

# *M*icroeconometrics

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Philipp Eisenhauer



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# Regression discontinuity design

Philipp Eisenhauer

I heavily draw on the material presented in:

- ▶ Lee, D. S., & Lemieux, T. (2010). Regression discontinuity designs in economics. *Journal of economic literature*, 48(2), 281–355.

## Issues

- ▶ intuition
- ▶ identification
- ▶ interpretation
- ▶ estimation

## Key points

- ▶ RD designs can be invalid if individuals can precisely manipulate the assignment variable.
  - ▶ discontinuity rules might generate incentives
- ▶ If individuals - even while having some influence - are unable to **precisely** manipulate the assignment variable, a **consequence** of this is that the variation in treatment near the threshold is randomized as though from a randomized experiment.
  - ▶ contrast to IV assumption

## Key points

- ▶ RD designs can be analyzed - and tested - like randomized experiments.
- ▶ Graphical representation of an RD design is helpful and informative, but the visual presentation should not be tilted toward either finding an effect or finding no effect.
- ▶ Nonparametric estimation does not represent a "solution" to functional form issues raised by RD designs. It is therefore helpful to view it as a complement to - rather than a substitute for - parametric estimation.

## Key points

- ▶ Goodness-of-fit and other statistical tests can help rule out overly restrictive specifications.

## Baseline

A simple way to estimating the treatment effect  $\tau$  is to run the following linear regression.

$$Y = \alpha + D\tau + X\beta + \epsilon,$$

where  $D \in [0, 1]$  and we have  $D = 1$  if  $X \geq c$  and  $D = 0$  otherwise.



# Baseline setup



Figure 1. Simple Linear RD Setup

# Potential outcome framework



Figure 2. Nonlinear RD

## Potential outcome framework

$$E[Y_i(1) - Y_i(0) \mid X = c]$$

⇒ average treatment effect at the cutoff

## Alternatives

Consider the standard assumptions for matching:

- ▶ ignorability
  - ▶ trivially satisfied by research design
- ▶ common support
  - ▶ cannot be satisfied and replaced by continuity

## **Alternatives**

Lee and Lemieux (2010) emphasize the close connection of RDD to randomized experiments.

- ▶ How does the graph in the potential outcome framework change?



Figure 3. Randomized Experiment as a RD Design

- ▶ *Continuity*, the key assumption of RDD, is a **consequence** of the research design and not simply imposed.

# Identification



## Question

- ▶ How do I know whether an RD design is appropriate for my context? When are the identification assumptions plausible or implausible?

## Answers

- ✗ An RD design will be appropriate if it is plausible that all other unobservable factors are "continuously" related to the assignment variable.
- ✓ When there is a continuously distributed stochastic error component to the assignment variable - which can occur when optimizing agents do not have *precise* control over the assignment variable - then the variation in the treatment will be as good as randomized in a neighborhood around the discontinuity threshold.

## Question

- ▶ Is there any way I can test those assumptions?

## Answers

- × No, the continuity assumption is necessary so there are no tests for the validity of the design.
- ✓ Yes. As in randomized experiment, the distribution of observed baseline covariates should not change discontinuously around the threshold.

## Simplified setup

$$Y = D\tau + W\delta_1 + U$$

$$D = I[X \geq c]$$

$$X = W\delta_2 + V$$

- ▶  $W$  is the vector of all predetermined and observable characteristics.

What are the source of heterogeneity in the outcome and assignment variable?

## Simplified setup

The setup for an RD design is more flexible than other estimation strategies.

- ▶ We allow for  $W$  to be endogenously determined as long as it is determined prior to  $V$ .
- ▶ We take no stance as to whether some elements  $\delta_1$  and  $\delta_2$  are zero (exclusion restrictions)
- ▶ We make no assumptions about the correlations between  $W$ ,  $U$ , and  $V$ .



Figure 4. Density of Assignment Variable Conditional on  $W = w, U = u$

## Local randomization

**Definition** We say individuals have imprecise control over  $X$  when conditional on  $W = w$  and  $U = u$  the density of  $V$  (and hence  $X$ ) is continuous.



## Applying Baye's rule

$$\begin{aligned} \Pr[W = w, U = u \mid X = x] \\ = f(x \mid W = w, U = u) \frac{\Pr[W = w, U = u]}{f(x)} \end{aligned}$$

**Local randomization** If individuals have imprecise control over  $X$  as defined above, then  $\Pr[W = w, U = u \mid X = x]$  is continuous in  $x$ : the treatment is "as good as" randomly assigned around the cutoff.

