Boolean Algebra

Boolean Algebra Part-1

Content:

1. **Introduction to K-map**
2. **2-variable K-Map**
3. **3-varibale K-Map**
4. **4-Variable K-Map**

The **Karnaugh map** method is a graphical technique which provides a simple straightforward procedure for simplification of Boolean expressions of two, three or four variables, it can also be extended to functions of five, six, or more variables. But as the number of variables increases, the excessive number of squares prevents a reasonable selection of adjacent squares and it is difficult to be sure that the best selection has been made.

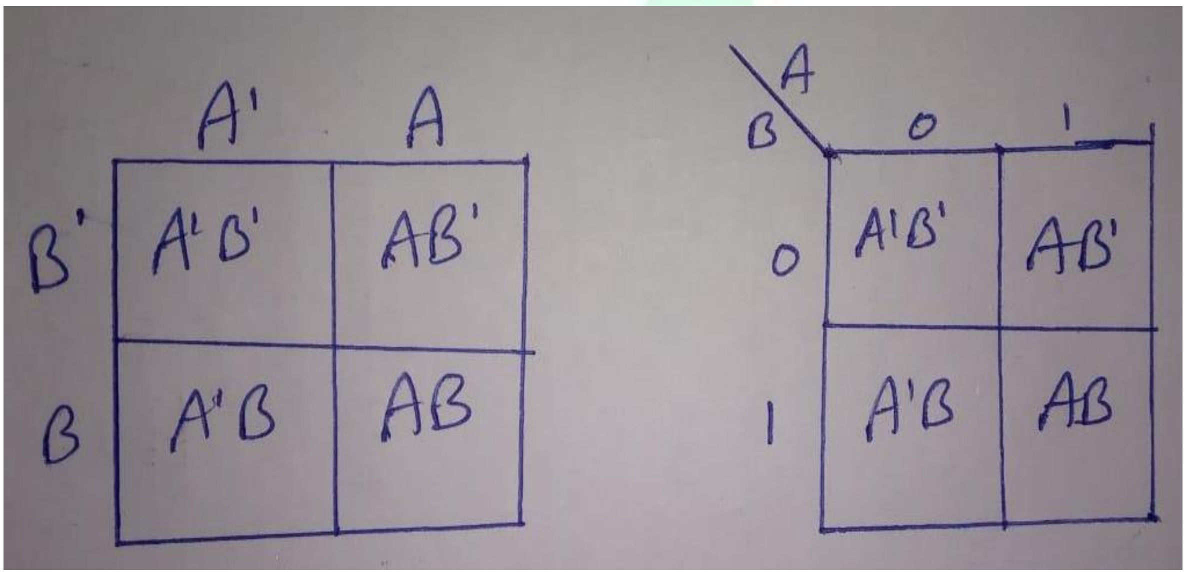
A Karnaugh map (K-map is a diagram made up of a number of squares. If the expression contains n variables, the map will have 2" squares. Each square represents a minterm and Is are written in the corresponding squares for the minterms present in the expression and Os are written in those squares which correspond to the minterms not present in the expression. Once the map is filled with Os and 1s, the canonical sum of products expression for the output can be obtained by ORing together those squares that contain 1. It is a two dimensional representation of a truth table. as it can display in a visual form all the information contained in a truth table.

2-varibale K-Map

Since the number of variables are 2, the map will have 2^2 = 4 squares. The values of one variable, say A, are listed above the top horizontal line and the values of other variable, say B, are listed on the left side. Four possible min terms with two variables A and B

AB, AB', A'B, A'B'

are represented by the four squares in the map and alterative representation in figure. Squares are said to be adjacent if the minterms that they represent differ in exactly one literal. For instance the square representing A B is adjacent to the squares representing AB and A'B'.



The expression can be simplified by properly combining those squares in the K-map which contains 1s. The process for combining 1s is called looping. Whenever there are Is in two adjacent squares in the K-map, the minterms represented by these squares can be looped and it eliminates the variable that appear in complemented and un-complemented form.

Example: Find Karnaugh maps and simplify the expressions (i) AB' + A'B' (ii) AB' + A'B (iii) AB'+ A'B+ A'B'

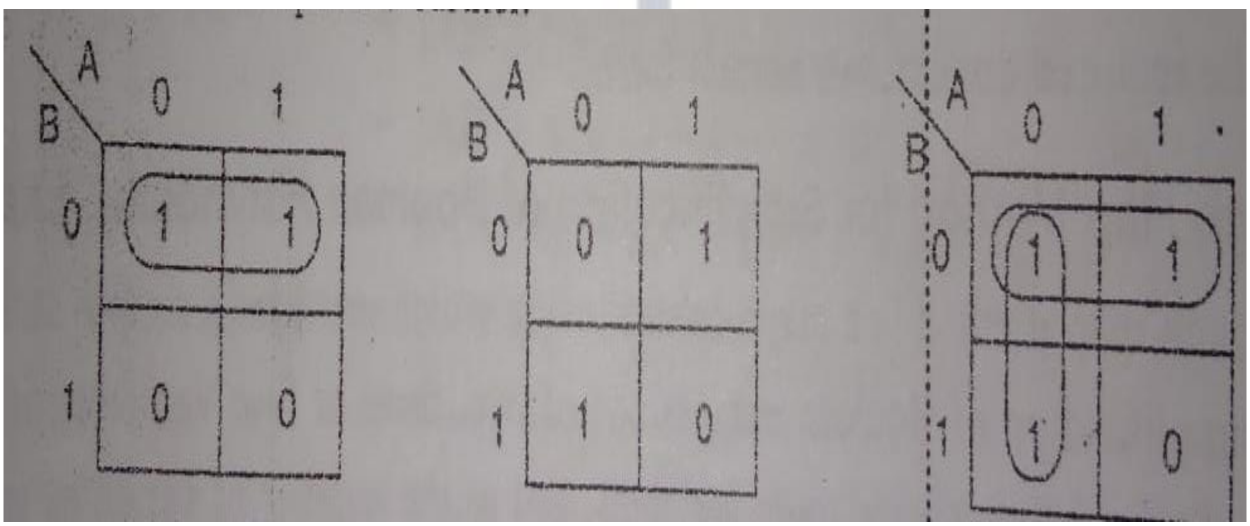
Solution. Since each expression contains two variables, three K-maps of four squares are shown below

(i) two adjacent squares A'B' and AB' containing 1 have been grouped together . They have been encircled. These two terms can be looped that eliminates the A variable since it appears both in complemented and un-complemented forms. This can be verified algebrically as follows:

AB + A'B'= (A+A')B

=1.B' = B'

(II) K-map does not contain any adjacent square of minterms containing 1, hence the expression can not be simplified further.



