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Radiocommunication Systems

Report 2

Synthetic Aperture Radar - Image Processing

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1 Introduction

In this project, we've dived into Synthetic Aperture Radar (SAR) technology to process radar images over Vilar de Luz Airport in Portugal. SAR's unique ability to capture high-resolution aerial views (images/pictures) provides a valuable lens for studying the airport's features and changes over time.

This report walks you through our SAR data processing journey, focusing on steps like range and azimuth compression, and resolution enhancement. These processes help turn raw radar data into clear, actionable insights. Additionally, a link budget analysis offers a peek into the gains and losses in our radar system.

2 System Overview

Figure 1 illustrates the system used to collect the data used to create the image. A plane flew over the airport of Vilar de Luz, Portugal equipped with an antenna tilted 60º. The antenna had a radius (A) of 0.2 m and was emitting a signal with a bandwidth (B) of 50 MHz, a sample frequency (f_s) of 60 MHz and a carrier frequency (f_c) of 2340 MHz, therefore, a wavelength: $\lambda = \frac{c}{f_c} = 0.1281$ m. We also know that the azimuth resolution is 5 cm, and the altitude of the plane is 5000 m.

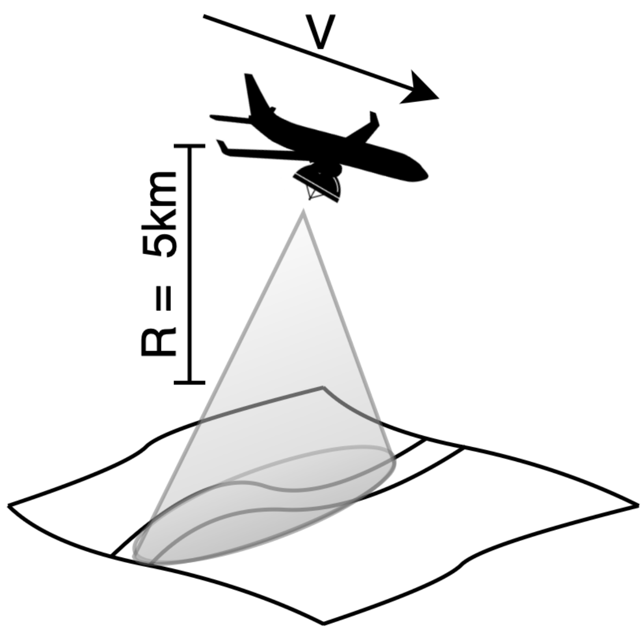


Figure 1: Representation of the system relative to the terrain

3 Data preprocessing

The data referred as "raw data" thought this report isn't in fact "raw". This would mean that the data came out of the system mentioned in section 2 without any prior processing, which isn't the case.

First of all, the plane collecting the data did not travel in a straight line, it had some deviation in it's trajectory, both altitude and azimuth. The available data already had this in consideration and compensated the deviation with the navigation record, thus we can consider that the plane travelled in a perfect straight line. This process is often referred as motion compensation or MoCo.

We are considering our Azimuth resolution to be 5cm, assuming that there was no change in speed from the aeroplane or loss of samples. In fact, we can assume this fact since the sample frequency used was 3 times the one we are considering and the samples obtained were corrected according to these markers, resulting in our data with a new sample frequency of $60MHz$. In simple words, is a velocity compensation by integrating the signal in azimuth.

Lastly, the acquired data was also correlated with the transmitted signal to implement a range compression. This compensation is one the SAR processing gain mentioned in 5, and will be better explained in that section.

The result of these processes is presented in figure 2. Unfortunately we cannot provide a "before" image, since it was not provided. Even after this prepossessing, the image is completely meaningless. Yet, there are some conclusion we can take that will be analysed in section 5.4.

