Corona Test: Application of Bayes' rule

MSE_FTP_MachLe

The following probabilities are given:

- the **prior probability** of getting infected: $p(c) = 10^{-3}$
- the **conditional probability** of having a positive test result given you are infected (true positive rate): $p(t|c) = \frac{9}{10}$
- the **conditional probability** of having a positive test result given you are not infected (false positive): $p(t|\bar{c}) = \frac{5}{100}$

$$\begin{aligned} p(c) &= 10^{-3} = 0.1\% \\ p(t|c) &= \frac{9}{10} = 90\% \\ p(t|\bar{c}) &= \frac{1}{100} = 1\% \end{aligned}$$

What is the probability that you actually are infected, given a positive test result? We can use Bayes' rule to invert the conditionals:

$$\begin{aligned} p(c|t) &= \frac{p(t|c) \cdot p(c)}{p(t)} = \frac{p(t|c) \cdot p(c)}{\sum_{c_i} p(t,c)} && (1) \\ &= \frac{p(t|c) \cdot p(c)}{\sum_{c_i} p(t|c_i) \cdot p(c_i)} = \frac{p(t|c) \cdot p(c)}{p(t|c) \cdot p(c) + p(t|\bar{c}) \cdot p(\bar{c})} && (2) \\ &= \frac{p(t|c) \cdot p(c)}{p(t|c) \cdot p(c) + p(t|\bar{c}) \cdot (1 - p(c))} && (3) \end{aligned}$$

In [5]:

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# CORONA: for binary variables, this nomenclature is practical.
p_1c    =1e-3
p_0c    =1-p_1c
p_1t_1c =9/10
p_1t_0c =1/100

p_1c_1t=(p_1t_1c*p_1c)/(p_1t_1c*p_1c+p_1t_0c*(1-p_1c))

print(p_1c_1t)
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0.08264462809917356
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In [ ]: