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Optimization Part -2

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Content:

1. **Assignment Problem**

It arises in a variety of situations. Typical assignment problems are:

assigning jobs to machines

assigning sales personnel to sales territories

assigning contracts to bidders

assigning agents to tasks assigning taxis to customers

The distinguishing feature of assignment problems is that one agent is assigned to one and only one task. For example, if we have three taxis, A, B and C, and three customers, 1, 2 and 3, who want a taxi then A →3, B→ 1, C→ 2 is a possible assignment. We will, initially, assume that

(number of agents) = (number of tasks)

Problems satisfying this condition are known as balanced problems. We will consider how to deal with unbalanced problems later in the chapter.

In proving the assignment problem we are looking for a set of pairings, for example agents to tasks, which optimizes a stated objective such as minimising total cost or distance or time.

Assignment problem

First check whether the number of rows is equal to the numbers of columns, if it is so, the assignment problem is said to be balanced.

Step :1 Choose the least element in each row and subtract it from all the elements of that row.

Step :2 Choose the least element in each column and subtract it from all the elements of that column. Step 2 has to be performed from the table obtained in step 1.

Step:3 Check whether there is at-least one zero in each row and each column and make an assignment as follows.

(i) consider the rows successively until a row with exactly one zero is found. Mark that zero by , that means an assignment is made there . Cross (×) all other zeros in its column. Continue this until all the rows have been examined.

(ii) consider the columns successively until a columns with exactly one zero is found. Mark that zero by , that means an assignment is made there . Cross (×) all other zeros in its row. Continue this until all the columns have been examined

Step :4 If every row and every column contains exactly one assignment, then the solution is optimal.

Example: Solve the following assignment problem. Cell values represent cost of assigning job A, B, C and D to the machines I, II, III and IV.

jobs \ machines	I	II	III	IV
A	10	12	19	11
B	5	10	7	8
C	12	14	13	11
D	8	15	11	9

Solution: Here the number of rows and columns are equal.

∴ The given assignment problem is balanced. Now let us find the solution.

Step 1: Select a smallest element in each row and subtract this from all the elements in its row.

	I	II	III	IV
A	0	2	9	1
B	0	5	2	3
C	1	3	2	0
D	0	7	3	1

Look for at-least one zero in each row and each column. Otherwise go to step 2.

Step 2: Select the smallest element in each column and subtract this from all the elements in its column.

	I	II	III	IV
A	0	0	7	1
B	0	3	0	3
C	1	1	0	0
D	0	5	1	1

Since each row and column contains atleast one zero, assignments can be made.

Step 3 (Assignment):

consider the rows with exactly one zero. First three rows contain more than one zero. Go to row D. There is exactly one zero. Mark that zero by (i.e) job D is assigned to machine I. Mark other zeros in its column by x.

	I	II	III	IV
A	~~0~~	[0]	7	1
B	~~0~~	3	0	3
C	1	1	0	0
D	[0]	5	1	2

Step 4: Now consider the columns with exactly one zero. Already there is an assignment in column I. Go to the column II. There is exactly one zero. Mark that zero by . Mark other zeros in its row by ×.

	I	II	III	IV
A	~~0~~	0	7	1
B	~~0~~	3	0	3
C	1	1	0	0
D	0	5	1	2

Column III holds more than one zero. Therefore proceed to Column IV, there is exactly one zero. Mark that zero by . Mark other zeros in its row by × .

	I	II	III	IV
A	~~0~~	0	7	1
B	~~0~~	3	0	3
C	1	1	~~0~~	0
D	0	5	1	2

Step 5: Again examine the rows. Row B contains exactly one zero. Mark that

	I	II	III	IV
A	~~0~~	0	7	1
B	~~0~~	3	0	3
C	1	1	~~0~~	0
D	0	5	1	2

zero .

Thus all the four assignments have been made. The optimal assignment schedule and total cost is

Job	Machine	cost
A	II	12
B	III	7
C	IV	11
D	I	8
Total cost		38

The optimal assignment (minimum) cost

= ₹ 38

Example: Solve the following assignment problem.

		Men		
		1	2	3
Task	P	9	26	15
	Q	13	27	6
	R	35	20	15
	S	18	30	20

Solution: Since the number of columns is less than the number of rows, given assignment problem is unbalanced one. To balance it , introduce a dummy column with all the entries zero. The revised assignment problem is

Task	Men 1	2	3	d
P	9	26	15	0
Q	13	27	6	0
R	35	20	15	0
S	18	30	20	0

Here only 3 tasks can be assigned to 3 men.

Step 1: is not necessary, since each row contains zero entry. Go to Step 2.

