

Markov?      memoriless

$$P(x_{t+1} | x_0 \dots x_t) = P(x_{t+1} | x_t)$$

Today

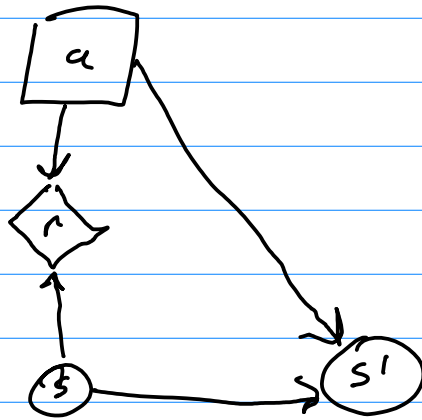
What is an MDP?

What is a policy?

How to evaluate policies

Policy Search

MDPs



$$\text{maximize } \sum_{t=0}^{\infty} \gamma^t r_t$$

## "Tuple Definition"

$$(S, A, R, T, \gamma) + p_0$$

$$\begin{array}{l} \downarrow \\ s \in S \\ s = (x, y) \quad S = \mathbb{R}^4 \end{array}$$

$S$  - state space - set of states  $\left\{ \begin{array}{l} \text{e.g. } \{1, 2, 3\}, \mathbb{R}^4 \\ \{ \text{working}, \text{malfunctioning} \} \\ [0, 1]^4 \end{array} \right\}$

$A$  - action space - set of actions

$$A(s)$$

$R$  - reward function  $R: S \times A \times S \rightarrow \mathbb{R}$

$$R(s, a) \equiv E_{s'} [R(s, a, s')]$$

$T$  - "transition kernel"

Explicit or Implicit ("Generative Model")

$$\uparrow T(s' | s, a)$$

$$s' = G(s, a)$$

$\gamma$  - discount  $\gamma \in [0, 1)$

$p_0$  - initial state distribution

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## Breakout Rooms

Cooking a pot of Pasta

$$(S, A, R, T, \gamma)$$

## Team 5

$$S = \underbrace{\{1 \dots 10\}}_{\text{water temp}} \times \underbrace{\{1 \dots 10\}}_{\text{softness}} \times \underbrace{\{0 \dots 5\}}_{\text{saltiness}}$$

$$A = \{\text{up, down, add salt, eat, taste, add sauce,}\}$$

$$R = + \text{fastness} - \frac{(\text{softness} - 5)^2}{2} \text{ timestep}$$

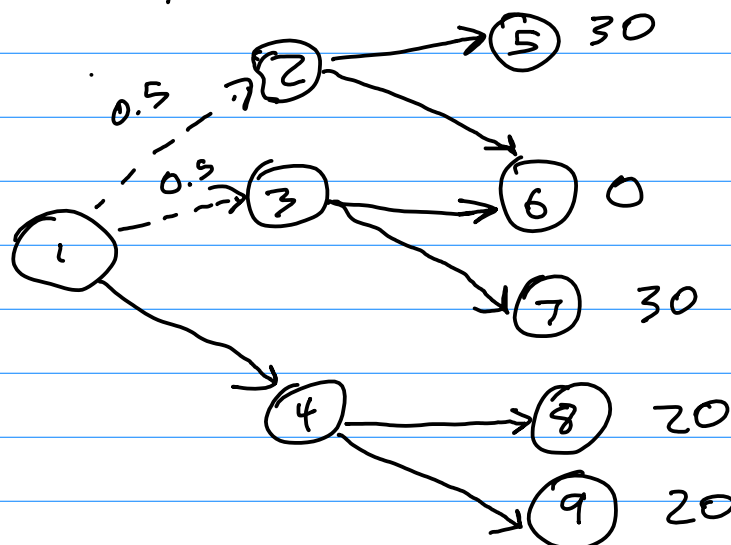
$$T = \text{if } a = \text{high temp increases, if temp} = 10 \text{ softness} += 1$$

$$\gamma = 0.99$$

Policies determine what actions are taken

Open Loop: sequence

Closed Loop:  $\pi: S \rightarrow A$



$$A = \{\text{up, down}\}$$

open loop policies

$$(\uparrow, \uparrow) \quad 0.5 \cdot 30 + 0.5 \cdot 0 = 15$$

$$(\uparrow, \downarrow) \quad 15$$

$$(\downarrow, \downarrow) \quad 20$$

$$(\downarrow, \uparrow) \quad 20$$

Closed Loop  $(\uparrow, \begin{cases} \uparrow & \text{if } 2 \\ \downarrow & \text{otherwise} \end{cases}) \quad 0.5 \cdot 30 + 0.5 \cdot 30 = 30$

## Evaluation

estimate  $u \equiv \mathbb{E}_{s_0 \sim p_0} \left[ \sum_{t=0}^{\infty} \gamma^t r_t \mid a_t = \pi(s_t) \right]$

$$\hat{u} = \sum_{t=0}^{T-1} \gamma^t r_t$$

Simulation

$s \leftarrow \text{sample}(p_0)$

$\hat{u} \leftarrow 0$

for  $t$  in  $0 \dots T-1$

$s', r \leftarrow G(s, a)$

$\hat{u} += \gamma^t r$

$s \leftarrow s'$

return  $\hat{u}$

$$u \approx \bar{u}_m = \frac{1}{m} \sum_{i=1}^m \hat{u}_i$$

← from simulation

↑  
Monte Carlo Estimate