gradeup

Optimization Part -1

Optimization Part-1
Content:
1. Transportation Problem
a. North West Corner Cell Method.
b. Least Call Cell Method.
c. Vogel's Approximation Method (VAM).

Transportation Problem

Introduction
It is a type of **Linear Programming Problem (LPP)** in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the sources and destination respectively such that the total cost of transportation is minimized. It is also sometimes called as Hitchcock problem.

Types of Transportation problems:
Balanced: When supplies and demands are equal then the problem is said to be a balanced transportation problem.
Unbalanced: When the supply and demand are not equal then it is said to be an unbalanced transportation problem. In this type problem, either a dummy row or a dummy column is added according to the requirement to make it a balanced problem. It can be solved similar to the balanced problem.

Methods to Solve:
To notice the initial basic feasible solution there are three methods:
1. North West Corner Cell Method.
2. Least Call Cell Method.
3. Vogel's Approximation Method (VAM).

Basic structure of transportation problem:

Destination

Source	D1	D2	D3	D4	Supply(s_i)
O1	C_{11}	C_{12}	C_{13}	C_{14}	s_1
O2	C_{21}	C_{22}	C_{23}	C_{24}	s_2
O3	C_{31}	C_{32}	C_{33}	C_{34}	s_3
O4	C_{41}	C_{42}	C_{43}	C_{44}	s_4
Demand (d_j):	d_1	d_2	d_3	d_4	

In the above table D1, D2, D3 and D4 are the destinations where the products/goods are to be delivered from different sources S1, S2, S3 and S4. S_i is the supply from the source O_i. d_j is the demand of the destination D_j. C_{ij} is the cost when the product is delivered from source S_i to destination D_j.

North-West Corner Method
An introduction to this problem has been discussed in the previous article, in this article, finding the initial basic feasible solution using the NorthWest Corner Cell Method will be discussed.

Destination

Source	D1	D2	D3	D4	Supply
O1	3	1	7	4	300
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand:	250	350	400	200	1200

Explanation: Given three sources **O1**, **O2** and **O3** and four destinations **D1**, **D2**, **D3** and **D4**. For the sources **O1**, **O2** and **O3**, the supply is **300**, **400** and **500** respectively. The destinations **D1**, **D2**, **D3** and **D4** have demands **250**, **350**, **400** and **200** respectively.

Solution: According to this method, **(O1, D1)** has to be the starting point i.e. the north-west corner of the table. Each value in cell is considered as the cost per transportation. Compare the demand for column **D1** and supply from the source **O1** and allocate the minimum of two to the cell **(O1, D1)** as shown in the figure.
The demand for Column **D1** is completed so the entire column **D1** will be cancelled. The supply from the source **O1** remains **300 – 250 = 50**.

Destination

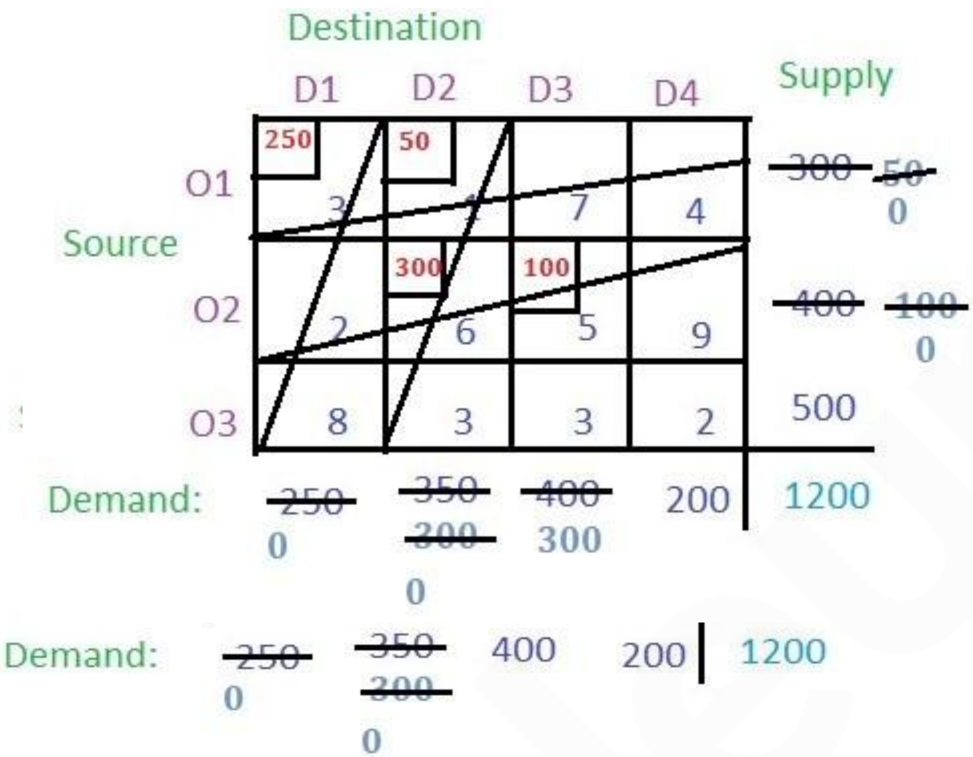
Source	D1	D2	D3	D4	Supply
O1	250 3	1	7	4	~~300~~ 50
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand:	~~250~~ 0	350	400	200	1200

