Ideas for a paper on multi-resolution localized voltage embedding

1 Introduction

We will for this project put the implementation of the epsilon cover on hold. Instead, we use the existing cover tree as a preprocessing step, which we use to construct the epsilon cover and gather statistics in advance, and store to file. We will then focus on how we can construct the multi-resolution voltage scheme.

We need to demonstrate that the multi-resolution scheme gives an efficient embedding of large point clouds. We will work on the following contributions.

1.1 Numerical contribution

- Implement multi-resolution scheme for voltage propagation
- Demonstrate time efficiency
- Show numerically how our scaling with densities makes the epsilon cover voltages converge to the voltage constructed with access to all data points
- Toy examples
  - Demonstrate embedding on the swiss roll data set, minst dataset, cells
  - Aim at data sets of sizes in the 100 000 data points range.
  - Show that we have injectivity for the datasets we embedd. (All datapoints have a unique representation, or at least approximately)
  - Compare with Laplacian eigenmaps embedding

1.2 Theoretical contribution

(i) Show theoretically how scaling with densities makes epsilon cover voltages converge to the voltage in the infinite data limit

- Not too difficult. Use Reiman integral arguments
- Would knowing the continuum operator make this proof easier?

(ii) Can we derive injectivity proofs for more advanced point clouds than the sphere?

(iii) Can we show that the multi-resolution scheme gives an embedding that is injective on the sphere? (Or is this ”obvious”?)

References