Министерство науки и высшего образования Российской Федерации

Федеральное государственное бюджетное образовательное учреждение высшего образования
«МОСКОВСКИЙ ГОСУДАРСТВЕННЫЙ ТЕХНИЧЕСКИЙ УНИВЕРСИТЕТ имени Н.Э.БАУМАНА
(национальный исследовательский университет)»

Факультет: Информатика и системы управления

Кафедра: Теоретическая информатика и компьютерные технологии

Лабораторная работа №2

«Методы оптимизации нейронных сетей»

По дисциплине «Теория искусственных нейронных сетей»

Работу выполнил

студент группы ИУ9-72Б

Владиславов А.Г.

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```python
import json
import random
import math
import pickle
from copy import deepcopy
import time

import numpy as np
import matplotlib.pyplot as plt

FILE_NAME = 'model.json'


class Sample:
    def __init__(self, input, expected):
        self.input = input
        self.expected = expected

    def show(self):
        plt.imshow(self.input.reshape(28, 28), cmap='gray')
        plt.show()


def combine_samples(samples):
    inpt = np.array([x.input for x in samples]).transpose()
    expect = np.array([x.expected for x in samples]).transpose()
    return Sample(inpt, expect)


def get_samples(train):
    samples = []
    x_train, t_train, x_test, t_test = mnist_load()
    if train:
        data = zip(x_train, t_train)
    else:
        data = zip(x_test, t_test)
    for x, y in data:
        one_hot = np.array([int(i == y) for i in range(10)])
        samples.append(Sample(np.vectorize(lambda x : x / 255)(x),
one_hot))
    return samples


def mnist_load():
    with open("../datasets/mnist/mnist.pkl", "rb") as f:
        mnist = pickle.load(f)
    return (
        mnist["training_images"],
        mnist["training_labels"],
```

```python
        mnist["test_images"],
        mnist["test_labels"],
    )


def show_random_wrong_sample(score):
    wrong = []
    for pred in score.predictions:
        if pred.sample.expected.argmax() != pred.prediction.argmax():
            wrong.append(pred)
    w = wrong[random.randint(0, len(wrong))]
    print('Expected: ', w.sample.expected.argmax())
    print('Got: ', w.prediction.argmax())
    w.sample.show()


class Prediction:
    def __init__(self, sample, prediction):
        self.sample = sample
        self.prediction = prediction


class ScoreResult:
    def __init__(self, accuracy, loss, predictions):
        self.accuracy = accuracy
        self.loss = loss
        self.predictions = predictions

    def __str__(self):
        return f"accuracy: {self.accuracy * 100}%, loss: {self.loss}"


class RunResult:
    def __init__(self, activations, weighted_inputs):
        self.activations = activations
        self.weighted_inputs = weighted_inputs

train_data = get_samples(True)
test_data = get_samples(False)

class SGD():
    def __init__(self, lr=0.01):
        self.lr = lr

    def init_params(self, model):
        self.model = model

    def step(self, grad_w, grad_b, batch):
        for i in range(1, len(self.model.layers)):
```

```
            self.model.weights[i] -= self.lr * grad_w[i]
            self.model.biases[i] -= self.lr * grad_b[i]

class Momentum():
    def __init__(self, lr=0.01, momentum=0.6):
        self.m = momentum
        self.lr = lr

    def init_params(self, model):
        self.model = model

        self.velocity_w = [None]
        self.velocity_b = [None]
        for i in range(1, len(self.model.layers)):

self.velocity_w.append(np.zeros_like(self.model.weights[i]))

self.velocity_b.append(np.zeros_like(self.model.biases[i]))

    def step(self, grad_w, grad_b, batch):
        for i in range(1, len(self.model.layers)):
            self.velocity_w[i] = self.m * self.velocity_w[i] + self.lr
* grad_w[i]
            self.velocity_b[i] = self.m * self.velocity_b[i] + self.lr
* grad_b[i]
            self.model.weights[i] -= self.velocity_w[i]
            self.model.biases[i] -= self.velocity_b[i]

class Nesterov():
    def __init__(self, lr=0.01, momentum=0.6):
        self.m = momentum
        self.lr = lr

    def init_params(self, model):
        self.model = model

        self.velocity_w = [None]
        self.velocity_b = [None]
        for i in range(1, len(self.model.layers)):

self.velocity_w.append(np.zeros_like(self.model.weights[i]))

self.velocity_b.append(np.zeros_like(self.model.biases[i]))

    def step(self, grad_w, grad_b, batch):
        model_ahead =  deepcopy(self.model)

        for i in range(1, len(model_ahead.layers)):
            model_ahead.weights[i] -= self.m * self.velocity_w[i]
            model_ahead.biases[i] -= self.m * self.velocity_b[i]
```

