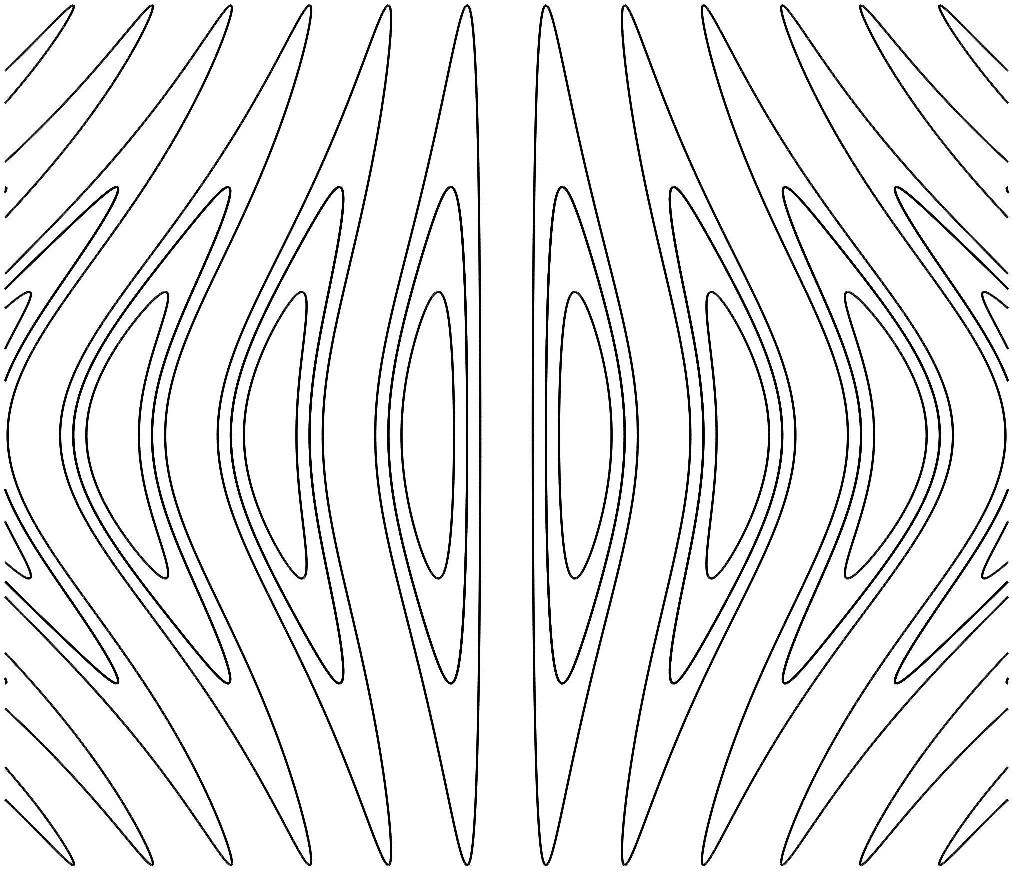
INDUCED SUPERCONDUCTIVITY IN A QUANTUM HALL EDGE STATE



ANDREAS BOCK MICHELSEN

PHD THESIS

Induced superconductivity in a quantum Hall edge state

Andreas Nicolai Bock Michelsen

This thesis is submitted in partial fulfilment for the degree of

Doctor of Philosophy (PhD)

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Abstract

In the search for non-Abelian anyonic zero modes for inherently fault-tolerant quantum computing, the hybridized superconductor – quantum Hall edge system plays an important role. Inspired by recent experimental realizations of this system, we describe it through a microscopic theory based on a BCS superconductor with Rashba spin-orbit coupling and Meissner effect at the surface, which is tunnel-coupled to a spin-polarized integer or fractional quantum Hall edge. By integrating out the superconductor, we arrive at an effective theory of the proximitized edge state and establish a qualitative description of the induced superconductivity.

We predict analytical relations between experimentally available parameters and the key parameters of the induced superconductivity, as well as the experimentally relevant transport signatures. Extending the model to the fractional quantum Hall case, we find that both the spin-orbit coupling and the Meissner effect play central roles. The former allows for transport across the interface, while the latter controls the topological phase transition of the induced p-wave pairing in the edge state, allows for particle-hole conversion in transport for weak induced pairing amplitudes, and determines when pairing dominates over fractionalization in the proximitized fractional quantum Hall edge.

Further experimental indicators are predicted for the system of a superconductor coupled through a quantum point contact with an integer or fractional quantum Hall edge, with a Pauli blockade which is robust to interactions and fractionalization as a key indicator of induced superconductivity. With these predictions we establish a more solid qualitative understanding of this important system, and advance the field towards the realization of anyonic zero modes.

Acknowledgements

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I would first like to thank my supervisors Thomas Schmidt and Bernd Braunecker for giving me the opportunity to work on this project, for their close collaboration on all the work I have done, and for the wisdom concerning physics, research and life that they have imparted on me. I would also like to thank my close collaborators Patrik Recher, Edvin Idrisov and Nathan Harshman for the countless invaluable insights they have shared with me.

