

# MF2007 - Workshop A

Adam Lang  
861110-3956

Gabriel Andersson Santiago  
910706-4538

Andreas Fröderberg  
880730-7577

April 3, 2016

## Parameter identification

This section covers the methods behind finding the parameters to enter into the model to get the biggest similarity to the physical plant.

### Level 1

To identify the parameters of the DC motor, firstly an idealized model is studied theoretically to understand which physical parameters that affect the performance of the motor. In the Laplace frequency domain, a model of the motor without inductance can be expressed as

$$X = \frac{k(1 + \frac{1}{Rd})}{s \frac{J_t}{\frac{k^2}{R} + d} + 1} U, \quad (1)$$

where  $X$  is the rotational speed,  $U$  is the input voltage,  $k$ ,  $R$  and  $d$  are the electric motor constant, internal resistance and the viscous friction. The variable  $J_t$  represents the total inertia of the motor and the load. With the load inertia after a gear with gear ratio  $n$ , the total inertia is

$$J_t = J + \frac{J_{load}}{n^2}. \quad (2)$$

Here, the load inertia and the inertia of the motor are separated and the motor inertia from the datasheet is presumed to be correct, leaving only the load inertia as an unknown. Now the relations between the motor parameters and the time performance can be seen. The time constant is given by the term

$$\frac{J_t}{\frac{k^2}{R} + d} \quad (3)$$

and as such contains all the parameters of the motor. According to the final value theorem, the steady state value from a step input is given by setting  $s = 0$  and therefore the steady state gain is given by

$$k(1 + \frac{1}{Rd}). \quad (4)$$

This contains only one unknown parameter. Therefore, to estimate the viscous friction parameter  $d$ , it is suitable to examine and match the steady state rotational speed of the real system and the model. Since this model contains no static friction which is probably present in the real system, the test is conducted at maximum permissible input voltage to minimize the disturbances in the test data. Since the desired value does not depend on frequency, only a square wave input is used to simulate steps in both directions.

Now that the friction parameter is presumed to be correct and fixed, the time constant in Equation (3) only has one unknown, the load inertia. This is estimated from taking the time constant from step inputs, simulated by a square wave signal. Again, to minimize model disturbances caused by the static friction present in the real system, the square wave is run at a full 24 V.

## Level 2

As could be seen in the earlier friction model, it did not incorporate the effects of static friction which led to inaccuracy in the model when varying the rotational speed. A model which captures the real behaviour of the system is used here, called the Karnop friction model. In addition to the linear friction, the static friction is taken into account by adding a stick-slip zone where the friction torque is equal to the applied torque. The model is given by the equation

$$M_f = d\dot{\phi} + F_c \text{sgn}(\dot{\phi}), \quad (5)$$

with the friction torque  $M_f$  and the static friction  $F_c$ . This friction is the maximum static friction torque that can be applied in the stick slip zone. There is now one more design parameter that needs to be tuned,  $F_c$ .

The linear friction is still most prominent when at high velocities and the static friction is the strongest when at low velocities. The inertia is decided solely from the rise time and should not differ much from the previous model. Firstly, the step response to a 5 V square wave is examined to determine the static friction. When the static friction is set, the response to a square wave at 24 V is examined and  $d$  is adjusted to fit the static friction. A sine of low amplitude is then run to test the behaviour around the stick slip area, i.e. at low speeds.

## Velocity control

The parameters generated in the previous section are used to design the controller for the motor. For the design, a simplified model without motor inductance is used in MATLAB to place the poles and estimate the behaviour of the system. Furthermore, a non-linear model with Karnop friction is used in SIMULINK to simulate the behaviour of the system with the controller implemented. Finally, the design is run on the actual motor for verification of plant model and controller.

### Level 1

Controlling the velocity is done with a PI controller. To best control the performance, the controller is designed in discrete time.

The thumb rule for sample time of a system states that the system should be sampled at 4-10 times the rise time of the plant in open loop. Using the simplified model in MATLAB and the commando `stepinfo`, the rise time is extracted and the sampling time is set to 10 times the rise time.

To get the desired performance of the closed loop system, the poles are placed directly in the discrete plane. First, the plant is converted from continuous to discrete space by Zero Order Hold (ZOH), with the sampling time derived earlier. The new system matrices are given by the equations

$$\Phi = e^{AT_s} \quad (6)$$

and

$$\Gamma = \int_0^{T_s} e^{A(t-s)} B ds. \quad (7)$$

Output feedback is used to create the closed loop system. The model of the plant is given in (1) and is here rewritten as

$$\frac{B}{A} = \frac{a}{s+b} \quad (8)$$

to simplify the equations. Firstly, the feedback polynomial  $\frac{S}{R}$  is derived. This is given by

$$\frac{S}{R} = \frac{s_1 z + s_0}{z - 1}. \quad (9)$$

The system is then closed, yielding the pole polynomial

$$AR + BS = z^2 + (as_1 + b - 1)z + as_0 - b. \quad (10)$$

The system is of order one so one closed loop pole given by

$$A_m = z - p_1 \quad (11)$$

is placed with an additional pole given by the observer polynomial

$$A_o = z - p_2 \quad (12)$$

This yields the Diophantine equation

$$A_m A_o = AR + BS. \quad (13)$$

Solving for (13) the unknown controller parameters  $s_0$  and  $s_1$  are found to be

$$s_0 = \frac{p_1 p_2 + b}{a} \quad (14)$$

and

$$s_1 = -\frac{b + p_1 + p_2 - 1}{a}. \quad (15)$$

The  $T$  polynomial is given by

$$T = t_0 A_0 \quad (16)$$

where  $t_0$  is a static value so that the steady state gain of the closed loop system is 1. Using the final-value theorem for discrete systems, one yields

$$t_0 = \left( \frac{B}{A_m} \right)^{-1} \quad (17)$$

evaluated in  $z = 1$ .