EL2450 Homework 1

Andreas Froderberg - 19880730-4577
Martin Favre - 19920130-0010

Q 1: A gain named Tap exists in the Tank 1 model, what is its function?

A: The gain Tap models the bypass tap from the upper tank to the main tank. The value 0 means that it is currently closed. A value of 1 means fully opened.

Q 2: Edit *pid_design.m* and fill in the transfer functions for the upper and lower tank.

A: The figure below shows the result.

Q 3: What does the reference signal look like?

A: The signal starts at 40 and recieves a step by 10 at 100 s which sets it to 50.

Q 4: Use the parameter generator to get values and fill in the transfer function F.

A: Done.

Q 5: Use the parameters to get different responses. Which is best?

A: The input parameters and their respecitve system parameters are:

Table 1: Parameter values and performance.

χ	ζ	ω_0	T_r [s]	M[%]	T_s [s]
0.5	0.7	0.1	6.38	6.67	38.1
0.5	0.7	0.2	3.31	22.6	18.8
0.5	0.8	0.2	3.19	20.9	18.4

The last parameter configuration works best. It is the fastest and even though it has quite significant overshoot, it is still within the given tolerance and is thus acceptable.

Q 6: What is the cutoff frequency for the open loop system? How was this derived?

A: The open loop system is $G_o = FG$ and the cutoff frequency is the frequency when the amplitude gain is 0 dB. Using the bode plots this frequency is found to be around $\omega_c = 0.35$rad/s. This is confirmed by the MatLab command *allmargin(Go)* which gives $\omega_c = 0.343$rad/s.

Q 7: A ZOH block is placed after the controller. What is the effect?

A: The effect of the system for different ZOH times is shown below.

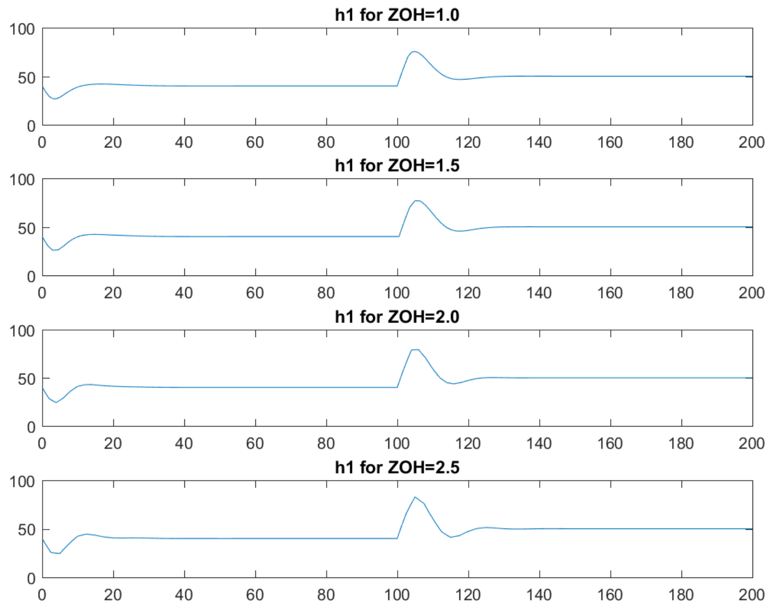


Figure 1: Upper tank level for different sampling times.

