ASEN 5519-003 Decision Making under Uncertainty
Quiz 1: Probabilistic Models and MDPs

Show all work and box answers.
You may consult any source, but you may NOT communicate with any person except the instructor.

Question 1. (30 pts) Let A, B, and C be three binary-valued random variables (binary-valued means the support is $\{0, 1\}$). You are given the following information: $P(C = 1) = 0.6$, $P(A = 1 \mid C = 1) = 0.1$, $P(A = 1 \mid C = 0) = 0.7$, $P(B = 1 \mid A = 0) = 0.5$, $P(B = 1 \mid A = 1) = 0.6$, and $P(B \mid A, C) = P(B \mid A)$.

a) What is $P(A = 1)$?
b) Write the joint distribution of A and B.
c) Are B and C independent?

Question 2. (15 pts) You are playing hide and seek with your three-year-old niece. You have noticed that she chooses her three favorite hiding places, under the table, in the closet, and behind the door with equal probability. You also know that if she is hiding behind the door, there is a 50% chance that her leg will be sticking out. If you walk by the door and **DO NOT** see her leg sticking out. Given this observation, **what is the probability that she is under the table?**

Question 3. (15 pts) Consider the MDP $\mathcal{S} = \{1, 2, 3\}$, $\mathcal{A} = \{-1, 1\}$, $R(s, a) = s - a$, $\gamma = 0.95$, and

$$T(s' \mid s, a) = \begin{cases} 1 & \text{if } s' = \text{clamp}(s + a, 1, 3) \\ 0 & \text{otherwise} \end{cases}, \tag{1}$$

where $\text{clamp}(x, a, b) = \min(\max(x, a), b)$ (intuitively, the state cannot be pushed beyond 1 or 3). Suppose we are using policy iteration to solve this problem, starting with an initial policy of $\pi_0(s) = -1$ for all states. The value function vector for this initial policy is $V^{\pi_0} = [40, 41, 42.95]$. **What is the new improved policy, π', that will be extracted from this value function?**

Question 4. (40 pts) You are considering possible (simplified) routes from Boulder to Denver International Airport. The following table shows the start and destination of various highways along with the travel times:

Highway Number	Start	End	Travel Time (Minutes)
470	Boulder	Airport	40
36	Boulder	Denver	20
76	Denver	Airport	25
70	Denver	Airport	15 (80%) or 30 (20%) (see below)

As you are traveling from Boulder to Denver, the traffic on highway 70 from Denver to the airport may change: with 80% chance the traffic stays light so that the highway 70 link will take 15 minutes to traverse, or with 20% chance it becomes heavy so that it will take 30 minutes. Your phone will give you perfect knowledge of this traffic. Your objective is to minimize the average amount of time that it will take to get from Boulder to the airport.

a) Formulate this problem as an MDP (specify $(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathcal{T}, \gamma)$).

b) What is the optimal highway to take from Boulder? Justify your answer by calculating the optimal state-action values (Q-values) for actions that you can take from Boulder.