

Reinforcement Learning

HSE, autumn - winter 2022

Lecture 5: Policy Gradient



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Background

1. Practical RL course by YSDA, week 6
2. Past iteration course, lecture 5
3. Sutton & Barto, Chapter 13
4. DeepMind course, Lecture 9

Recap: Value-based Methods

Approximate action-value function with a neural network: $Q^*(s, a) \approx Q(s, a; \theta)$

Take action which maximises $Q(s, a; \theta)$

Recap: Value-based Methods

Approximate action-value function with a neural network: $Q^*(s, a) \approx Q(s, a; \theta)$

Take action which maximises $Q(s, a; \theta)$

- + Easy to generate policy
- + Close to true objective
- + Fairly well-understood, good algorithms exist
- Still not the true objective
- May focus capacity on irrelevant details
- Small value error can lead to larger policy error

Recap: Value-based Methods

Approximate action-value function with a neural network: $Q^*(s, a) \approx Q(s, a; \theta)$

Take action which maximises $Q(s, a; \theta)$

“When solving a problem of interest, do not solve a more general problem as an intermediate step. Try to get the answer that you really need but not a more general one.”

—Vladimir Vapnik

Recap: Objective

Suppose that since now we are living in the class of parametrised policies:

$\pi_\theta(a \mid s) = \mathbb{P}(A_t = a \mid S_t = s, \theta_t = \theta)$, where θ is some parameter.

$$\theta^* = \operatorname{argmax}_\theta J(\theta) = \operatorname{argmax}_\theta \mathbb{E}_{p_\theta(\tau)} \left[\sum_{t=0}^T \gamma^t R_t \right] = \operatorname{argmax}_\theta \mathbb{E}_{p_\theta(\tau)} [G(\tau)]$$

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$$J(\theta) = \mathbb{E}_{p_\theta(\tau)} [G(\tau)] = \int p_\theta(\tau) G(\tau) d\tau$$

$$p_\theta(\tau) = p(s_0) \pi_\theta(a_0 \mid s_0) p(r_0, s_1 \mid s_0, a_0) \dots$$

Objective's Gradient

$$p_{\theta}(\tau) = p(s_0)\pi_{\theta}(a_0 | s_0)p(r_0, s_1 | s_0, a_0)\dots$$

$$\nabla J(\theta) = \nabla \mathbb{E}_{p(\tau|\pi_{\theta})}[G(\tau)] = \int \nabla p_{\theta}(\tau)G(\tau)d\tau$$

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$$\nabla p(\tau | \pi_{\theta}) = p(\tau | \pi_{\theta})\frac{\nabla p(\tau | \pi_{\theta})}{p(\tau | \pi_{\theta})} = p(\tau | \pi_{\theta}) \nabla \log p(\tau | \pi_{\theta})$$

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Objective's Gradient

$$\nabla J(\theta) = \nabla \mathbb{E}_{p_{\theta}(\tau)}[G(\tau)] = \mathbb{E}_{p_{\theta}(\tau)}[\nabla \log p_{\theta}(\tau) G(\tau)]$$

REINFORCE

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Estimate gradient using Monte-Carlo estimator:

$$\nabla J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \left[\sum_{t=0}^T \nabla \log \pi_{\theta}(a_{i,t} | s_{i,t}) G(\tau_i) \right] = \frac{1}{N} \sum_{i=1}^N \left[\sum_{t=0}^T \nabla \log \pi_{\theta}(a_{i,t} | s_{i,t}) \sum_{t=0}^T r_{i,t} \right]$$

Make gradient ascent step:

$$\theta_{k+1} = \theta_k + \alpha \nabla J(\theta_k)$$

REINFORCE

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Make gradient ascent step:

$$\theta_{k+1} = \theta_k + \alpha \nabla J(\theta_k)$$

Note that the algorithm is on-policy
so old samples can not be used for
gradient update



No Replay Buffer

Connection with Behavioural Cloning

$$\nabla J_{BC}(\theta) = \mathbb{E}_{\tau \sim D} \left[\sum_{t=0}^T \nabla \log \pi_{\theta}(A_t | S_t) \right],$$

where D is a buffer contains samples collected by an expert

VS

$$\nabla J_{PG}(\theta) = \mathbb{E}_{p_{\theta}(\tau)} \left[\sum_{t=0}^T \nabla \log \pi_{\theta}(A_t | S_t) G(\tau) \right]$$

Entropy Regularisation

We would still like to sustain the exploration-exploitation trade-off.