```
        result = model_ahead.run(batch)
        errors = model_ahead.backprop(batch, result)
        grad_ahead_w, grad_ahead_b =
model_ahead.calculate_grad(result.activations, errors)

        for i in range(1, len(model_ahead.layers)):
            self.velocity_w[i] = self.m * self.velocity_w[i] + self.lr
* grad_ahead_w[i]
            self.velocity_b[i] = self.m * self.velocity_b[i] + self.lr
* grad_ahead_b[i]

            self.model.weights[i] -= self.velocity_w[i]
            self.model.biases[i] -= self.velocity_b[i]

class AdaGrad():
    def __init__(self, lr=1):
        self.lr = lr
        self.eps = 1e-10


    def init_params(self, model):
        self.model = model

        self.N_w = [None]
        self.N_b = [None]
        for i in range(1, len(self.model.layers)):
            self.N_w.append(np.zeros_like(self.model.weights[i]))
            self.N_b.append(np.zeros_like(self.model.biases[i]))

    def step(self, grad_w, grad_b, batch):
        for i in range(1, len(self.model.layers)):
            self.N_w[i] += grad_w[i] ** 2
            self.N_b[i] += grad_b[i] ** 2

            self.model.weights[i] -= self.lr * grad_w[i] /
(np.sqrt(self.N_w[i]) + self.eps)
            self.model.biases[i] -= self.lr * grad_b[i] /
(np.sqrt(self.N_b[i]) + self.eps)

class RMSprop():
    def __init__(self, lr=0.01, decay=0.9):
        self.lr = lr
        self.decay = decay
        self.eps = 1e-10

    def init_params(self, model):
        self.model = model

        self.N_w = [None]
```

```
        self.N_b = [None]
        for i in range(1, len(self.model.layers)):
            self.N_w.append(np.zeros_like(self.model.weights[i]))
            self.N_b.append(np.zeros_like(self.model.biases[i]))

    def step(self, grad_w, grad_b, batch):
        for i in range(1, len(self.model.layers)):
            self.N_w[i] = self.decay * self.N_w[i] + (1-self.decay) *
grad_w[i] ** 2
            self.N_b[i] = self.decay * self.N_b[i] + (1-self.decay) *
grad_b[i] ** 2

            self.model.weights[i] -= self.lr * grad_w[i] /
(np.sqrt(self.N_w[i]) + self.eps)
            self.model.biases[i] -= self.lr * grad_b[i] /
(np.sqrt(self.N_b[i]) + self.eps)

class AdaDelta():
    def __init__(self, lr=0.01, decay=0.9):
        self.lr = lr
        self.decay = decay
        self.eps = 1e-10

    def init_params(self, model):
        self.model = model

        self.N_w = [None]
        self.N_b = [None]

        self.P_w = [None]
        self.P_b = [None]
        for i in range(1, len(self.model.layers)):
            self.N_w.append(np.zeros_like(self.model.weights[i]))
            self.N_b.append(np.zeros_like(self.model.biases[i]))

            self.P_w.append(np.zeros_like(self.model.weights[i]))
            self.P_b.append(np.zeros_like(self.model.biases[i]))


    def step(self, grad_w, grad_b, batch):
        for i in range(1, len(self.model.layers)):
            self.N_w[i] = self.decay * self.N_w[i] + (1-
self.decay)*grad_w[i] ** 2
            self.N_b[i] = self.decay * self.N_b[i] + (1-
self.decay)*grad_b[i] ** 2

            d_w = grad_w[i] * np.sqrt(self.P_w[i] + self.eps) /
np.sqrt(self.N_w[i] + self.eps)
            d_b = grad_b[i] * np.sqrt(self.P_b[i] + self.eps) /
np.sqrt(self.N_b[i] + self.eps)
```