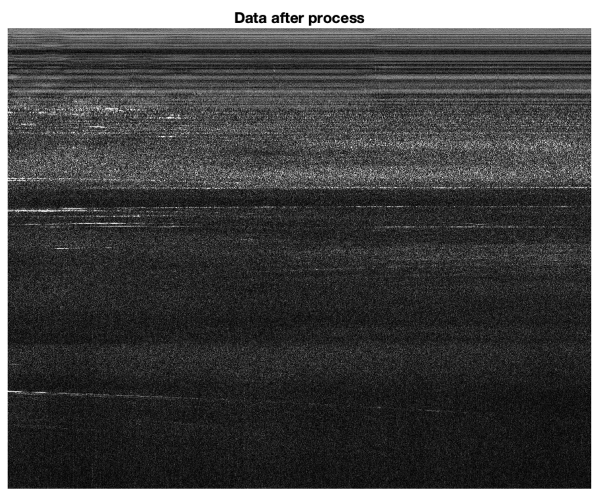


Figure 2: Data after the prepossessing

4 Link Budget

Firstly, let's define some key parameters, expressed in 1, where B, L_A, L_B, ϵ, f_c, λ, R, $NESZ$, L, and P_{TX} are the Bandwidth, antenna's aperture, efficiency, carrier frequency, wave length, altitude of the plane, Noise Equivalent Signal Zero and the transmitted power.

$$\begin{cases} B = 50\ MHz \\ L_A = L_B = 0.25\ m \\ \epsilon = 0.5 \\ f_c = 2349\ MHz \\ \lambda = 0.128\ m \\ R = 5\ km \\ NESZ = -25\ dB\ (assumed) \\ L = 700\ m \\ P_{tx} = 40\ W = 46\ dBm \end{cases} \tag{1}$$

4.1 Gains ans losses

We can now define the gains of the transmitter and receiver, equations 2a and 2b, the free space losses (FSL), equation 3 and the noise power by equation 4.

$$G_{tx} = \frac{\epsilon L_A L_B}{\lambda^2/4\pi} = 13.8\ dB \tag{2a}$$

$$G_{tx} = \frac{\epsilon L_A L_B}{\lambda^2/4\pi} = 13.8\ dB \tag{2b}$$

$$FSP = (\frac{\lambda^2}{4\pi R})^2 = -113.8\ dB \tag{3}$$

$$N_{power} = kB_0T = -97\ dBm \tag{4}$$

Furthermore, we consider other losses and noise figure to be $-5\ dB$. Not to forget that the FSL occur before and after reflection, thus we need to count it twice. The N_{power} improves our SNR, since it represents the magnitude of the noise in the system, therefore, the lowest the better. in equation 4, k is the Boltzmann's constant, B_0 is the bandwidth and T is the temperature of the system. The gains of the antennas have equal values since we use the same antenna to transmit and receive. So far, we have an SNR of $46\ dBm - (-97\ dBm) + 13.8\ dB + 13.8\ dB + 2 \times (-113.8\ dB) - 5\ dB = -62.0\ dB$

4.2 Radar crossing section

Let's now estimate radar cross section, R_{cs}. This corresponds to the equivalent area of an object that acts as a perfect reflector. To estimate this value given by 7 we first need to calculate τ_{min}, which consists of the reflectivity of the material, i.e. the minimum area of an element that SAR can isolate. This value depends on the resolution in both range and azimuth. We define τ_{min} in equation 6, where α is an attenuation coefficient that we will consider equal to be 1 (this value is assumed, another frequent assumption is to consider $\alpha = \frac{NESZ}{10}$) and the angle θ is described in figure 3 which consists on the aperture of the antenna. We know that the green section of figure 3 is $\frac{c}{2B}$ because it represents the minimum distance that we can detect with a straight line from the antenna. Thus, the resolution in range and azimuth as in equations 5a and 5b, respectively, since the resolution of our SAR system is $Resolution_{range} \times Resolution_{Azimuth}$.

$$\begin{cases} R_{range} = \dfrac{c}{2Bsin(\theta)} = 3.5\ m & (5a) \\ R_{azimuth} = \dfrac{\lambda R}{2L} = 0.46\ m & (5b) \end{cases}$$

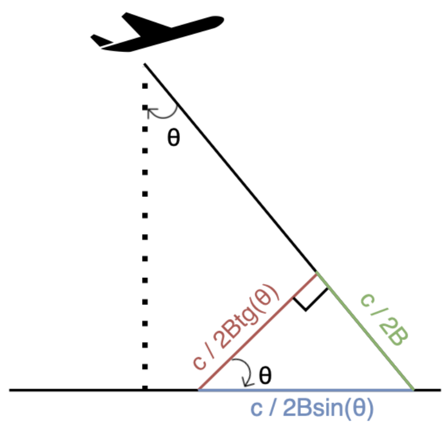


Figure 3: Resolution in range and azimuth

$$\tau_{min} = \alpha \frac{c}{2Btg(\theta)} \times \frac{\lambda R}{2L} = 1 \times 1.73 \times\ 0.46 = 0.796 \quad (6)$$