Now from the remaining table i.e. excluding column **D1**, check **(O1, D2)** and assign the minimum among the supply for the respective column and the rows. The supply from **O1** is **50** which is less than the demand for **D2** (i.e. 350), so allocate **50** to the cell **(O1, D2)**. Since the supply from row **O1** is completed cancel the row **O1**. The demand for column **D2** remain **350 – 50 = 300**.

Destination

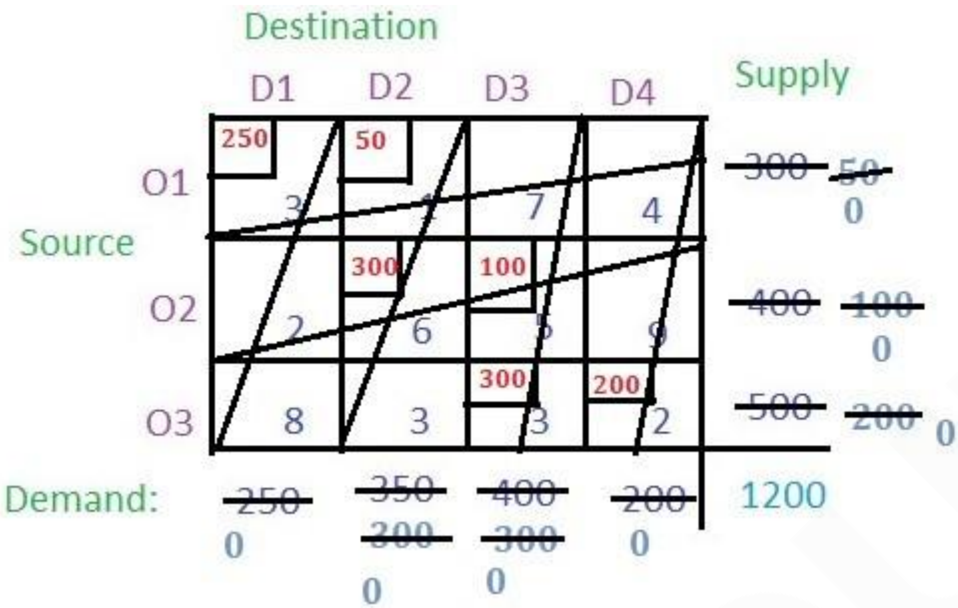
Source	D1	D2	D3	D4	Supply
O1	250 3	50 1	7	4	~~300~~ ~~50~~ 0
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand:	~~250~~ 0	~~350~~ 300	400	200	1200

From the remaining table the north-west corner cell is **(O2, D2)**. The minimum supply from source **O2** (i.e 400) and demand for column **D2** (i.e 300) is **300**, so allocate **300** to the cell **(O2, D2)**. The demand for the column **D2** is completed so cancel the column and the remaining supply from source **O2** is **400 – 300 = 100**.



Now from remaining table find **(O2, D3)** and compare the **O2** supply (i.e. 100) and the demand for **D2** (i.e. 400) and assign the smaller (i.e. 100) to the cell **(O2, D2)**. The supply from **O2** is completed so cancel the row **O2**. The remaining demand for column **D3** remains **400 – 100 = 300**.