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$$H(\pi_\theta(\cdot | S_t)) = - \mathbb{E}_{\pi_\theta} \log \pi_\theta(\cdot | S_t) \quad \text{General case}$$

$$H(\pi_\theta(\cdot | S_t)) = - \sum_a \pi_\theta(a | S_t) \log \pi_\theta(a | S_t) \quad \text{Discrete case}$$

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Recall that uniform distribution has largest entropy while deterministic distribution has the lowest one.

We can add regularisation term $-\rho H(\pi_{\theta}(\cdot | S_t))$ to our objective:

- To encourage an agent to increase curiosity

Connection with Behavioural Cloning

$$\nabla J_{BC}(\theta) = \mathbb{E}_{\tau \sim D} \left[\sum_{t=0}^T \nabla \log \pi_{\theta}(A_t | S_t) \right],$$

Maximise log-likelihood (minimise cross-entropy loss) to take the similar actions as an expert.

where D is a buffer contains samples collected by an expert

VS


$$\nabla J_{PG}(\theta) = \mathbb{E}_{p_{\theta}(\tau)} \left[\sum_{t=0}^T \nabla \log \pi_{\theta}(A_t | S_t) G(\tau) \right]$$

Learn actions which lead to higher returns

Policy-based RL

- + Optimise true objective
- + Easy extended to high-dimensional or even continuous action spaced
- + Learn stochastic policies
- + No prior knowledge regarding the MDP dynamics
- + Sometimes it's easy to learn policy directly instead of value function. Moreover, it seems more natural.
 - Could get stuck in local optima
 - Less sample efficient in comparison with value-based methods
 - High variance

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- Could get stuck in local optima
- Less sample efficient in comparison with value-based methods
- High variance  Let's decrease it

Variance Reduction

$$\nabla J_{PG}(\theta) = \mathbb{E}_{p_{\theta}(\tau)} \left[\sum_{t=0}^T \nabla \log \pi_{\theta}(A_t | S_t) G(\tau) \right] = \mathbb{E}_{p_{\theta}(\tau)} \left[\sum_{t=0}^T \nabla \log \pi_{\theta}(A_t | S_t) \sum_{k=0}^T \gamma^k R_k \right]$$

Variance Reduction

Current action A_t influences
only future rewards

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Variance Reduction

Current action $a_{i,t}$ influences
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$$\begin{aligned}\nabla J_{PG}(\theta) &= \mathbb{E}_{p_{\theta}(\tau)} \left[\sum_{t=0}^T \nabla \log \pi_{\theta}(A_t | S_t) G(\tau) \right] = \mathbb{E}_{p_{\theta}(\tau)} \left[\sum_{t=0}^T \nabla \log \pi_{\theta}(A_t | S_t) \sum_{k=0}^T \gamma^k R_k \right] \\ &= \mathbb{E}_{p_{\theta}(\tau)} \left[\sum_{t=0}^T \nabla \log \pi_{\theta}(A_t | S_t) \sum_{k=t}^T \gamma^k R_k \right] \\ &= \mathbb{E}_{p_{\theta}(\tau)} \left[\sum_{t=0}^T \nabla \log \pi_{\theta}(A_t | S_t) \gamma^t \sum_{k=t}^T \gamma^{k-t} R_k \right]\end{aligned}$$

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$$= \mathbb{E}_{p_{\theta}(\tau)} \left[\sum_{t=0}^T \nabla \log \pi_{\theta}(A_t | S_t) \gamma^t Q_{\pi_{\theta}}(S_t, A_t) \right] \quad \longrightarrow \quad \text{Let's ignore } \gamma^t$$

Variance Reduction: Baseline

$$\nabla J_{PG}(\theta) = \mathbb{E}_{p_{\theta}(\tau)} \left[\sum_{t=0}^T \nabla \log \pi_{\theta}(A_t | S_t) Q_{\pi_{\theta}}(S_t, A_t) \right]$$

Consider some baseline $b(S_t)$ and compute $\mathbb{E}_{p_{\theta}(\tau)} [b(S_t) \nabla \log \pi(A_t | S_t)]$:

