ASEN 5519-003 Decision Making under Uncertainty
Homework 1: Probabilistic Models

January 14, 2021

1 Conceptual Questions

Question 1. (20 pts) Consider the following joint distribution of three binary-valued random variables, A, B, and C:

A	B	C	$P(A, B, C)$
0	0	0	0.08
0	0	1	0.15
0	1	0	0.05
0	1	1	0.10
1	0	0	0.14
1	0	1	0.18
1	1	0	0.19
1	1	1	0.11

a) What is the marginal distribution of A?

b) What is the conditional distribution of A given $B = 1$ and $C = 1$?

Question 2. (20 pts) 1% of women at age forty who participate in routine screening have breast cancer. 80% of those with breast cancer will get positive mammographies. 9.6% of those without breast cancer will also get positive mammographies. A woman in this age group had a positive mammography in a routine screening. What is the probability that she actually has breast cancer?

Question 3. (10 pts) Suppose that a stochastic process $\{x_t\}$ is defined by the following equation: $x_{t+1} = x_t + x_{t-1} + v_t$ where v_t are i.i.d. noise random variables.

(a) Is this process Markov if the state is defined as x_t?

(b) What would need to be included in the state at time t to make this a Markov process?

2 Exercises

Question 4. (30 pts) Consider two stochastic processes, $\{x_t\}$ and $\{y_t\}$, defined by $x_t = f_x(x_1, x_2..., x_{t-1}, v_t)$ and $y_t = f_y(y_1, y_2..., y_{t-1}, v_t)$ where v_t are independent, identically distributed random variables that introduce noise. The `HW1` module in `DMUStudent` contains two Julia functions, `fx` and `fy`, which can sample from the stochastic processes, i.e. `fx([x1, x2])` will return a sample of x_3 given that $x_1 =$ `x1` and $x_2 =$ `x2`.

a) Plot a sample trajectory of $\{x_t\}$ with $x_1 = 1$ and a sample trajectory of $\{y_t\}$ with $y_1 = 1$.

b) Suppose you know that one of these processes is Markov and the other is not. By drawing samples with the Julia functions, determine which one is Markov.

3 Challenge Problem

Question 5. (20 pts) Write a function in Julia that takes in two arguments and uses the Pythagorean Theorem to compute the hypotenuse of a right triangle with side lengths specified by the arguments. Evaluate

this function with `DMUStudent.HW1.evaluate` and submit the resulting json file *along with a listing of the code in your pdf*. A score of 1 will receive full credit.[1]

[1]This particular "Challenge Problem" is not meant to be challenging; it is meant to test that everyone can download the code and submit. It should require only 2 lines. Future problems in this section will be quite challenging.