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Mathematical Logic Part-1

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Proposition (Statement)

A **Proposition** or **Statement** is a declarative sentence that is either true or false, but not both. For example, "Three plus three equals six". and "Three plus three equals seven " are both statement, the first because it is true and the second because it is false. Similarly "x+y> 1" is not a statement because for some values of x and y the sentence is true, whereas for others it is false. For instance, if x=1 and y=2, the sentence is true , if x=-3 and y=1, this is false. The truth or falsity of a statement is called its **truth value.** Since only two possible truth values are admitted this logic is sometimes called **two—valued logic.** Questions , exclamation and commands are not propositions.

For example, consider the following sentences:

a) The sun rises in the west.
b) 2+4=6
c) (5,6) c(7,6,5)
d) Do you speak Hindi?
e) 4-x=8
f) Close the door.
g) What a hot day!
h) We shall have chicken for dinner.

The sentences (a), (b) and (c) are statements, the first and second and third are true.
(d) is a question, not a declarative sentence, hence it is not a statement.
(e) is a declarative sentence, but not a statement, since it is true or false depends on the value of x.
(f) is not a statement, it is a command.
(g) is not a statement, it is exclamation.
(h) is a statement since it is either true or false but not both, although one has to wait until dinner to find out if it is true or false.

It is customary to represent simple propositions by letters p, q, r…………. known as **proposition variables** (Note that usually a real variable is represented by the symbol x. This means that x is not a real number but can take a real value. Similarly, a propositional variable is not a proposition but can be replaced by a proposition) Propositional variables can only assume two values: 'true' denoted by T or 1 and 'false' denoted by F or 0. If p denotes the proposition "The sun sets in the east", then instead of saying the proposition is false, one can simply say the value of p is F.

Compound Proposition

A proposition consisting of only a single propositional variable or a single propositional constant is called an **atomic (primary, primitive)** proposition or simply proposition; that is they can not be further subdivided. A proposition obtained from the combinations of two or more propositions by means of logical operators or connectives of two or more propositions or by negating a single proposition is referred to **molecular or composite or compound proposition.**

Connectives

The words and phrases (or symbols) used to form compound proposition are called **connectives.** There are five basic connectives called Negation, Conjunction, Disjunction, Conditional and Bi conditional. The following symbols are used to represent connectives.

Symbol used	Connective word	Nature of the compound statement formed by the connective	Symbolic form	Negation
$\sim, \neg, ⅂$, N	not	Negation	$\sim p$	$\sim(\sim p) = p$
$\wedge$	and	Conjunction	$p \wedge q$	$(\sim p) \vee (\sim q)$
$\vee$	or	Disjunction.	$p \wedge q$	$(\sim p)\ (\sim q)$
$\rightarrow$	if.....then	Conditional	$p \rightarrow q$	$p \wedge (\sim q)$
$<->$	if and only if	Bi- Conditional.	$p \longleftrightarrow q$	$[p \wedge (\sim q)] \vee [q \wedge (\sim p)]$

Negation

If *p* is any proposition, the negation of *p,* denoted by ~*p* and read as not *p*, is a proposition which is false when *p* is true and true when *p* is false. Consider the statement

p : Paris is in France.

Then the negation of *p* is the statement

~*p* : It is not case that Paris is in France.

Normally it is written as

~*p* : Paris is not in France.

Strictly speaking, not is not a connective, since it does not join two statements and ~*p* is not really a compound statement. However, not is a unary operation for the collection of statements, and ~*p* is a statement if *p* is considered a statement.

Note: 1**.** The following propositions all have the same meaning:

p: All people are intelligent.

q: Every person is intelligent.

r: Each person is intelligent.

s: Any person is intelligent.

2. The negation of the proposition

p: All student are intelligent.

is

~p: Some student are not intelligent.

~p: There exists a student who is not intelligent.
~p: At least one student is not intelligent.

The negation of
q: No student is intelligent.
is
~q: Some students are intelligent

Note that "No student is intelligent" is not the negation of p, " All students are intelligent" is not the negation of q.