Step 2 :

Task	Men 1	2	3	d
P	0	6	9	0
Q	4	7	0	0
R	26	0	9	0
S	9	10	14	0

Step 3 (Assignment) :

		Men 1	2	3	d
Task	P	[0]	6	9	~~0~~
	Q	4	7	[0]	~~0~~
	R	26	[0]	9	~~0~~
	S	9	10	14	[0]

Since every row and every column contains exactly one assignment, all the three men have been assigned a task. But task S is not assigned to any Man. The optimal assignment schedule and total cost is

Task	Men	cost
P	1	9
Q	3	6
R	2	20
S	d	0
Total cost		35

The optimal assignment (minimum) cost = ₹ 35

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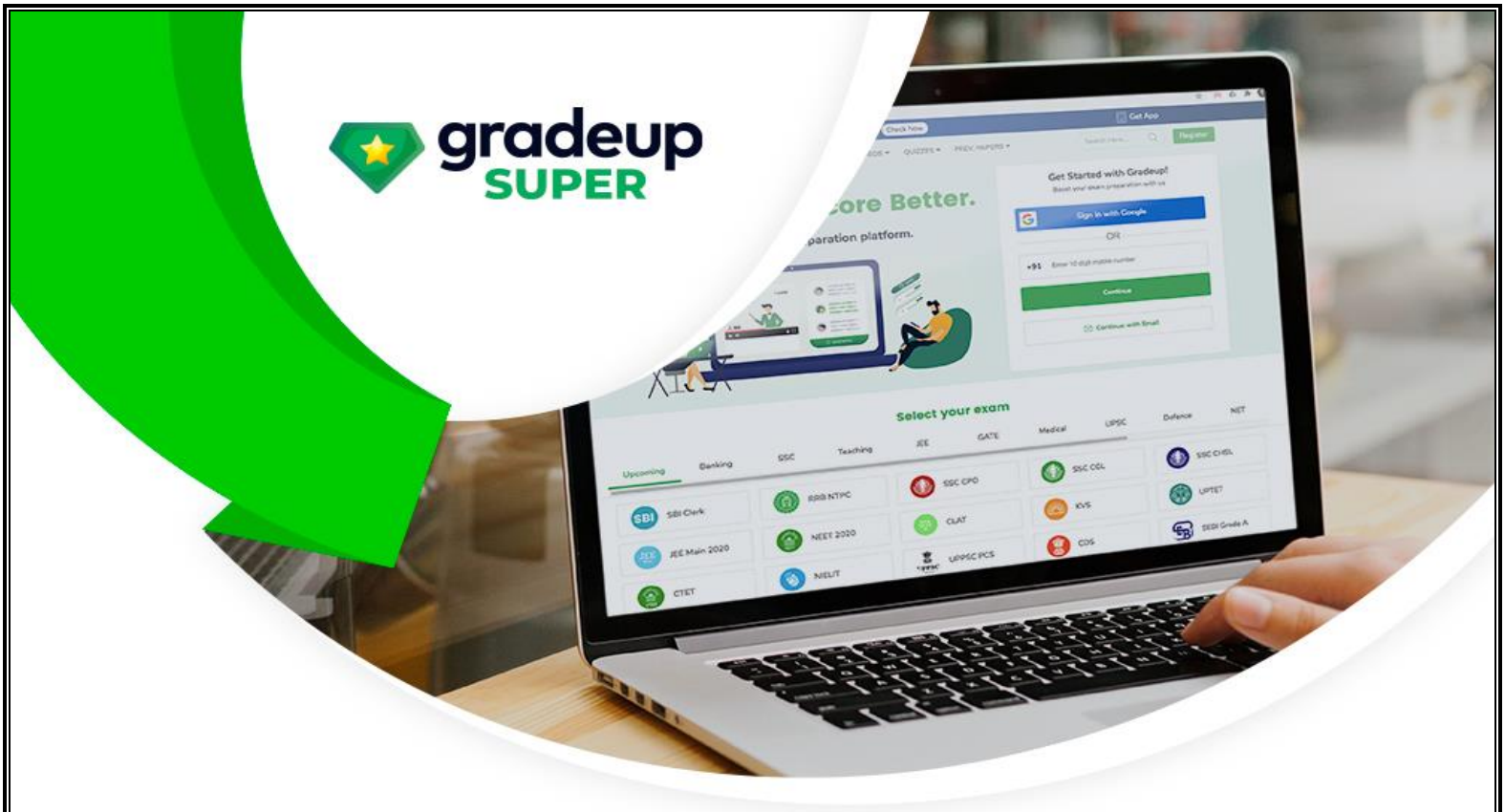

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