EL2450 Homework 1

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Task 1

The gain Tap models the bypass tap from the upper tank to the main tank. The value 0 means that it is currently closed. A value of 1 means fully opened.

Task 2

The figure below shows the result.

```
uppertank=tf([K],[Tau 1]);          % Transfer function for upper tank
lowertank=tf([Gamma],[Gamma*Tau 1]); % Transfer function for upper tank
```

Task 3

The signal starts at 40 and recieves a step by 10 at 100 s which sets it to 50.

Task 4

Done.

Task 5

The input parameters and their respecitve system parameters are:

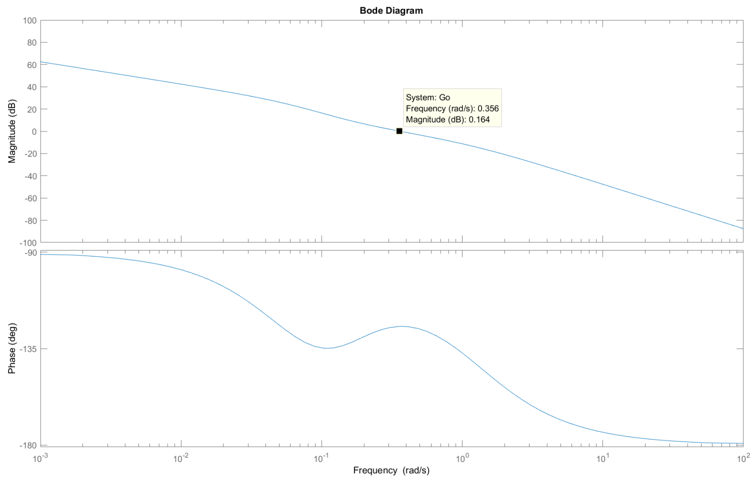
Table 1: Parameter values and performance.

χ	ζ	ω_0	T_r [s]	M[%]	T_s [s]
0.5	0.7	0.1	6.38	6.67	38.1
0.5	0.7	0.2	3.31	22.6	18.8
0.5	0.8	0.2	3.19	20.9	18.4

The last parameter configuration works best. It is the fastest and even though it has quite significant overshoot, it is still within the given tolerance and is thus acceptable.

Task 6

The open loop system is $G_o = FG$ and the cutoff frequency is the frequency when the amplitude gain is 0 dB. Using the bode plots this frequency is found to be around $\omega_c = 0.35$rad/s, see figure below.



This is confirmed by the MatLab command *allmargin(Go)* which gives $\omega_c =$ 0.343rad/s.

Task 7

The effect of the system for different ZOH times is shown below.

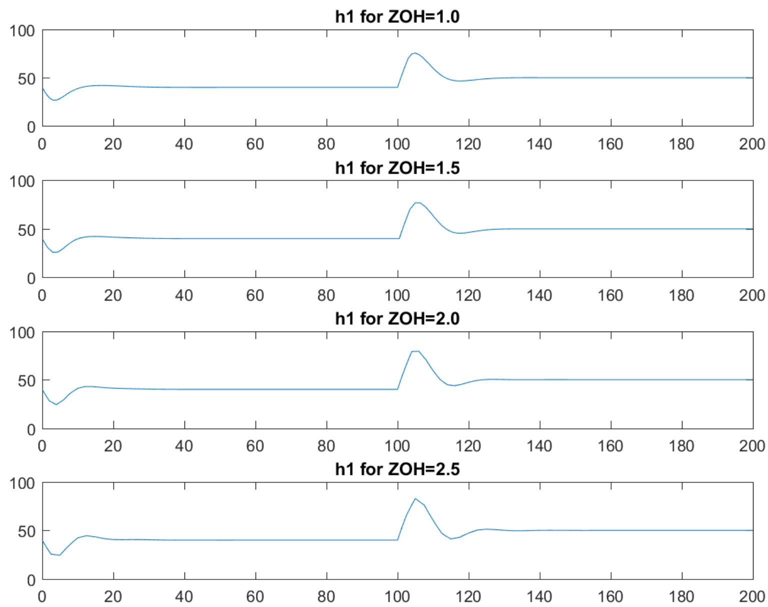


Figure 1: Upper tank level for different sampling times.