Proceeding in the same way, the final values of cells will be:



Note: In the end remaining cell the demand for the respective columns and rows are equal which was cell **(O3, D4)**. In this case, the supply from **O3** and the demand for **D4** was **200** which was allocated to this cell. At last, nothing remained for any row or column.
Now just multiply the assigned value with the respective cell value (i.e. the cost) and add all of them to get the basic solution i.e. **(250 * 3) + (50 * 1) + (300 * 6) + (100 * 5) + (300 * 3) + (200 * 2) = 4400**

Least Cost Cell Method

Destination

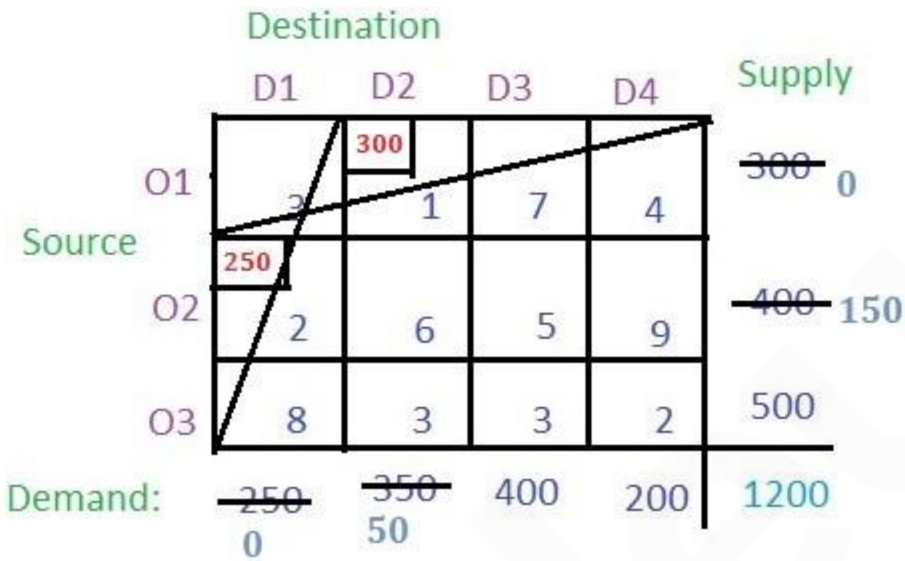
Source	D1	D2	D3	D4	Supply
O1	3	1	7	4	300
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand:	250	350	400	200	1200

Solution: According to this process, the least cost among all the cells in the table has to be found which is **1** (i.e. cell **(O1, D2)**).
Now check the supply from the row **O1** and demand for column **D2** and assign the smaller value to the cell. The smaller value is **300** so allocate this to the cell. The supply from **O1** is completed so cancel this row and remaining demand for the column **D2** is **350 – 300 = 50**.

Destination

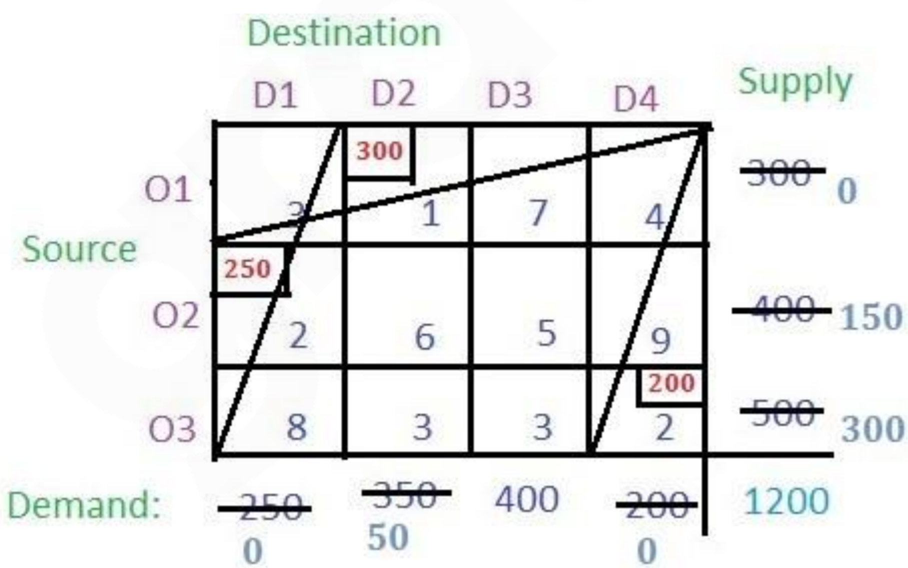
Source	D1	D2	D3	D4	Supply
O1	3	300 / 1	7	4	~~300~~ 0
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand:	250	~~350~~ 50	400	200	1200

Now find the cell with the least cost among the remaining cells. There are two cells with least cost i.e. **(O2, D1)** and **(O3, D4)** with cost **2**. Lets select **(O2, D1)**. Now find the demand and supply for the respective cell and assign the minimum among them to the cell and cancel the row or column whose supply or demand becomes **0** after allocation.

