Last Time

Trying Actions
Adjusting θ so that better actions are more likely

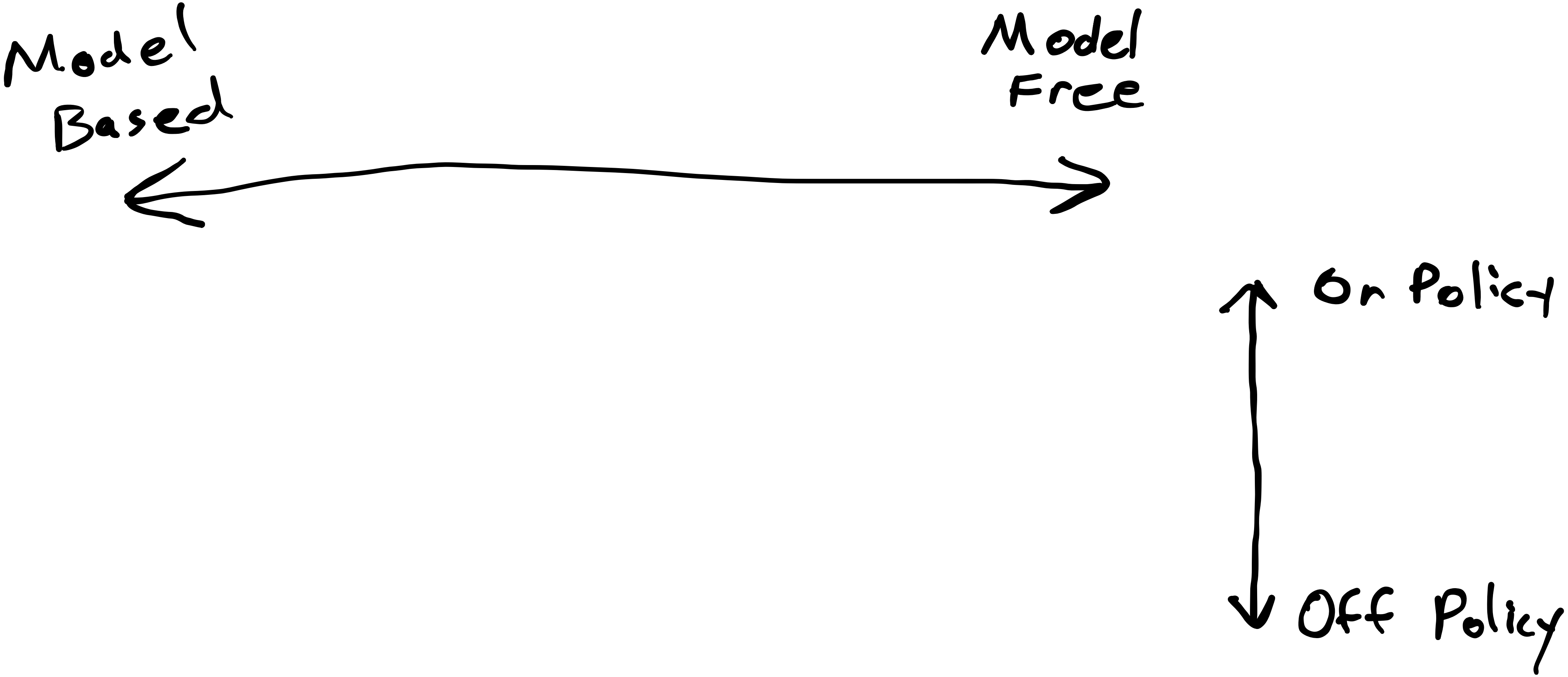
- What is Policy Gradient?
- What tricks are needed to make Policy Gradient work?

- Log Derivative
- Causality
- Baseline Subtraction

Map

Map

Map



Map

Model Based

Model Free

On Policy

Off Policy

MLMBTRL
(learn T,R)

Map

Model Based

Model Free

learn Q

learn π

On Policy

Off Policy

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Map

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Policy Gradient

On Policy

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Model Based

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SARSA

learn π

Policy Gradient

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Map

Model Based

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Q-learning

Off Policy

Today

Today

- Basic On- and Off-Policy **value based** model free RL algorithms

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- Basic On- and Off-Policy **value based** model free RL algorithms
- Tricks for tabular value based RL algorithms

Today

- Basic On- and Off-Policy **value based** model free RL algorithms
- Tricks for tabular value based RL algorithms
- What makes it hard for some approaches to use Off-Policy data?

Review: Why learn Q?

$V(s)$ $R(s,a)$ $Q(s,a)$

$$\pi^*(s) = \arg\max_a Q^*(s,a)$$

Incremental Mean Estimation

Incremental Mean Estimation

$$\hat{x}_m = \frac{1}{m}\sum_{i=1}^{m} x^{(i)}$$

Incremental Mean Estimation

$$
\begin{aligned}
\hat{x}_m &= \frac{1}{m} \sum_{i=1}^{m} x^{(i)} \\
&= \frac{1}{m} \left(x^{(m)} + \sum_{i=1}^{m-1} x^{(i)} \right)
\end{aligned}
$$