To estimate R_{cs} we use the value obtained for τ_{min} and the assumed $NESZ$, which is the threshold from where we assume that the reflected signal is not detected by the receiver.

$$R_{cs} = \frac{\tau_{min}}{\lambda^2/4\pi} - NESZ = 10\log_{10}\left(\frac{0.791}{0.128^2/4\pi}\right) - 25\ dB = 2.9\ dB \quad (7)$$

As so, the SNR is now equal to $-62\ dB + 2.9\ dB = -54.1\ dB$

4.3 SAR Processing Gain

In SAR we have two processing gains: Range compression and the Azimuth compressing. The first one consists on the number of sample per echo and it's give by the bandwidth times the pulse duration (equation 8a). The Azimuth compression is the number of pulses per echo, equation 8b. A better reasoning of this process will given in section 5.

$$Range\ compression = BT = 50 \times 10^6 * 10 \times 10^-6 = 500 = 27\ dB \quad (8a)$$

$$Azimuth\ compression = \#pulse = 14001 = 41.5\ dB \tag{8b}$$

In total, the SAR processing gain gives us plus 68.5 dB, resulting in an SNR $= -54.1\ dB + 27\ dB + 41.5\ dB = 14.4\ dB$. This is already good value, at this point we should be able to see a clear image of the terrain.

4.4 Anti-Speckle Filter

In the previous section we calculate the range and azimuth resolution. These values refer to the distance each pixel represent in either direction. The value obtain was $3.5\ m \times 0.46\ m$. We have a much higher resolution in azimuth than in range and we can compensate that by applying a low pass filter. In section 5 we provide a better explanation about the implementation of this filter. Regarding it's contribute for the SNR, it adds a gain of $\frac{3.5}{0.46} = 7.6 = 8.8\ dB$. The final SNR is $14.4\ dB + 8.8\ dB = 23.2\ dB$. The new resolution of the data is $3.5\ m \times 3.5\ m$.

4.5 Resume of link budget

Table 1 has the values of the main contribution to the link budget of our system in order to provide an overview of the elementary contribution of the different components.

There is an alternative way to calculate the processing gain based on characterisation of the transmitted signal. Equation 9a express the total SAR gain, note that the equation is not in dB. As $T_{pulse} \times \#pulses$ is equal to the total transmitted time, which we refer as T_{on} we get equation 9b. The total transmitted time is in fact the total time of the capture times the percentage of time the signal is actually on, therefore $\delta \times T_{capture}$. With this consideration we get the SAR gain described as in equation 9c.

$$Gain_{SAR} = B \times T_{pulse} \times \#pulses \tag{9a}$$

$$Gain_{SAR} = B \times T_{on} \tag{9b}$$

$$Gain_{SAR} = B \times \delta \times T_{capture} \tag{9c}$$

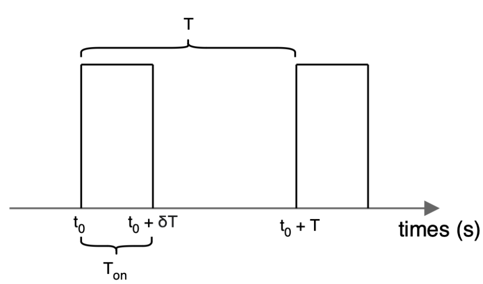


Figure 4: Estimation of the duty cycle

Equation 9c depends on the duty cycle which we don't know the value, but we can calculate. Consider the figure 4. We know that the number of pulses is 14000 and that the capture time was 15 seconds with a pulse width (T_{pulse}) is $10\mu s$. The definition of duty cycle is the time the signal is on over the period (equation 10a). this is the same if we consider all the periods, therefore the capture time and the pulse width times the number of pulses (equation 10b). With this result, we can estimate the SAR gain using equation 9c, obtaining a gain of 78.7 dB. Note that this value is in dB and equation 9c is not and the value for the bandwidth is from section 4. This value is very similar to the SAR processing gain calculated in 4.3.

$$\delta = \frac{T_{pulse}}{T} \tag{10a}$$

$$\delta = \frac{T_{pulse \times \#pulses}}{T_{capture}} = \frac{10 \times 10^-6 \times 14000}{15} = 9.33 \times 10^{-3} \approx 1\% \tag{10b}$$

