

Imitation and Inverse Reinforcement Learning

- Last time:
 - Turn-taking zero sum games
 - Markov Games
 - Incomplete Information Games
- Today:

Imitation and Inverse Reinforcement Learning

- **Last time:**
 - Turn-taking zero sum games
 - Markov Games
 - Incomplete Information Games
- **Today:**
 - What if you don't know the reward function and just want to act like an expert?
 - Imitation Learning
 - Inverse Reinforcement Learning

Trivia: When was the first car driven with a Neural Network?

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Dean Pomerleau
@deanpomerleau

...

Replying to @GTARobotics

GPU? Gez, ALVINN ran on 100 MFLOP CPU, ~10x slower than iWatch; Refrigerator-size & needed 5000 watt generator. @olivercameron

What's Hidden in the Hidden Layers?

The contents can be easy to find with a geometrical problem, but the hidden layers have yet to give up all their secrets

David S. Touretzky and Dean A. Pomerleau

AUGUST 1989 • BYTE 231

tions, we fed the network road images taken under a wide variety of viewing angles and lighting conditions. It would be impractical to try to collect thousands of real road images for such a data set. Instead, we developed a synthetic road-image generator that can create as many training examples as we need.

To train the network, 1200 simulated road images are presented 40 times each, while the weights are adjusted using the back-propagation learning algorithm. This takes about 30 minutes on Carnegie Mellon's Warp systolic-array supercomputer. (This machine was designed at Carnegie Mellon and is built by General Electric. It has a peak rate of 100 million floating-point operations per second and can compute weight adjustments for back-propagation networks at a rate of 20 million connections per second.)

Once it is trained, ALVINN can accurately drive the NAVLAB vehicle at about 3½ miles per hour along a path through a wooded area adjoining the Carnegie Mellon campus, under a variety of weather and lighting conditions. This speed is nearly twice as fast as that achieved by non-neural-network algorithms running on the same vehicle. Part of the reason for this is that the forward pass of a back-propagation network can be computed quickly. It takes about 200

milliseconds on the Sun-3/160 workstation installed on the NAVLAB.

The hidden-layer representations ALVINN develops are interesting. When trained on roads of a fixed width, the net-

work chooses a representation in which hidden units act as detectors for complete roads at various positions and orientations. When trained on roads of variable

continued



Photo 1: The NAVLAB autonomous navigation test-bed vehicle and the road used for trial runs.

Trivia: When was the first car driven with a Neural Network?



Dean Pomerleau
@deanpomerleau

...

1995: 2797/2849 miles (98.2%)

Replying to @GTARobotics

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continued



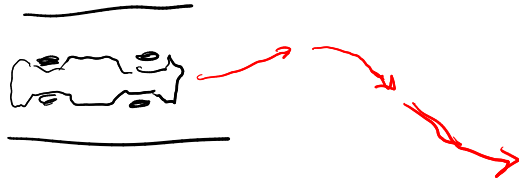
Photo 1: The NAVLAB autonomous navigation test-bed vehicle and the road used for trial runs.



Behavioral Cloning

Behavioral Cloning

$$\underset{\theta}{\text{maximize}} \prod_{(s,a) \in D} \pi_{\theta}(a \mid s)$$



Behavioral Cloning

$$\underset{\theta}{\text{maximize}} \prod_{(s,a) \in D} \pi_{\theta}(a \mid s)$$

Problem: Cascading Errors

How did ALVINN do it?

3.2. TRAINING "ON-THE-FLY" WITH REAL DATA

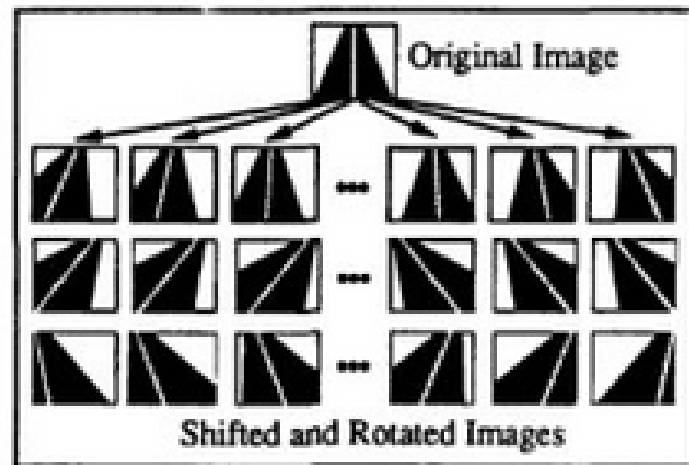


Figure 3.4: The single original video image is shifted and rotated to create multiple training exemplars in which the vehicle appears to be at different locations relative to the road.

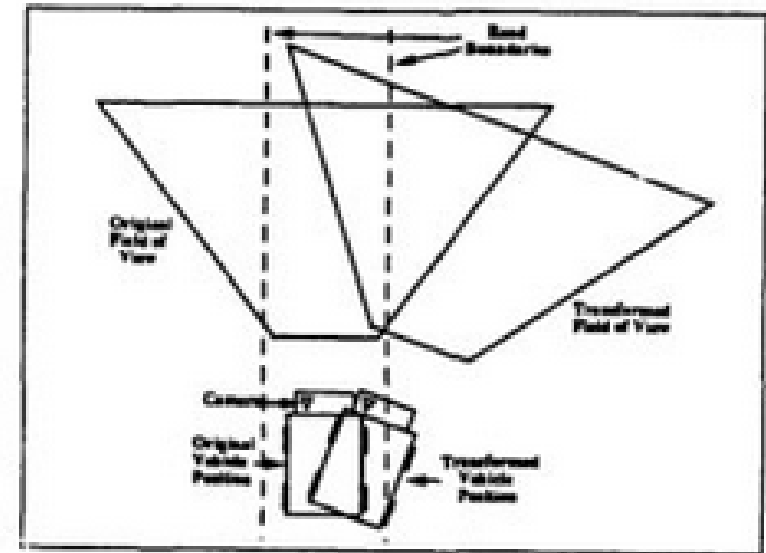


Figure 3.5: An aerial view of the vehicle at two different positions, with the corresponding sensor fields of view. To simulate the image transformation that would result from such a change in position and orientation of the vehicle, the overlap between the two field of view trapezoids is computed and used to direct resampling of the original image.