Variance Reduction: Baseline

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$$\begin{aligned} \mathbb{E}_{p_{\theta}(\tau)} [b(S_t) \nabla \log \pi(A_t | S_t)] &= \int p_{\theta}(\tau) b(S_t) \nabla \log p_{\theta}(\tau) d\tau = \\ &= \int b(S_t) \nabla p_{\theta}(\tau) d\tau = b(S_t) \nabla \mathbb{E}_{p_{\theta}(\tau)}[1] = b(S_t) \nabla[1] = 0 \end{aligned}$$

$$\text{Var}(Q(s, a) - b(s)) = \text{Var}(Q(s, a)) + \text{Var}(b(s)) - 2\text{cov}(Q(s, a), b(s))$$

Variance Reduction: Baseline

$$\nabla J_{PG}(\theta) = \mathbb{E}_{p_{\theta}(\tau)} \left[\sum_{t=0}^T \nabla \log \pi_{\theta}(A_t | S_t) [Q_{\pi_{\theta}}(S_t, A_t) - b(S_t)] \right]$$

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Typically we take $V_{\pi_{\theta}}(S_t)$ as a baseline so $Q_{\pi_{\theta}}(S_t, A_t) - V_{\pi_{\theta}}(S_t) = A_{\pi_{\theta}}(S_t, A_t)$ is an advantage function considered in the previous lecture among DQN modification.

Actor-Critic

$$\nabla J_{AC}(\theta) = \mathbb{E}_{p_{\theta}(\tau)} \left[\sum_{t=0}^T \nabla \log \pi_{\theta}(A_t | S_t) [A_{\pi_{\theta}}(S_t, A_t)] \right]$$

We can approximate $A_{\pi_{\theta}}(S_t, A_t)$ with a neural network $A(S_t, A_t; \phi)$

... but we can make slightly better

Actor-Critic

$$\nabla J_{AC}(\theta) = \mathbb{E}_{p_{\theta}(\tau)} \left[\sum_{t=0}^T \nabla \log \pi_{\theta}(A_t | S_t) [A_{\pi_{\theta}}(S_t, A_t)] \right]$$

We can approximate $A_{\pi_{\theta}}(S_t, A_t)$ with a neural network $A(S_t, A_t; \phi)$

... but we can make slightly better

$$A_{\pi_{\theta}}(s, a) = Q_{\pi_{\theta}}(s, a) - V_{\pi_{\theta}}(s) = \mathbb{E}_{r, s' \sim p(\cdot | s, a)} [r + \gamma V(s')] \approx r + \gamma V_{\pi_{\theta}}(s') - V_{\pi_{\theta}}(s)$$

for the transition (s, a, r, s')

Advantage Actor-Critic

- Generate trajectories $\{\tau_i\}$ following $\pi_\theta(a | s)$

- Policy improvement:

Estimate gradient and make gradient ascent step:

