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- How is a **Markov decision process** defined?
- What is a **policy**?
- How do we **evaluate** policies?

Guiding Questions

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- How do we reason about the **future consequences** of actions in an MDP?
- What are the basic **algorithms for solving MDPs**?

Value-Based Policy Evaluation

MDP Example: Up-Down Problem

For this lecture, => is same as ->> (distinguishes from Bayes Net)

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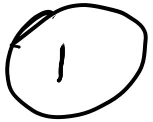

MDP Example: Up-Down Problem

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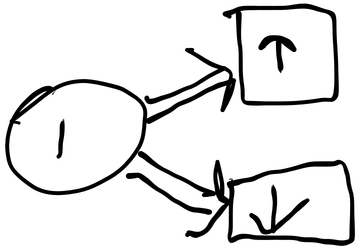

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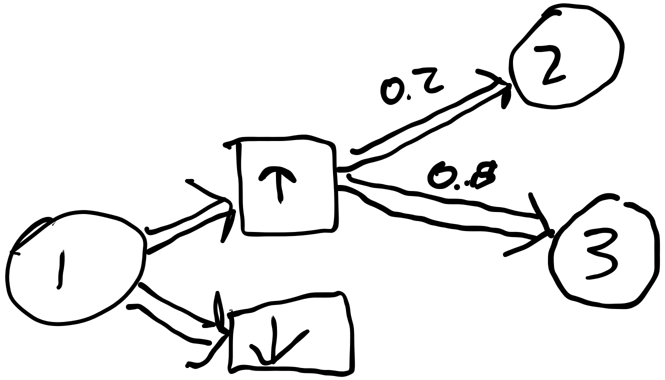

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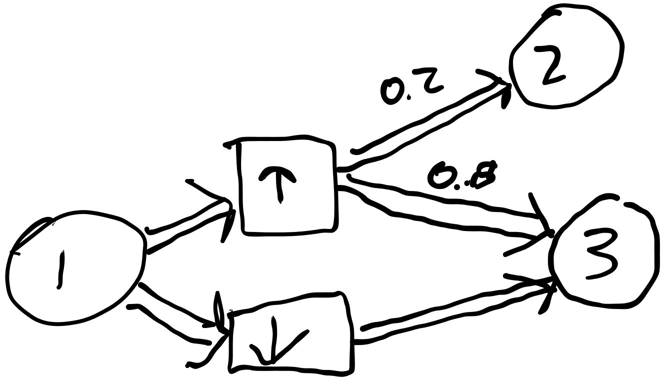

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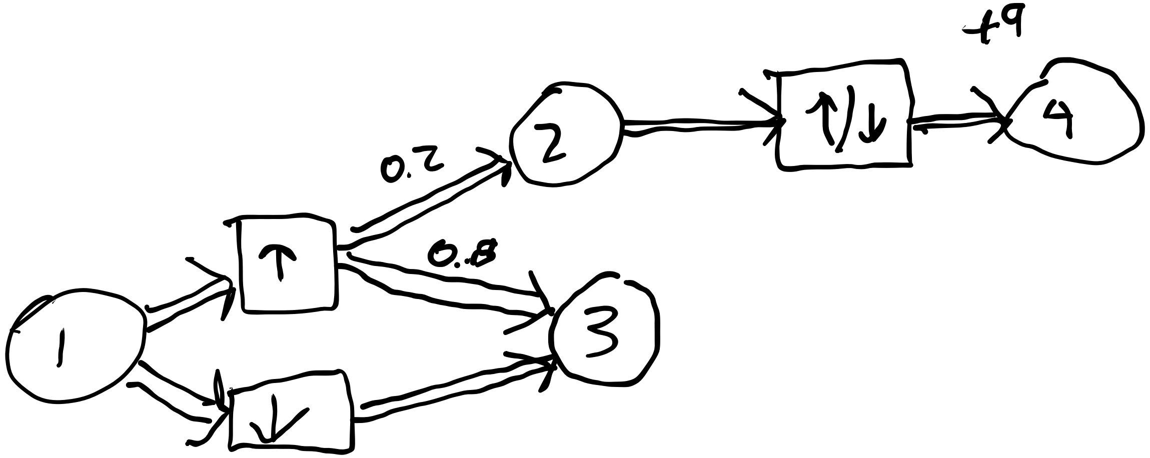