How did NVIDIA do it in 2016?

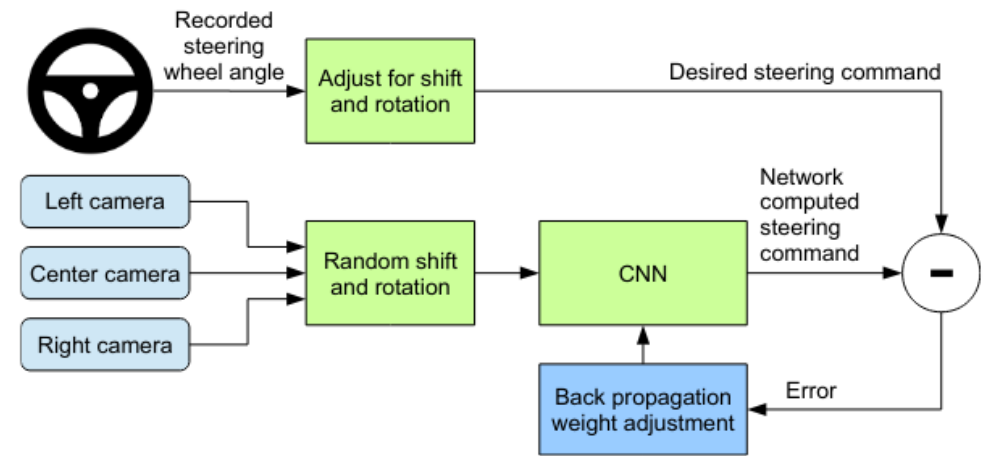
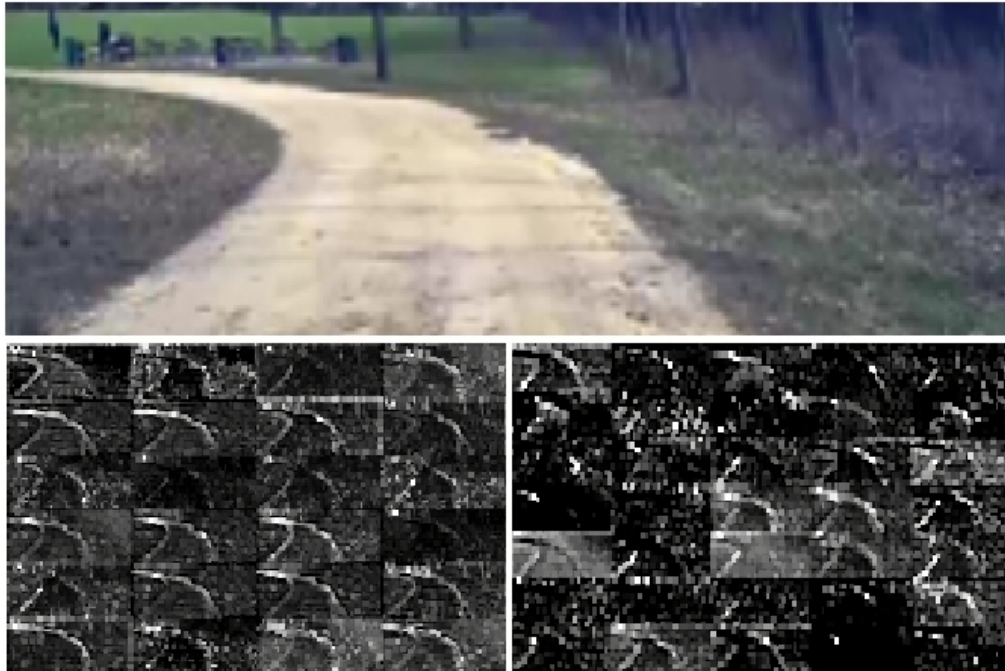


Figure 2: Training the neural network.

Dataset Aggregation (DAgger)

```
function optimize(M::DataSetAggregation, D,  $\theta$ )
     $\mathcal{P}$ , bc, k_max, m = M. $\mathcal{P}$ , M.bc, M.k_max, M.m
    d, b,  $\pi_E$ ,  $\pi_\theta$  = M.d, M.b, M. $\pi_E$ , M. $\pi_\theta$ 
     $\theta$  = optimize(bc, D,  $\theta$ )
    for k in 2:k_max
        for i in 1:m
            s = rand(b)
            for j in 1:d
                push!(D, (s,  $\pi_E(s)$ ))
                a = rand( $\pi_\theta(\theta, s)$ )
                s = rand( $\mathcal{P}.T(s, a)$ )
            end
        end
         $\theta$  = optimize(bc, D,  $\theta$ )
    end
    return  $\theta$ 
end
```

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                a = rand( $\pi_\theta(\theta, s)$ )  
                s = rand( $\mathcal{P}.T(s, a)$ )  
            end  
        end  
         $\theta$  = optimize(bc, D,  $\theta$ )  
    end  
    return  $\theta$   
end
```

} rollout

Dataset Aggregation (DAgger)

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            for j in 1:d  
                push!(D, (s,  $\pi_E(s)$ ))  
                a = rand( $\pi_\theta(\theta, s)$ )  
                s = rand( $\mathcal{P}.T(s, a)$ )  
            end  
        end  
         $\theta$  = optimize(bc, D,  $\theta$ )  
    end  
    return  $\theta$   
end
```

