Last Time

- What is Policy Gradient?
- What tricks are needed to make Policy Gradient work?

Map

Model Based			Model Free	
	learn Q	learn π		On Policy
	SARSA	Policy Gradient		
MLMBTRL (learn T,R)	Q-learning			Off Policy

Today

- Basic On- and Off-Policy **value based** model free RL algorithms
- Tricks for tabular value based RL algorithms
- Understanding of On- vs Off-Policy

Why learn Q?

Incremental Mean Estimation

$$
\begin{aligned}
\hat{x}_m &= \frac{1}{m}\sum_{i=1}^{m} x^{(i)} \\
&= \frac{1}{m}\left(x^{(m)} + \sum_{i=1}^{m-1} x^{(i)}\right) \\
&= \frac{1}{m}\left(x^{(m)} + (m-1)\,\hat{x}_{m-1}\right) \\
&= \hat{x}_{m-1} + \frac{1}{m}\left(x^{(m)} - \hat{x}_{m-1}\right)
\end{aligned}
$$

```julia
function simulate!(π::MonteCarloTreeSearch, s, d=π.d)
    if d ≤ 0
        return π.U(s)
    end
    𝒫, N, Q, c = π.𝒫, π.N, π.Q, π.c
    𝒜, TR, γ = 𝒫.𝒜, 𝒫.TR, 𝒫.γ
    if !haskey(N, (s, first(𝒜)))
        for a in 𝒜
            N[(s,a)] = 0
            Q[(s,a)] = 0.0
        end
        return π.U(s)
    end
    a = explore(π, s)
    s′, r = TR(s,a)
    q = r + γ*simulate!(π, s′, d-1)
    N[(s,a)] += 1
    Q[(s,a)] += (q-Q[(s,a)])/N[(s,a)]
    return q
end
```

loop

$$\hat{x} \leftarrow \hat{x} + \alpha\left(x - \hat{x}\right)$$

"Temporal Difference (TD) Error"

Q Learning

Q learning and SARSA

Q-Learning

$Q(s,a) \leftarrow 0$

$s \leftarrow s_0$

loop

$a \leftarrow \text{argmax}\, Q(s,a) \text{ w.p. } 1-\epsilon, \quad \text{rand}(A) \text{ o.w.}$

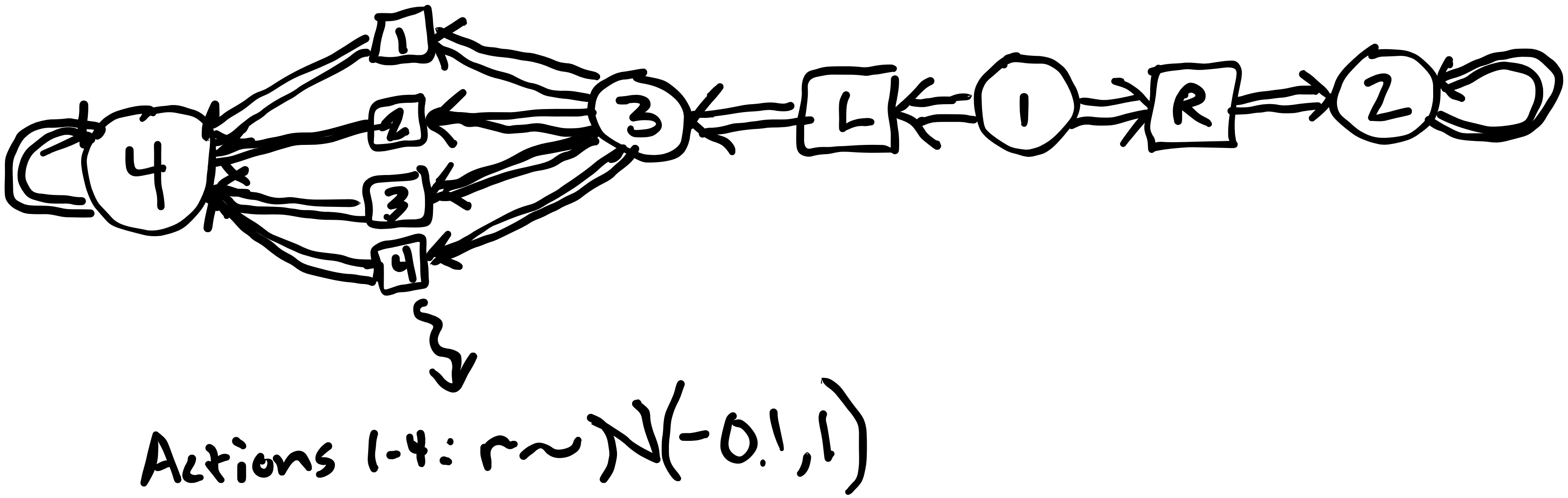
$r \leftarrow \text{act!}(\text{env}, a)$

$s' \leftarrow \text{observe}(\text{env})$

$$Q(s,a) \leftarrow Q(s,a) + \alpha\ (\underbrace{r + \gamma \max_{a'} Q(s',a') - Q(s,a)}_{TD})$$

