Part II

Dynamics and motion control

Modelling of actuators and sensors 2016

Modelling, simulation and analysis of DC-motor with load

The DC-motor with load depicted in Figure 1 is the main modelling and control task of the exercises. It consist of a DC-motor with an incremental sensor mounted on the left hand side of the motor, the output of the motor is via a gearbox connected to a shaft which transfers the torque to the load on the right hand side. The load position is measured with a second incremental encoder. The first exercises will assume that the shaft can be modelled as a rigid body, where it later will be modelled as a flexible link between motor and load.

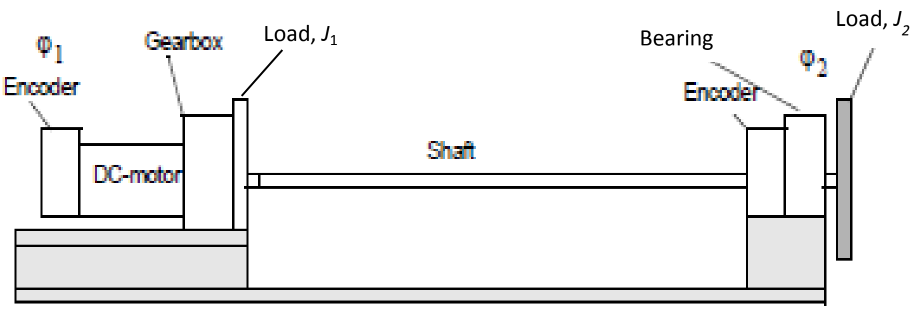


Figure 1. DC-motor with a gearbox and two flywheels connected by a shaft

Exercise 1 Model including inductance

Build a state space model using the motor data from the data sheet in Appendix 1 for the motor with winding data 949. The input to the motor is voltage, both position and velocity of the second flywheel can be used as outputs. The two flywheels are considered as two loads. Assume that the shaft is stiff and without inertia.

Data:

Ratio of gearbox	n = 5
Load 1 inertia	Jl = 1.8e-5 kgm2
Load 2 inertia	J2 = 1.8e-5 kgm2
Viscous friction in the motor	dm = 3.8e-6 Nms/rad
Viscous friction of the two loads	d1 = d2 = 0.0 Nms/rad
Supply voltage range	Vmax = 24 V, Vmin = -24 V
Maximum current	Imax = 430 mA

- Write a Matlab script to obtain the following: (1) the state space model with the voltage as the input and the position and velocity of the second flywheel as the outputs, (2) poles & zeros of the two transfer functions, (3) step responses of the two transfer functions, (4) frequency responses of the two transfer functions..
- Compare the step response for all states and compare with the poles and draw conclusions.
- Build a Simulink model based on simple blocks, e.g., integrators, gains and summation blocks.
- Simulate the state space block in the same Simulink model and verify that the result is the same as for the model based on simple blocks (compare all states).
- (*Optional*) Build a Simscape model for the DC motor system. The "DC Motor" block is in the library "Simscape/SimElectronics/Actuators & Drivers/Rotational Actuators". The "Gear Box" block is in the library "Simscape/Foundation Library/Mechanical/Mechanisms". Compare the Simscape model with the Simulink model on model complexity and accuracy.

Exercise 2 Simplified model without inductance

Simplify the model by neglecting the inductance. The system outputs are still the position and velocity of the second flywheel. Find the following:

- States space and transfer function models both in Matlab and Simulink and a block diagram model in Simulink. Consider two different inputs: (1) voltage and (2) motor current.
- Compare the results of the model with voltage as input with those from Exercise 1 and identify the changes in the dynamics.
- Analysis: poles & zeros, step response and frequency response
- (*Optional*) Modify the Simscape model from Exercise 1 to neglect the motor inductance. Verify the new Simscape model has the same result as the new Simulink model.

Exercise 3 Include static friction in the motor and gearbox

Include a Coulomb friction in the Simulink model of Exercise 2.

The total Coulomb friction torque for the motor and the gearbox is $T_c = 1e\text{-}3\ Nm$

The Coulomb friction is a constant force/torque at any velocity. When the velocity is zero or its absolute value is below a velocity threshold, the friction force/torque is equal to the applied force/torque if its absolute value is less than or equal to Coulomb friction force/torque. If the applied force/torque is larger than Coulomb friction or the absolute value of the velocity is larger than the velocity threshold, then the actual friction is equal to Coulomb friction.

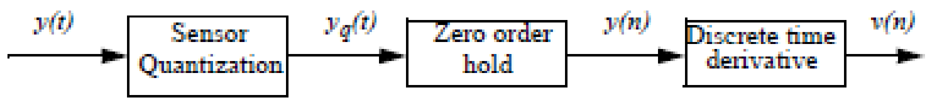
Simulate both with and without Coulomb friction to show the effects of Coulomb friction.