Gather
from expert

} rollout

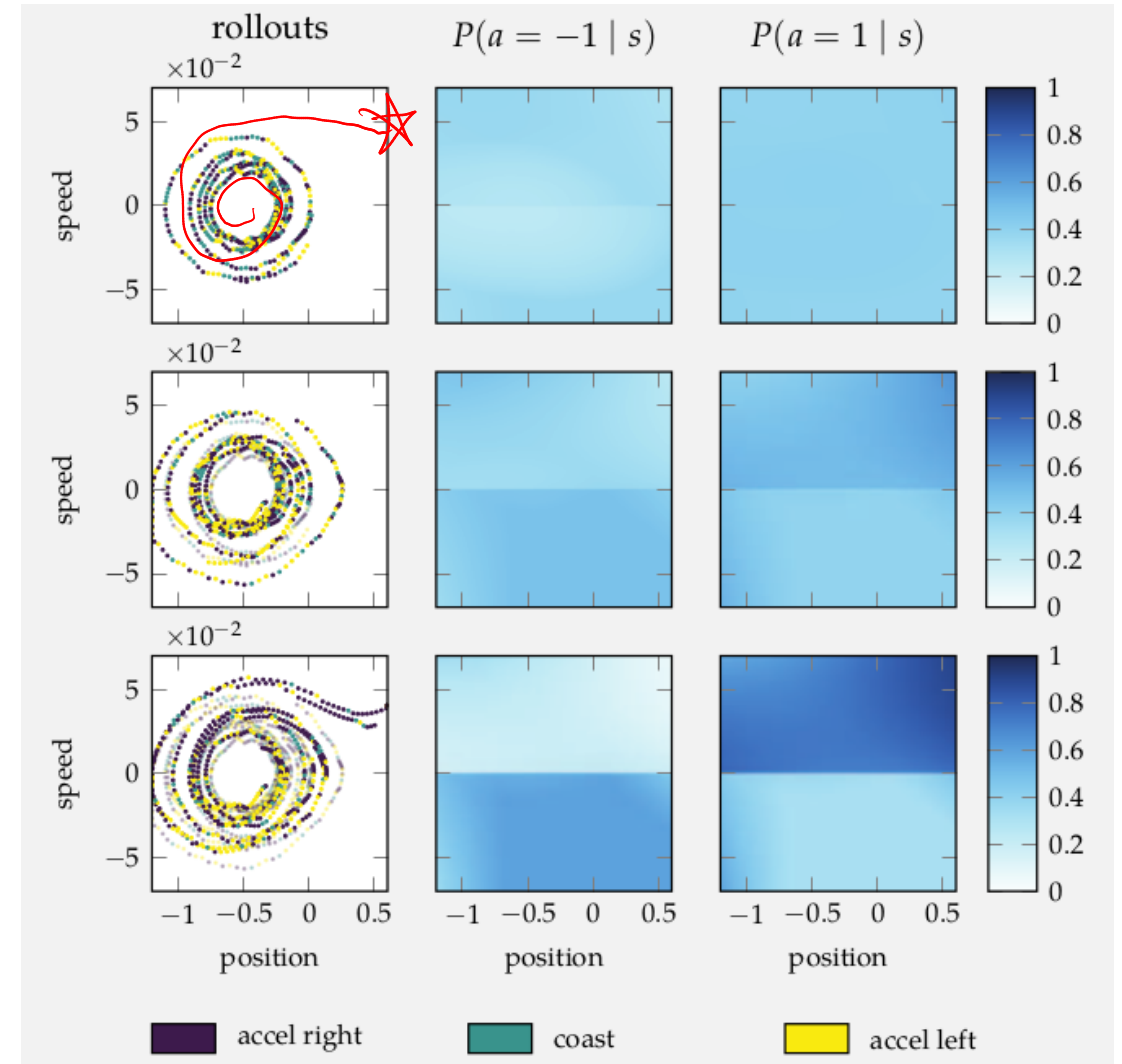
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            for j in 1:d  
                push!(D, (s,  $\pi E(s)$ ))  
                a = rand( $\pi\theta(\theta, s)$ )  
                s = rand( $\mathcal{P}.T(s, a)$ )  
            end  
        end  
         $\theta$  = optimize(bc, D,  $\theta$ )  
    end  
    return  $\theta$   
end
```

Gather from expert

rollout



Stochastic Mixing Iterative Learning (SMILe)