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Research Data/Digital Outputs access statement

The work presented in this paper is theoretical. No data were produced, and supporting research data are not required.

Contents

CHAPTER 1

Introduction

"The old particles were the particles of the problem, and the new particles are the particles of the solution."

Jainendra Jain on quasiparticles,
in Composite Fermions *(2007)*

The work is introduced.

Outline

The thesis is structured as follows.

In **Chapter 2** we ...

List of publications

This thesis is based on the following papers co-authored by myself:

- Ref. [1]: A. B. Michelsen, T. L. Schmidt, E. G. Idrisov, *Current correlations of Cooper-pair tunneling into a quantum Hall system*, Phys. Rev. B **102** 125402 (2020)
- ...

Furthermore, the following paper was published during my work on this thesis, but is not included in the thesis itself:

- ...

Acronyms

QH quantum Hall

SC superconductor

CHAPTER 2

Macroscopic quantum states

The main system of concern in this thesis consists of two elements: a material exhibiting the quantum Hall (QH) effect, and a superconductor (SC). Both of these are examples of "macroscopic quantum states", in the sense that they are both expressions of a macroscopic number of electrons condensing into a degenerate ground state at low temperatures.

...

APPENDIX A

The renormalized Numerov method

This appendix is based on a more in-depth explanation in Ref. [2]. We reintroduce $\hbar \neq 1$ for clarity. The Numerov method integrates one-dimensional second order differential equations with no first order derivative. Assume that such an equation is of the form

$$\left[\frac{\partial^2}{\partial x^2} + Q(x)\right]\psi(x) = 0, \tag{A.1}$$

which is the Schrödinger equation when

$$Q(x) = \frac{2m}{\hbar}(E - V(x)). \tag{A.2}$$

We discretize space into N small intervals of length a starting at a point x_0, continuing in points $x_n = x_0 + na$ with $n \in \mathbb{N}$ and ending at a point x_N. The discretized wave function is then $\psi_n = \psi(x_n)$, and $Q_n = Q(x_n)$. The Numerov formula, which is derived through a Taylor expansion in a, says that

$$\Big(1 + \frac{a^2}{12}Q_{n+1}\Big)\psi_{n+1} + \Big(1 + \frac{a^2}{12}Q_{n-1}\Big)\psi_{n-1} - \Big(2 - \frac{5a^2}{6}Q_n\Big)\psi_n + \mathcal{O}(a^6) = 0 \tag{A.3}$$

where Q_n is known for all n. For increased numerical stability we introduce the ratio

$$R_n = \frac{\psi_{n+1}}{\psi_n} \tag{A.4}$$

which is usually on order one. If we ignore terms of order a^6 and higher, we find

$$R_n = \Big(1 + \frac{a^2}{12}Q_{n+1}\Big)^{-1}\Big[\Big(2 - \frac{5a^2}{6}Q_n\Big) - \Big(1 + \frac{a^2}{12}Q_{n-1}\Big)R_{n-1}^{-1}\Big]. \tag{A.5}$$

Given $R_0 = \psi_1/\psi_0$, we can use this formula to iteratively construct the wave function forwards from x_0. We can also construct the wave function backwards from x_N by defining

$$\tilde{R}_n = \frac{\psi_{n-1}}{\psi_n} \tag{A.6}$$

and use the formula

$$\tilde{R}_n = \left(1 + \frac{a^2}{12} Q_{n-1}\right)^{-1} \left[\left(2 - \frac{5a^2}{6} Q_n\right) - \left(1 + \frac{a^2}{12} Q_{n+1}\right) R_{n+1}^{-1}\right]. \tag{A.7}$$

The numerically optimal strategy is to choose a point x_c between x_0 and x_N. The wave function is then iterated from left and right up to x_c. At this point, the matching function

$$G(E) = \left(\frac{\psi(x_N + a)}{\psi(x_N)}\right)_{\text{right}} - \left(\frac{\psi(x_N + a)}{\psi(x_N)}\right)_{\text{left}} \tag{A.8}$$

is evaluated. G is zero if and only if E is an eigenvalue of the Hamiltonian $\hat{H}$ corresponding to $Q(x, E)$. While the choice of x_c is arbitrary, it can be advantageous to use the first extremum of the wave function as evaluated from x_N when the more complex behaviour occurs in x_0.

The task is then to choose x_0 and x_N as points with well known wave function behaviour (and thus $R_0, \tilde{R}_N$). Then a root of $G(E)$ can be found through optimization in E, with the resulting energy being an eigenvalue of the Hamiltonian. The wave function can then be reconstructed from the boundary conditions and the ratios R and $\tilde{R}$.

Bibliography

[1] A. B. Michelsen, T. L. Schmidt, and E. G. Idrisov, "Current correlations of Cooper-pair tunneling into a quantum Hall system," *Physical Review B*, vol. 102, no. 12, p. 125402, Sep. 2020.

[2] H. Doerk, "An atom-ion quantum gate," Ph.D. dissertation, Universität Ulm, 2008.