- Step responses with a very small voltage, e.g., 1 V, and a large voltage, e.g., 10 V. Observe the difference when Coulomb friction is present.
- Sine wave input, $u = a\sin(\omega t)$. Vary amplitude and frequency and show when the Coulomb friction has most influence. At which voltage does the motor start?
- Plot the applied force, friction force and velocity.
- (*Optional*) Add Coulomb friction torque to the Simscape model of Exercise 2. The Coulomb friction can be added through the block “Rotational Friction” at the library “Simscape/Foundation Library/Mechanical/Rotational Elements”. Add two torque sensors to measure the torques of the motor and the friction torque. Check if your Simscape model agrees with the Simulink model.

Exercise 4 Sensor simulation

Include a model of the incremental encoder in Simulink, it should be simple to change the number of steps of the sensor.

To simulate the behavior of the real position sensor and how the velocity is calculated on the embedded computer, the discrete time derivative of the position sensor signal should be included as shown in the figure below.



When you plot discrete time signals in Matlab you should use the command *stairs* instead of *plot*. *Stairs* shows the signal level as it is seen by the embedded computer whereas *plot* interpolates between samples.

- Simulate with two different sensor resolutions, 512 and 8192 pulses per revolution, and three different sampling intervals, 100 ms, 10 ms, and 1 ms.
- Zoom in on the velocity plot and compare the simulated quantization level with the theoretical quantization level. **Tip**, you may have to do a simulation which gives low velocities to get the correct simulated quantization. Why?
- (*Optional*) Add the same Simulink blocks for the discrete sensors into the Simscape model.

Exercise 5 Gearbox and flexible shaft

When the stiffness of the shaft between the two flywheels, as illustrated in Figure 1, is low with respect to the closed loop bandwidth of the controller it has to be modelled.

Shaft Parameters:

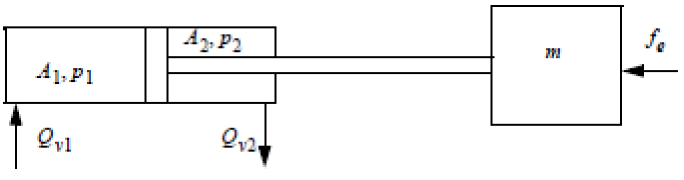
Shaft stiffness	kf = 0.05 Nm/rad
Viscous friction (damping) in the shaft	df = 0.0001 Nms/rad

Deliver the following material:

- Derive a state space model with both motor and load 2's velocities as outputs. The input is the voltage and the motor inductance is ignored.
- Poles and zeros of the model. The zeros should be for both motor velocity and load 2 velocities. Why are they different?
- Step and frequency response. (compare with Exercise 2)
- Divide the model into two parts: a stiff part corresponds to the model from Ex 2 and the other part corresponds to the resonance and anti-resonance frequencies from this new model. Verify that the divided model and the complete state space two models identical.
- (Optional) Improve the Simscape model from Exercise 3 to change the rigid shaft to a flexible shaft. Does the Simscape model produce the same step response as the state space model? If not, why? Change the parameters of the "Rotational Friction" block to so that the Simscape model produces identical result as the state space model.

Exercise 6 Hydraulic Actuator

Derive the nonlinear model for the non-symmetric valve driven hydraulic cylinder shown in the figure. Build a Simulink model which can move the mass in both directions that is, so that either the first cylinder chamber is connected to the pump and the second to the tank or opposite. In the figure are the flow directions for extracting motion shown (to the right in the figure). The other direction with opposite flows as in the figure is called retraction (to the left).



Data: Cylinder:

Cylinder inner diameter 5.0 [cm].
Piston rod diameter 2.0 [cm].
Maximum piston stroke 0.7 [m]
Total moving mass 100 [kg]

Linear friction coefficient 200 [Ns/m]

Valve:

Flow constant Rv = 1e-4
Valve opening range -1.0..1.0 [cm]

Pump:

Supply pressure 20 [MPa]
Tank return pressure 0.0 [MPa]

Simulate the nonlinear model for different constant valve openings for both positive and negative valve signals.
Apply a external force of $f_e = 10000$ [N]when the oscillations have stopped.
Plot both velocity and pressures, explain the difference in steady state pressures p_1 and p_2.

Linearization:

Derive a linear model of the cylinder, set the cross sectional cylinder chamber areas equal. Take the mean value of A_1 and A_2. Try to linearize around different operating points, i.e., different external force f_e and different valve openings x_v.
Compare the simulation results between the nonlinear and the linear models, look at both velocity and load pressure. Where are the poles and zeros located and what influences them the most?
The linear model is an approximation of the nonlinear model, what seems to make the difference between the models the largest?