Conjunction

If p and q are two statements, then conjunction of p and q is the compound statement denoted by $p \wedge q$ and read as *"p and q".* The compound statement $p \wedge q$ is true when both p and q are true, otherwise it is false. The truth values of $p \wedge q$ are given in the truth table shown in Table 2.1 (a).

Example . Form the conjunction of *p and q* for each of the following.

(a) p : Ram is healthy q : He has blue eyes
(b) p : It is cold. q : It is raining
(c) p : $5x + 6 = 26$ q : $x > 3$,

Solution: (a) $p \wedge q$ Ram is healthy and he has blue eyes.
(b) $p \wedge q$: It is cold and raining.
(c) $p \wedge q$: $5x + 6 = 26$ and $x > 3$.

Remark

In logic we many combine any two sentences to form a conjunction, there is no requirement that the two sentences be related in content or subject matter. Any combinations, however absurd, are permitted, of course, we are usually not interested in sentences like 'Tapas lovers Rini, and 4 is divisible by 2'.

Disjunction

If *p and q* are two statements, the disjunction of *p and q* is the compound statement denoted by $p \vee q$ and read as *"p or q"* The statement $p \vee q$ is true if at least one of *p or q* is true. It is false when both *p and q* are false. The truth of $p \vee q$ are given in the truth table shown in Table 2.1 (b).

Example. Assign a truth value to each of the following statements.
(i) $5 < 5 \vee 5 < 6$
(ii) $5 \times 4 = 21 \vee 9 + 7 = 17$
(iii) $6 + 4 = 10 \vee 0 > 2$

Solution: (i) True, since one of its component viz, $5 < 6$ is true.
(ii) False, since both of its components are false.
(iii) True, since one of its component viz, $6 + 4 = 10$ is true

Example . If p It is cold and q: It is raining.
Write simple vertical sentence which describes each of the following statements
(a) ~p (b) p ^ q (c) p v q (d) p v -q.

Solution. (a) ~p : It is not cold.
(b) $P \wedge q$: It is cold and raining.
(c) $p \vee q$: It is cold or raining.
(d) $p \vee {\sim}p$: It is cold or it is not raining.

Propositions and Truth Tables

A truth table is a table that shows the truth value of a compound proposition for all possible cases.

For example, consider the conjunction of any two proposition p and q. The compound statement *p ^ q* is true when both p and q are true, otherwise it is false. There are four possible cases.

1. *p* is true and *q* is true.
2. *p* is true and *q* is false.
3. *p* is false and *q* is true.
4. *p* is false and *q* is false.

These four cases are listed in the first two columns and the truth values of *p ^ q* are shown in the third column of Table 2.1 (a). The truth tables for the other two connectives disjunction and negation discussed above are shown in Table 2.1 (b) and 2.1 (c).

p	q	$(p \wedge q)$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$(q \vee q)$
T	T	T
T	F	T
F	T	T
F	F	F

p	$\sim p$
T	F
F	T

(a) (b) (c)

The first columns of the table are for the variables *p, q*, ……… and the number of rows depends on the number of variables. For 2 variables, 4 rows are necessary; for 3 variables , 8 rows are necessary ; and in general , for n variables, 2^n rows are required. There is then a column for each elementary stage of the construction of the proposition. The truth value at each step is determined from the previous by the definition of connectives. The truth value of the proposition appears in the last column.

Example . Construct a truth table for each compound proposition.
(i) p ^ (~q v q) *(ii)* ~ (p v q) v (~p ^ ~q)
Solution. (i) Make columns labelled *p, q, ~q, (~q v q)*. Fill in the *p and q* columns with all the logically possible combinations of Ts and Fs. Then fill in the *~q* and *~q v q* columns with the appropriate truth values. Complete the table by considering the truth values of *p ^ (~q v*

p	q	$\sim q$	$\sim q \vee q$	$p \wedge (\sim q \vee q)$
T	T	F	T	T
T	F	T	T	T
F	T	F	T	F
F	F	T	T	F

q).