	Gain/Loss	Contribution
System Gains ($-54.1\ dB$)	Power Transmitted	16 dB
	Transmitter Gain	13.8 dB
	Free Space Losses	-227.6 dB
	Radar Crossing Section	2.9 dB
	Receiver Gain	13.8 dB
	Losses + Noise Figure	-5 dB
	Noise Power	-67 dB
Processing Gains ($+77.3\ dB$)	Range Compression	27 dB
	Azimuth Compression	41.5 dB
	Resolution Enhancement	8.8 dB
	Total SNR	**23.2 dB**

Table 1: Link budget contributions

5 SAR processing

The challenge of SAR processing is to acquire the gains defined in section 4 while staying computation feasible. State-of-the-art algorithms can even process data in real time which present a major advance since it is possible to evaluate the quality of the reading while flying and if needed alter parameters mid flight or even abort it.

5.1 Range compression

In section 2, we referred that the the range compression was already applied to our data, therefore, we do not have an image of before this processing. Figure 2 presents the data with this compression already applied. As so, we will use this subsection to explain what this procedure is and to present the reader with data we stated working on.

In range compression, the signal reaching the radar is processed using a matched filter. The concept of a matched filter is to correlate the incoming signal with the conjugate of the pulse that was sent out. In case of a perfect reflection of the pulse on the target of the radar, this would result in a autocorrelation of the sent pulse. The pulses used for radars are constructed in a way that their autocorrelation function resembles a Dirac pulse, meaning a single strong peak. The incoming signal consists of more that the perfect reflection of the pulse, but the correlation of the matched filter with noise yields no strong results. Therefore, the matched filter can extract the reflection of the pulse in the noise of the incoming signal. This strong peak in time can be related to the distance the pulse traveled to the target and back. This way the image can be sharpened in range. This matched filter gets used on every incoming pulse, meaning on every row of data.

As seen in figure 2, this operation results in an image with stripes. These stripes result from the image being sharp only in one dimension, the range, but still not sharpened in the other dimension, the azimuth. This is taken care of in the next step, the azimuth compression.

The formula 8a stating that the gain is composed by the bandwidth multiplied with the pulse duration emphasises that the energy spread out over the whole pulse in bandwidth and time gets compressed into one point by the autocorrelation leading to this gain of signal power.

5.2 Azimuth Compression

The idea behind azimuth compression is the same as in range compression. A convolution is conducted to focus the energy of the signal in the correct azimuth value. This time the correlations is not conducted with the conjugate of the chirp, but a function showing the modulation profile of the data in azimuth. This function is comprised of the patterns of the antenna used in the radar system and a function representing the phase shift caused by the azimuth modulation.

The antenna pattern (A) is modulated using a sinc function with a main lobe of a specified angle. This results in a function $s(x)$ shown in equation 11 the data needs to be correlated with along the azimuth direction. Not in direction of the columns like in range compression but along the rows of the data, which are the individual received pulses of the radar. So we combine the information from all sent radar pulses. This is also shown in the formula used to calculate the gain from azimuth compression. Formula 8b shows that the gain is the number of pulses because we combine the information/power of all pulses into one image.

$$s(x) = A \cdot e^{-\frac{j4\pi}{\lambda}x},\ x \text{ is the distance to the target} \tag{11}$$

Correlation is a very demanding computational task, since it consists on comparing each data point with every other data point, resulting in a quadratic growth in computations according to the size of the data. Thus, after an empirical trial, we conclude that this operation is unfeasible in this situation due to the size of the data, which would result in the usage of an absurd among of RAM and huge processing time. As a solution, instead of making a correlation in time, we make a multiplication in frequency by the conjugate of $s(x)$ and then we do an inverse Fourier transform to reconstruct the signal in time. This process results in figure 5. Through out this chapter the capital letter of a given signal will be used to identify the fast Fourier transform of that signal. For example, the data $ch \xrightarrow{\mathscr{F}} CH$. Equation 12 represents the azimuth compression.

