

# ASEN 5519-003 Decision Making under Uncertainty

## Homework 1: Probabilistic Models

January 15, 2021

### 1 Conceptual Questions

**Question 1.** (20 pts) Consider the following joint distribution of three binary-valued random variables,  $A$ ,  $B$ , and  $C$ :

$A$	$B$	$C$	$P(A, B, C)$
0	0	0	0.08
0	0	1	0.15
0	1	0	0.05
0	1	1	0.10
1	0	0	0.14
1	0	1	0.18
1	1	0	0.19
1	1	1	0.11

a) What is the marginal distribution of  $A$ ?

b) What is the conditional distribution of  $A$  given  $B = 1$  and  $C = 1$ ?

**Question 2.** (20 pts) 1% of women at age forty who participate in routine screening have breast cancer. 80% of those with breast cancer will get positive mammographies. 9.6% of those without breast cancer will also get positive mammographies. A woman in this age group had a positive mammography in a routine screening. What is the probability that she actually has breast cancer?

**Question 3.** (10 pts) Suppose that a stochastic process  $\{x_t\}$  is defined by the following equation:  $x_{t+1} = x_t + x_{t-1} + v_t$  where  $v_t$  are i.i.d. noise random variables.

(a) Is this process Markov if the state is defined as  $x_t$ ?

(b) What would need to be included in the state at time  $t$  to make this a Markov process?

### 2 Exercises

**Question 4.** (30 pts) Consider two stochastic processes,  $\{x_t\}$  and  $\{y_t\}$ , defined by  $x_t = f_x(x_1, x_2, \dots, x_{t-1}, v_t)$  and  $y_t = f_y(y_1, y_2, \dots, y_{t-1}, v_t)$  where  $v_t$  are independent, identically distributed random variables that introduce noise. The HW1 module in `DMUStudent` contains two Julia functions, `fx` and `fy`, which can sample from the stochastic processes, i.e. `fx([x1, x2])` will return a sample of  $x_3$  given that  $x_1 = \mathbf{x1}$  and  $x_2 = \mathbf{x2}$ .

a) Plot sample trajectories of 100 steps of  $\{x_t\}$  with  $x_1 = 1$  and  $\{y_t\}$  with  $y_1 = 1$ .

b) Suppose you know that one of these processes is Markov and the other is not. By drawing samples with the Julia functions, determine which one is Markov.

### 3 Challenge Problem

**Question 5.** (20 pts) Write a function in Julia that takes in two arguments and uses the Pythagorean Theorem to compute the hypotenuse of a right triangle with side lengths specified by the arguments. Evaluate

this function with `DMUStudent.HW1.evaluate` and submit the resulting json file *along with a listing of the code in your pdf*. A score of 1 will receive full credit.<sup>1</sup>

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<sup>1</sup>This particular “Challenge Problem” is not meant to be challenging; it is meant to test that everyone can download the code and submit. It should require only 2 lines. Future problems in this section will be quite challenging.