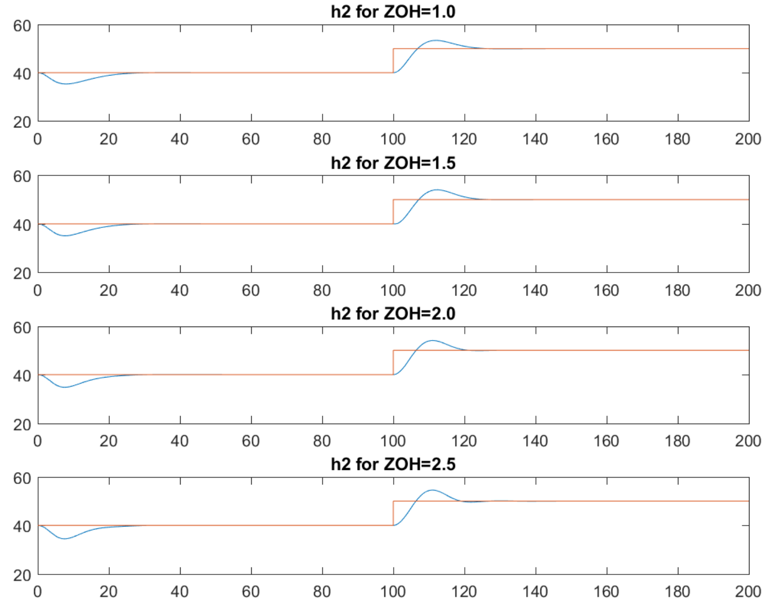


Figure 2: Lower tank level for different sampling times.

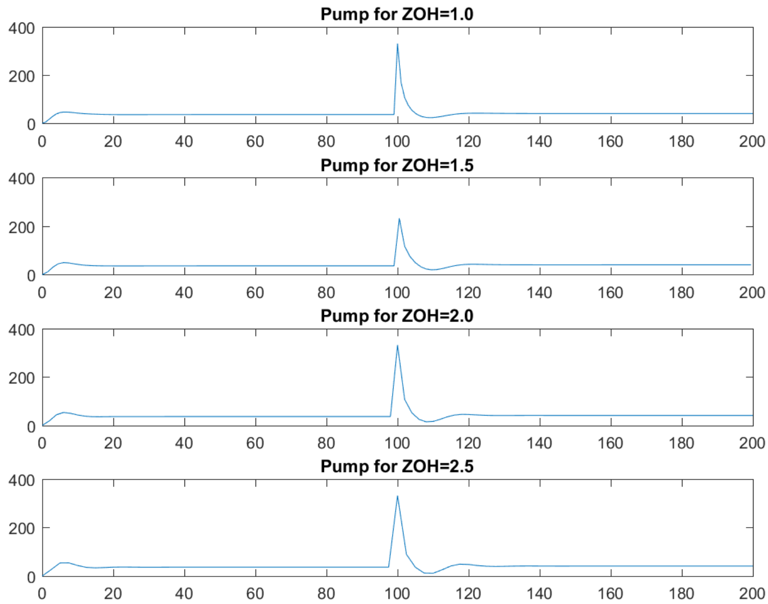


Figure 3: Pump input for different sampling times.

From the figures it can be seen that the stabilility and performance of the system decreases with higher hold time. For a hold time of about 1 second, the system is stable and the performance is smooth. At 9 second hold time, it is oscillating and is unstable.

Task 8

The performance of the sampled system is displayed below.

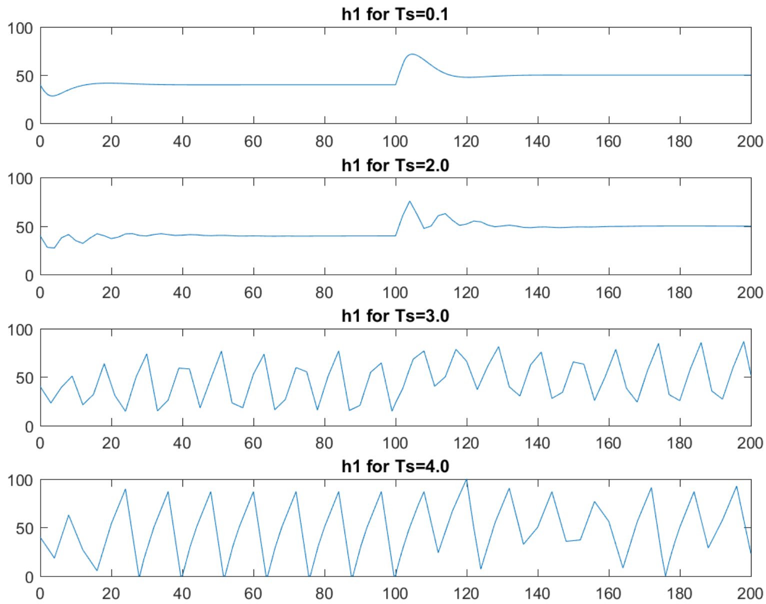


Figure 4: Upper tank level for different sampling times.