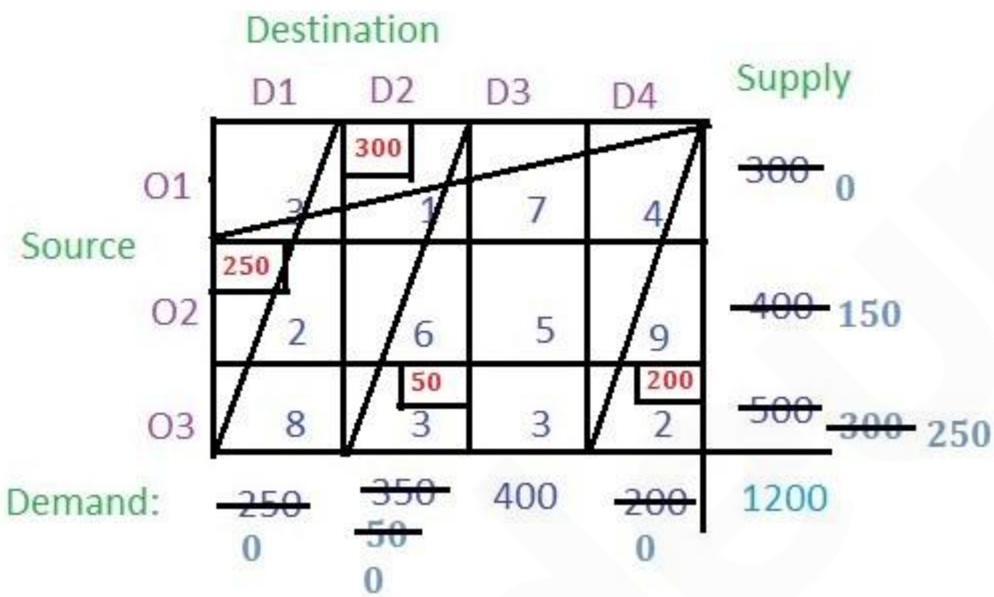


Now the cell with the least cost is **(O3, D4)** with cost **2**. Assign cell with **200** as the demand is smaller than the supply. So the column gets cancelled.

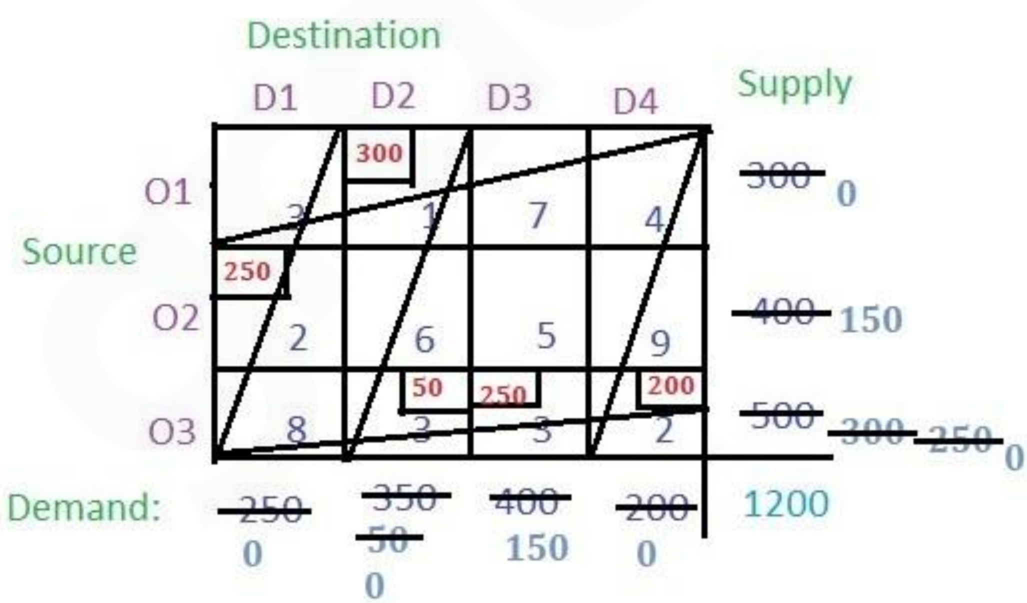
.



There are two cells among the unallocated cells that have the least cost. Choose any at random say **(O3, D2)**. Allocate cell with a minimum among the supply from the respective row and the demand of the respective column. Cancel the row or column with zero value.



Now the cell with the least cost is **(O3, D3)**. Allocate minimum of supply and demand and cancel the row or column with zero value.



The only remaining cell is **(O2, D3)** with cost **5** and its supply is **150** and demand is **150** i.e. demand and supply are equal. Allocate it to cell.