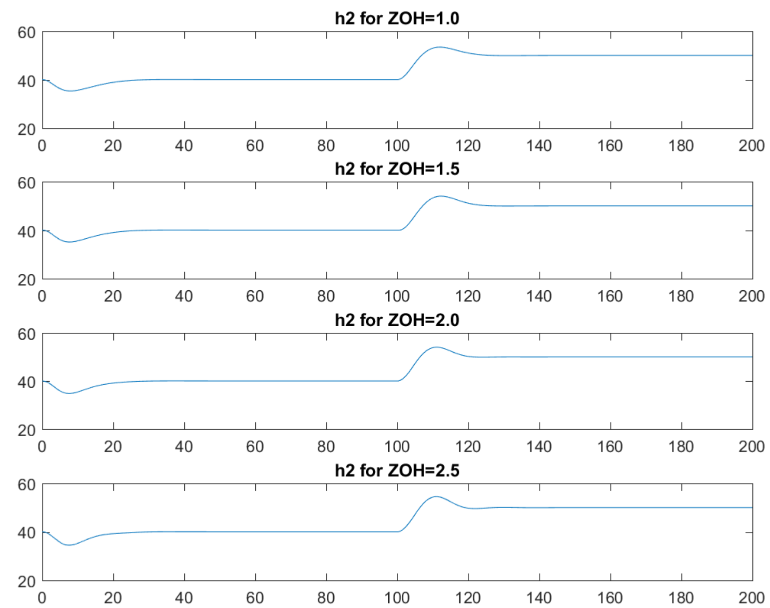


Figure 2: Lower tank level for different sampling times.

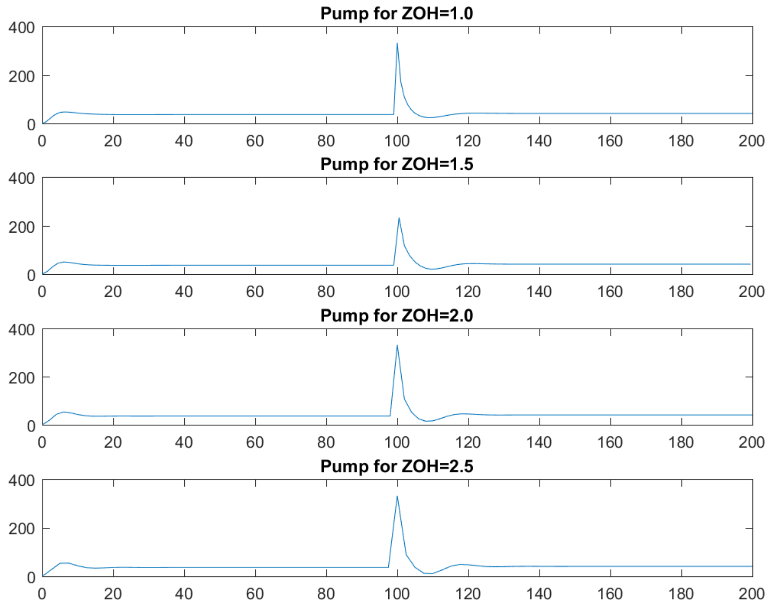


Figure 3: Pump input for different sampling times.

From the figures it can be seen that the stabilility and performance of the system decreases with higher samplnig time. For a sampling time of about 1 second, the system is stable and the performance is smooth. At 3 second sampling time, it is oscillating and is on the verge of becoming unstable.

Q 8: Discretize the continuous controller into state space form. Replace the simulink controller block with this new discrete controller. What differences can be seen?

A: The performance of the sampled system is displayed below.

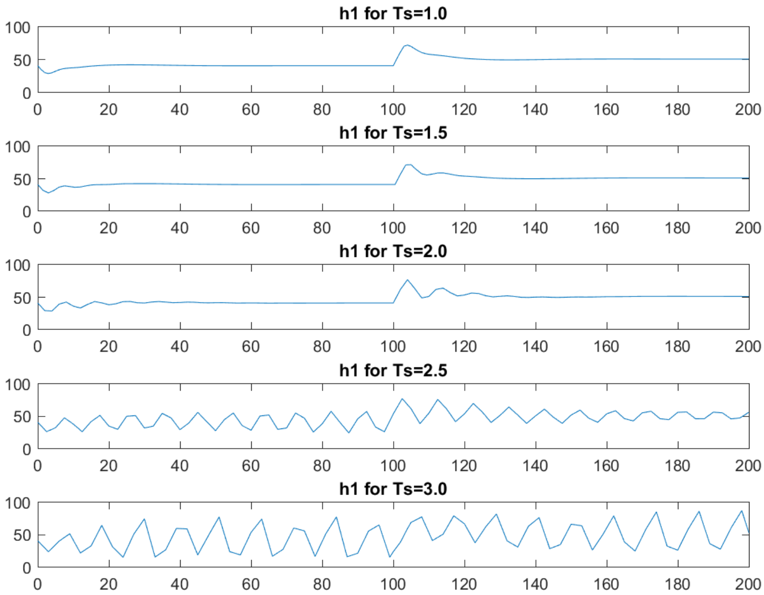


Figure 4: Upper tank level for different sampling times.