```
            self.P_w[i] = self.decay * self.P_w[i] + (1 - self.decay)
* d_w ** 2
            self.P_b[i] = self.decay * self.P_b[i] + (1 - self.decay)
* d_b ** 2
            self.model.weights[i] -= self.lr * d_w
            self.model.biases[i] -= self.lr * d_b

class Adam():
    def __init__(self, lr=0.01, beta1=0.9, beta2=0.999):
        self.lr = lr
        self.beta1 = beta1
        self.beta2 = beta2
        self.eps = 1e-10

    def init_params(self, model):
        self.model = model

        self.M_w = [None]
        self.M_b = [None]

        self.N_w = [None]
        self.N_b = [None]
        for i in range(1, len(self.model.layers)):
            self.M_w.append(np.zeros_like(self.model.weights[i]))
            self.M_b.append(np.zeros_like(self.model.biases[i]))

            self.N_w.append(np.zeros_like(self.model.weights[i]))
            self.N_b.append(np.zeros_like(self.model.biases[i]))

    def step(self, grad_w, grad_b, batch):
        t = self.model.current_epoch + 1
        for i in range(1, len(self.model.layers)):
            self.M_w[i] = self.beta1 * self.M_w[i] + (1 - self.beta1)
* grad_w[i]
            self.M_b[i] = self.beta1 * self.M_b[i] + (1 - self.beta1)
* grad_b[i]

            self.N_w[i] = self.beta2 * self.N_w[i] + (1 - self.beta2)
* grad_w[i] ** 2
            self.N_b[i] = self.beta2 * self.N_b[i] + (1 - self.beta2)
* grad_b[i] ** 2

            m_w_hat = self.M_w[i] / (1 - self.beta1 ** t)
            m_b_hat = self.M_b[i] / (1 - self.beta1 ** t)

            n_w_hat = self.N_w[i] / (1 - self.beta2 ** t)
            n_b_hat = self.N_b[i] / (1 - self.beta2 ** t)

            self.model.weights[i] -= self.lr * m_w_hat /
```

```
(np.sqrt(n_w_hat) + self.eps)
            self.model.biases[i] -= self.lr * m_b_hat /
(np.sqrt(n_b_hat) + self.eps)

class Network:
    def __init__(self, layers, optimizer, batch_size):
        self.current_epoch = 0
        self.layers = layers
        self.weights = [None]
        self.biases = [None]
        self.batch_size = batch_size
        for i in range(1, len(layers)):
            self.weights.append(np.random.uniform(-1, -1, (layers[i],
layers[i - 1])) * np.sqrt(1/layers[i - 1]))
            self.biases.append(np.random.uniform(-1, 1, layers[i]))
        self.optimizer = optimizer
        optimizer.init_params(self)

    def dump(self, path):
        d = {
            "layers": self.layers,
            "weights": [x.tolist() for x in self.weights[1:]],
            "biases": [x.tolist() for x in self.biases[1:]],
        }
        with open(path, 'w') as f:
            json.dump(d, f)

    def load(self, path):
        with open(path, 'r') as f:
            d = json.load(f)
        self.layers = d["layers"]
        self.weights = [None] + [np.array(x) for x in d["weights"]]
        self.biases = [None] + [np.array(x) for x in d["biases"]]

    def train(self, samples, epochs, monitor_dataset=None):
        samples_copy = samples[:]
        res = []
        for epoch in range(epochs):
            self.current_epoch += 1
            random.shuffle(samples_copy)
            for i in range(0, len(samples_copy), self.batch_size):
                batch =
combine_samples(samples_copy[i:i+self.batch_size])
                result = self.run(batch)
                errors = self.backprop(batch, result)
                grad_w, grad_b =
self.calculate_grad(result.activations, errors)
                self.optimizer.step(grad_w, grad_b, batch)
            if monitor_dataset is not None:
                score = self.score(monitor_dataset)
```

```
                res.append(score)
        return res

    def run(self, sample):
        activations = [None for _ in self.layers]
        activations[0] = sample.input
        weighted_inputs = [None for _ in self.layers]
        for i in range(1, len(self.layers)):
            weighted_inputs[i] = self.weights[i] @ activations[i - 1]
+ self.biases[i].reshape(self.layers[i], 1)
            activations[i] = self.calc_activation(weighted_inputs[i])
        return RunResult(activations, weighted_inputs)

    def backprop(self, sample, result):
        errors = [None for _ in self.layers]
        errors[-1] = self.calc_delta(sample.expected,
result.activations[-1])
        for i in reversed(range(1, len(self.layers) - 1)):
            errors[i] = (
                (np.transpose(self.weights[i + 1]) @ errors[i + 1])
                *
self.calc_activation_derivative(result.weighted_inputs[i])
            )
        return errors