```
function optimize(M::SMILe,  $\theta$ )
     $\mathcal{P}$ , bc, k_max, m = M. $\mathcal{P}$ , M.bc, M.k_max, M.m
    d, b,  $\beta$ ,  $\pi E$ ,  $\pi\theta$  = M.d, M.b, M. $\beta$ , M. $\pi E$ , M. $\pi\theta$ 
     $\mathcal{A}$ , T =  $\mathcal{P}.\mathcal{A}$ ,  $\mathcal{P}.T$ 
     $\theta s$  = []
     $\pi$  =  $s \rightarrow \pi E(s)$ 
    for k in 1:k_max
        # execute latest  $\pi$  to get new data set D
        D = []
        for i in 1:m
            s = rand(b)
            for j in 1:d
                push!(D, (s,  $\pi E(s)$ ))
                a =  $\pi(s)$ 
                s = rand(T(s, a))
            end
        end
        # train new policy classifier
         $\theta$  = optimize(bc, D,  $\theta$ )
        push!( $\theta s$ ,  $\theta$ )
        # compute a new policy mixture
        P $\pi$  = Categorical(normalize([(1- $\beta$ )^(i-1) for i in 1:k],1))
         $\pi$  =  $s \rightarrow$  begin
            if rand() < (1- $\beta$ )^(k-1)
                return  $\pi E(s)$ 
            else
                return rand(Categorical( $\pi\theta$ ( $\theta s$ [rand(P $\pi$ )], s)))
            end
        end
    end
    end
    Ps = normalize([(1- $\beta$ )^(i-1) for i in 1:k_max],1)
    return Ps,  $\theta s$ 
end
```

Stochastic Mixing Iterative Learning (SMILe)

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function optimize(M::SMILe,  $\theta$ )
     $\mathcal{P}$ , bc, k_max, m = M. $\mathcal{P}$ , M.bc, M.k_max, M.m
    d, b,  $\beta$ ,  $\pi E$ ,  $\pi\theta$  = M.d, M.b, M. $\beta$ , M. $\pi E$ , M. $\pi\theta$ 
     $\mathcal{A}$ , T =  $\mathcal{P}.\mathcal{A}$ ,  $\mathcal{P}.T$ 
     $\theta s$  = []
     $\pi$  =  $s \rightarrow \pi E(s)$ 
    for k in 1:k_max
        # execute latest  $\pi$  to get new data set D
        D = [] ← reset D
        for i in 1:m
            s = rand(b)
            for j in 1:d
                push!(D, (s,  $\pi E(s)$ ))
                a =  $\pi(s)$ 
                s = rand(T(s, a))
            end
        end
        # train new policy classifier
         $\theta$  = optimize(bc, D,  $\theta$ )
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     $\mathcal{A}$ , T =  $\mathcal{P}.\mathcal{A}$ ,  $\mathcal{P}.T$ 
     $\theta s$  = []
     $\pi$  =  $s \rightarrow \pi E(s)$ 
    for k in 1:k_max
        # execute latest  $\pi$  to get new data set D
        D = [] ← reset D
        for i in 1:m
            s = rand(b)
            for j in 1:d
                push!(D, (s,  $\pi E(s)$ )) } Gather data
                 $a = \pi(s)$ 
                s = rand(T(s, a))
            end
        end
        # train new policy classifier
         $\theta$  = optimize(bc, D,  $\theta$ )
        push!( $\theta s$ ,  $\theta$ )
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         $P\pi$  = Categorical(normalize([(1- $\beta$ )^(i-1) for i in 1:k], 1))
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            else
                return rand(Categorical( $\pi\theta$ ( $\theta s$ [rand( $P\pi$ )], s)))
            end
        end
    end
     $P s$  = normalize([(1- $\beta$ )^(i-1) for i in 1:k_max], 1)
    return  $P s$ ,  $\theta s$ 
end
```

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     $\theta s$  = []
     $\pi = s \rightarrow \pi E(s)$ 
    for k in 1:k_max
        # execute latest  $\pi$  to get new data set D
        D = [] ← reset D
        for i in 1:m
            s = rand(b)
            for j in 1:d
                push!(D, (s,  $\pi E(s)$ )) } Gather data
                a =  $\pi(s)$ 
                s = rand(T(s, a))
            end
        end
        # train new policy classifier ← train only on D
         $\theta$  = optimize(bc, D,  $\theta$ )
        push!( $\theta s$ ,  $\theta$ )
        # compute a new policy mixture
        P $\pi$  = Categorical(normalize([(1- $\beta$ )^(i-1) for i in 1:k], 1))
         $\pi = s \rightarrow$  begin
            if rand() < (1- $\beta$ )^(k-1)
                return  $\pi E(s)$ 
            else
                return rand(Categorical( $\pi\theta$ ( $\theta s$ [rand(P $\pi$ )], s)))
            end
        end
    end
    end
    Ps = normalize([(1- $\beta$ )^(i-1) for i in 1:k_max], 1)
    return Ps,  $\theta s$ 
end
```

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     $\mathcal{A}$ , T =  $\mathcal{P}.\mathcal{A}$ ,  $\mathcal{P}.T$ 
     $\theta_s$  = []
     $\pi = s \rightarrow \pi_E(s)$ 
    for k in 1:k_max
        # execute latest  $\pi$  to get new data set D
        D = [] ← reset D
        for i in 1:m
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                s = rand(T(s, a))
            end
        end
        # train new policy classifier ← train only on D
         $\theta$  = optimize(bc, D,  $\theta$ )
        push!( $\theta_s$ ,  $\theta$ )
        # compute a new policy mixture
        P $\pi$  = Categorical(normalize([(1- $\beta$ )^(i-1) for i in 1:k], 1)) ← Mix Policies
         $\pi = s \rightarrow$  begin
            if rand() < (1- $\beta$ )^(k-1)
                return  $\pi_E(s)$ 
            else
                return rand(Categorical( $\pi_\theta(\theta_s[\text{rand}(P\pi)])$ , s)))
            end
        end
    end
    end
    Ps = normalize([(1- $\beta$ )^(i-1) for i in 1:k_max], 1)
    return Ps,  $\theta_s$ 
end
```

Stochastic Mixing Iterative Learning (SMILe)

```
function optimize(M::SMILe, θ)
    ℙ, bc, k_max, m = M.ℙ, M.bc, M.k_max, M.m
    d, b, β, πE, πθ = M.d, M.b, M.β, M.πE, M.πθ
    ℳ, T = ℙ.ℳ, ℙ.T
    θs = []
    π = s → πE(s)
    for k in 1:k_max
        # execute latest π to get new data set D
        D = []
        for i in 1:m
            s = rand(b)
            for j in 1:d
                push!(D, (s, πE(s)))
                a = π(s)
                s = rand(T(s, a))
            end
        end
        # train new policy classifier
        θ = optimize(bc, D, θ)
        push!(θs, θ)
        # compute a new policy mixture
        Pπ = Categorical(normalize([(1-β)^(i-1) for i in 1:k], 1))
        π = s → begin
            if rand() < (1-β)^(k-1)
                return πE(s)
            else
                return rand(Categorical(πθ(θs[rand(Pπ)]), s)))
            end
        end
    end
    Ps = normalize([(1-β)^(i-1) for i in 1:k_max], 1)
    return Ps, θs
end
```