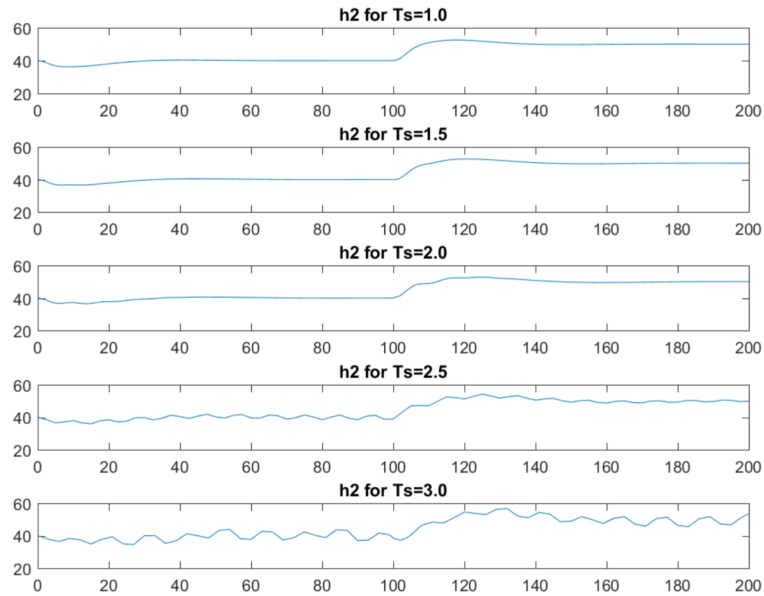


Figure 5: Lower tank level for different sampling times.