    def calculate_grad(self, activations, errors):
        grad_w = [None]
        grad_b = [None]
        for i in range(1, len(self.layers)):
            grad_w.append(errors[i] @ activations[i - 1].transpose() /
self.batch_size)
            grad_b.append(errors[i].sum(axis=1) / self.batch_size)
        return grad_w, grad_b

    def score(self, samples):
        cost = 0
        accurate = 0
        predictions = []
        for sample in samples:
            res = self.run(combine_samples([sample]))
            out = res.activations[-1][:, 0]
            pred = Prediction(sample, out)
            predictions.append(pred)
            cost += self.calc_cost(sample.expected, out)
            if out.argmax() == sample.expected.argmax():
                accurate += 1
        return ScoreResult(accurate / len(samples), cost /
len(samples), predictions)

    def calc_cost(self, expected, out):
```

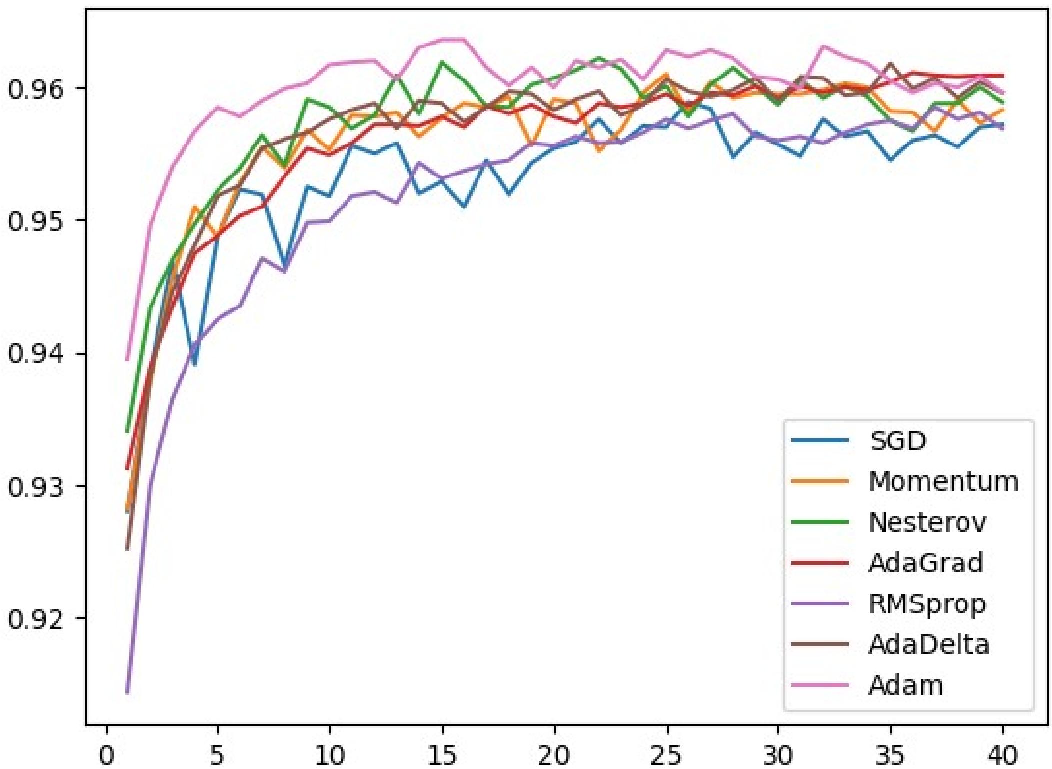
```
        return np.sum(np.nan_to_num(-expected*np.log(out)) - (1 -
expected) * np.log(1 - out))

    def calc_delta(self, expected, activations):
        return (activations - expected)

    def calc_activation(self, x):
        return 1 / (1 + np.exp(-x))

    def calc_activation_derivative(self, x):
        return self.calc_activation(x) * (1 - self.calc_activation(x))

optimizers = [
    SGD(1),
    Momentum(0.1, 0.8),
    Nesterov(0.1, 0.8),
    AdaGrad(0.1),
    RMSprop(0.001, 0.9),
    AdaDelta(100),
    Adam(0.01, 0.9, 0.999),
]