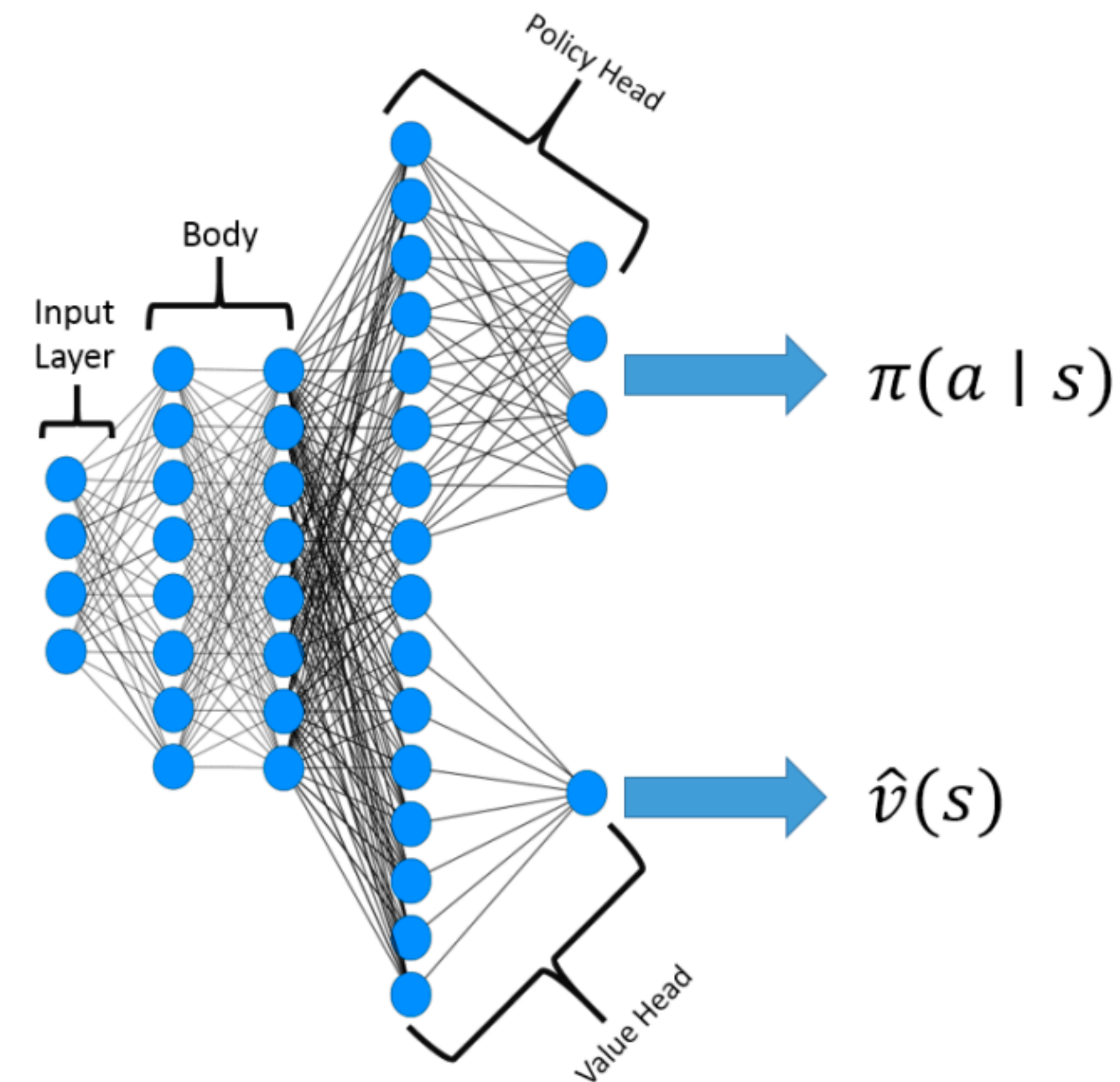
$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \left[\sum_{t=0}^T \nabla \log \pi_\theta(a_{i,t} | s_{i,t}) A_{\pi_\theta}(s_{i,t}, a_{i,t}) \right]$$

- Policy evaluation:

Estimate gradient and make gradient descent step:

$$\nabla_\phi L(\phi) \approx \frac{1}{N} \sum_{i=1}^N \left[\sum_{t=0}^T \nabla_\phi (r_{i,t} + \gamma \boxed{V_\phi(s_{i,t+1})} - V_\phi(s_{i,t}))^2 \right]$$

Not target network, just fixate parameters from the previous step



Asynchronous Advantage Actor-Critic (A3C)