⇒ the **behavioral** assumption of imprecise control of  $X$  around the threshold has the **prediction** that treatment is locally randomized.

## Consequences

- ▶ testing prediction that  $\Pr[W = w, U = u \mid X = x]$  is continuous in  $x$
- ▶ irrelevance of including baseline covariates

# Interpretation

## Questions

- ▶ To what extent are results from RD designs generalizable?

## Answers

- ✗ The RD estimate of the treatment effect is only applicable to the subpopulation of individuals at the discontinuity threshold and uninformative about the effect everywhere else.
- ✓ The RD estimand can be interpreted as a weighted average treatment effect, where the weights are relative ex ante probability that the value of an individual's assignment variable will be in the neighborhood of the threshold.

## Accounting for treatment effect heterogeneity

$$Y = D\tau(W, U) + W\delta_1 + U$$

What is creating treatment effect heterogeneity?

## Accounting for treatment effect heterogeneity

$$\lim_{\epsilon \downarrow 0} E(Y | X = c + \epsilon) - \lim_{\epsilon \uparrow 0} E(Y | X = c + \epsilon) = ?$$



## **Alternative evaluation strategies**

- ▶ randomized experiment
- ▶ regression discontinuity design
- ▶ matching on observables
- ▶ instrumental variables

How do the (assumed) relationships between the observables and unobservable differ?

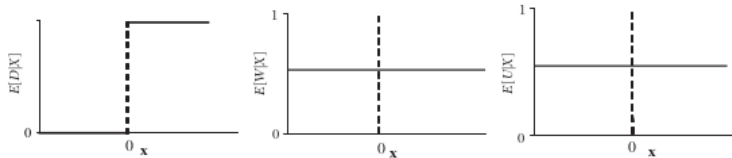
## Endogenous dummy variable

$$Y = D\tau + W\delta_1 + U$$

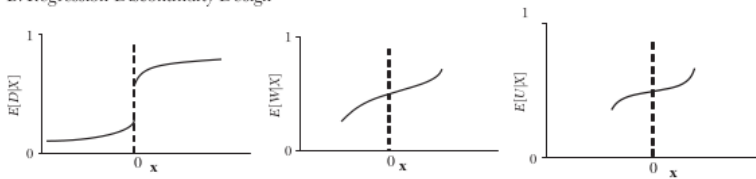
$$D = I[X \geq c]$$

$$X = W\delta_2 + V$$

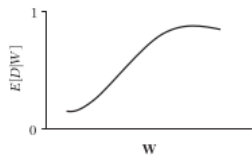
# A. Randomized Experiment



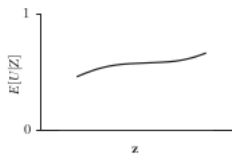
## B. Regression Discontinuity Design



### C. Matching on Observables



#### D. Instrumental Variables



# Estimation

We will explore issues in estimation using a Python notebook.

<http://bit.ly/2WGjWNI>



# Checklist

## Recommendations

- ▶ To assess the possibility of manipulations of the assignment variable, show its distribution.
- ▶ Present the main RD graph using binned local averages.
- ▶ Graph a benchmark polynomial specification.

## Recommendations

- ▶ Explore the sensitivity of the results to a range of bandwidth, and a range of orders to the polynomial.
- ▶ Conduct a parallel RD analysis on the baseline covariates.
- ▶ Explore the sensitivity of the results to the inclusion of baseline covariates.

# Resources

## Technical

- ▶ Hahn, J., Todd, P. E., & van der Klaauw, W. (2001). Identification and estimation of treatment effects with a regression-discontinuity design. *Econometrica*, 69(1), 201–209.

## Applications

- ▶ Lee, D. S. (2008). Randomized experiments from non-random selection in US House elections. *Journal of Econometrics*, 142(2), 675–697.
- ▶ Thistlethwaite, D. L., & Campbell, D. T. (1960). Regression-discontinuity analysis: An alternative to the ex-post facto experiment. *Journal of Educational Psychology*, 51(6), 309–317.

# Appendix

# *References*



- Hahn, J., Todd, P. E., & van der Klaauw, W. (2001). Identification and estimation of treatment effects with a regression-discontinuity design. *Econometrica*, 69(1), 201–209.
- Lee, D. S. (2008). Randomized experiments from non-random selection in US House elections. *Journal of Econometrics*, 142(2), 675–697.
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