(III) These are two pairs of 1s is and they can be combined .Notice that 1 in first column and first row has been enclosed twice, as it is permissible to use the same 1 more than once. Looping of horizontal 1-squares gives the result B' and vertical 1-squares gives A. Hence the given expression reduces to the simplified form as A'+ B' This can also be verified algebraically as follows:

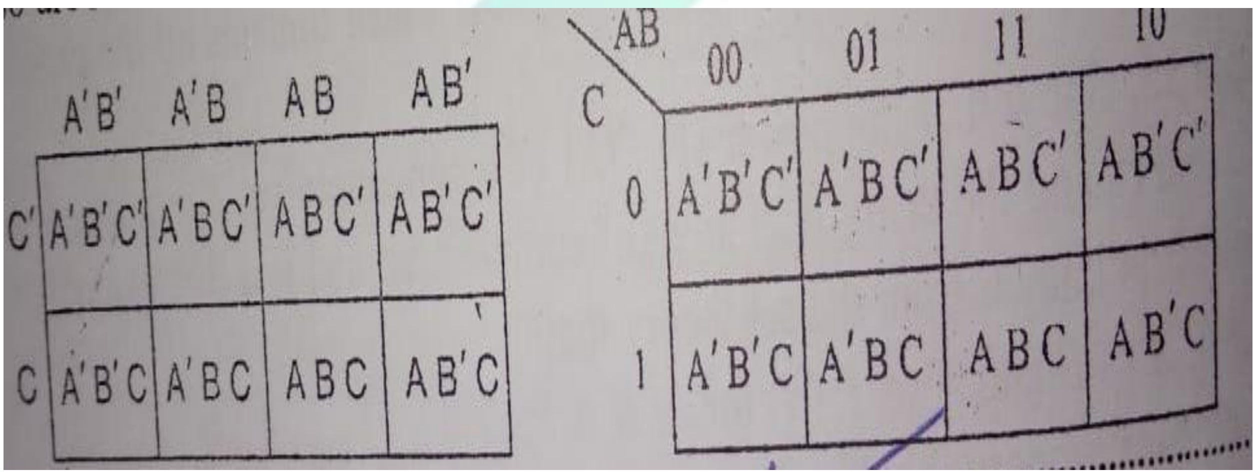
AB' A’B+ A'B'= AB' + A' (B + B')

=AB' + A

=A ‘+B’

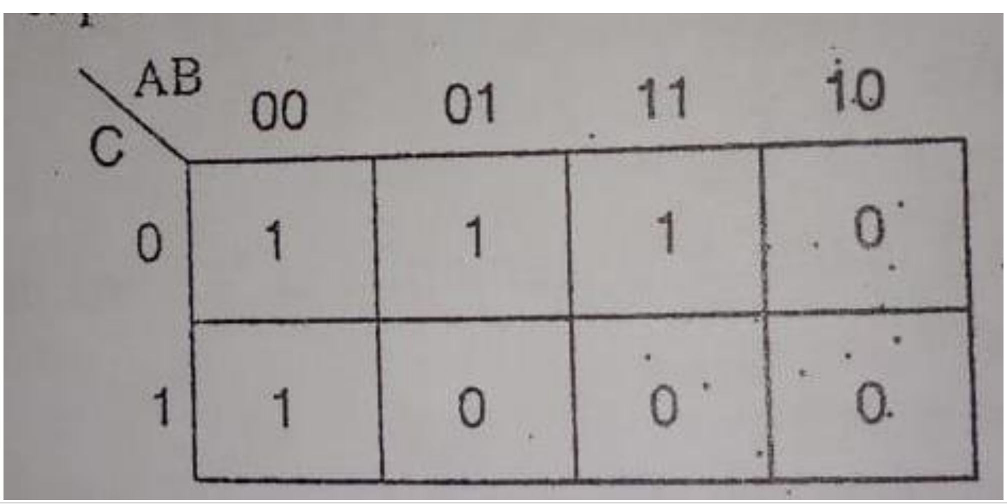
3- Variable K- Map

A Karnaugh map in three variables is a rectangle divided into eight squares, One of the ways that eight possible mintena are labeled in squares .Show the alternate way of representing three variables. In addition to square which are physically adjacent, leftrast and rightmost columns of K-map differ in only one variable. Thus A'B'C' and AB’C’ adjacent, and so are A’ B’ C and AB’C.



	A'B'	A'B	AB	AB'
C'	A'B'C'	A'BC'	ABC'	AB'C'
C	A'B'C	A'BC	ABC	AB'C

C \ AB	00	01	11	10
0	A'B'C'	A'BC'	ABC'	AB'C'
1	A'B'C	A'BC	ABC	AB'C



C \ AB	00	01	11	10
0	1	1	1	0
1	1	0	0	0

Given a minterm expansion of a function, it can be plotted on a map by placing Is in the Squares which corresponds to minterms present in the expressions and 0s in the remaining squares. Kamaugh map of the Boolean expression A'B'C' + A'B'C + A'BC' + ABC '.

To simplify a sum-of-products expansion in three variables, one has to identify groups of terms that can be combined. While forming groups of squares containing Is the following considerations must be kept in mind.

(i) The number of squares in a group must be equal to 2" such that 2, 4, 8, 16.

(ii) A square containing can be included in as many groups as desired.

(iii) Group must be largest possible groups, a group of two squares containing 1 should not be made if these squares can be included in a group of four squares. A K-map that contains group of four Is that are adjacent to each other is called a quad. Looping a quad of Is eliminates the two variables that appear in both complemented and un-complemented form.