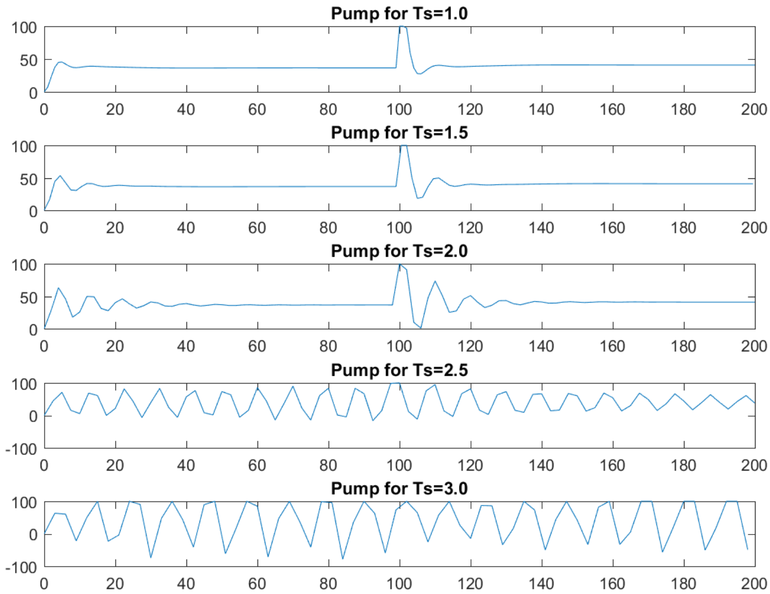


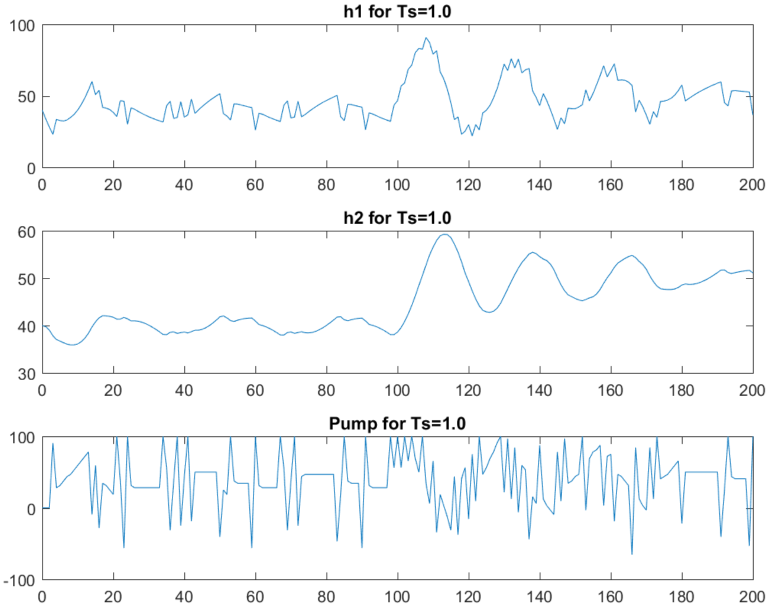
Figure 6: Pump input for different sampling times.

Comparison shows that ZOH method yields overall better results. This may be due to that the continuous system on which the ZOH method is based is more precise.

Q 9: What sampling time should be chosen?
A: The crossover frequency of the open-loop system is $\omega_c = 0.36$rad/s. The thumbrule states that the sampling frequency should be approximately $20\omega_c = 7.2rad/s$. This gives a samplingtime of $\frac{2\pi}{20\omega_c} = 0.87$ seconds

Q 10: How long can the samplingtime be without affecting the performance?
A: From question 8 it can be seen that the system is performing well up until a sampling time of about 2 seconds which is a little more than twice as fast as the rule of thumb in question 9.

Q 11: Simulate system with samplingtime at 4 seconds and estimate the control performance.
A: As can be seen below, the system is asymptotically stable but oscillating heavily. Controller performance is not good.



Q 12: Sample G, make Gd

A: The coefficients are shown in the table below.

a_1	0.092
a_2	0.074
b_1	-1.4
b_2	0.52

Q 13: Where should discrete poles be placed?

A: For a stable discrete system, the poles should be located within the unit circle.

Q 14: What are the poles corresponding to the continuous poles in the discrete case? Also, what is the pole polynomial for discrete?

A: After removing pole/zero pairs that cancel out, the continuous system contains one double pole on the real axis at -0.5 and a pair of complex conjugate poles at $0.16 \pm 0.12i$. Remapping to discrete poles $z_i = e^{T_s p_i}$ where T_s is the sampling time of 4 s, the poles in the discrete case are as displayed in the table below. Using matlabs *poly* command, the pole polynomial becomes

Poles	Comment
0.13	Real, double
$0.47 + 0.24i$	Complex
$0.47 - 0.24i$	Complex

$$0 = z^4 - 1.2z^3 + 0.55z^2 - 0.094z + 0.0051 \tag{1}$$

Q 15: Show that the closed system equations turn into the equation system.

A: For the system

$$G = \frac{a_1 z + a_2}{z^2 + b_1 z + b_2} \tag{2}$$

and the desired regulator

$$F = \frac{c_0 z^2 + c_1 z + c_2}{(z-1)(z+r)}. \tag{3}$$

The closed loop system has the equation

$$\frac{FG}{1+FG}. \tag{4}$$

Using 3, 2 and 4 and looking only at the denominator yields the pole equation

$$z^4+(b_1+r-1+c_0a_1)z^3+(b_2+b_13-b_1-r+c_0a_2+c_1a_1)z^2+(b_2r-b_2-b_1r+c_1a_2-c_2a_1)z+c_2a_2-rb_2 = 0. \tag{5}$$

Setting 5 equal to the pole polynomial calculated in task 14 and solving for each power of z separately yields the equation system in task 15.

Q 16: Solve the equation to get a discrete controller.

A: Solving the linear equation system gives

$$F_d = \frac{20.4 - 35.7 * z^{-1} + 15.7 * z^{-2}}{1 - 1.20z^{-1} + 0.197z^{-2}} \tag{6}$$

Poles	Gc	Gdc
1	1.0000 + 0.0000i	1.0000 + 0.0000i
2		
3		

Comparing the two sets poles leads to the conclusion that they are equal.