Destination

Source	D1	D2	D3	D4	Supply
O1	3	300 / 1	7	4	~~300~~ 0
O2	250 / 2	6	150 / 5	9	~~400~~ ~~150~~ 0
O3	8	50 / 3	250 / 3	200 / 2	~~500~~ ~~300~~ ~~250~~ 0
Demand:	~~250~~ 0	~~350~~ ~~50~~ 0	~~400~~ ~~150~~ 0	~~200~~ 0	1200

Just multiply the cost of the cell with their respective allocated values and add all of them to get the basic solution i.e. **(300 * 1) + (250 * 2) + (150 * 5) + (50 * 3) + (250 * 3) + (200 * 2) = 2850**

Vogel's Approximation Method

Destination

Source	D1	D2	D3	D4	Supply
O1	3	1	7	4	300
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand:	250	350	400	200	1200

Solution:

• For each row find the least value and then the second least value and take the absolute difference of these two least values and write it in the corresponding row difference as shown in the image below. In row O1, 1 is the least value and 3 is the second least value and their absolute difference is 2. Similarly, for row O2 and O3, the absolute differences are 3 and 1 respectively.
• For each column find the least value and then the second least value and take the absolute difference of these two least values then write it in the corresponding column difference as shown in the figure. In column D1, 2 is the least value and 3 is the second least value and their absolute difference is 1. Similarly, for column D2, D3 and D3, the absolute differences are 2, 2 and 2 respectively.

Destination

Source	D1	D2	D3	D4	Supply	Row Difference
O1	3	1	7	4	300	2
O2	2	6	5	9	400	3
O3	8	3	3	2	500	1
Demand:	250	350	400	200	1200	
Column Difference:	1	2	2	2		

• These value of row difference and column difference are also called as penalty. Now select the maximum penalty. The maximum penalty is 3 i.e. row O2. Now find the cell with the least cost in row O2 and allocate the minimum among the supply of the respective row and the demand of the respective column. Demand is smaller than the supply so allocate the column's demand i.e. 250 to the cell. Then cancel the column D1.

Destination

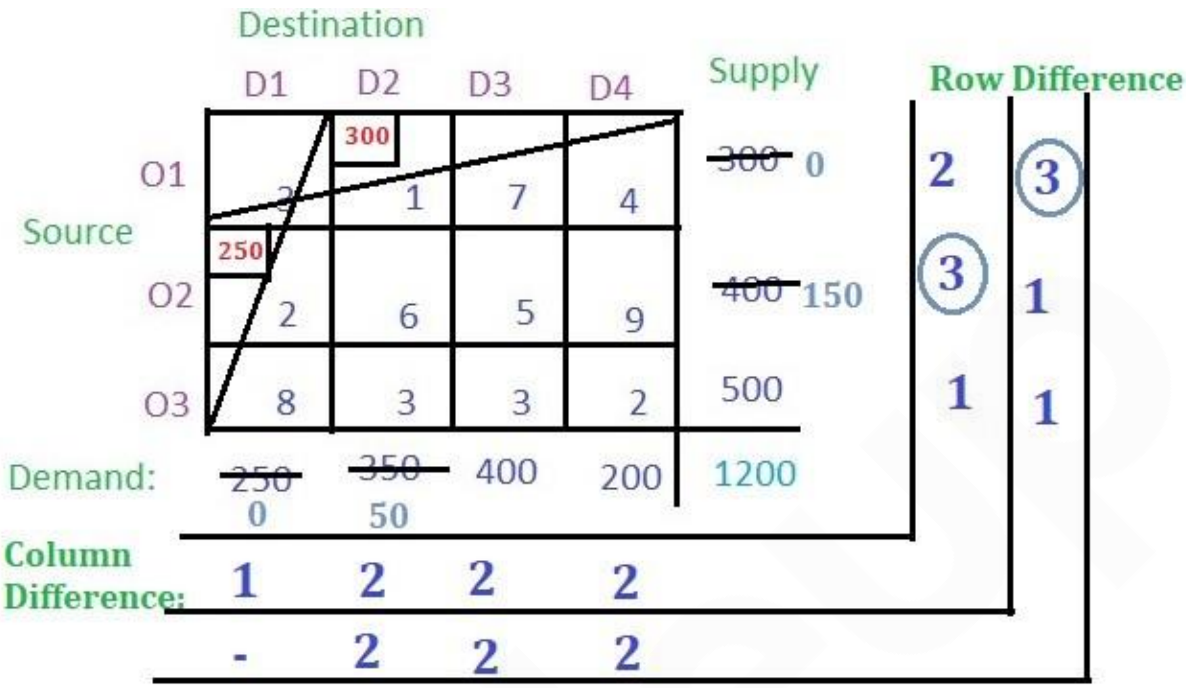
Source	D1	D2	D3	D4	Supply	Row Difference
O1	3	1	7	4	300	2
O2	250 2	6	5	9	~~400~~ 150	3
O3	8	3	3	2	500	1
Demand:	~~250~~ 0	350	400	200	1200	
Column Difference:	1	2	2	2		

- From the remaining cells, find out the row difference and column difference.