Asynchronous Methods for Deep Reinforcement Learning

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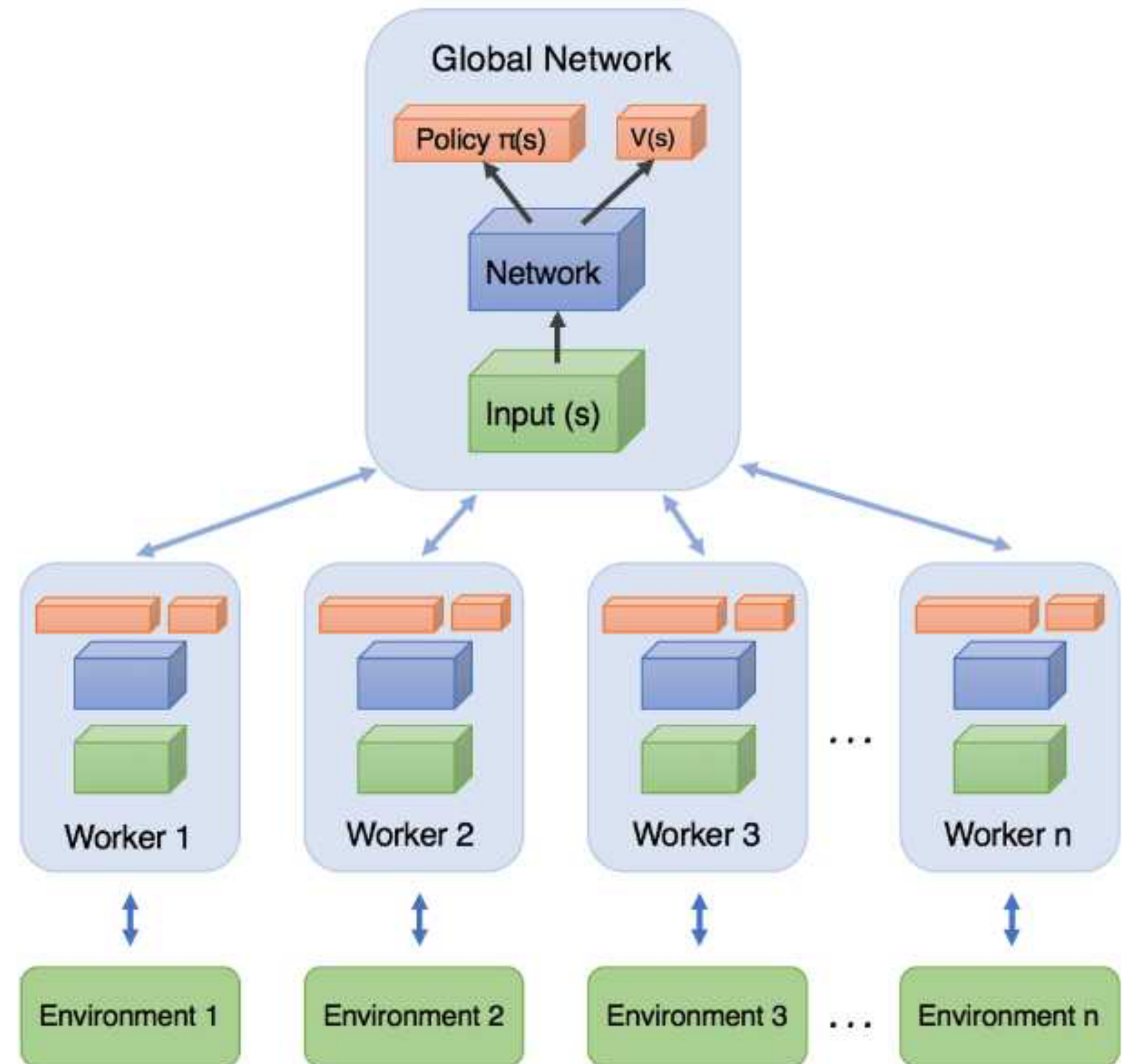
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[Original paper](#)