(ii) Set up columns labelled p, q, ~q p v q, ~ (p v q), (~p ^ -q). Fill in the p and q columns with all the logically possible combinations of Ts and Fs. Then fill in the ~p , ~q , p v q, ~(p v q), (~p ^ ~q) columns with appropriate truth values. Complete the table by considering the truth values of ~ *(p v q) v (~ p ^ ~q).*

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$(\sim p \wedge \sim q)$	$\sim(p \vee q) \vee (\sim p \wedge \sim q)$
T	T	F	F	T	F	F	F
T	F	F	T	T	F	F	F
F	T	T	F	T	F	F	F
F	F	T	T	F	T	T	T

Table 2.3 Truth table for ~ $(p \vee q) \vee (\sim p \wedge \sim q)$

Logical Equivalence

If two proposition *P (p, q ,…….)* and *Q (p, q, ……)* where *p, q* ……. are propositional variables have the same truth values in every possible case or $P \leftrightarrow Q$ is a tautology, then the propositions are called logically equivalent or simply equivalent, and denoted by

P (p, q ,…….) = Q (p, q, ………) or P (p, q, ……….) ⇔ Q (p, q, ……….)

It is always permissible, and sometimes desirable to replace a given proposition by an equivalent one.

To test whether two proposition *P* and *Q* are logically equivalent the following steps are followed.

1. Construct the truth table for *P.*
2. Construct the truth table for Q using the same propositional variables.
3. Check each combinations of truth values of the propositional variables to see whether the value of *P* is the same as the truth value of *Q*. If in each row the truth value of *P* is the same as the truth value of *Q*, then *P* and *Q* are logically equivalent.

Algebra of Propositions

Propositions satisfy various lows which are listed in Table 2.4. These laws are useful to simplify expression. Note that, with the exception of the Involution law, all the laws of Table come in pairs, called **dual pairs.** For each expression, one finds the dual by replacing all T by F and all F by T and replacing all ^ by v and all v by ^.

Idempotent laws	
(1a) $p \vee p \equiv p$	(1b) $p \wedge p \equiv p$
Associative laws	
(2a) $(p \vee q) \vee r \equiv p \vee (q \vee r)$	(2b) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Commutative laws	
(3a) $p \vee q \equiv q \vee p$	(3b) $p \wedge q \equiv q \wedge p$
Distributive laws	
(4a) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	(4b) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Identity laws	
(5a) $p \vee F \equiv p$	(5b) $p \wedge T \equiv p$
(6a) $p \vee T \equiv T$	(6b) $p \wedge F \equiv F$
Complement laws	
(7a) $p \vee \sim p \equiv T$	(7b) $p \wedge \sim p \equiv F$
(8a) $\sim T \equiv F$	(8b) $\sim F \equiv T$
Involution law	
(9) $\sim(\sim p) \equiv p$	
De Morgan's laws	
(10a) $\sim(p \vee q) \equiv \sim p \wedge \sim q$	(10b) $\sim(p \wedge q) \equiv \sim p \vee \sim q$

From the laws given in Table 2.4 one can derive further laws. Of particular importance are the absorption laws, which are

P v (p ^ q) = p 11 (a)

P ^ (p v q) = p 11 (b)

Equivalence in 11 (a) can be proved as follows:

P v (p ^ q) = (p ^ T) v (p ^ q)

= *(p ^(T v q)*

= *p ^ T* using Identity law

= *p* using Identity law

The proof of 11 (b) is similar. The absorption laws are very useful when expression need in be simplified.

All the laws given in Table 2.4 can be proved with the help of truth table.

Example . Use truth table to prove the distribute law

p v (q ^r) = (p v q) ^(pV r)

Solution. The truth table of the compound proposition is shown in the table. Since the entries in the 5th and last column of the table are the same, the two proposition are logically equivalent.

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

Conditional Proposition

If p and q are proposition, the compound proposition " if p then q " denoted by $p \rightarrow q$ is called a **conditional proposition** and the connective is the **conditional connective.** The proposition p is called **antecedent** or **hypothesis,** and the proposition q is called the **consequent** or **conclusion.**

The only circumstances under which $p \rightarrow q$ is false when p is true and q is false.

Examples:

1. If tomorrow is Sunday then today is Saturday.
2. If it rains then I will carry an umbrella.

Here p : Tomorrow is Sunday is antecedent .

q : Today is Sunday is consequent.

and p : It rains is antecedent.

q : I will carry an umbrella is consequent.

The connective if ……………… then can also be read as follows:

1. p is sufficient for q.
2. p only if q.
3. q is necessary for p.
4. q if p.
5. q follows from p.
6. q is consequence of p.

The truth of $p \rightarrow q$ is given in table 2.6

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Note: Some authors call $p \to q$ as an implication.

Example . Which of the following proposition are true and which are false?

(a) If the earth is round then the earth travels round the sun.
(b) If Alexander Graham Bell invented telephone, then tiger have wings.
(c) If tigers have wings, then RDX is dangerous.

Solution. (i) True. Let

p : The earth is round
q : The earth travels round the sun

Here p is true and q is true and hence the conditional proposition is true.

(ii) False. Let

p : Alexander Graham Bell invented telephone.
q : Tigers have wings.

Here p is true and q is false and hence the conditional proposition is false.

(iii) True. Let

p : Tigers have wings.
q : RDX is dangerous.

Here p is false and q is true and hence the conditional proposition is true.

Example . Construct truth table for

(i) $p \vee \sim q \to p$
(ii) $(\sim(p \wedge q) \vee r) \to \sim p$

Solution. (i) The truth of the given compound statement is shown below.

p	q	$\sim q$	$p \vee \sim q$	$p \vee \sim q \rightarrow p$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	F

Table 2.7

(ii) The truth table of the given compound statement is shown below.

p	q	r	$(p \wedge q)$	$\sim(p \wedge q)$	$(\sim(p \wedge q) \vee r)$	$(\sim(p \wedge q) \vee r) \rightarrow \sim p$
T	T	T	T	F	T	F
T	T	F	T	F	F	T
T	F	T	F	T	T	F
T	F	F	F	T	T	F
F	T	T	F	T	T	T
F	T	F	F	T	T	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

Example . Use truth table to show that
p → *q* = ~ *p* v *q*

Solution. The truth table of $p \rightarrow q$ and $\sim p \vee q$ is shown below

p	q	$p \rightarrow q$	$\sim p$	$\sim p \vee q$
F	F	T	T	T
F	T	T	T	T
T	F	F	F	F
T	T	T	F	T

The logical equivalence is established by the 3rd and 5th column of the table which are identical.

Converse, Contrapositive and Inverse

If p and q are any two propositions, then some other conditional proposition related to $p \rightarrow q$ are

(i) **Converse :** The converse of $p \rightarrow q$ *is* $q \rightarrow p$.

(ii) **Contrapositive :** The contrapositive of $p \rightarrow q$ *is* $\sim q \rightarrow \sim p$.

(iii) **Inverse :** The inverse of $p \rightarrow q$ *is* $\sim p \rightarrow \sim q$.

- The truth of the four proposition follow:

Consider the statement

p	q	Conditional $p \rightarrow q$	Converse $q \rightarrow p$	Inverse $\sim p \rightarrow \sim q$	Contrapositive $\sim q \rightarrow \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

p : It rains.

q: The crops will grow

The conditional proposition $p \rightarrow q$ states that,

$p \rightarrow q$: If it rains then the crops will grow.

The conserve of $p \rightarrow q$ namely $q \rightarrow p$ states that,

$q \rightarrow p$: If the crops grow, then there has been rain.

The contrapositive of $p \rightarrow q$, namely $\sim q \rightarrow \sim p$ state that,

$\sim q \rightarrow \sim p$ *:* If the crops do not grow then there has been no rain.

The inverse of $p \rightarrow q$, namely $\sim p \rightarrow \sim q$ states that,

$\sim p \rightarrow \sim q$: If it does not rain then the crops will not grow.

Notice that a conditional proposition and its converse or inverse are not logically equivalent. On the other hand, a conditional proposition and its contrapositive are logically equivalent.