Example: Use Karnaugh map to simplify the followings

a) X = ABC '+ ABC

b) X = A'B'C'+ AB'C'

c) X = A'B'C'+ A'BC' + ABC' + AB'C

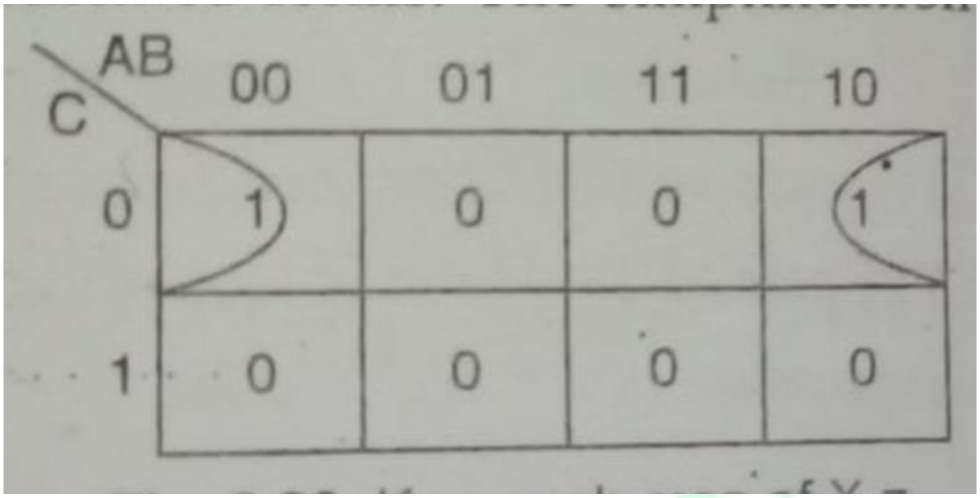
d) X = A'B'C+ A'B'C + A'BC + A'BC' + AB'C +ABC

Solution: a) it represents three variables Karnaugh map of the given Boolean function.

C \ AB	00	01	11	10
0	1	1	1	0
1	1	0	0	0

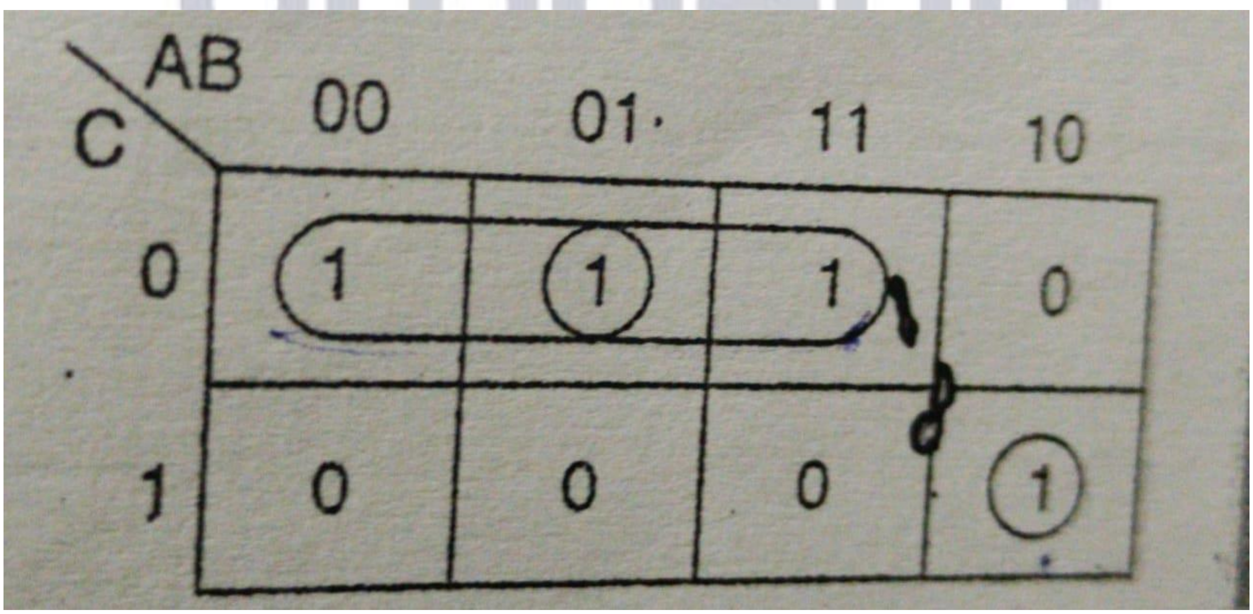
The adjacent squares representing ABC’ and ABC are grouped together. This eliminates the C variable since it appears in both un-complemented and complemented form. The simplification function will be X=AB.

b) The Boolean function X = A’B'C’ +AB'C’. In a K-map, the left and right column of squares are considered to be adjacent. Then, the two 1s in this map can be looped to provide a simplified result X= B’C’ (variable A is eliminated).