$$ch = CH^* \times S(x) \tag{12}$$

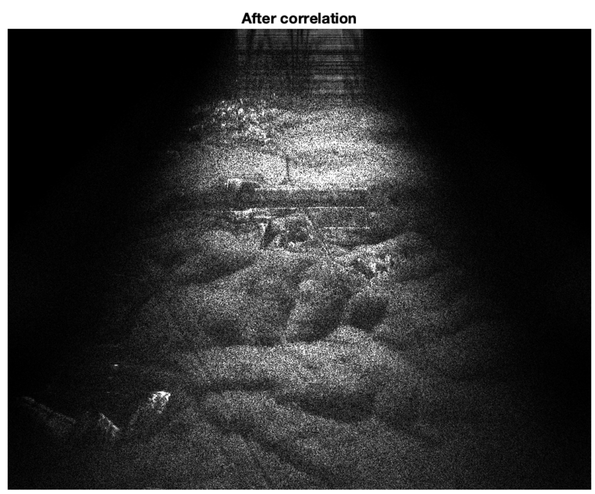


Figure 5: Result of Azimuth Compression

We can clearly see an improvement of the image comparing with figure 2 (represents a gain of 41.5 dB after all!). This is also the first time we present an image with a positive SNR, thus we can clearly distinguish some elements of the terrain like the tree and the track. After performing both range and azimuth compression the image is sharpened in both dimensions.

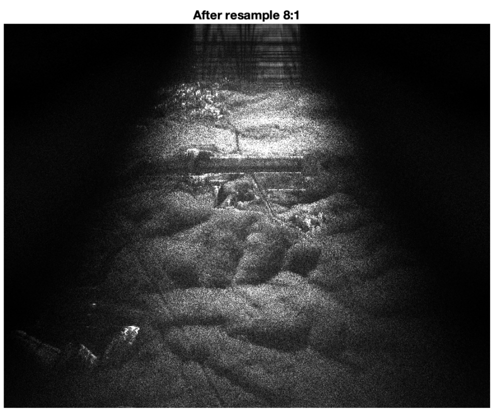
This image is presented in sort of a cone shape. This is due to the antenna aperture with we simulates by filtering every data over 30 degrees.

5.3 Azimuth decimation

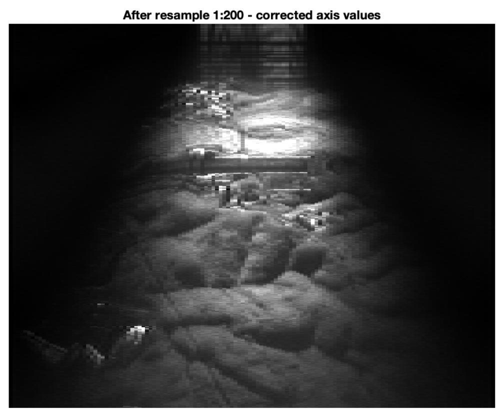
The goal of this process is to remove the speckle noise present in the figure 5. This noise is common in image processing and one way to remove is to apply an average filter. This replaces a pixel for the average of it's neighbours, therefore, removes the high frequencies which is the noise component.

The implementation we did, consists of applying a resample of 1/8, which will take 8 azimuth point, i.e, 8 points of a row and transform then into 1 point that is the result on the average of the 8. This will smooth the transitions between points, therefore remove rough variation. Thus, it's a low-pass filter, often know as anti-speckle filter. The value 8 was chosen according to the nearest integer of the resolution gain calculated in 4.4. This will cause the new image to maintain the number of points in range (y axis) and decrease the points in azimuth (x axis) by a factor of 8. This goes accordingly to the change in azimuth resolution that represents a gain of 7.8 (section 4.4), in this case as the resample was a factor of 8, the gain would be of 9 dB.

The result of this transformation is represented in figure 6a. Comparing with figure 5, some speckle noise has been removed. In order to remove more noise we could have used a more aggressive resample like in figure 6b at the cost of losing resolution in azimuth. The value 8 grants the same resolution for both range and azimuth.



(a) Resample in azimuth of 1:8



(b) Resample in azimuth of 1:500

5.4 Range Migration

As can be seen in 8a, all supposed straight lines on the image in horizontal direction are warped towards the edges. This is caused by the movement of the plane relative to the objects on the ground, as shown in figure 7. An object might be closer to the antenna during one pulse but further away in another pulse when the plane is flying away from it. Because of this the pulse travels further, therefore takes a longer time and the same object appears at a different distance. This causes the distance variation to behave in like an hyperbolic function.

