

Last Time

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- How do we reason about the **future consequences** of actions in an MDP?

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- How do we reason about the **future consequences** of actions in an MDP?
- What are the basic **algorithms for solving MDPs**?

Guiding Questions

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- Does value iteration always converge?
- Is the value function unique?

Value Iteration: The Bellman Operator

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Algorithm: Value Iteration

while $\|V - V'\|_\infty < \epsilon$

$V \leftarrow V'$

$V' \leftarrow B[V]$

return V'

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$$B[V](s) = \max_{a \in A} (R(s, a) + \gamma E[V(s')])$$

Value Iteration Convergence

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Theorem 1: Let $\{V_1, \dots, V_\infty\}$ be a sequence of value functions for a discrete MDP generated by the recurrence $V_{k+1} = B[V_k]$. If $\gamma < 1$, then $\lim_{k \rightarrow \infty} V_k = V^*$.

Metrics

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Contraction Mappings

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for some α , $0 \leq \alpha \leq 1$ and all x and y in M .

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Script: contraction_mapping.jl

Banach's Theorem

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By Banach's theorem (part 1), $\hat{V} = B[\hat{V}]$. Since \hat{V} satisfies Bellman's equation, it is optimal and $\hat{V} = V^*$.

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2. The policy will stay the same if and only if $V^\pi = V^*$
3. There are a finite number of possible policies
4. By (1), (2), and (3), the policy will improve until it finds the optimal policy, and it will always find the optimal policy.

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Break

Conservation MDP