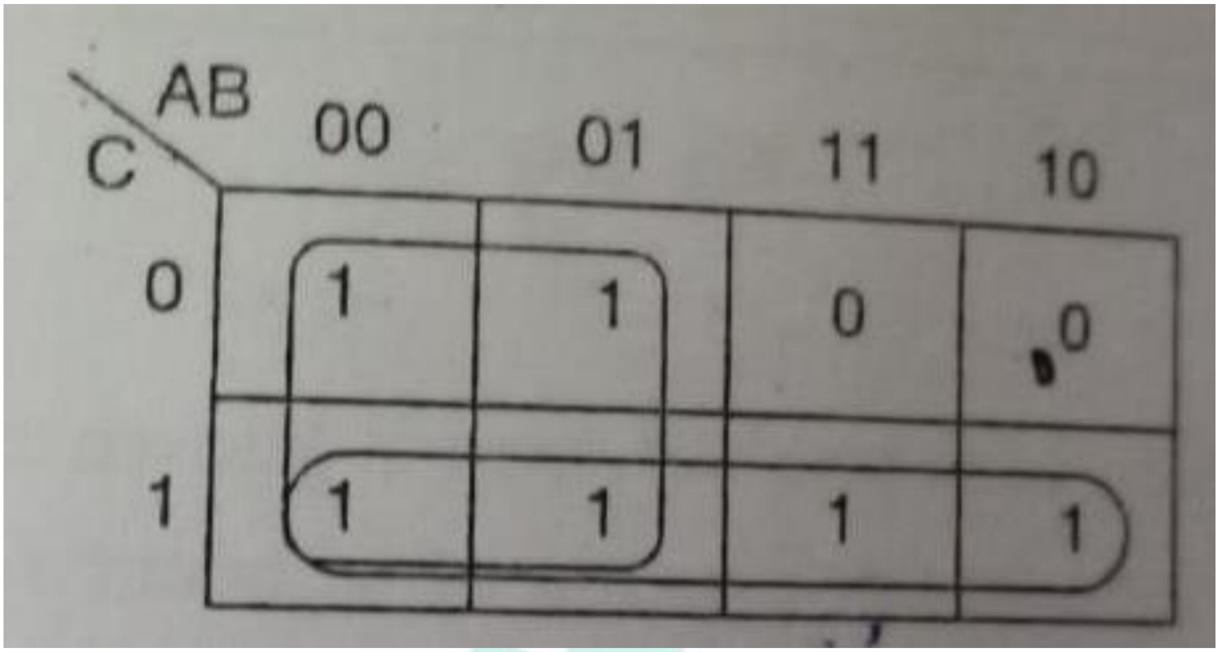


C) it represents three variables Karnaugh map of the given Boolean function There are two pairs of Is and they can be combined. Only I representing the term AB'C is isolated. The group of first two horizontal I square gives A'C' and the group second and third horizontal I square gives BC'. Hence, the simplified result is

X = A'C' + BC' + AB'C



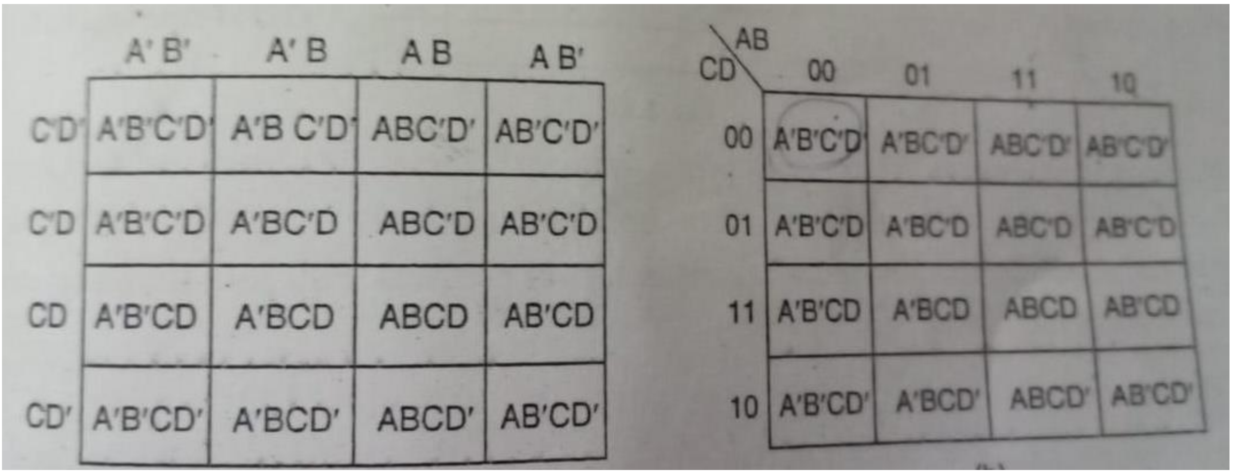
d) The Kamaugh map of the given function



Here two quads have been formed. Notice that 1s are a part of both the quads. The qand formed by A’B’C’, A’B’C’, A'BC’ and A'BC produces the resultant as A' and the quad formed by A'B'C, A’BC, ABC and AB'C produces the resultant as C. Hence the final resultant expression is X= A’ +C.

4- Variable K- Map

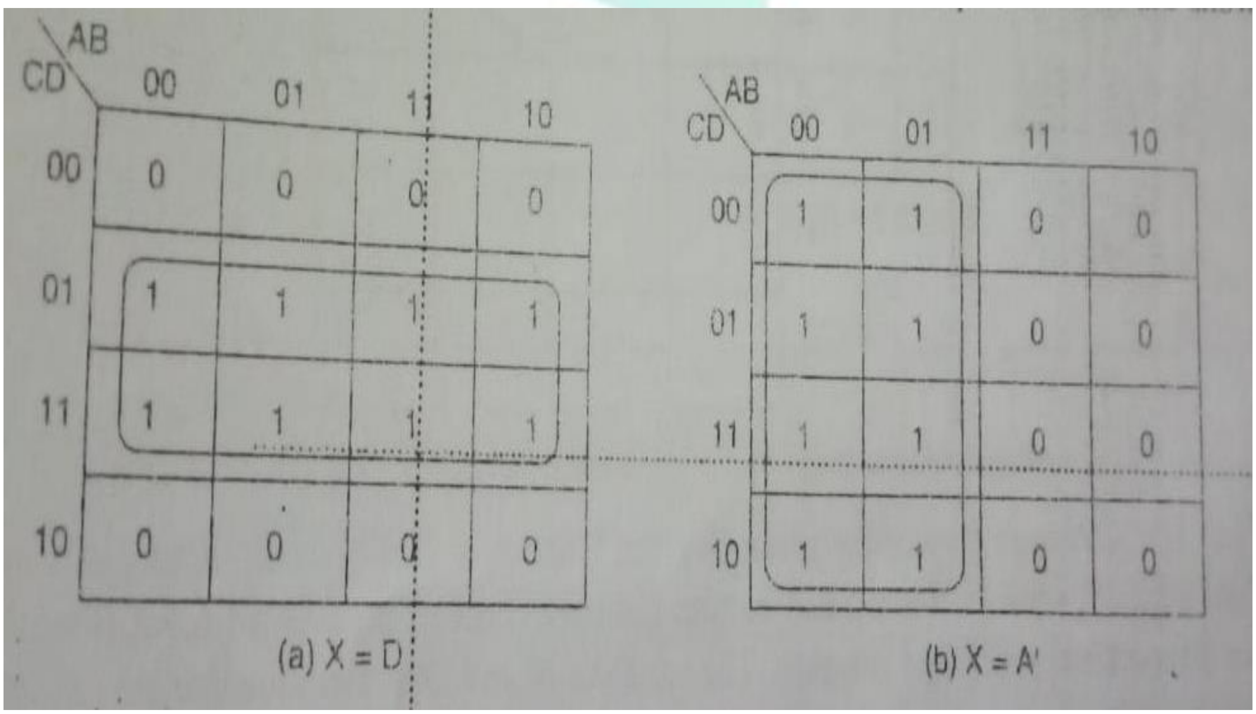
A Karnaugh map in four variables is a square divided into 16($=2^4$) squares. The squares represent the 16 possible minterms in four variables. The definition of adjacent squares to be extended so that not only are leftmost and rightmost column adjacent as in the 3-variable map, but also the first and last rows are adjacent (a) shows Karnaugh map for four variables A, B, C and D and (b) shows the alternate way of representing four variable. Note that A’B'C’D’ square in the first row is adjacent to the square A'B'C’D’ in the last row, since they differ only in the variable C. One can think of the top of the map as being wrapped around to touch the bottom of the map.