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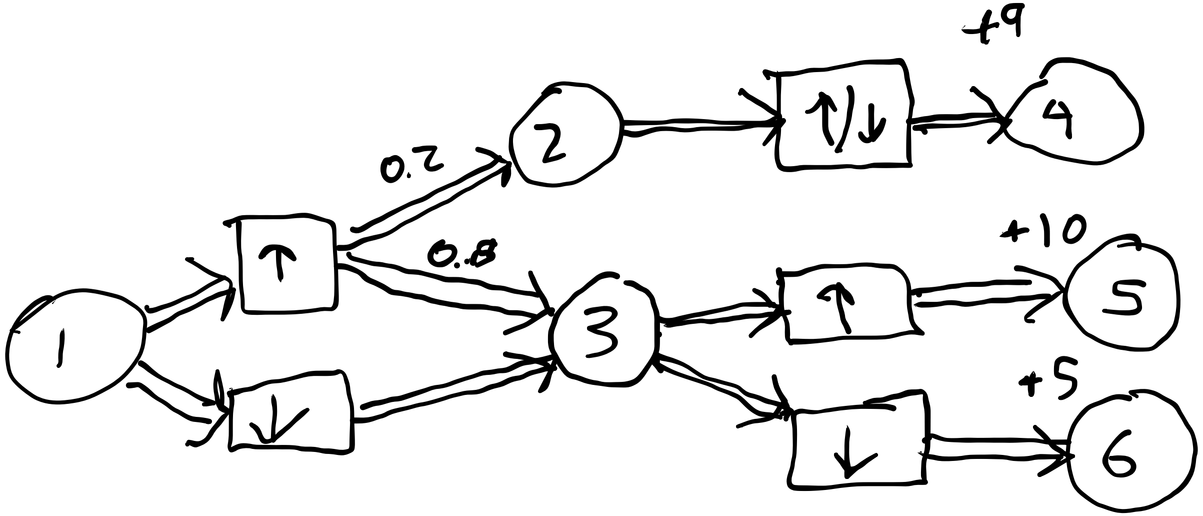
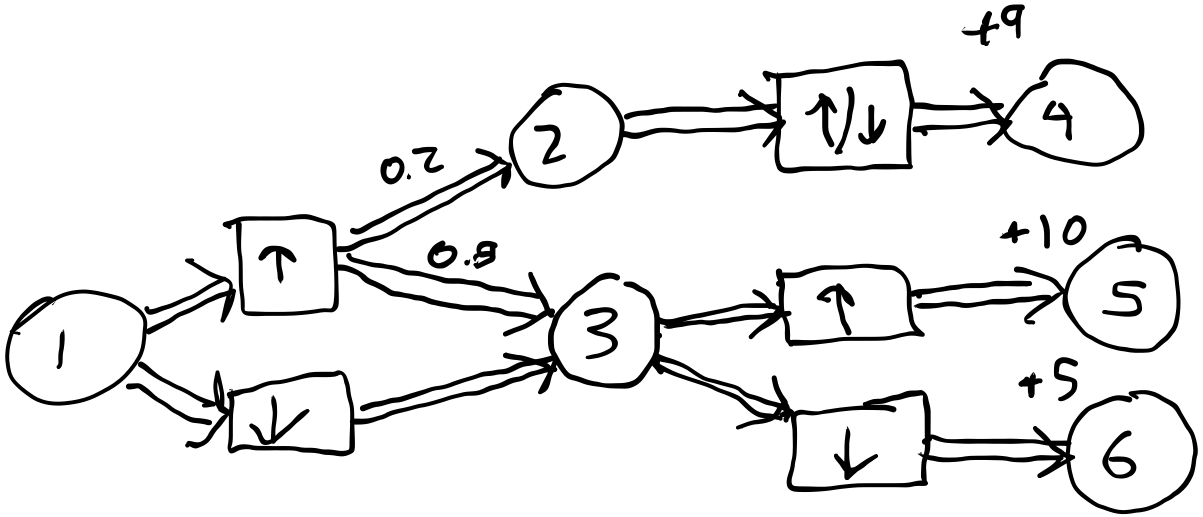
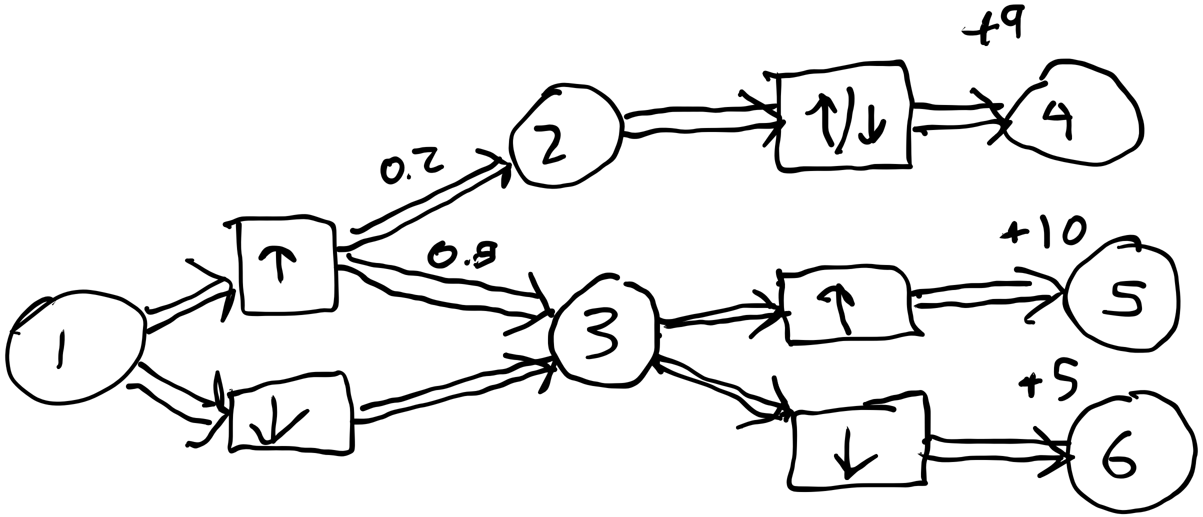