$s \leftarrow s'$

Illustrative Problem



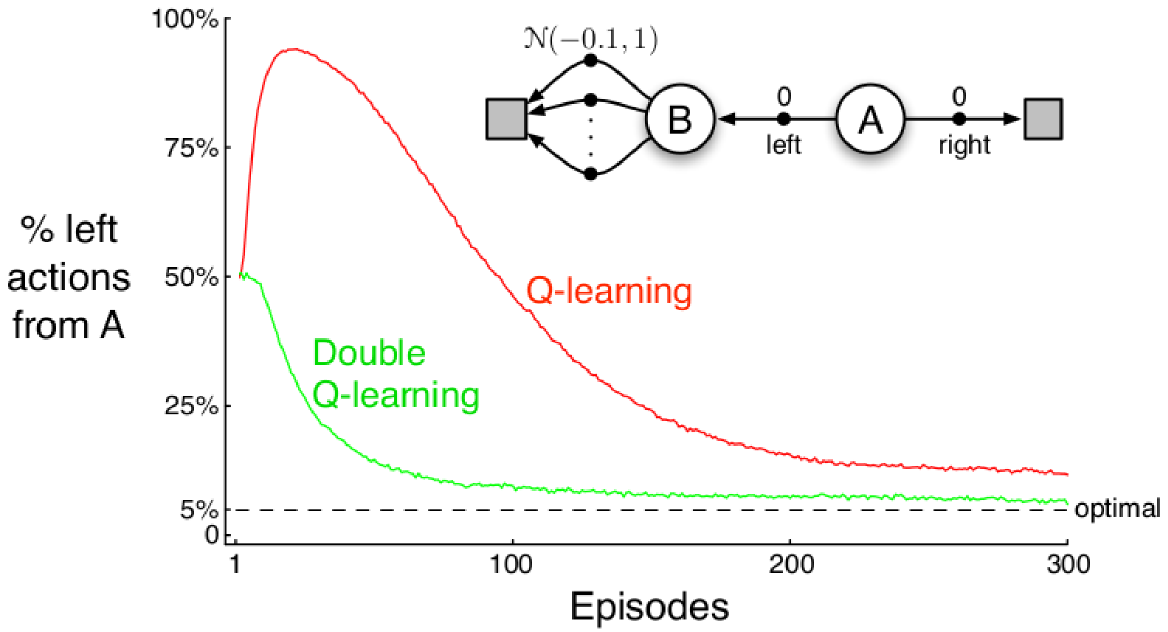
1. After a few episodes, what is Q(3, a) for a in 1-4?
2. After a few episodes, what is Q(1, L)?
3. Why is this a problem and what are some possible solutions?

Big Problem: Maximization Bias

Even if all $Q(s', a')$ unbiased, $\max_{a'} Q(s', a')$ is biased!

Solution: Double Q Learning Q_1, Q_2

$$Q_1(s,a) \leftarrow Q_1(s,a) + \alpha \left(r + \gamma\, Q_2 \left(s', \underset{a'}{\operatorname{argmax}} Q_1(s', a') \right) - Q_1(s,a) \right)$$



Eligibility Traces

SARSA-λ

Games

Half-Life at 20: why it is the most important shooter ever made

From its opening scenes, Valve's pioneering sci-fi horror game reinvented storytelling and universe building - what made it such a terrifying success?

'It taught a whole generation of big-budget game developers how to tell stories' ... the Half-Life box art. Illustration: Valve

$Q(s,a), N(s,a) \leftarrow 0$

initialize s, a, r, s'

loop

$$a' \leftarrow \operatorname{argmax} Q(s', a) \text{ w.p. } 1-\epsilon, \quad \text{rand}(A) \text{ o.w.}$$

$$N(s,a) \leftarrow N(s,a) + 1$$

$$\delta \leftarrow r + \gamma Q(s', a') - Q(s,a)$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha\delta\, N(s,a) \quad \forall s, a$$

$$N(s,a) \leftarrow \gamma\lambda N(s,a)$$

$$s \leftarrow s', \quad a \leftarrow a'$$

$$r \leftarrow \text{act!}(\text{env}, a)$$

$$s' \leftarrow \text{observe}(\text{env})$$

Convergence

- Q learning converges to optimal Q-values w.p. 1
  (Sutton and Barto, p. 131)
- SARSA converges to optimal Q-values w.p. 1 ***provided that***

  $\pi \to \text{greedy}$

  (Sutton and Barto, p. 129)

On vs Off-Policy

On Policy

SARSA:

$$Q(s,a) \leftarrow Q(s,a) + \alpha\ (r + \gamma Q(s',a') - Q(s,a))$$

Off Policy

Q-learning:

$$Q(s,a) \leftarrow Q(s,a) + \alpha\ (r + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

Will eligibility traces work with Q-learning?

Not easily

Policy Gradient:

$$\theta \leftarrow \theta + \alpha \sum_{k=0}^{d} \nabla_\theta \log \pi_\theta(a_k \mid s_k) R(\tau)$$

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