MF2007 Workshop Part1

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1 Ex. 1

1.1 State Space Model and Transfer Function

The model can be described with the following differential equation,

$$f - d\dot{x} = m\ddot{x} \Leftrightarrow \ddot{x} = \frac{1}{m}(f - d\dot{x}) \tag{1}$$

In this system we have one energy storing element, the mass, m. The transfer function can be derived by first Laplace transforming the differential equation,

$$\mathscr{L}\{\ddot{x} = \frac{1}{m}(f - d\dot{x})\} \Rightarrow s^2 Y = (\frac{1}{m}(U - dsY) \tag{2}$$

Then can the transfer function be found as,

$$G(s) = \frac{1}{ms^2 + ds} \tag{3}$$

We can also derive a state space model from the differential equations where,

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}, \tag{4}$$

so that,

$$\dot{\mathbf{x}} = \begin{bmatrix} x_2 \\ \frac{1}{m}(F - dx_2) \end{bmatrix}. \tag{5}$$

This will then give us the state space model as,

$$\begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{d}{m} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \mathbf{u} \\ \\ \mathbf{y} = \begin{bmatrix} 0 & 1 \end{bmatrix} \mathbf{x} \end{cases} \tag{6}$$

1.2 Matlab and System Characteristics

The system is put into MATLAB where the frequency response, the pole zero plot and the step response is plotted, these can be found in figure

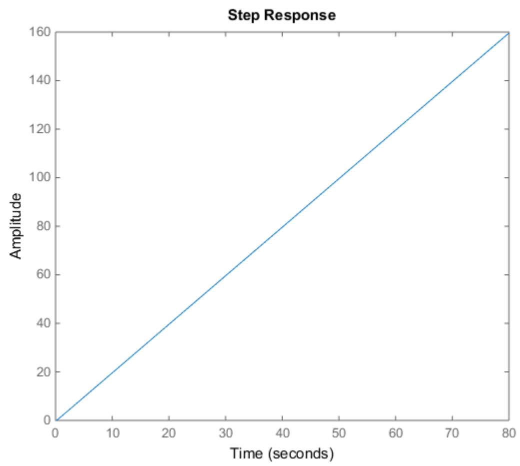

Figure 1: The step response for the system

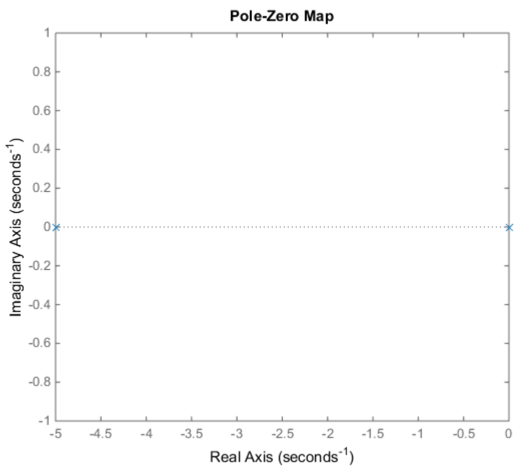

Figure 2: The pole zero plot for the system

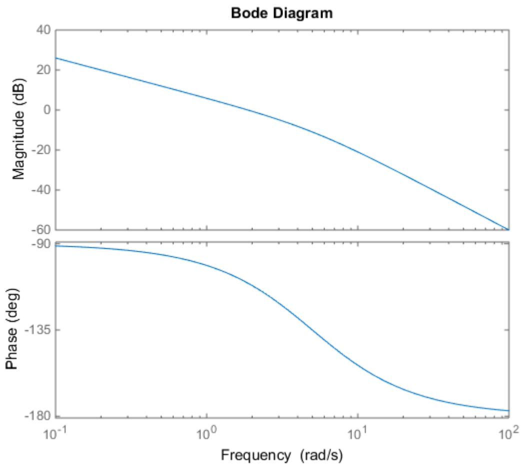

Figure 3: The frequency response for the system

2 Ex. 3

When applying a force to a weightless spring, newtons second law ($F = ma$) states that if there is no weight there will be infinite acceleration. If we have infinite acceleration with a inertia in form of a mass at the other end, there will be full compression of the spring infinitely fast. Thus we can model the input force as the spring force as input on the mass. We obtain the differential equations as

$$\begin{cases} f = m\dot{v}_2 + d\dot{v}_2 \\ f = k(v_2 - v_1) \end{cases} \tag{7}$$

After set to equal and Laplace transformation can the first transfer function be obtained as

$$G(s)_1 = \frac{1}{\frac{m}{s}s^2 + \frac{d}{k}s + 1} \tag{8}$$

and

$$G(s)_2 = \frac{1}{sm + d} \tag{9}$$

which will give the last transfer function as

$$G(s)_3 = \frac{G(s)_2}{G(s)_1} = \frac{\frac{m}{s}s^2 + \frac{d}{k}s + 1}{sm + d} \tag{10}$$

An improper transfer function represents a pure integration. Till will imply that the output is a derivation of the input. If this is true, then this will

mean that the systems input knows future characteristics of the system as implied by the defintion of the derivative $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$. Where $f(x+h)$ is the future value.
It will be hard, if not impossible, to model a real systems this way and one should proceed with caution.