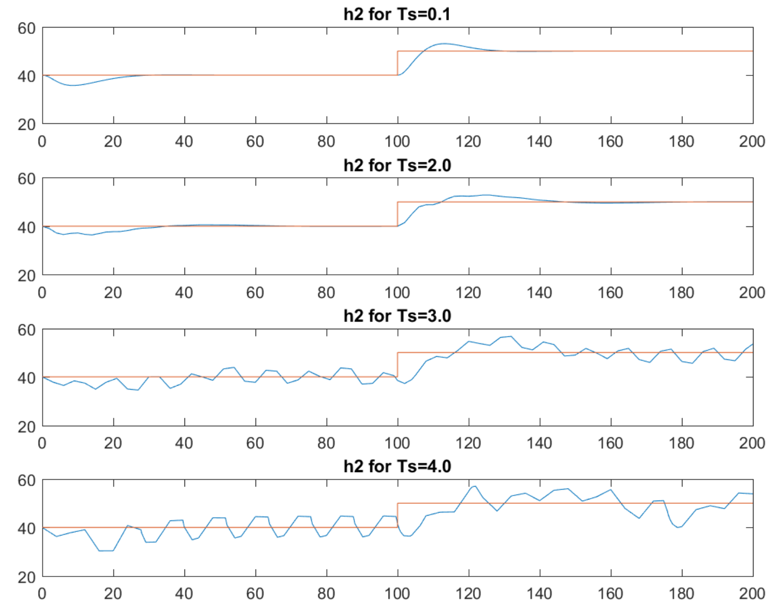


Figure 5: Lower tank level for different sampling times.

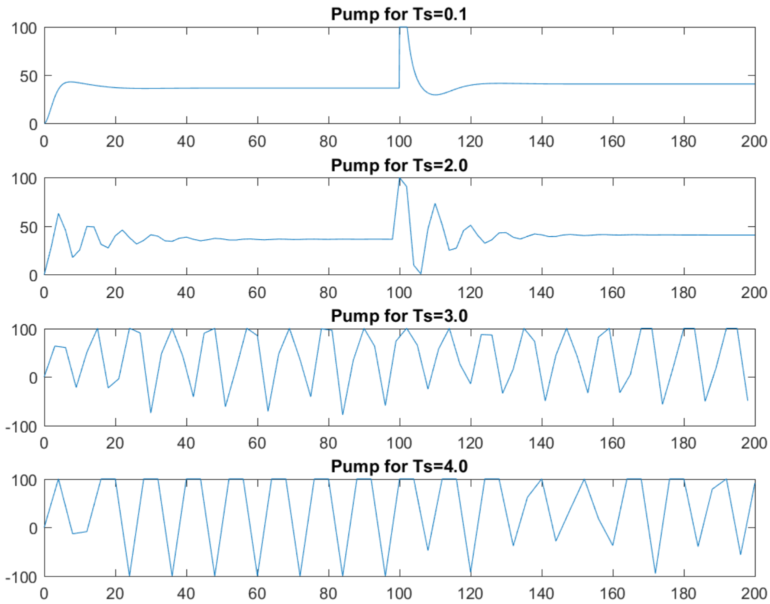


Figure 6: Pump input for different sampling times.

Comparison shows that ZOH method yields overall better results. This may be due to that the continuous system on which the ZOH method is based is more precise.