EPOCHS = 40

scores = []
labels = ["SGD", "Momentum", "Nesterov", "AdaGrad", "RMSprop",
"AdaDelta", "Adam"]

for i, optimizer in enumerate(optimizers):
    start_time = time.time()
    network = Network(layers=[784, 30, 10], optimizer=optimizer,
batch_size=10)
    score = network.train(train_data, EPOCHS,
monitor_dataset=test_data)
    print(f"{labels[i]}: {score[-1]}")
    print(f"{time.time() - start_time}")
    scores.append(score)
```

```
SGD: accuracy: 95.72%, loss: 0.33273904947070304
42.52070593833923
Momentum: accuracy: 95.83%, loss: 0.32346239906508806
47.275773763656616
Nesterov: accuracy: 95.89%, loss: 0.32140317102912935
77.97539782524109
AdaGrad: accuracy: 96.09%, loss: 0.2666548896643032
49.052425146102905
RMSprop: accuracy: 95.69%, loss: 0.4622452222234267
55.96908211708069
AdaDelta: accuracy: 95.96000000000001%, loss: 0.4471318212923637
69.02415299415588
```

```
Adam: accuracy: 95.96000000000001%, loss: 0.34356949469522235
68.53807091712952
```

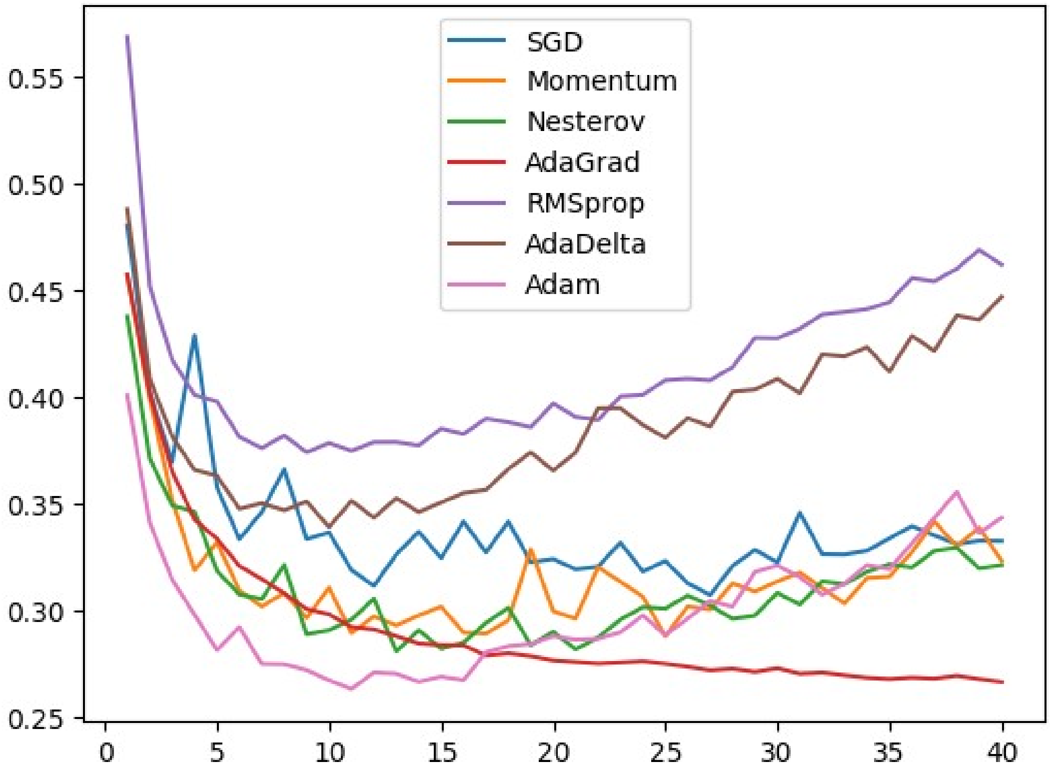
```
xs = list(range(1, EPOCHS + 1))
for i in range(len(optimizers)):
    ys = [x.accuracy for x in scores[i]]
    plt.plot(xs, ys)
plt.legend(labels)
```

```
<matplotlib.legend.Legend at 0x2eaec2880>
```




```
xs = list(range(1, EPOCHS + 1))
for i in range(len(optimizers)):
    ys = [x.loss for x in scores[i]]
    plt.plot(xs, ys)
plt.legend(labels)
```

```
<matplotlib.legend.Legend at 0x2b5c24be0>
```


SGD
Momentum
Nesterov
AdaGrad
RMSprop
AdaDelta
Adam
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0
5
10
15
20
25
30
35
40