Rel. Design Quiz 2 Answers

Multivalued Functional Dependencies

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Introduction

In this document I'll provide the solutions for the Relational Design Multivalued Functional Dependencies quiz questions from the (infamous) dbclass MOOC kindly provided by Prof. Jennifer Widom.

Since markdown does not convey maths as it should, I created this document in order to preserve the intended notation as well as maintain cohesion; thus, since I got into all this trouble anyway solutions are a bit more detailed than the other (plain markdown) solutions in most chapters.

An **important** node is that the provided answers are the ones that were generated in my instance but the system is smart enough to generate *different* possible answers and mixes up the order as well, so your mileage may vary.

Finally you should use this documents (and my solutions for the dbcourse in general) as a *reference* and just copy-paste; you're just hurting yourself if you do so. Now without further ado, off we go!

Question 1

Let's consider a relation $\mathcal{R}(A, B, C)$ and has the following snapshot which is shown in Table 1.

Table 1

$\mathcal{R}$		
A	**B**	**C**
1	2	3
1	3	2
1	2	2
3	2	1
3	2	3

Given the following options, which one multivalued functional dependency does **not** $\mathcal{R}$ satisfy?

Q1 Options

In my instance the options to select the answer from, were the following:

a) $B \twoheadrightarrow A$

b) $AB \twoheadrightarrow A$

c) $BC \twoheadrightarrow C$

d) $C \twoheadrightarrow A$

Q1 Answer

The answer to this question is tedious in general to find, if you want to check all of the options exhaustively... Although, thankfully since we know that only one does not fit the bill we can do that easily. Now recall from the lectures that iff $A \twoheadrightarrow B$ then the following should hold.

Table 2

$\mathcal{R}$		
A	**B**	**rest**
a	b_1	r_1
a	b_2	r_2
a	b_1	r_2
a	b_2	r_1

For tuples in cyan color (the first two rows), then iff $A \twoheadrightarrow B$ then *both* yellow and orange need to be present in order to satisfy that multivalued dependency.

Taking a careful look into $\mathcal{R}$, we can easily see that for tuples $\mathcal{T}_1(1,2,3)$ & $\mathcal{T}_2(1,2,2)$ the formula mentioned above *does not* hold true as tuple $\mathcal{T}_3(3,2,1)$ and $\mathcal{T}_4(3,2,3)$ have the **same** b cell values. So the correct answer is option **a**, $B \twoheadrightarrow A$.

Question 2

Let's consider a relation $\mathcal{R}(A, B, C, D)$ which has the snapshot shown in Table 3.

Table 3

$\mathcal{R}$			
A	**B**	**C**	**D**
1	2	3	4
1	3	3	3
1	3	3	4
1	2	3	3
2	2	4	4
2	4	2	4
2	4	4	4
2	2	2	4

Given the following options, which one of the following multivalued functional dependencies this instance of $\mathcal{R}$ satisfy?

Q2 Options

In my instance the options to select the answer from, were the following:

a) $D \twoheadrightarrow C$

b) $BD \twoheadrightarrow C$

c) $A \twoheadrightarrow CD$

d) $B \twoheadrightarrow AD$

Q2 Answer

This answer need a bit of work, first of all in order to *completely satisfy* a multivalued dependency we need to have *at least* four tuples, two initials and two that are implied. Let's start by evaluating the first option, $D \twoheadrightarrow C$. To do that we have to first filter our all of the tuples which have more than 4 times the same value in D column, these are shown in Table 4.

Table 4

$\mathcal{R}_1$			
A	**B**	**C**	**D**
1	2	3	4
1	3	3	4
2	2	4	4
2	4	2	4
2	4	4	4
2	2	2	4

Let's take the first two tuples as they have a violation of that MVD already; recall from lectures as well as from Table 2 that $b_1 \neq b_2$ but clearly in this case they have the same value, equal to 3.
Moving on let's examine in a similar fashion the second option which is $BD \twoheadrightarrow C$, the resulting table after filtering out the suitable tuples is shown in Table 5.

Table 5

$\mathcal{R}_1$			
A	**B**	**C**	**D**
1	2	3	4
2	2	4	4
2	2	2	4

Aside from already existing violations, as we previously said we need *at least* four tuples to completely satisfy this MVD, so this is again rejected.

Continuing on let's examine in a similar fashion the third option which is $A \twoheadrightarrow CD$, the resulting table after filtering out the suitable tuples is shown in Table 6.

Table 6

$\mathcal{R}_1$			
A	**B**	**C**	**D**
1	2	3	4
1	3	3	3
1	3	3	4
1	2	3	3
2	2	4	4
2	4	2	4
2	4	4	4
2	2	2	4

Now if you look carefully this is essentially $\mathcal{R}$ in its entirety, and we have eight (8) tuples in total, this is starting to look like a promising option. Let's start by breaking it into two subtables one that contains the tuples which have $A = 1$ and in the other those which have $A = 2$ shown in Table 7 and Table 8 respectively.