Dynamic Programming and Value Backup



Dynamic Programming and Value Backup



Bellman's Principle of Optimality: Every sub-policy in an optimal policy is locally optimal

Break: DIA Run

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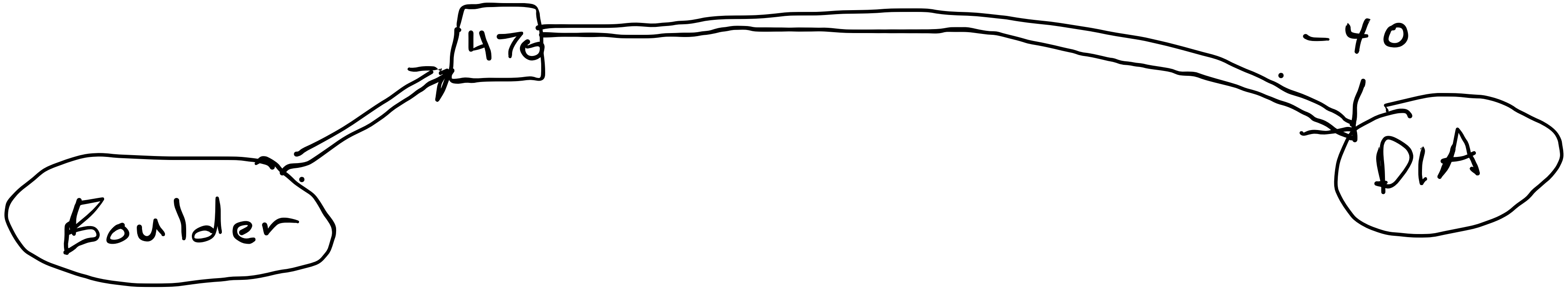