A3C

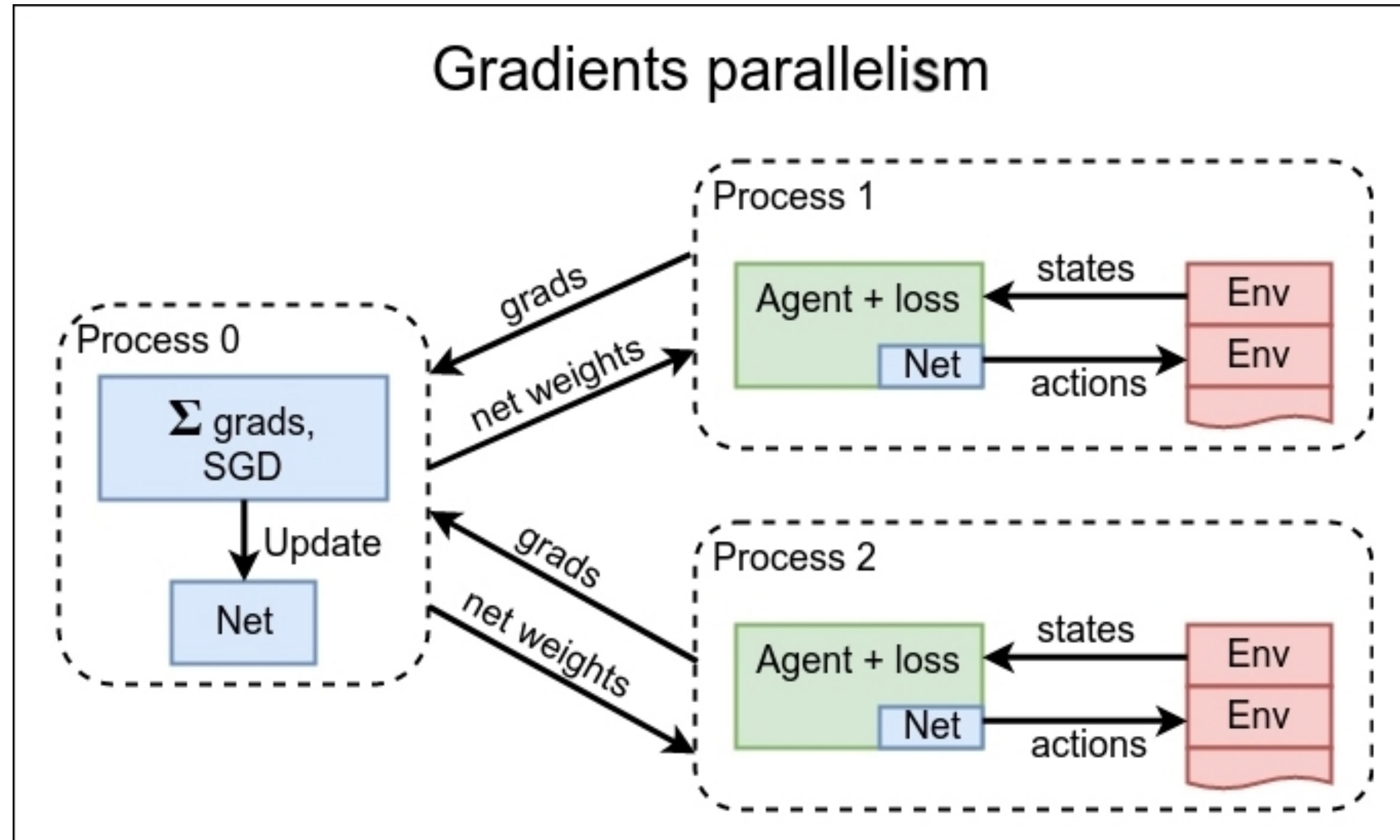
- N-step advantage estimation
- LSTM network
- No experience replay
- Entropy regularisation