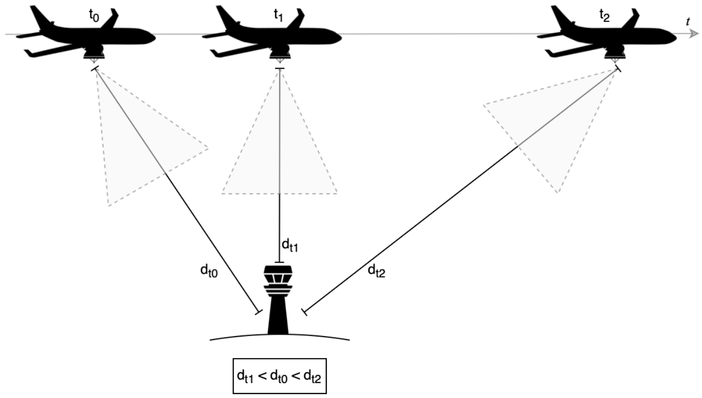
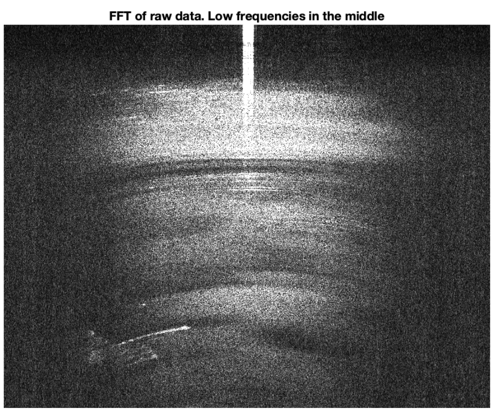
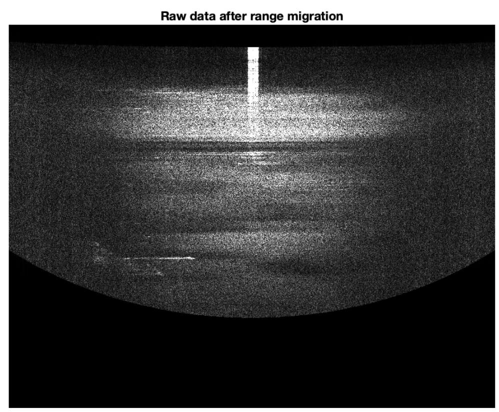


Figure 7: Range migration diagram

To compensate it the columns of data get compressed to fewer number of entries according to the function in equation 13.



(a) Raw data before range migration



(b) Raw data after range migration

$$compression\ factor = \frac{1300}{\sqrt{1 - (\lambda \cdot \omega / 4\pi)^2}} \tag{13}$$

In 13, ω is the spacial frequency here. The function describes a parabola, which means that the columns on the edge of the picture get compressed more because the range migration effect is stronger there. The correction gets less in the middle with the central column needing no correction at all.

This compression to a lower number of entries is done by using a FFT to transfer the columns into the frequency domain and cutting of the necessary number of entries on both sides. This realises a low pass filtering because the removed entries correspond with the high frequencies (positive and negative). Then the data is transformed back with an IFFT and now has the right number of entries with zeros padded to the end to reach the required column length.

After this final step, we can see that the lines are now straightened (figure 8b). This results in the image is shown 9. This represents the final result of our processing. We now have a clear image of the airport, where we can clearly distinguish the track from the trees. Observe the marks on the bottom left side and a some dwellings at the upper centre.