← reset D

} Gather data

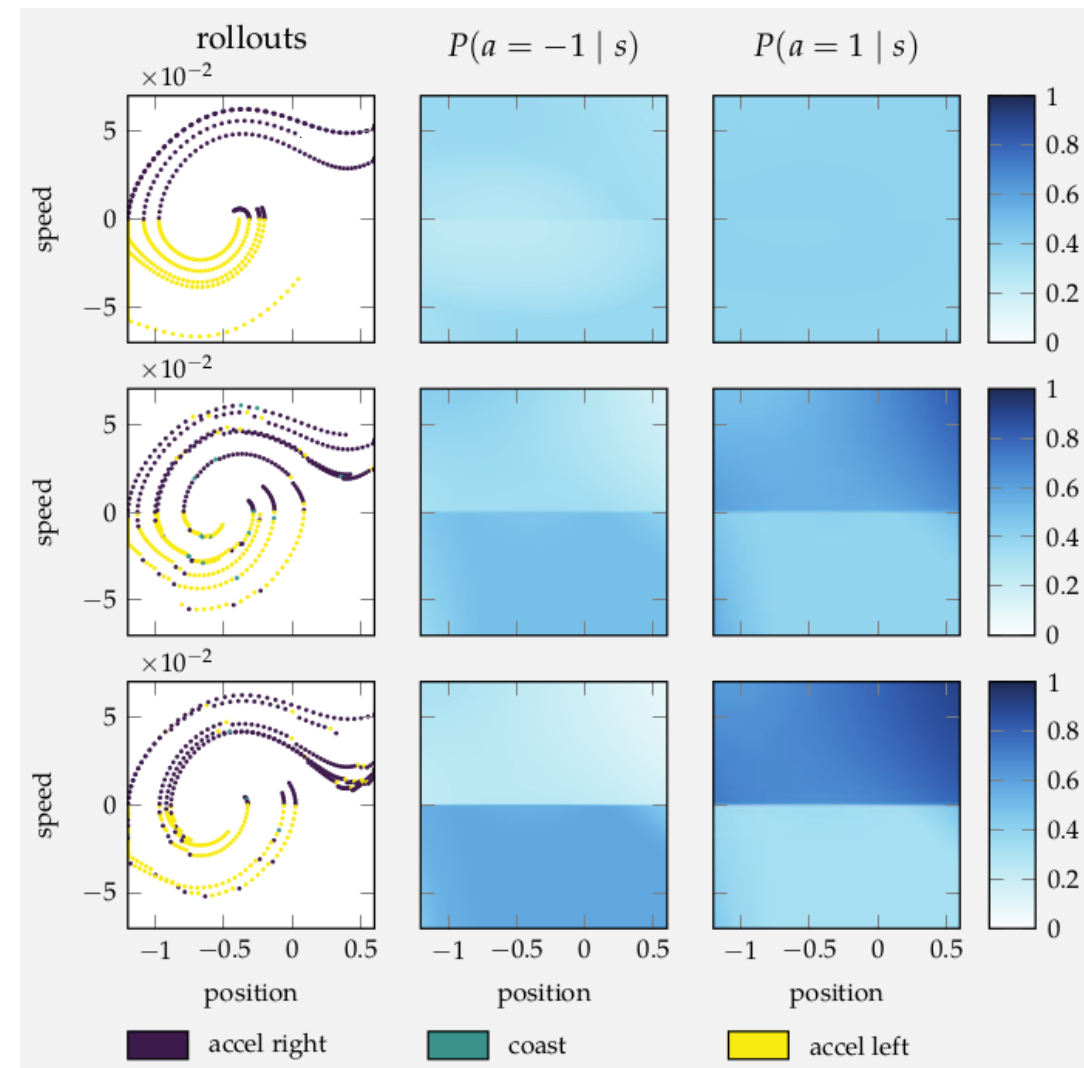
← train only on D

← Mix Policies

$$(1 - \beta)^k$$

Stochastic Mixing Iterative Learning (SMILE)

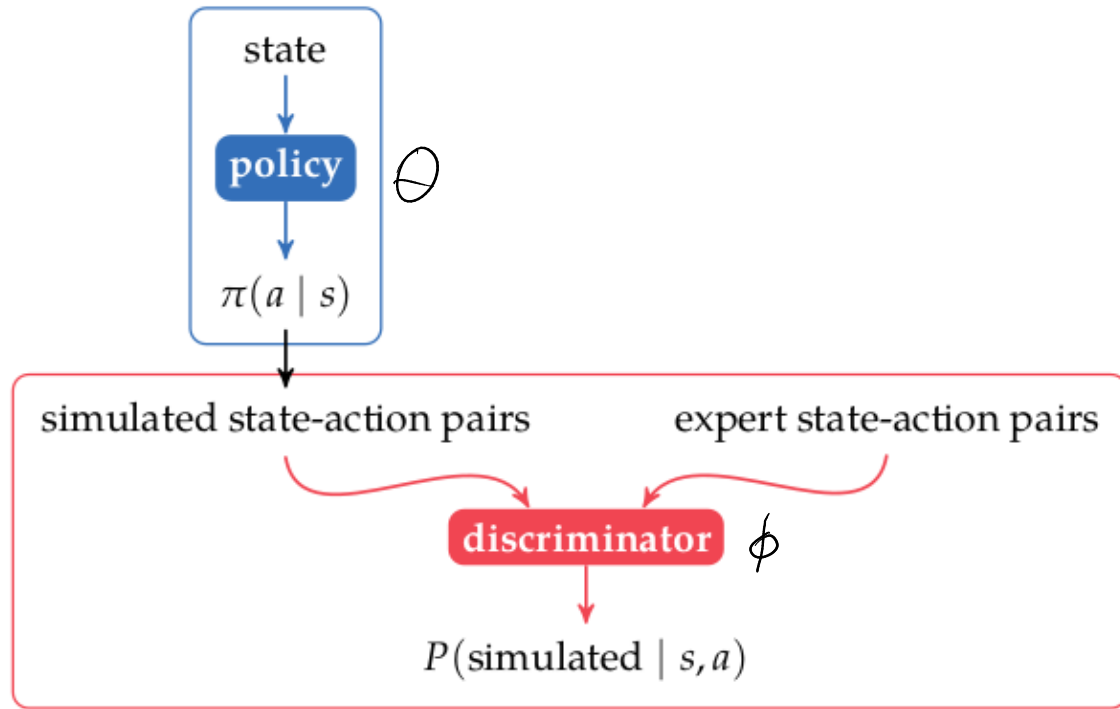
```
function optimize(M::SMILE, θ)
    P, bc, k_max, m = M.P, M.bc, M.k_max, M.m
    d, b, β, πE, πθ = M.d, M.b, M.β, M.πE, M.πθ
    A, T = P.A, P.T
    θs = []
    π = s → πE(s) ← reset D
    for k in 1:k_max
        # execute latest π to get new data set D
        D = []
        for i in 1:m
            s = rand(b)
            for j in 1:d
                push!(D, (s, πE(s))) } Gather data
                a = π(s)
                s = rand(T(s, a))
            end
        end
        # train new policy classifier ← train only on D
        θ = optimize(bc, D, θ)
        push!(θs, θ)
        # compute a new policy mixture
        Pπ = Categorical(normalize([(1-β)^(i-1) for i in 1:k], 1))
        π = s → begin
            if rand() < (1-β)^(k-1) ← Mix Policies
                return πE(s)
            else
                return rand(Categorical(πθ(θs[rand(Pπ)], s)))
            end
        end
    end
    Ps = normalize([(1-β)^(i-1) for i in 1:k_max], 1)
    return Ps, θs
end
```