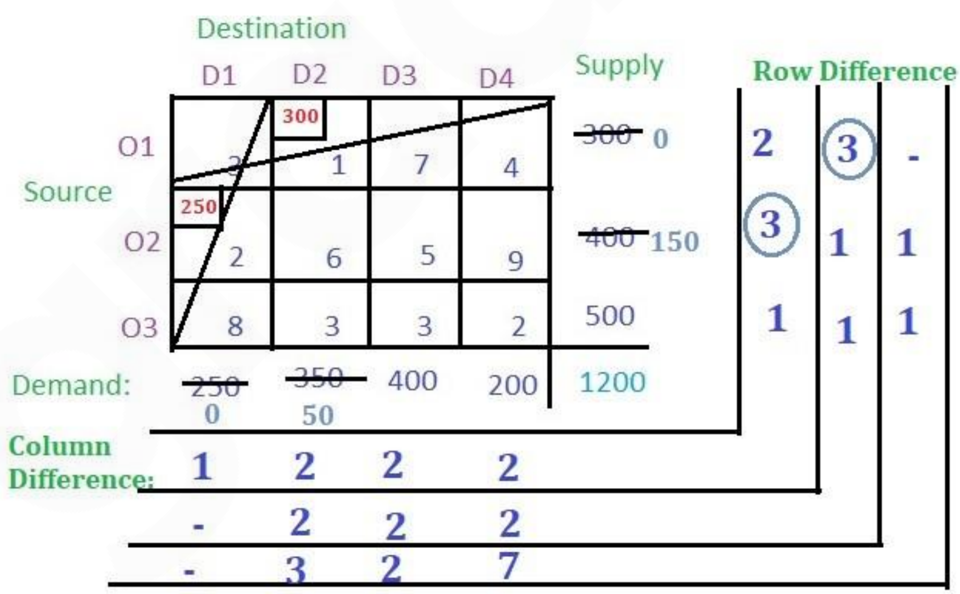
Destination

Source	D1	D2	D3	D4	Supply	Row Difference	
O1	3	1	7	4	300	2	3
O2	250 2	6	5	9	~~400~~ 150	3	1
O3	8	3	3	2	500	1	1
Demand:	~~250~~ 0	350	400	200	1200		
Column Difference:	1	2	2	2			
	-	2	2	2			

- Again select the maximum penalty which is 3 corresponding to row O1. The least-cost cell in row O1 is (O1, D2) with cost 1. Allocate the minimum among supply and demand from the respective row and column to the cell. Cancel the row or column with zero value.



• Now find the row difference and column difference from the remaining cells.



• Now select the maximum sanction which is 7 corresponding to column D4. The least cost cell in column D4 is (O3, D4) with cost 2. The demand is smaller than the supply for cell (O3, D4). Allocate 200 to the cell and cancel the column.

Destination

Source	D1	D2	D3	D4	Supply	Row Difference		
O1	3	300 1	7	4	~~300~~ 0	2	3	-
O2	250 2	6	5	9	~~400~~ 150	3	1	1
O3	8	3	3	200 2	~~500~~ 300	1	1	1
Demand:	~~250~~ 0	~~350~~ 50	400	~~200~~ 0	1200			
Column Difference:	1	2	2	2				
	-	2	2	2				
	-	3	2	7				