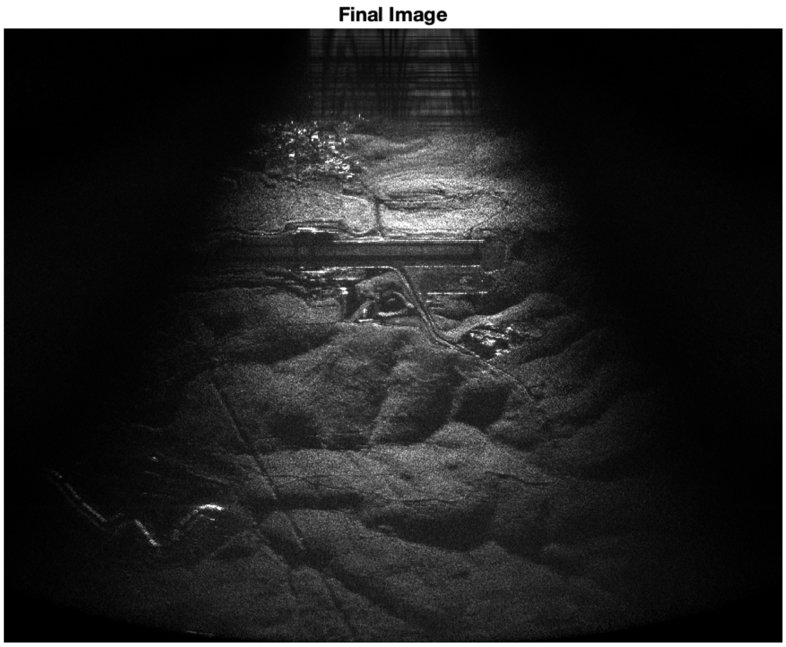


Figure 9: Result of range migration

5.5 Comparison to Google Earth

To show the accuracy of the picture obtained with the SAR and through the processing we conducted, a comparison can be made to a satellite image from Google Earth. The view angle on Google Earth can be adjusted to match the angle of the SAR, we choose 39°. The comparison can be seen in figure 10. This proves that SAR can deliver accurate, high quality images.

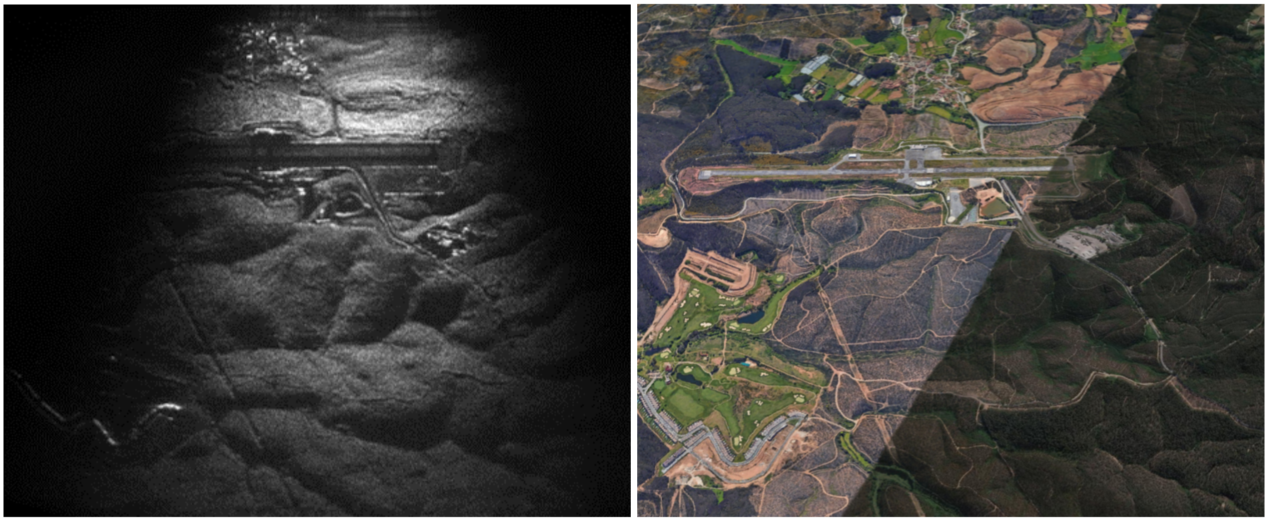


Figure 10: Comparison of Image from SAR and Google Earth

6 Conclusion

To summarize, the goals we set for the project were met successfully, with the image being processed to great detail and accuracy when compared to the satellite capture of the same location using Google Earth.

Originally we planned on altering the perspective of the image using Matlab, to make it similar to actual SAR Satellites, however Google Earth's feature of being able to angle the camera was able to provide a close enough comparison.

SAR proved to be a reliable processing method, possessing inherent advantages over more traditional photography, whose usefulness for this project was invaluable. Qualities such as its natural three-dimensional imaging, higher resolution, insensitivity to motion and ability to cover wider areas, with the added bonuses of not being affected by unfavourable weather conditions, lack of daylight brightness or obstruction due to clouds, were key factors that lead to the production of a clear image from the collected data.