Example . Show that contrapositive and conditional proposition are logically equivalent that is

$\sim q \rightarrow \sim p = p \rightarrow q$

Solution 9. The truth table of $\sim q \rightarrow \sim p$ *and* $\rightarrow q$ are shown below and the logical. Equivalence is established by the last two columns of the table, which are identical.

p	q	$\sim p$	$\sim q$	$\sim q \rightarrow \sim p$	$p \rightarrow q$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Example Prove that if x^2 is divisible by 4, then x is even.

Solution. Let *p and q* be the propositions such that

p : x^2 is divisible by 4.

and q : x is even.

The conditional is of the form $p \rightarrow q$. The contrapositive is $\sim q \rightarrow \sim p$, which states in words:

If x is odd, then x^2 is not divisible by 4.

The proof of contrapositive is easy.

Since x is odd, one can write x = 2k + 1, for some integer k. Hence

$$X^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 4\ (k^2 + k + 1/4)$$

Since $k^2 + k$ is an integer, $k^2 + k + 1/4$ is not an integer, therefore x^2 is not divisible by 4

Bi conditional Statement

If *p and q* are statement, then the compound statement *p* if and only if *q* , depend by *p* <-> *q* is called a bi-conditional statement and the connective if and only if is the **bi-conditional connective.** The bi-conditional statement *p* <-> *q* can also be stated as " *p* is a necessary and sufficient condition for *q*" Example include

1. He swims if and only if the water is warm.
2. Sales of houses fall if and only if interest rate rises.

The truth table of p <-> q is given in Table 2.12. It may be noted *p* <-> *q* is true when both *p* and *q* are true or both *p and q* are false.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Example Show that *p <-> q = (p → q) ^ (q → p).*
Solution. Table 2.13. shows that these two expression are logically equivalent ; the two columns corresponding to the given two expression have identical truth values.

p	q	$p \leftrightarrow q$	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

Table 2.13

Example Show that *p <-> q = (p v q) → (p ^ q)*, using
(a) Truth table
(b) algebra of propositions.
Solution. (a) Truth table shows that these two expressions are logically equivalent ; the two columns corresponding to the given two expression have identical truth value.

p	q	$p \leftrightarrow q$	$p \vee q$	$p \wedge q$	$(p \vee q) \rightarrow (p \wedge q)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	F	F
F	F	T	F	F	T

b. $p \leftrightarrow\ = (p \rightarrow q) \wedge (q \rightarrow p)$
$= (\sim p \vee q) \wedge (\sim q \vee p)$
$= [(\sim p \vee q) \wedge \sim q\} \vee [(\sim p \vee q) \wedge p]$ by Distributive law
$= [\sim q \wedge (\sim p \vee q)] \vee [p \wedge (\sim p \vee q)]$ by Commutative law
$= [(\sim q \wedge q) \vee (\sim q \wedge \sim p)] \vee [(p \wedge q) \vee (p \wedge \sim q]$
by Distributive law
$= [F \vee (\sim q \wedge \sim p)] \vee [(p \wedge q) \vee F]$ by Complement law
$= (\sim q \wedge \sim p) \vee (p \wedge q)$ by Identity law
$= [\sim (p \vee q)] \vee (p \wedge q)$ by De Morgan’s law
$= (p \vee q) \rightarrow (p \wedge q)$

Negation of Compound Statements
Negation of conjunction:
The negation of a conjunction $p \wedge q$ is the disjunction of the negation of p and the negation of q. Equivalent, we write
$\sim(p \wedge q) = \sim p \vee \sim q$
In order to prove the above equivalence, we prepare the following table.

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Table 2.15

Example Write the negation of each of the following conjunctions:
(a) Paris is in France and London is in England.
(b) $2 + 4 = 6$ and $7 < 12$.
Solution. (a) Let p : Paris is in France and q : London is in England.
Then, the conjunction in (a) is given by *(p ^ q)*.
Now
$\sim p$: Paris is not in France and
$\sim q$: London is not in England.

Therefore, negation of p ^ q is given by
~ $(p$ ^ $q)$: Paris is not in France or London is not in England.
(b) Let p : 2 + 4 = 6 and q : 7 < 12
Then the conjunction in (b) is given by p ^ q .
Now ~ p : 2 +4 = 6 and ~ q : 7 < 12

Negation of Disjunction:
The negation of a disjunction p v q is the conjunction of the negation of p and the negation of q. Equivalently, we write
~ $(p$ v $q)$ = ~ p ^ ~ q

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \vee q$	$\sim (p \vee q)$
T	T	F	F	F	T	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	T	F	T

In order to prove the above equivalence, we prepare the following table.