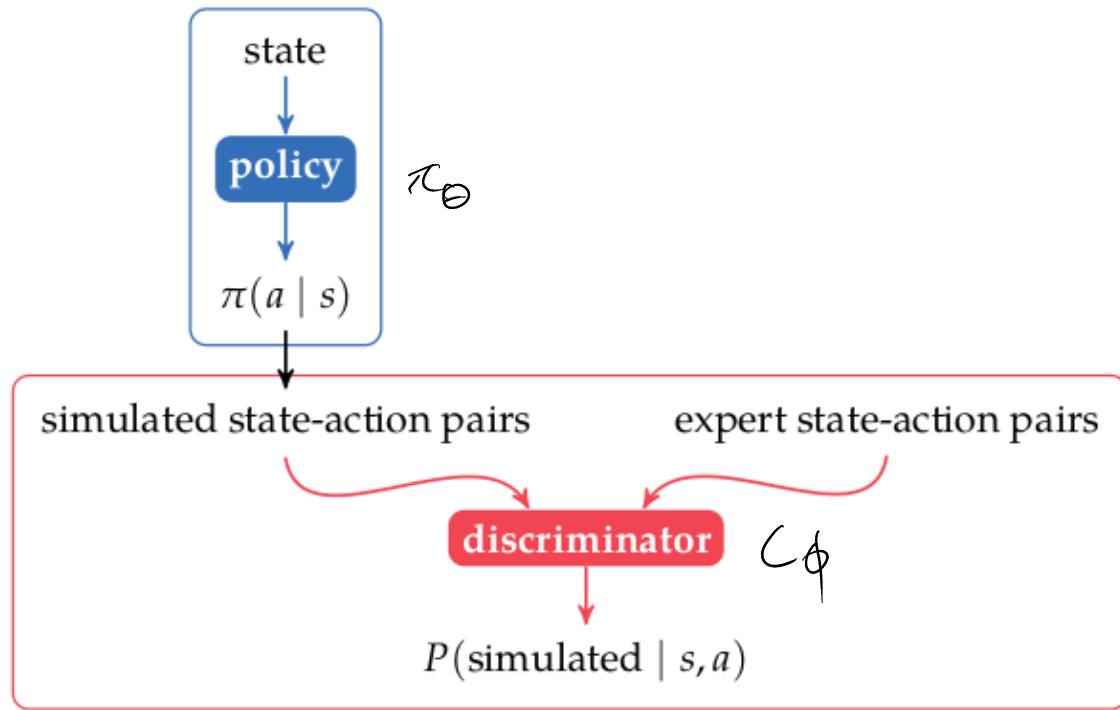
Generative Adversarial Imitation Learning (GAIL)

GAN

Generative Adversarial Imitation Learning (GAIL)

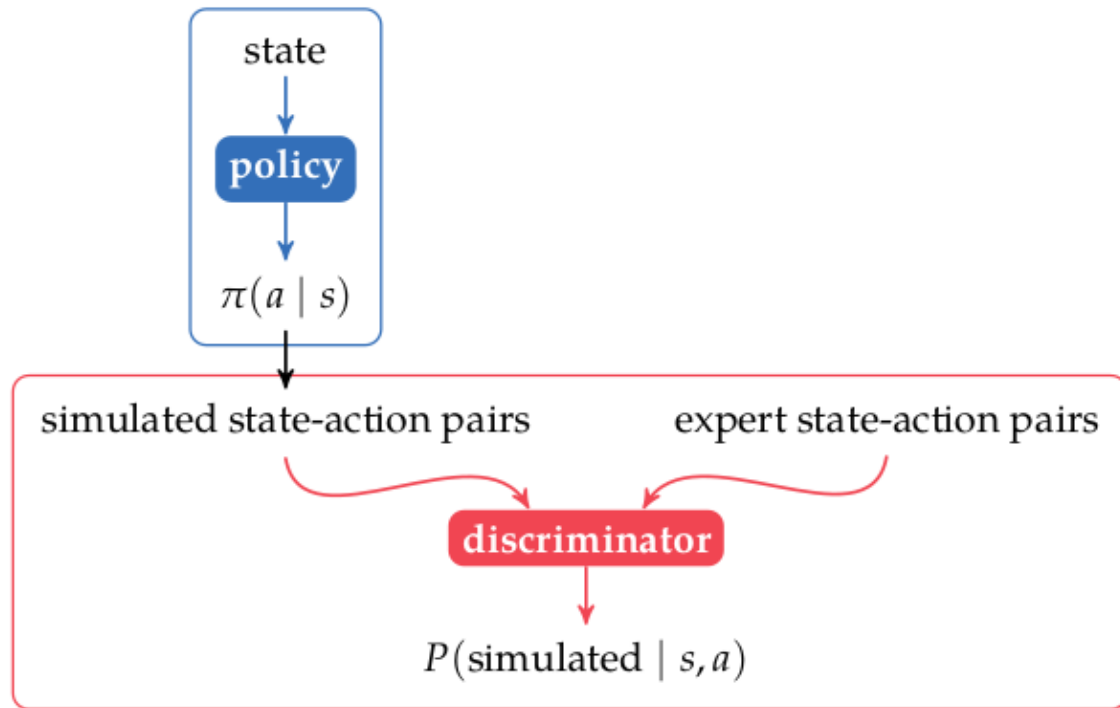


Generative Adversarial Imitation Learning (GAIL)



$$\max_{\phi} \min_{\theta} \mathbb{E}_{(s,a) \sim \pi_{\theta}} [\log(C_{\phi}(s,a))] + \mathbb{E}_{(s,a) \sim \mathcal{D}} [\log(1 - C_{\phi}(s,a))]$$