A Appendix

A.1 Matlab code used in processing

```
close all
clear all
clc


%% Raw data
% Load data
load CH_07_MoCo_5cm.mat

% Plot original image
figure; image(abs(ch(:,1:10:end))/100); title("Data after process");
colormap('gray'); axis("off");

%% Parameteres and importamt results
% Bandwidth = 60 MHz
% fs = 60 MHz (each sample 2.5 meters (horizontal))
% A = 0.2 m
% fc = 2340 MHz
% direct signal = -100
% sol = c = 3e8

sol = 299792458; % Speed of light
fs = 60e6; % Sample frequency
fc = 2.34e9; % Carrier frequency
lambda = sol/2340e6; % Wave length

R = ((0:1199)'+100)*(sol/(2*fs)); % Range resolution = c/2*B
x = (-32768:32767)*0.05; % Azimute resolution, samples separated by 5 cm
d = sqrt(R.^2*ones(1,65536) + ones(1200,1)*x.^2); % Distance to object
theta = atan2(ones(1200,1)*x, R*ones(1,65536)) * (180/pi); % beam angle of the
antenna - pointing direction


A = sinc(theta/30) .* (abs(theta)<=90) .* (abs(theta)>=1); % Antenna Pattern

% resises the image, adds zeros to left and right CH is now a 1200 by 65536
CH = single([zeros(1200,(2^16-14000)/2) conj(ch)
zeros(1200,(2^16-14002)/2)]);
figure; image(abs(CH(:,:))/500); title("Raw data but resised");

S = A .* exp(-4j*pi/lambda*d); % SAR signal
S = single(S);

%% SAR processing gain
% Azimuth compression
azimuthCompression = abs(ifft(conj(fft(fftshift(S,2), [], 2)).*fft(CH, [],2),
[],2)) / 5000;
figure; image(azimuthCompression); title("Azimuth Compression");
colormap('gray'); axis("off"); % Correlation of conjugated of S by CH,
represents a correlation in time

% Azimuth Decimation
azimuthDecimation = resample(double(abs(ifft(conj(fft(fftshift(S,2), [],
2)).*fft(CH, [],2), [],2))'), 1, 8)' / 5000;
figure; image(x, R, azimuthDecimation); title("After resample 8:1");
colormap('gray'); axis("off");% Same as before but with the correct axis values

%% Range Migration
%figure; image(abs(CH) / 500); title("raw data: line are not perfectly
%horizontal"); % Note that in raw data, the lines are not horizontal, we need
to compensate that. It's am hiperbolic because as the plane move, the distance
```

```
to the object increases
figure; image(abs(fftshift(fft( CH, [],2), 2)) / 1000); title("FFT of raw data.
Low frequencies in the middle"); colormap('gray'); axis("off"); % center of low
frequencies that correspond to the plane on top of the object. Smate as before
but with low freq in center

% distorce the line line in order to make them straight lines, R varies
% with an hiporbole, we want to make it staight by using an interpolation

w = 2*pi*(-32768:32767)/65536/0.05; % Frequency in space -> radiants per meter
(AKA doppler?)

CHF = fftshift(fft([zeros(100,65536); CH; zeros(400, 65536)], [], 2),2); % fft
of raw data with low frequencies in center, resized bu adding zeros on top and
bottom
%figure; image(abs(CHF)/20000); title("fft of raw data resized with low freq
in middle");

% Initial sample value
NN = round(1300./sqrt(1-(sol/2340e6*w/(4*pi)).^2)); % we have 1300 samples
times 1/ sqrt(...) we want N samples
% figure; plot(NN, '.-'); grid; title("Samples times the scale factor");

% Low pass filter along the columns | Compensating according to NN
for k= 1:65536
    dummy = fft(CHF(1: NN(k),k));
    dummy = dummy([1:650 end-649:end]); % Midle is fine, so we don't mess with
it
    CHF(1:1300,k) = ifft(dummy);
end

% Now we can consider the lines to be straight when in fact they are hiperbolic
figure; image(abs(CHF)/7000); title("Raw data after range migration");
colormap('gray'); axis("off");

%% Final Result
% Final image, plot the correlated and resampled signal, but with the ridge
regression compensation
figure; image(x, R, resample(double(abs(ifft(conj(fft(fftshift(S,2), [],
2)).*fftshift(CHF(101:1300,:),2), [],2))'), 1, 25)' / 7000); title("Final
Image") %estamos a fazer a correlação com a linha horizontal (,2) mas na verdade
esta linha é uma parábola
colormap('gray'); axis("off");
```