Task 9

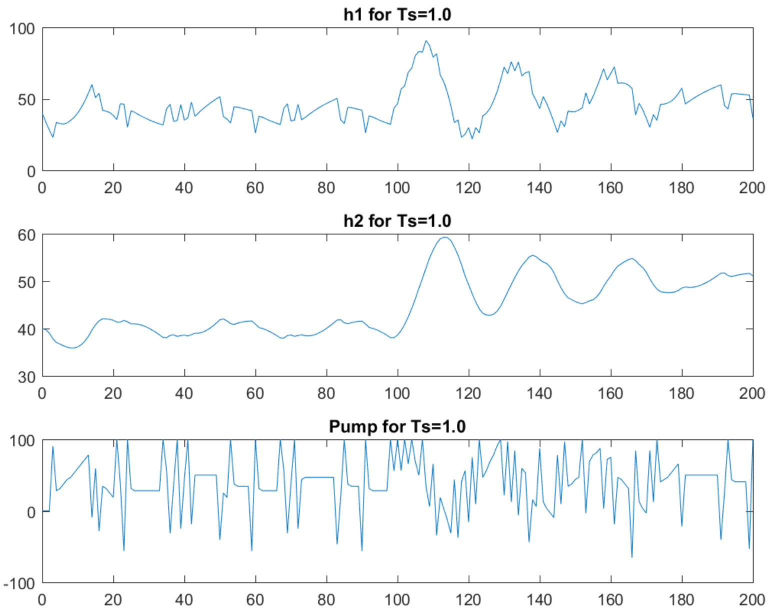
The crossover frequency of the open-loop system is $\omega_c = 0.36$rad/s. The thumbrule states that the sampling frequency should be approximately $20\omega_c = 7.2rad/s$. This gives a samplingtime of $\frac{2\pi}{20\omega_c} = 0.87$ seconds

Task 10

From question 8 it can be seen that the system is performing well up until a sampling time of about 2 seconds which is a little more than twice as fast as the rule of thumb in question 9. The rule of thumb is just a heuristic and just gives an idea of the values needed for stabilility and therefore this is considered reasonable.

Task 11

As can be seen below, the system is asymptotically stable but oscillating heavily. Controller performance is not good.



Task 12

The coefficients are shown in the table below.

a_1	0.092
a_2	0.074
b_1	-1.4
b_2	0.52

Task 13
For a stable discrete system, the poles should be located within the unit circle.

Task 14
After removing pole/zero pairs that cancel out, the continuous system contains one double pole on the real axis at -0.5 and a pair of complex conjugate poles at $0.16 \pm 0.12i$. Remapping to discrete poles $z_i = e^{T_s p_i}$ where T_s is the sampling time of 4 s, the poles in the discrete case are as displayed in the table below. Using matlabs *poly* command, the pole polynomial becomes

Poles	Comment
0.14	Real, double
$0.47 + 0.24i$	Complex
$0.47 - 0.24i$	Complex

$$0 = z^4 - 1.2z^3 + 0.55z^2 - 0.094z + 0.0051 \tag{1}$$

Task 15
For the system

$$G = \frac{a_1 z + a_2}{z^2 + b_1 z + b_2} \tag{2}$$

and the desired regulator

$$F = \frac{c_0 z^2 + c_1 z + c_2}{(z-1)(z+r)}. \tag{3}$$

The closed loop system has the equation

$$\frac{FG}{1+FG}. \tag{4}$$

Using 3, 2 and 4 and looking only at the denominator yields the pole equation

$$z^4 + (b_1 + r - 1 + c_0 a_1)z^3 + (b_2 + b_1 3 - b_1 - r + c_0 a_2 + c_1 a_1)z^2 + (b_2 r - b_2 - b_1 r + c_1 a_2 - c_2 a_1)z + c_2 a_2 - r b_2 = 0. \tag{5}$$

Setting (5) equal to the pole polynomial calculated in task 14 and solving for each power of z separately yields the equation system in task 15.

Task 16
Solving the linear equation system gives

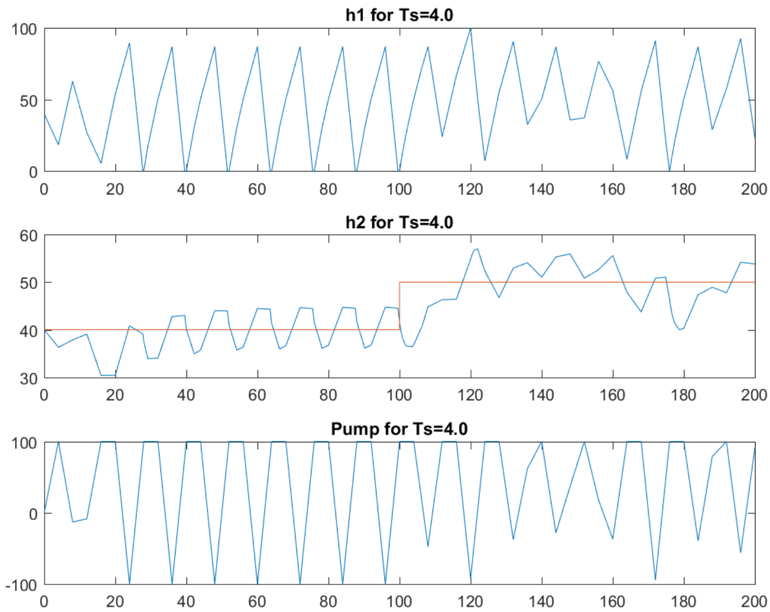
$$F_d = \frac{20.4 - 35.7 * z^{-1} + 15.7 * z^{-2}}{1 - 1.20z^{-1} + 0.197z^{-2}} \tag{6}$$

