Question 5

Let's consider a relation $\mathcal{R}(A, B, C, D, E)$ for which of the following functional dependencies $\mathcal{R}$ is in Boyce-Codd Normal Form (BCNF)?

6.0.1 Q5 Options

In my instance the options to select the answer from, were the following:

a) $\mathcal{F}_a : ABD \rightarrow C, ACD \rightarrow E, ACE \rightarrow B, BC \rightarrow E$

b) $\mathcal{F}_b : BCD \rightarrow E, BDE \rightarrow C, BE \rightarrow D, BE \rightarrow A$

c) $\mathcal{F}_c : AD \rightarrow B, ABC \rightarrow E, BD \rightarrow A, B \rightarrow A$

d) $\mathcal{F}_d : BDE \rightarrow A, AC \rightarrow E, B \rightarrow C, DE \rightarrow A$

6.0.2 Q5 Answer

As we previously did in Q3 have to compute the *closures* of the given set's and check whether **all** FD's are a key of $\mathcal{R}$. Similarly to the previous question once an violating FD is found the rest are *skipped* – off we go then.

Let's start by evaluating $\mathcal{F}_a$ set.

- $(ABD)^+ \stackrel{ABD \rightarrow C}{=} (ABCD)^+ \stackrel{BC \rightarrow E}{=} ABCDE = \mathcal{R}$
- $(ACE)^+ \stackrel{ACE \rightarrow B}{=} ACEB \neq \mathcal{R}$

Looks like this set is not the correct one, let's try $\mathcal{F}_b$.

- $(BCD)^+ \stackrel{BCD \rightarrow E}{=} (BCDE)^+ \stackrel{BE \rightarrow A}{=} ABCDE = \mathcal{R}$
- $(BDE)^+ \stackrel{BDE \rightarrow C}{=} (BCDE)^+ \stackrel{BE \rightarrow A}{=} ABCDE = \mathcal{R}$
- $(BE)^+ \stackrel{BE \rightarrow D}{=} (BDE)^+ \stackrel{BDE \rightarrow C}{=} (BCDE)^+ = \stackrel{BE \rightarrow A}{=} ABCDE = \mathcal{R}$

This set, looks like its the correct one – and indeed it is; thus the correct answer is option: **b**, $\mathcal{F}_b : BCD \rightarrow E, BDE \rightarrow C, BE \rightarrow D, BE \rightarrow A$.

7 Question 6

Let's consider a relation $\mathcal{R}(A, B, C, D)$ which has the both normal and multi-valued functional dependencies that are shown below.

- $A \rightarrow B$
- $C \rightarrow D$
- $B \twoheadrightarrow C$

Suppose we decompose $\mathcal{R}$ into 4^{rth} Normal Form. Depending on the order in which we deal with $4NF$ violations we can get different final decompositions. Which one of the following relation schemas could be in the final $4NF$ decomposition?

7.0.1 Q6 Options

In my instance the options to select the answer from, were the following:

a) $\mathcal{S}_a : BC$

b) $\mathcal{S}_b : ACD$

c) $\mathcal{S}_c : ABD$

d) $\mathcal{S}_d : BCD$

7.0.2 Q6 Answer

In order to find the result we have to *decompose* our relation $\mathcal{R}$ into $4NF$, so let's perform that.

$$\mathcal{R}(A, B, C, D) \overset{A \rightarrow B}{\equiv} \begin{cases} \mathcal{R}_1(A, B) \\ \\ \mathcal{R}_2(A, C, D) \end{cases}$$

We can easily see that $\mathcal{R}_2$ violates the $C \rightarrow D$ FD, so we have to break it even more.

$$\mathcal{R}_2(A, C, D) \overset{C \rightarrow D}{\equiv} \begin{cases} \mathcal{R}_3(C, D) \\ \\ \mathcal{R}_4(A, C) \end{cases}$$

Now we can easily see that the resulting relations $\mathcal{R}_1$, $\mathcal{R}_3$ and $\mathcal{R}_4$ are in $4NF$ but does not contain any of our options, so let's try the decomposition again but this time starting with another rule instead.

$$\mathcal{R}(A,B,C,D) \overset{B \to C}{\equiv} \begin{cases} \mathcal{R}_1(B,C) \\ \\ \mathcal{R}_2(A,B,D) \end{cases}$$

We can easily see that $\mathcal{R}_2$ violates the $A \to B$ FD, so we have to break it even more.

$$\mathcal{R}_2(A,B,D) \overset{A \to B}{\equiv} \begin{cases} \mathcal{R}_3(A,B) \\ \\ \mathcal{R}_4(A,D) \end{cases}$$

Finally! Now we can easily see that the resulting relations $\mathcal{R}_1$, $\mathcal{R}_3$ and $\mathcal{R}_4$ are in $4NF$ but this time we have one of our options within the resulting relations, which is option: **a**, $\mathcal{S}_a : BC$ and thus it's the correct answer.