Source

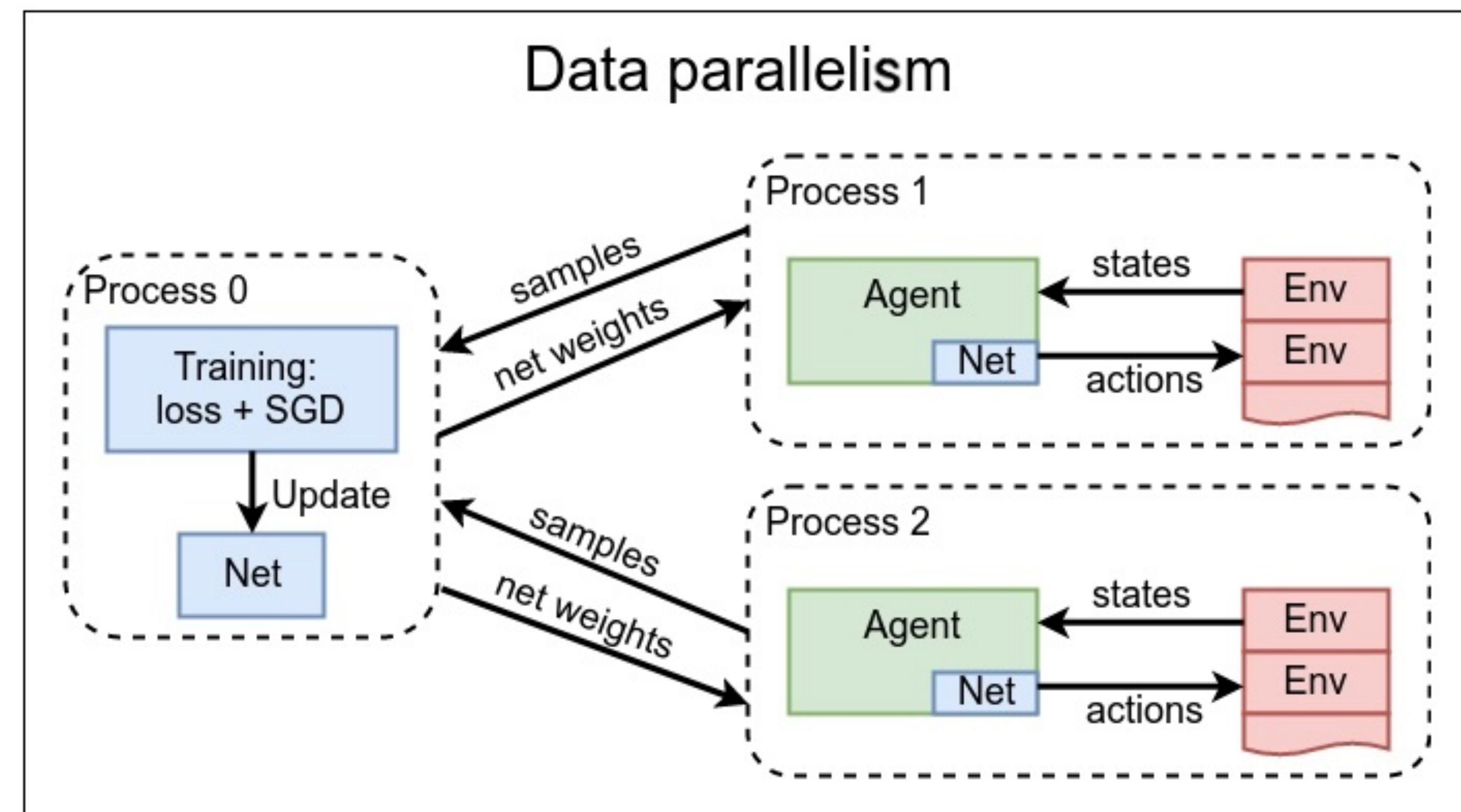
Asynchronous vs Parallel

A3C



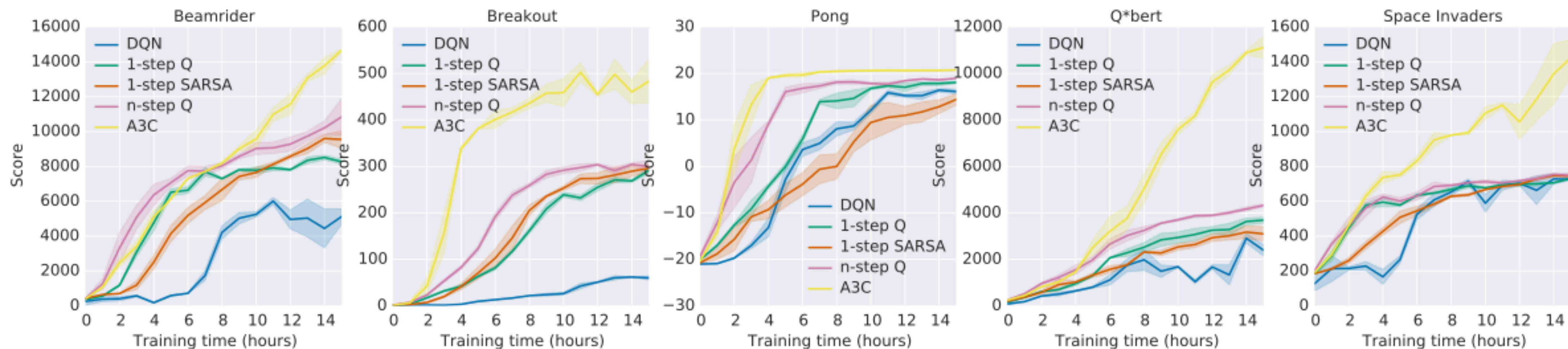
Source

A2C



Source

Comparison



Method	Training Time	Mean	Median
DQN	8 days on GPU	121.9%	47.5%
Gorila	4 days, 100 machines	215.2%	71.3%
D-DQN	8 days on GPU	332.9%	110.9%
Dueling D-DQN	8 days on GPU	343.8%	117.1%
Prioritized DQN	8 days on GPU	463.6%	127.6%
A3C, FF	1 day on CPU	344.1%	68.2%
A3C, FF	4 days on CPU	496.8%	116.6%
A3C, LSTM	4 days on CPU	623.0%	112.6%

Table 1. Mean and median human-normalized scores on 57 Atari games using the human starts evaluation metric. Supplementary Table S3 shows the raw scores for all games.

Source

Thank you for your attention!