This time: Decision Theory + Games

$A \succ B$ prefer A

$A \sim B$ indifferent

$A \succeq B$ prefer A or indifference

Lottery

$$[S_1 : p_1 ; \; S_2 : p_2 ; \; \ldots \; S_n : p_n]$$

von Neumann-Morgenstern Axioms "Rational"

- Completeness: Exactly 1 holds: $A \succ B$, $B \succ A$, $A \sim B$
- Transitivity: If $A \succeq B$ and $B \succeq C$, then $A \succeq C$
- Continuity: If $A \succeq C \succeq B$, $\exists \, p \in [0,1]$ s.t. $[A : p ; B : 1-p] \sim C$
- Independence: If $A \succ B$ then $\forall \, C, p$
  $[A : p ; C : 1-p] \succ [B : p ; C : 1-p]$

From these

$\exists \; U : S \to \mathbb{R}$ ("Utility Function") s.t. $U(A) > U(B)$ iff $A \succ B$

$U(A) = U(B)$ iff $A \sim B$

$$U([S_1 : p_1 \; \ldots \; S_n : p_n]) = \sum p_i U(S_i)$$

MEU

given model $P(s' \mid o, a)$

$$EU(a \mid o) = \sum_{s'} P(s' \mid a, o) \, U(s')$$

$$a^* = \underset{a}{\operatorname{argmax}} \; EU(a \mid o)$$

Utility Elicitation

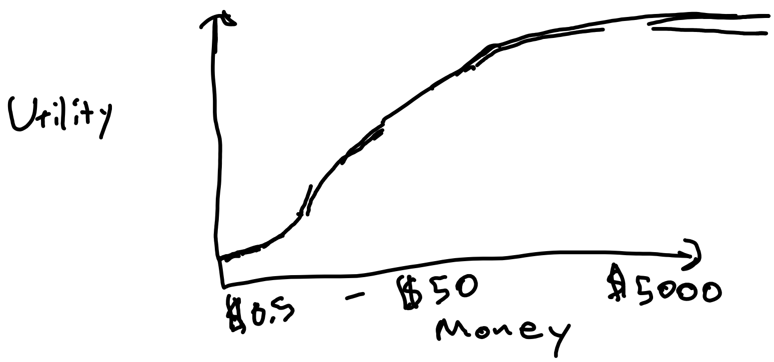
$U(s_\perp) = 0$ ← worst $\qquad U(s_\top) = 1$ ← best

to determine $U(s)$, find p s.t.

$s \sim [s_\top : p ; \; s_\perp : 1-p]$

Grand Unifying Utility Function

- Happiness
- Net Worth \$



1. Risk neutral
   prefer all equally, straight line
2. Risk Seeking ←
   prefer low odds, high payout, convex
3. Risk Averse ←
   prefer high odds, lower pay, concave

Tversky + Kahneman $\quad U(s) = \#$ lives saved

$U(1.A) = 25 \cdot 1.0 = 25$

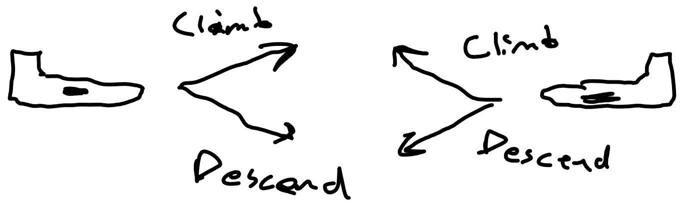
→ $U(1.B) = 100 \times 0.2 + 0.8 \cdot 0 = 20$

→ $U(2.A) = 25 \cdot 0.1 + 100 \cdot 0.9 = 92.5$

$U(2.B) = 0 \cdot 0.08 + 100 \cdot 0.92 = 92$

Games

A B



Collision -4
Climbing -1

Game Matrix

		B	
		Climb	Descend
A	Climb	-5, -5	-1, 0
	Descend	0, -1	-4, -4

Best Choice?

Equilibria

Pure strategy: action chosen deterministic

Mixed Strategy: actions chosen probabilistically

Strategy Profile: collection of strategies for all players
$s_{1:n}$

Best response: s_i^* such that $U_i(s_i^*, \underbrace{s_{-i}}_{\text{profile for all others}}) \geq U_i(s_i, s_{-i})$ for any other s_i
for s_{-i}

Nash Equilibrium: A strategy profile is a Nash Equilibrium if no agent can benefit by unilaterally switching strategy.

If mixed strategies are allowed there is at least one N.E. for every finite game.

		B		
		R	P	S
A	R	0,0	-1,1	1,-1
	P	1,-1	0,0	-1,1
	S	-1,1	1,-1	0,0

0 0 1

$\frac{1}{3}$

$\frac{1}{3}+\varepsilon$

$\frac{1}{3}-\varepsilon$

$\frac{1}{3}+\varepsilon$ +1

$\frac{1}{3}-\varepsilon$ -1

$\frac{1}{3}$ 0

Prisoner's Dilemma

Both Testify : 5 years
One Testifies : other gets 10 years
None Testify : both get 1 years

		B	
		T	R
A	Testify	-5,-5	0,-10
	Refuse	-10,0	-1,-1

Dominant Strategy Equilibrium

If $\tilde{s}$ is a best response to all possible strategy profiles, it is a <u>dominant strategy</u>

If all players have a dominant strategy, the profile is called a <u>DSE</u>

Every DSE is a Nash E.

{ Collision Avoidance

{ 1:

0,0	1,1
0,0	0,0

2: No