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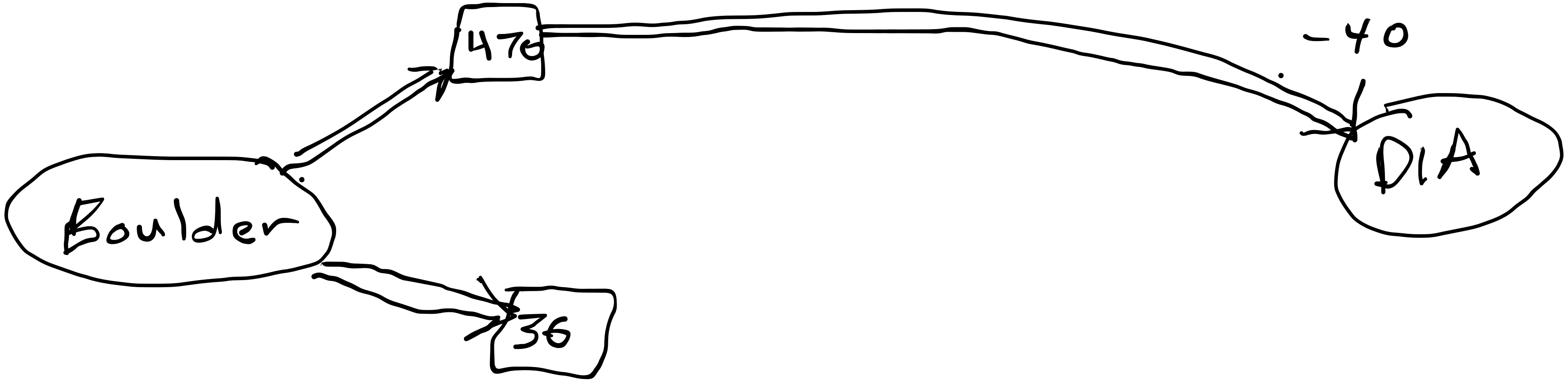

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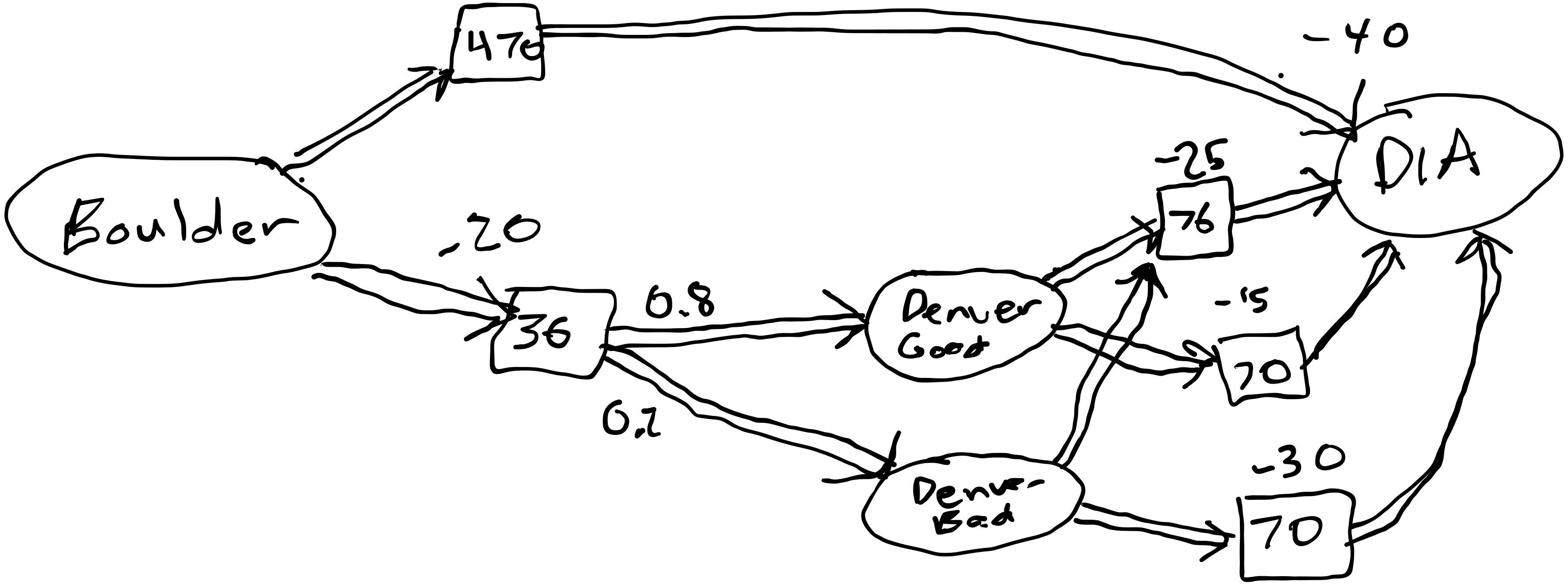

Policy Iteration

Algorithm: Policy Iteration

Given: MDP (S, A, R, T, γ, b)

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6. return $\pi$
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- Returned U' will be U^*!

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```

- Returned U' will be U^*!
- π^* is easy to extract: $\pi^*(s) = \arg\max(R(s,a) + \gamma E[U^*(s)])$

Bellman's Equations

Guiding Questions

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