	A' B'	A' B	A B	A B'
C'D'	A'B'C'D'	A'B C'D'	ABC'D'	AB'C'D'
C'D	A'B'C'D	A'BC'D	ABC'D	AB'C'D
CD	A'B'CD	A'BCD	ABCD	AB'CD
CD'	A'B'CD'	A'BCD'	ABCD'	AB'CD'

CD \ AB	00	01	11	10
00	A'B'C'D'	A'BC'D'	ABC'D'	AB'C'D'
01	A'B'C'D	A'BC'D	ABC'D	AB'C'D
11	A'B'CD	A'BCD	ABCD	AB'CD
10	A'B'CD'	A'BCD'	ABCD'	AB'CD'

To simplify a sum-of-product expansion in four variables, one has to identify group of minterms of squares 2 4, 8 or 16 containing 1s that can be combined. A group of eight 1s that are adjacent to one another is called an octet. Looping an octet of 1s eliminates three variables that appear in both compilemented and uncomilemented form . Several examples of octet are :

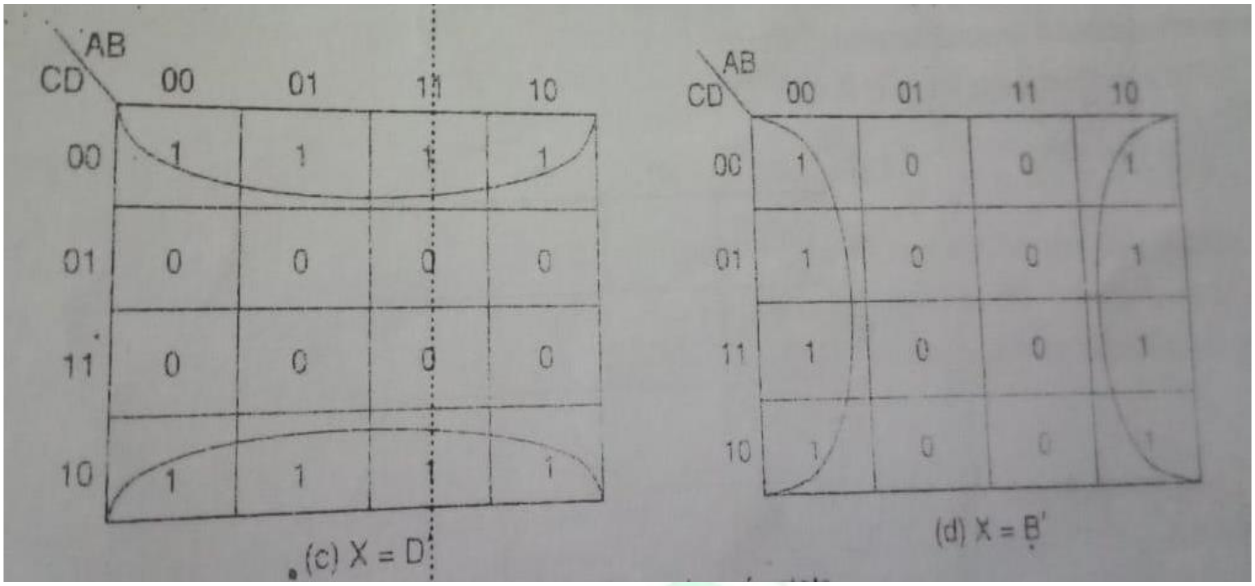


CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

(a) X = D

CD \ AB	00	01	11	10
00	1	1	0	0
01	1	1	0	0
11	1	1	0	0
10	1	1	0	0

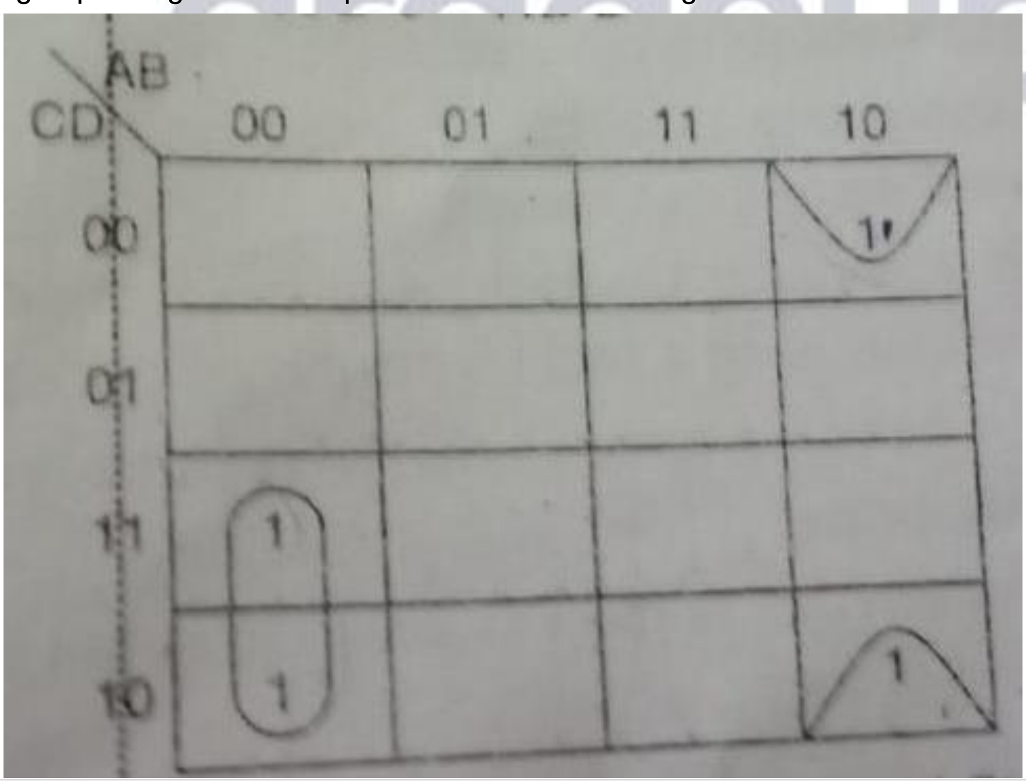
(b) X = A'