Table 7

$\mathcal{R}_1$			
A	**B**	**C**	**D**
1	2	3	4
1	3	3	3
1	3	3	4
1	2	3	3

Table 8

$\mathcal{R}_2$			
A	**B**	**C**	**D**
2	2	4	4
2	4	2	4
2	4	4	4
2	2	2	4

Now if you apply the same rules for the tuple generation in $\mathcal{R}_1, \mathcal{R}_2$ respectively you will find that this is indeed the one multivalued functional dependency that $\mathcal{R}$ adheres to.

Finally, for the same of sanity and completeness let us evaluate the final rule as well, $B \twoheadrightarrow AD$. The tuples from $\mathcal{R}$ which are eligible as shown in Table 9.

Table 9

$\mathcal{R}_1$			
A	**B**	**C**	**D**
1	2	3	4
1	2	3	3
2	2	4	4
2	2	2	4

We can easily see that we have four (4) distinct b values in these four tuples,

thus this MVD does not hold as well.

This leads us to easily select from the given answers option **c**, $A \twoheadrightarrow CD$.

Question 3

Let's consider a relation $\mathcal{R}(A, B, C, D, E)$ which has the following multivalued functional dependencies:

- $A \twoheadrightarrow B$
- $B \twoheadrightarrow D$

Suppose that $\mathcal{R}$ has the tuples $(0, 1, 3, 4, 4)$ and $(0, 5, 6, 7, 8)$, which of the following tuples must also be in $\mathcal{R}$?

Q3 Options

In my instance the options to select the answer from, were the following:

a) $\mathcal{T}_a(0, 1, 6, 7, 4)$

b) $\mathcal{T}_b(0, 1, 2, 7, 8)$

c) $\mathcal{T}_c(0, 1, 6, 4, 3)$

d) $\mathcal{T}_d(0, 5, 2, 3, 4)$

Q3 Answer

Recall from lectures that the transitive rule does **not** apply in multivalued functional dependencies in the same way that it did for normal functional dependencies. Concretely, if you had the following functional dependencies $A \to B$, $B \to C$ then using the transitive rule you could also infer that $A \to C$.

On the other hand in the case of multivalued functional dependencies you would have the following instead: $A \twoheadrightarrow B$, $B \twoheadrightarrow C$ then $A \twoheadrightarrow C - B$. Generally this is somewhat hard to follow, what we can do though is to apply each tuple step generation based on our given MVD. The initial relation is shown in Table 10 and the expansion step based on the first MVD is shown in Table 11.

Table 10

$\mathcal{R}$				
A	**B**	**C**	**D**	**E**
0	1	2	3	4
0	5	6	7	8

$\xrightarrow{A \twoheadrightarrow B}$

Table 11

$\mathcal{R}_1$				
A	**B**	**C**	**D**	**E**
0	1	2	3	4
0	5	6	7	8
0	1	6	7	8
0	5	2	3	4

Now let's apply the second MVD that we were given.

Table 12

$\mathcal{R}_1$				
A	**B**	**C**	**D**	**E**
0	1	2	3	4
0	5	6	7	8
0	1	6	7	8
0	5	2	3	4

$\xrightarrow{B \twoheadrightarrow C}$

Table 13

$\mathcal{R}_2$				
A	**B**	**C**	**D**	**E**
0	1	2	3	4
0	5	6	7	8
0	1	6	7	8
0	5	2	3	4
0	5	2	7	4
0	5	6	3	8
0	1	6	3	8
0	1	2	7	4

Based on the given options we can easily see that the correct answer is option **d** which is the tuple $\mathcal{T}_d(0, 5, 2, 3, 4)$.

Question 4

Let's consider a relation $\mathcal{R}(A, B, C, D)$ which has the multivalued functional dependencies shown in Table 14 and its snapshot is as shown in Table 15.

Table 14: MV Deps

- $AB \twoheadrightarrow C$
- $CD \twoheadrightarrow A$
- $D \twoheadrightarrow C$

Table 15

$\mathcal{R}$			
A	**B**	**C**	**D**
1	2	3	7
1	3	2	8
4	2	5	7
4	2	5	8

Given the following $(\mathcal{M}, n)$ option pairs, which say that in order to satisfy the multivalued dependency $\mathcal{M}$ ($\mathcal{M} = 1, \mathcal{M} = 2, \mathcal{M} = 3$) a *minimum* of n tuples must be added to the given instance of $\mathcal{R}$. Only one option is correct, which one?

Q4 Options

In my instance the options to select the answer from, were the following:

a) $(2, 1)$

b) $(1, 2)$

c) $(2, 0)$

d) $(2, 4)$

Q4 Answer

This is really tricky and confusing to solve. I'll explain the intuition behind it and then give the answer. Firstly, it has to be noted that in its current state $\mathcal{R}$ satisfies *all* the MVD's so you must not select an option which would result in adding a number of tuples that would produce a violation. Now, consider the first option where we have to add only one tuple to satisfy the second MVD, $CD \twoheadrightarrow A$. This is not correct as each one of the four CD tuples have *different* values thus we should leave it as is (which is the correct answer). Again, examining the second option it says that we have to add two tuples in the relation to satisfy $AB \twoheadrightarrow C$, which is *incorrect* as these tuples have the same b values in the given pairs, so they are in violation of the MVD – thus must be treated as separate tuples. Finally, as you might've guessed from what we already said the correct answer is option: **c**, $(2, 0)$ as it does not introduce any violations in our relation.