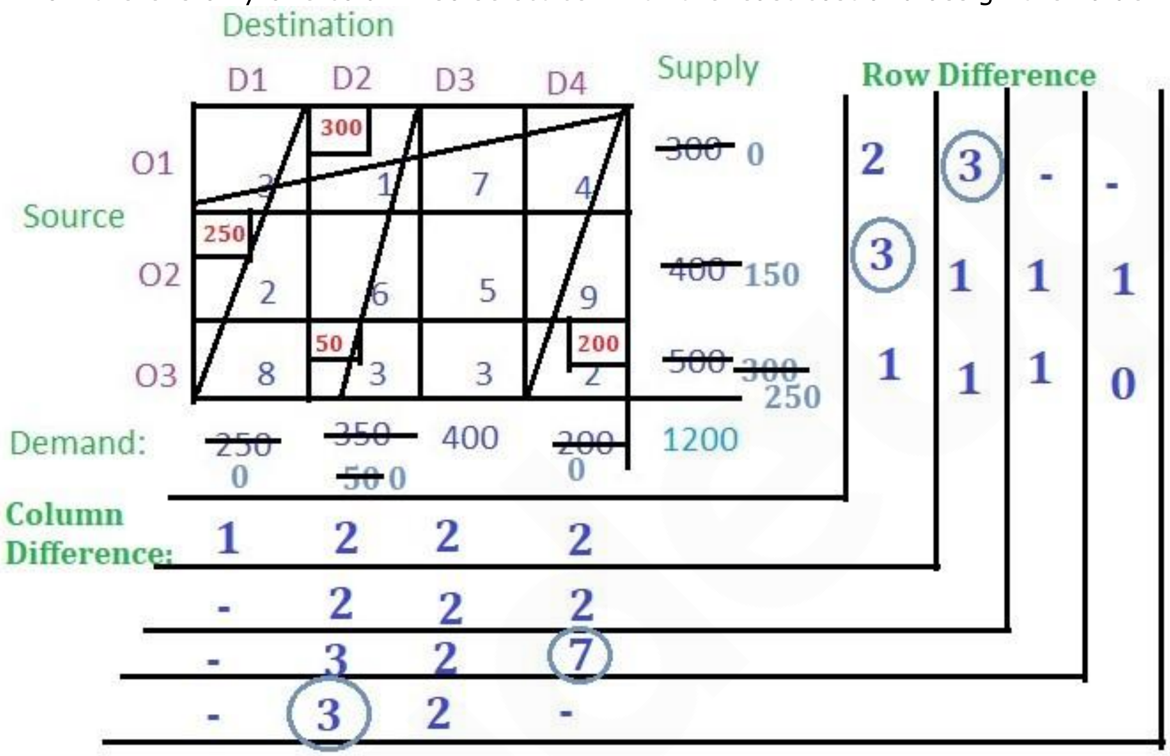
- Find the row difference and the column difference from the remaining cells.