Example: Use K- map to simplify the following expression

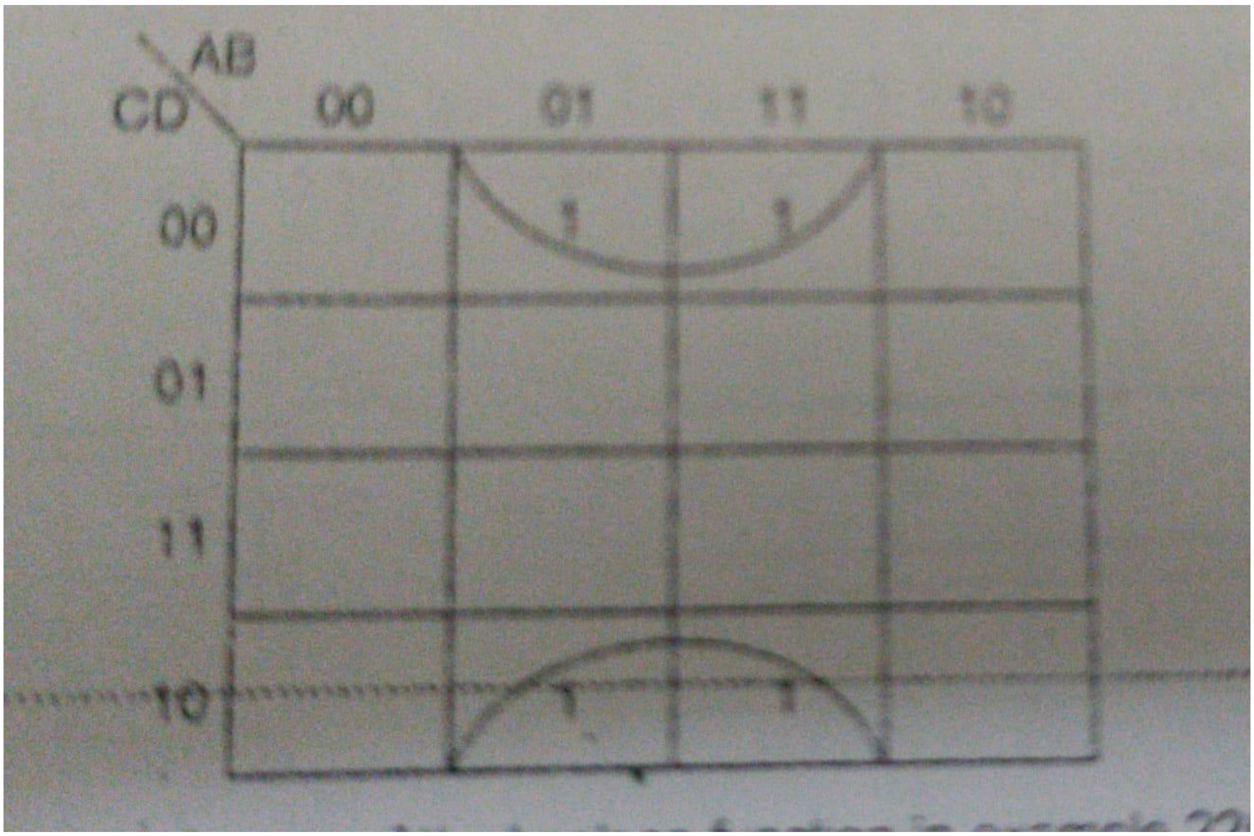
a) X= A'B'CD + A'B'CD +AB'C'D + AB'CD'
b) X=A'BC'D +ABC'D +A'BCD' +ABCD'
c) X= A'B'C'D' +AB'C'D +A'B'CD' +AB'CD'

Solution: a) The Karnaugh map of the given function .Two pairs of adjacent Is are grouped together. The pair in the first column gives term A'B'C and the pair of top



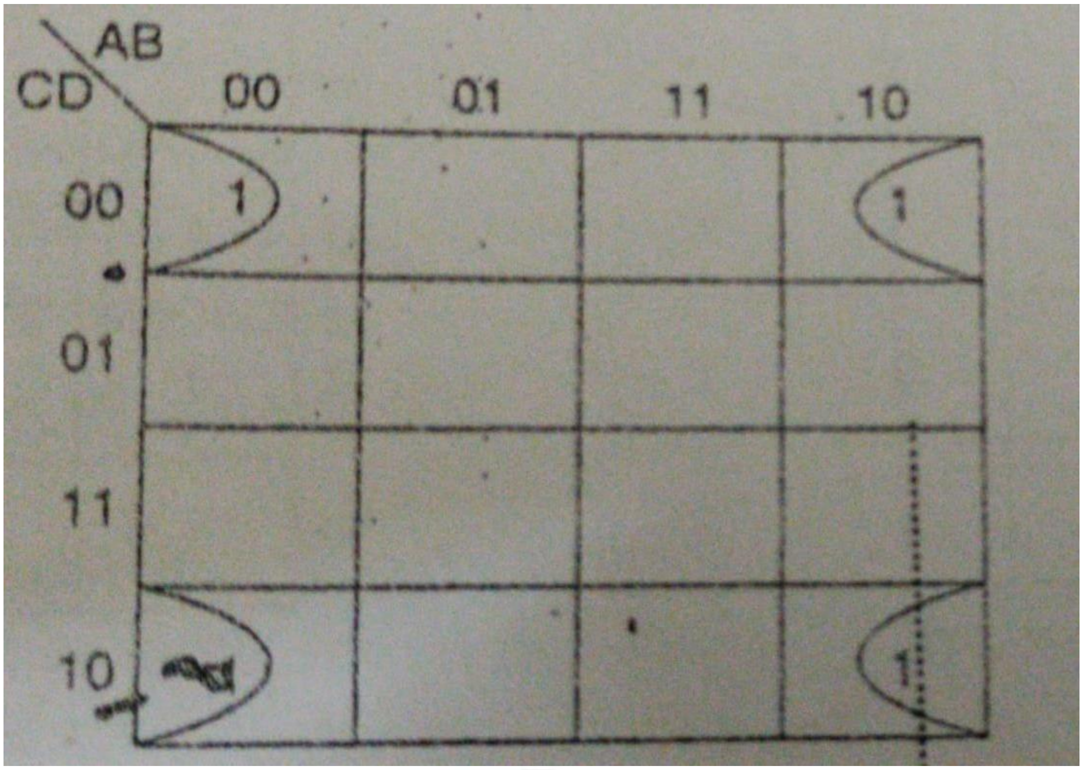
row and bottom row produces the term AB'D'. Hence, the simplified result is X= A'B'C + AB'D'.