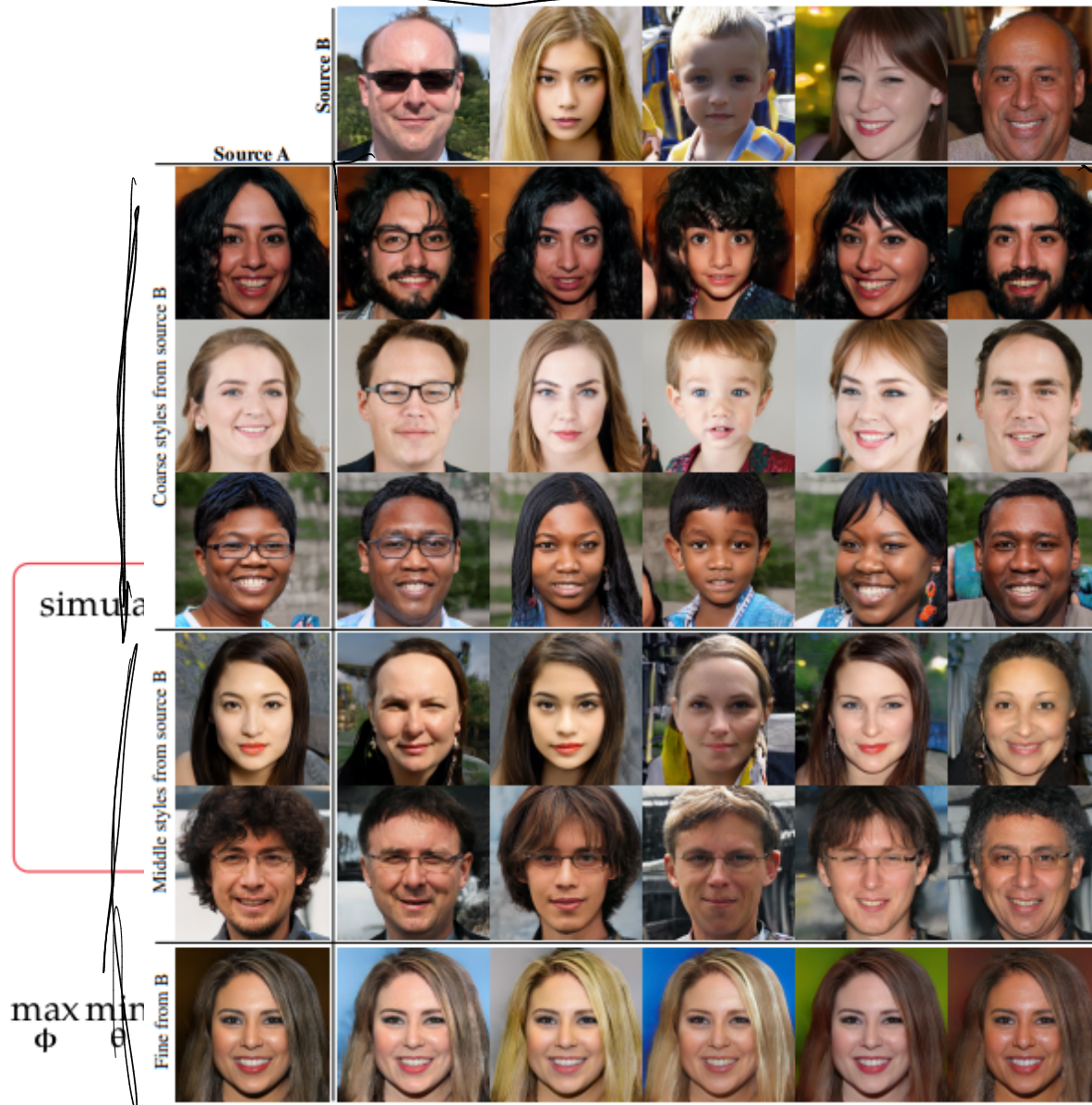
Generative Adversarial Imitation Learning (GAIL)



GANs are frighteningly good
at generating believable
synthetic things

$$\max_{\Phi} \min_{\Theta} \mathbb{E}_{(s,a) \sim \pi_{\Theta}} [\log(C_{\Phi}(s,a))] + \mathbb{E}_{(s,a) \sim \mathcal{D}} [\log(1 - C_{\Phi}(s,a))]$$

Adversarial Imitation Learning (GAIL)

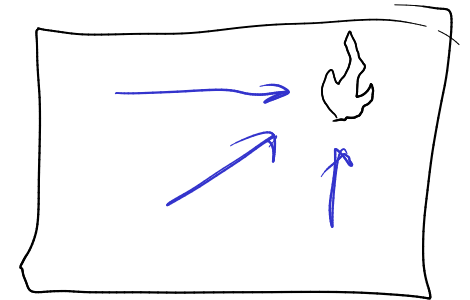
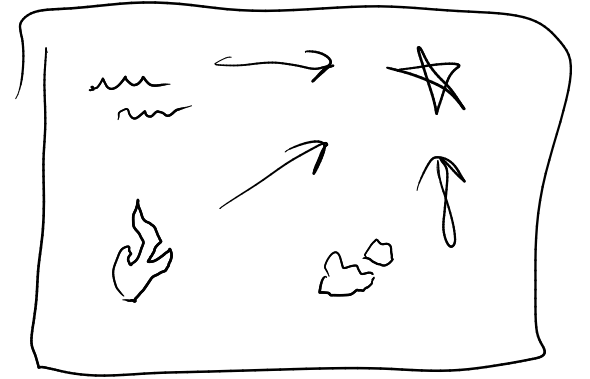


GANs are frighteningly good
at generating believable
synthetic things

Inverse Reinforcement Learning

Inverse Reinforcement Learning

What if we know the dynamics, but not the reward?