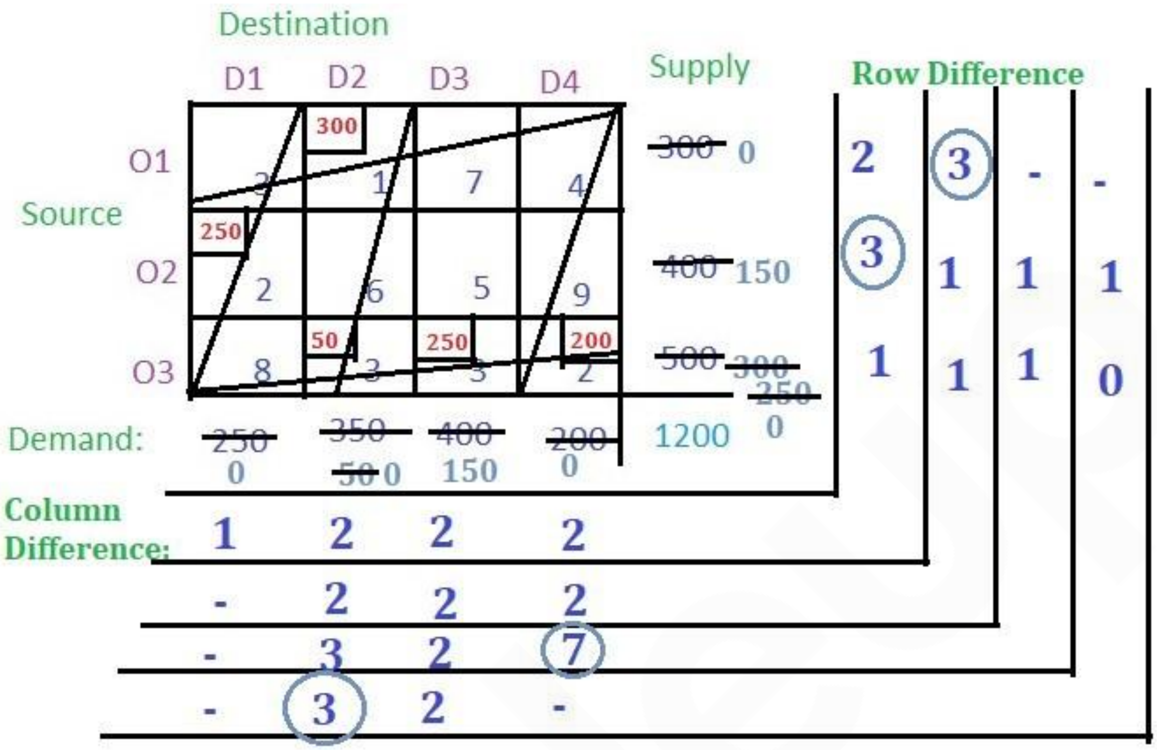
Destination

Source	D1	D2	D3	D4	Supply	Row Difference			
O1	3	300 1	7	4	~~300~~ 0	2	3	-	-
O2	250 2	6	5	9	~~400~~ 150	3	1	1	1
O3	8	3	3	200 2	~~500~~ 300	1	1	1	0
Demand:	~~250~~ 0	~~350~~ 50	400	~~200~~ 0	1200				
Column Difference:	1	2	2	2					
	-	2	2	2					
	-	3	2	7					
	-	3	2	-					

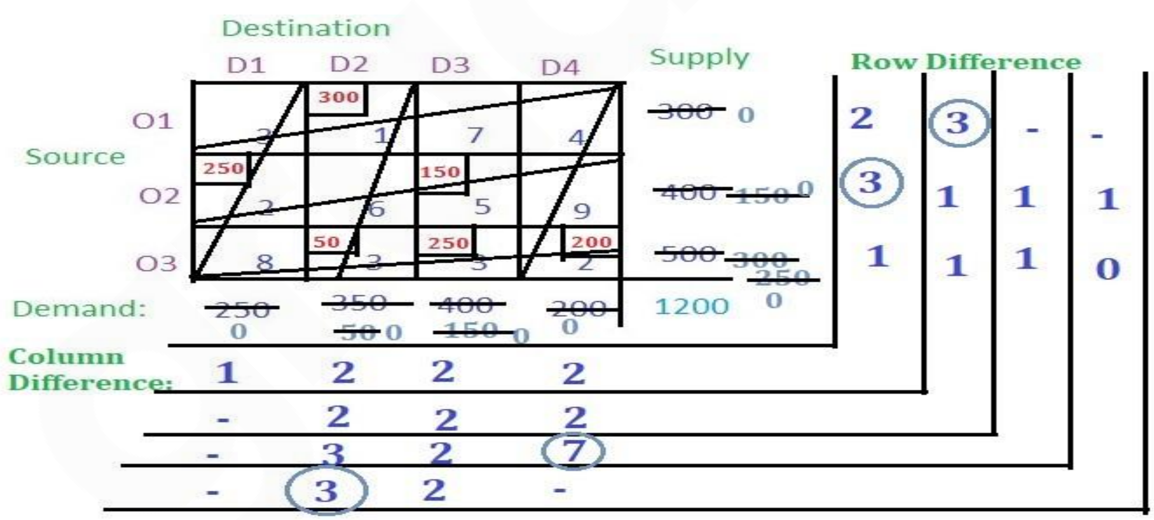
• Now the maximum penalty is 3 corresponding to the column D2. The cell with the least value in D2 is (O3, D2). Allocate the minimum of supply and demand and cancel the column.

• Now there is only one column so select cell with the least cost and assign the value.





• Now there is only one cell to allocate the remaining demand or supply to the cell



• No balance remains. So multiply the assigned value of the cells with their corresponding cell cost and add all to get the final cost i.e. (300 * 1) + (250 * 2) + (50 * 3) + (250 * 3) + (200 * 2) + (150 * 5) = 2850

Destination

Source	D1	D2	D3	D4	D5	Supply(S_i)
O1	5	1	8	7	5	15
O2	3	9	6	7	8	25
O3	4	2	7	6	5	42
O4	7	11	10	4	9	35
Demand(d_j):	30	20	15	10	20	117 / 95

Transportation Problem Unbalanced

The problem is unbalanced because the sum of all the supplies i.e. O1, O2, O3 and O4 is not equal to the sum of all the demands i.e. D1, D2, D3, D4 and D5.
Solution:
In this type of problem, the concept of a dummy row or a dummy column will be used. As in this case, since the supply is more than the demand so a dummy demand column will be added and a demand of (total supply – total demand) will be given to that column i.e. 117 – 95 = 22 as shown in the image below. If demand were more than the supply then a dummy supply row would have been added.

Destination

Source	D1	D2	D3	D4	D5		Supply(S_i)
O1	5	1	8	7	5	0	15
O2	3	9	6	7	8	0	25
O3	4	2	7	6	5	0	42
O4	7	11	10	4	9	0	35
Demand(d_j):	30	20	15	10	20	22	117 / 117

Now the problem has been modified to a balanced transportation problem, it can be proved using any one of the following methods to solve a balanced this problem.

1. North West Corner Method
2. Least cost cell method
3. Vogel's approximation method