(b) The Karnaugh tip of the given Boolean function .Two 1-Squares in the first row and two 1-square in the last row f are adjacent squares and hence they form a quad. The variables B and D' remain unchanged (A and C being complemented and uncompleted form). The resultant expression is X=B.D'.



C) The Karnaugh map of the given Boolean function. Four squares are adjacent squares, the top and bottom rows are considered to be adjacent to each other as are the leftmost and rightmost columns. The variables B' and D' remain unchanged (A and Care in complemented and uncomplemented form).

The resultant expression is X = B'.D'.



Complete Simplification Process

We have seen that on a K-map, a loop of two eliminates one variable, a loop of four eliminates two variables, are a loop of eight eliminates four variables. Therefore, to eliminate maximum number of variables, one must form the grouping with maximum number of consecutive terms having marked l. However, number of such consecutive terms, forming a group, must be expressed in powers of 2. The steps below are grouped in using the K-map method for simplifying a Boolean expression.

1. Construct the Karnaugh map and place in those squares corresponding to the minterm present in the expression and 0s in other squares

2. Find out the squares containing Is which cannot be grouped with any other square. Write the minterms corresponding to those squares.

3. Look for those squares containing 1s which are adjacent to only one square containing 1s. Loop for any pair containing such a 1 and write the simplified Corresponding terms.

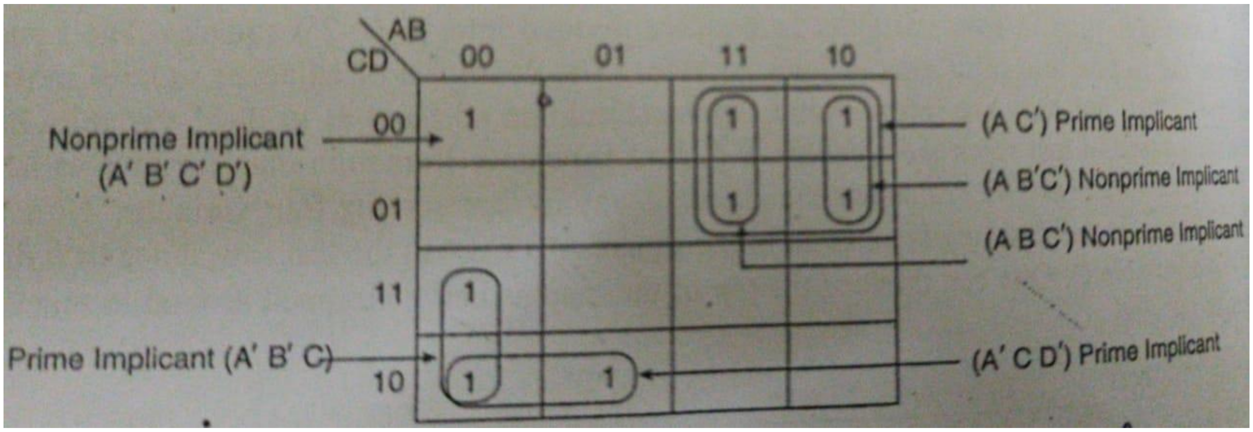
4. Loop any octet even it contains some 1s that have already been looped and write the simplified corresponding terms.

5. Loop any quad that contains one or more 1s which have not been looped, making sure to use the minimum number of loops.

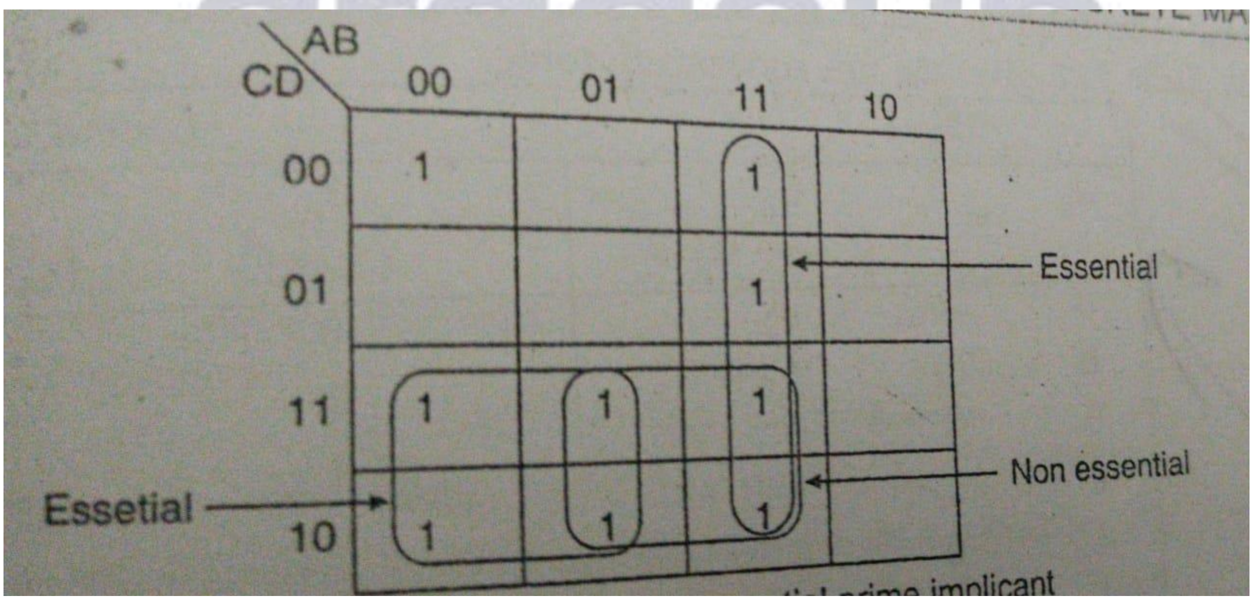
6. The simplified expression of the given Boolean expression is the sum of all the terms derived during the process from (2) to (4).