The two systems poles after minimal realization is as follows:

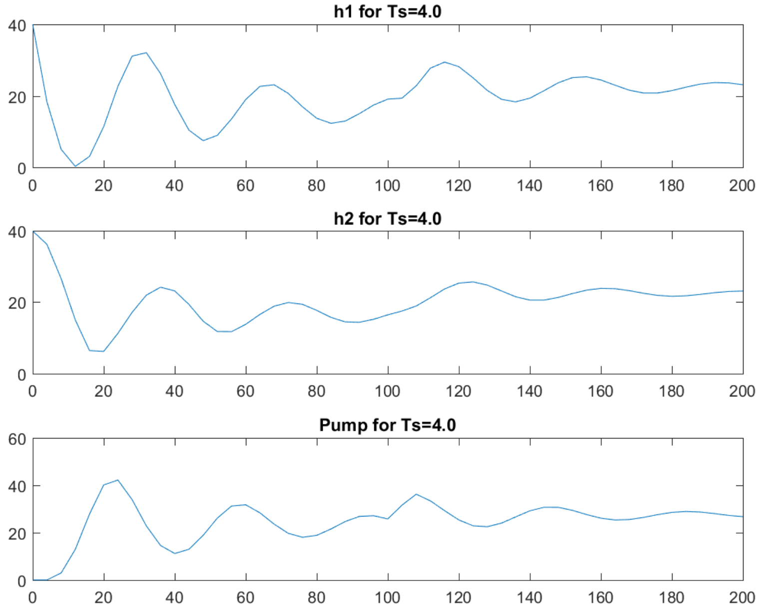
Poles	G_c	G_{dc}
1	$0.61 \pm 0i$	$0.61 \pm 0i$
2	$0.85 \pm 0.10i$	$0.85 \pm 0.10i$
3	-	$0.92 \pm 0i$

Four poles are exactly equal whilst G_{dc} has an additional real pole. This might be because of truncation errors in MatLab that cause pole/zero pair that should cancel out not to be exactly correct and thus not canceling. Since the other poles are correct, it is believed that the calculations are correct.

Task 17
The system from question 11



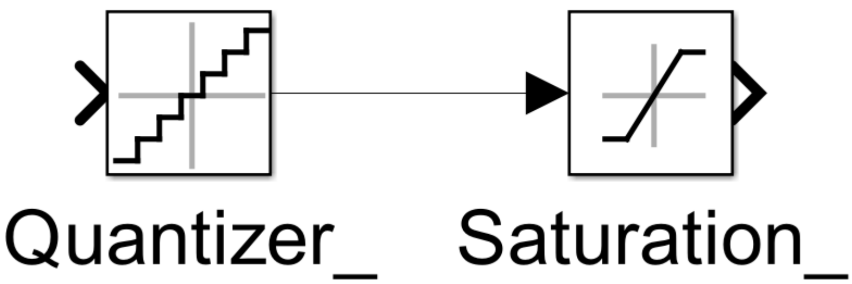
The current system



Here a great improvement can be seen.

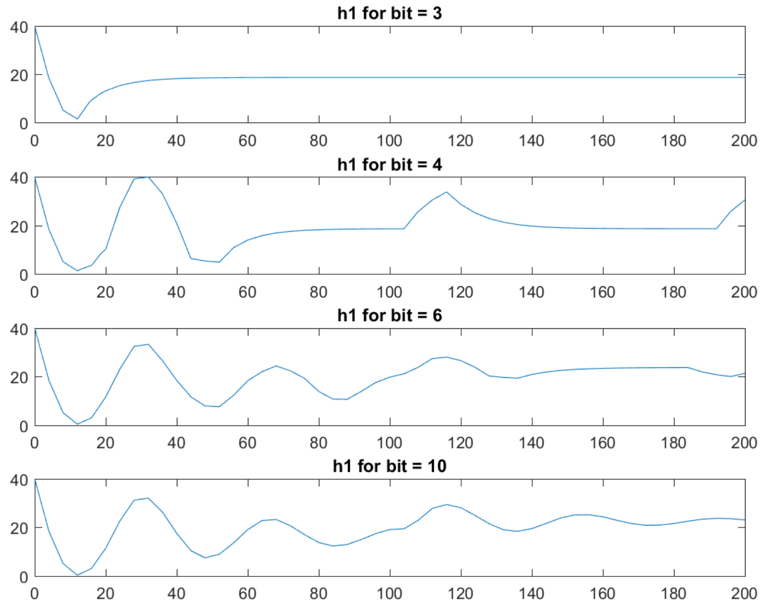
Task 18
Quantization with 10 bits is abs($\frac{0-100}{2^{10}}$) $= 9.77 \cdot 10^{-4}$