Example Write the negation of each of the following disjunction:
(a) Ram is in class XI or Arun is in Class XII.
(b) 9 is greater than 4 or 6 is less than 8.
Solution.(a) Let p Ram is in class XI and q : Arun is in class XII.
Then, the disjunction in (a) is given by p v q.
Now ~p : Ram is not in Class XI.
~q : Arun is not in class XII.

Then, negation of p v q is given by
~ $(p$ v $q)$: Ram is not in class X and Arun is not in Class XII.
(b) Let p : 9 is greater than 4 and q : 6 is less than 8.
Then negation of p v q is given by
~(p v q) : 9 is not greater than 4 and 6 is not less than 8.

Negation of a Negation: A negation of negation of a statement is the statement itself.
Equivalently, we write
~ (~ p) = p

Example Verify for the statement
p: Roses are red
Solution. The negation of p is given by
~p: Roses are not red.
Therefore, the negation of negation of p is ~(~p):
It is not the case that Roses are not red.
Or
It is false that Roses are not red.
Or
Roses are red.

Negation of Conditional: If *p and q* are two statement, then
$\sim (p \to q) = p \wedge \sim q$
In order to prove the above Equivalence, we prepare the following table.

p	q	$p \to q$	$\sim (p \to q)$	$\sim q$	$p \wedge \sim q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

Example. Write the negation of each of the following statements:
(a) If it is raining , then the game is cancelled.
(b) If he studies, he will pass the examination.
Solution. (a) Let p *:* It is raining. q: The game is cancelled .
The given statement can be written as $p \to q$. The negation of $p \to q$ is written as
$\sim (p \to q) = p \wedge q$
Hence the negation of the given statement is it is raining and the game is not cancelled
(b) He studies and q : He will pass the examination.
The given statement can be written as $p \to q$. The negation of $p \to q$ is written as
$\sim (p \to q) = p \wedge \sim q$
Hence the negation of the given statement is he studies and he will not pass the examination
Negation of Bi-conditional : If *p and q* are two statement, then
$\sim (p \leftrightarrow q) = p \leftrightarrow \sim q = \sim p \leftrightarrow q$
In order to prove the above equivalence, we prepare the following table.

p	q	$p \leftrightarrow q$	$\sim(p \leftrightarrow q)$	$\sim p$	$\sim p \leftrightarrow q$	$\sim q$	$p \leftrightarrow \sim q$
T	T	T	F	F	F	F	F
T	F	F	T	F	T	T	T
F	T	F	T	T	T	F	T
F	F	T	F	T	F	T	F

Note that $\sim (p \leftrightarrow q) = \sim [(p \to q) \wedge (q \to p)] = \sim [(\sim p \vee q) \wedge (\sim q \vee p)]$
$= (p \wedge q) \vee (q \wedge \sim p)$
Example Write the negation of each of the following statements:
(a) He swims if and only if the water is warm.
(b) This computer program is correct if and only if , it produces the correct answer for possible sets of input data.
Solution. (a) Let *p* : He swims and *q* : The water is warm.
The given statement can be written as $p \leftrightarrow q$*.* The negation of $p \leftrightarrow q$ is written as
$\sim (p \leftrightarrow q) = p \leftrightarrow \sim q = \sim p \leftrightarrow q$.
Hence the negation of the given statement is either of the following :
He swims if and only if the water is not warm.

He does not swim if and only if the water is warm.
(b) Let p : This computer program is correct and
q : It produces the correct answer for all possible sets of input data.
The given statement can be written as $p <-> q$. The negation of $p <-> q$ is written as
$\sim (p <-> q) = p <-> \sim q = \sim p <-> q$
Hence the negation of the given statement is either of the following
This program is correct is and only if it does not produce the correct answer for all possible sets of input data.
This program is not correct if and only if it produces the correct answer for all possible sets of input data.