Inverse Reinforcement Learning

What if we know the dynamics, but not the reward?

Reinforcement Learning

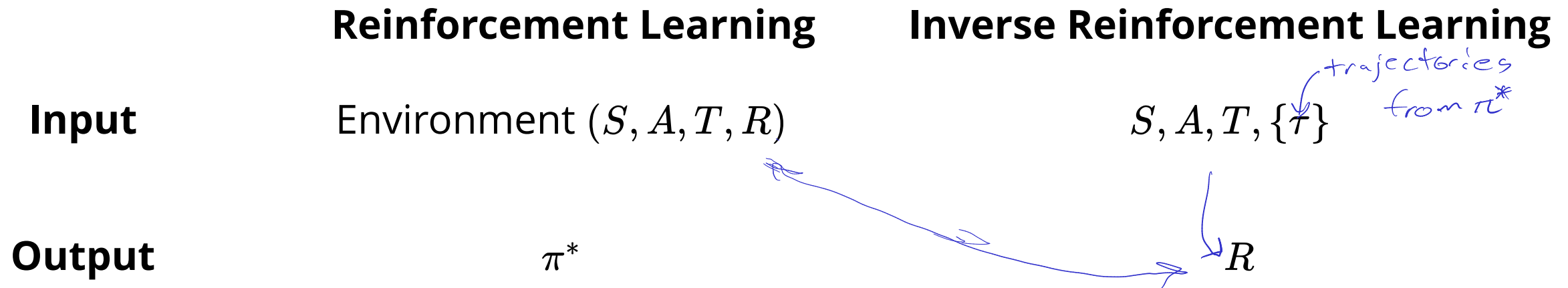
Inverse Reinforcement Learning

Input

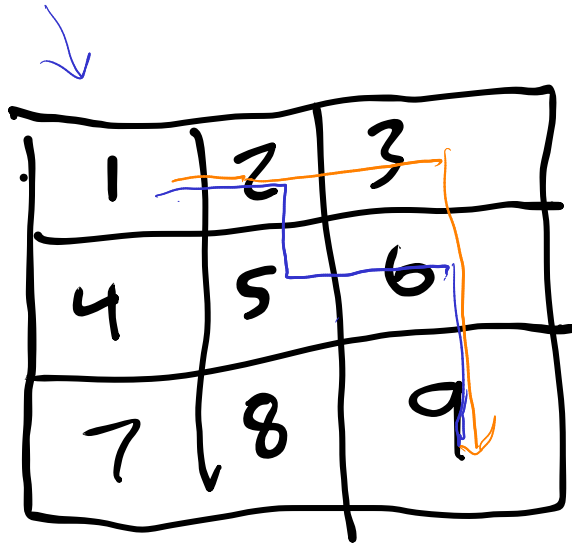
Output

Inverse Reinforcement Learning

What if we know the dynamics, but not the reward?



Exercise



τ

1 →

2 →

3 ↓

6 ↓

9

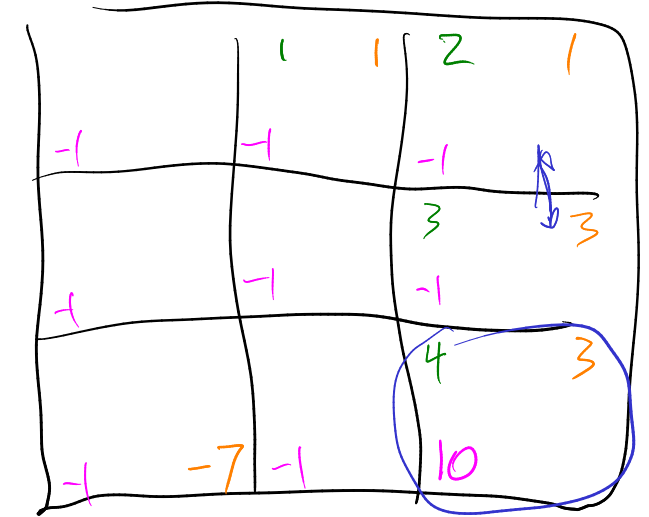
1 →

2 ↓

5 →

6 ↓

9



What is the reward function?

Maximum Margin Inverse Reinforcement Learning

Maximum Entropy Inverse Reinforcement Learning

$$P_{\boldsymbol{\phi}}(\tau) = \frac{1}{Z(\boldsymbol{\phi})} \exp(R_{\boldsymbol{\phi}}(\tau)) \quad Z(\boldsymbol{\phi}) = \sum_{\tau} \exp(R_{\boldsymbol{\phi}}(\tau))$$

$$\begin{aligned} \max_{\boldsymbol{\phi}} f(\boldsymbol{\phi}) &= \max_{\boldsymbol{\phi}} \sum_{\tau \in \mathcal{D}} \log P_{\boldsymbol{\phi}}(\tau) \\ &= \left(\sum_{\tau \in \mathcal{D}} R_{\boldsymbol{\phi}}(\tau) \right) - |\mathcal{D}| \log \sum_{\tau} \exp(R_{\boldsymbol{\phi}}(\tau)) \end{aligned}$$

Recap

Recap

- Behavioral cloning is supervised learning to match the actions of an expert

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- IRL is an underspecified problem
- Maximum entropy and maximum margin IRL solve this problem by regularizing the reward function