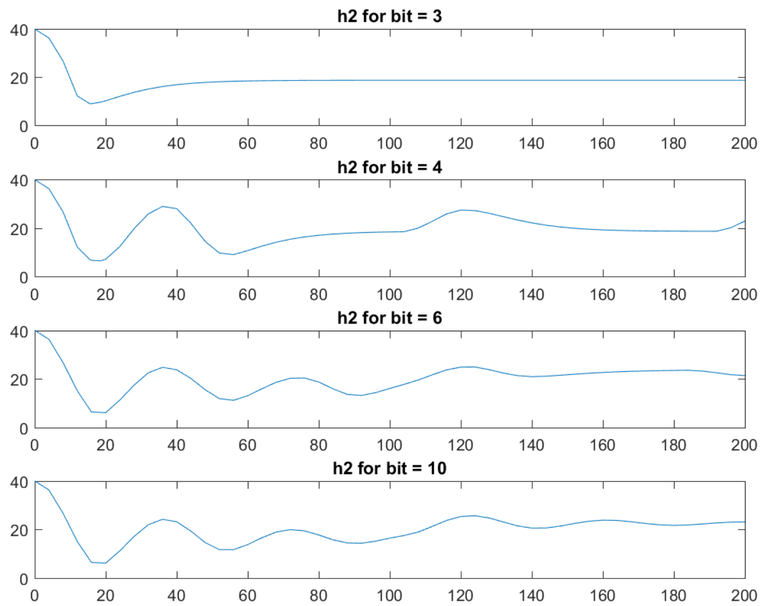
Task 19

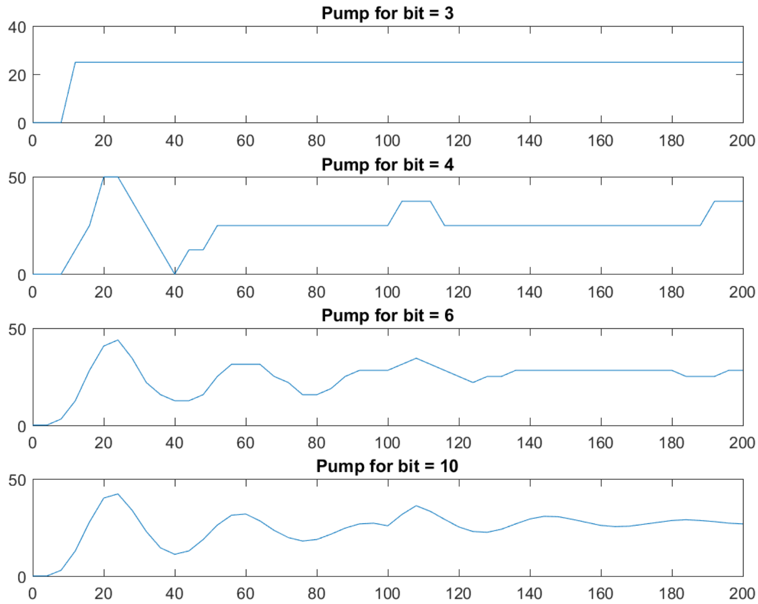


Done!

Task 20
The system was tested with multiple quantizationlevels determined by $\frac{100-(-100)}{s^n} = quantization$


h1 for bit = 3
h1 for bit = 4
h1 for bit = 6
h1 for bit = 10
0
20
40
0
20
40
60
80
100
120
140
160
180
200


h2 for bit = 3
h2 for bit = 4
h2 for bit = 6
h2 for bit = 10
0
20
40
0
20
40
60
80
100
120
140
160
180
200



Exactly when the system performance starts to degrade is subjective, but it is clear that when the system goes below a 4-bit A\D converter the performance is seriously hampered.