Prime Implicant

In a sum of product expression each product term is known as implicant. On a Karnaugh map each implicant relates to a single 1-square or a group of adjacent 1-squares. A prime implicant is an implicant if it can not be combined with another term to eliminate a variable. In Fig A'B'C, A'CD',and AC' are prime implicants because they can not be combined with other terms to eliminate a variable. All of the prime implications of a function can be obtained from a Karnaugh map. A single I on a map represents a prime implicant if it is not adjacent to any other Is. Two adjacent 1s on a map form a prime implicant if they are not contained in a group of four Is., four adjacent 1s form a prime implication if they are not contained in a group of eight Is etc:



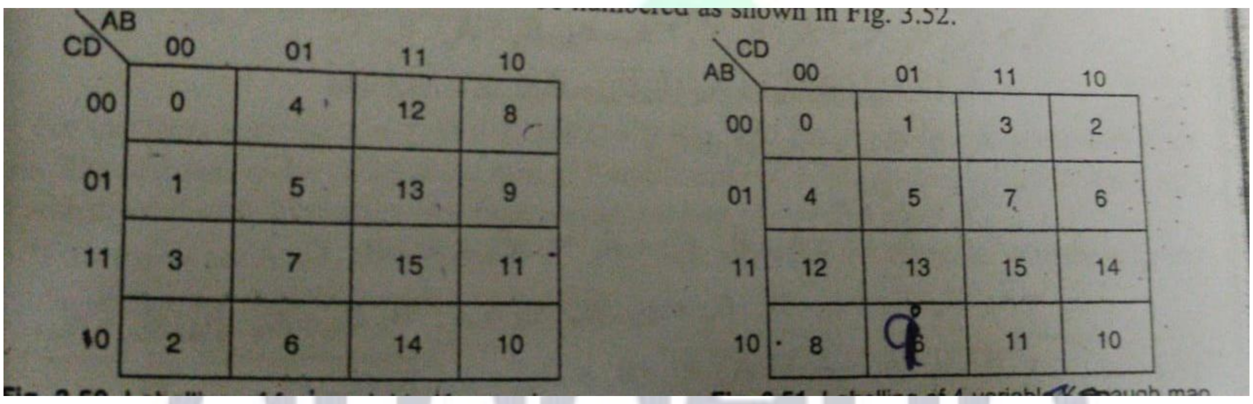
If among the minterms subsuming a prime implicant, there is at least one that is covered by this and only by this prime implicant, then the prime implicant is called an essential prime Implicant and it must be included in the minimum sum of products.



Labeling of Karnaugh-map squares

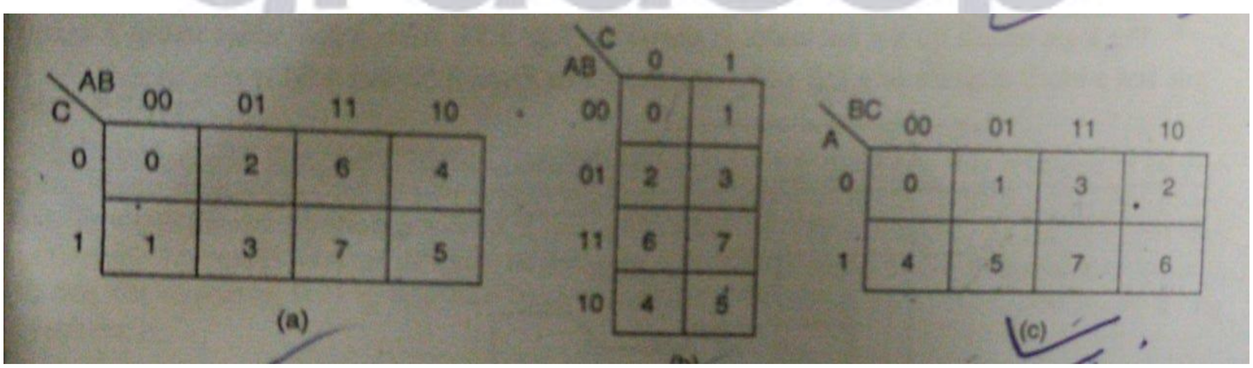
The sequence of a Kamaugh map can be numbered and these numbers are equal to the binary equivalent of logic values of the corresponding minterm variables. The numbering of 4-variable Karnaugh. The number in third column and second row is 13, which corresponds to minterm ABCD whose logic value 1101 equals to 13 in decimal system.

It is to be noted that if AB and CD are interchanged, the numbers in square will also change. Three-variable Karnaugh maps can also be numbered .



CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10



C \ AB	00	01	11	10
0	0	2	6	4
1	1	3	7	5

(a)

AB \ C	0	1
00	0	1
01	2	3
11	6	7
10	4	5

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

(c)

Alternate way of Representation: The alternate way of representing the sum of products is explained with the help of an example.

Consider the Boolean function

X = A’B’C’ + ABC’ +A’BC

This function can be represented as

X (A, B, C) = ∑(0, 6, 3) or ∑m (0, 6, 3)

Where m stands for minterms.

This means the minterms corresponding to squares 0, 6 and 3 of the Karnaugh map are present in the Boolean function and marked by 1. Other squares are marked by 0.

C \ AB	00	01	11	10
0	1	0	1	0
1	0	1	0	0

Example: Simplify the Boolean function F(A. B, C, D) = ∑ (0,1,2.3,4,5,7,6,8,9,11)

Solution: it shows the Karnaugh map of the given Boolean function. The looping of one octet and two quads are shown in the map. The looping octet will give the minterm as A'. The looping quad formed by 0, 1, 8,9 produces the minterm as B'C' and the quad formed by 1, 3,9, 11 gives the minterm as B'D. Hence, the simplified function is F=A'+ B'C' + B'D.