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```
function simulate!(π::MonteCarloTreeSearch, s, d=π.d)
    if d ≤ 0
        return π.U(s)
    end
    𝒫, N, Q, c = π.𝒫, π.N, π.Q, π.c
    𝒜, TR, γ = 𝒫.𝒜, 𝒫.TR, 𝒫.γ
    if !haskey(N, (s, first(𝒜)))
        for a in 𝒜
            N[(s,a)] = 0
            Q[(s,a)] = 0.0
        end
        return π.U(s)
    end
    a = explore(π, s)
    s′, r = TR(s,a)
    q = r + γ*simulate!(π, s′, d-1)
    N[(s,a)] += 1
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```

loop

learning rate

$$\hat{x} \leftarrow \hat{x} + \alpha\,(\underbrace{x - \hat{x}})$$

Incremental Mean Estimation

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```

loop

$$\hat{x} \leftarrow \hat{x} + \alpha\,(\underbrace{x - \hat{x}}_{\text{"Temporal Difference (TD) Error"}})$$

Q Learning

$\hat{q}(r, s')$

Q Learning

Want: $Q(s,a) \leftarrow Q(s,a) + \alpha\,(\hat{q} - Q(s,a))$

$\hat{q}(r, s')$

Q Learning

Want: $Q(s,a) \leftarrow Q(s,a) + \alpha\left(\hat{q}(r,s') - Q(s,a)\right)$

$\hat{q}(r,s')$

Q Learning

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$$Q(s,a) = \underbrace{R(s,a) + \gamma E[V(s')]}_{\hat{q}(r,s')}$$

Q Learning

Want: $Q(s,a) \leftarrow Q(s,a) + \alpha\left(\hat{q}(r,s') - Q(s,a)\right)$

$$\begin{aligned} Q(s,a) &= R(s,a) + \gamma E[V(s')] \\ &= R(s,a) + \gamma E\left[\max_{a'} Q(s',a')\right] \end{aligned}$$

$\hat{q}(r,s')$

Q Learning

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&= R(s,a) + \gamma E\left[\max_{a'} Q(s',a')\right] \\
&= E\left[\underbrace{r + \gamma \max_{a'} Q(s',a')}_{\hat{q}(r,s')}\right]
\end{aligned}
$$

Q learning

TD

Q learning

$Q(s,a) \leftarrow 0$

$s \leftarrow s_0$

loop

$$a \leftarrow \overbrace{\operatorname{argmax} Q(s,a) \text{ w.p. } 1-\epsilon, \quad \operatorname{rand}(A) \text{ o.w.}}^{\epsilon\text{-greedy exploration}}$$

$$r \leftarrow \operatorname{act!}(\text{env}, a)$$

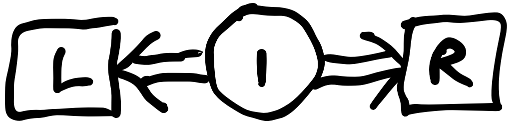
$$s' \leftarrow \operatorname{observe}(\text{env})$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha\, (\underbrace{r + \gamma \overset{\text{aggressive}}{\max_{a'}} Q(s',a') - Q(s,a)}_{\text{TD}})$$

$$s \leftarrow s'$$

Illustrative Problem

Illustrative Problem



Illustrative Problem

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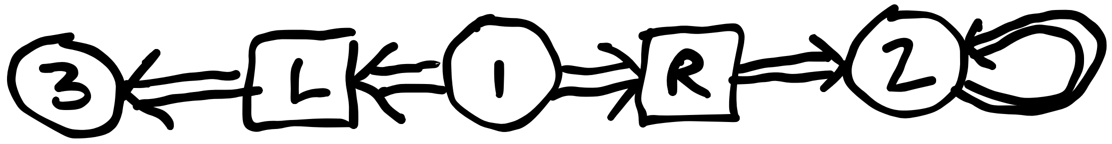

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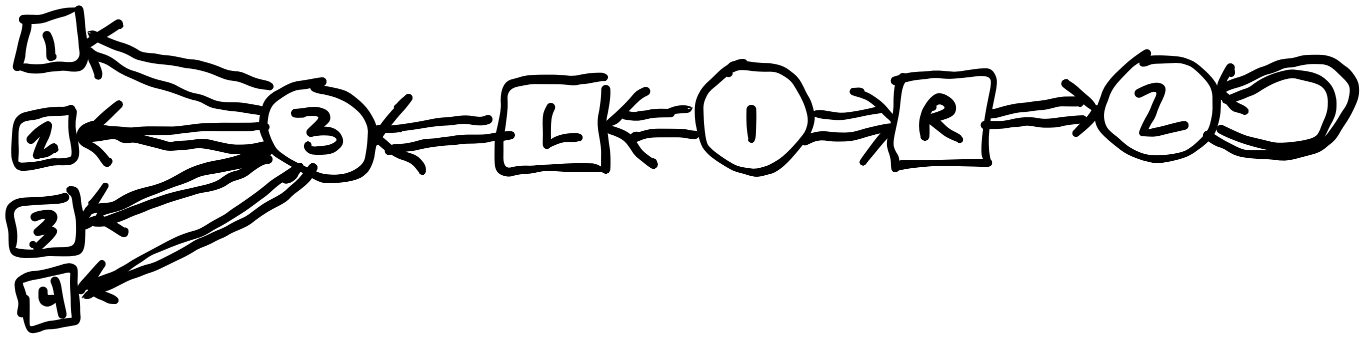

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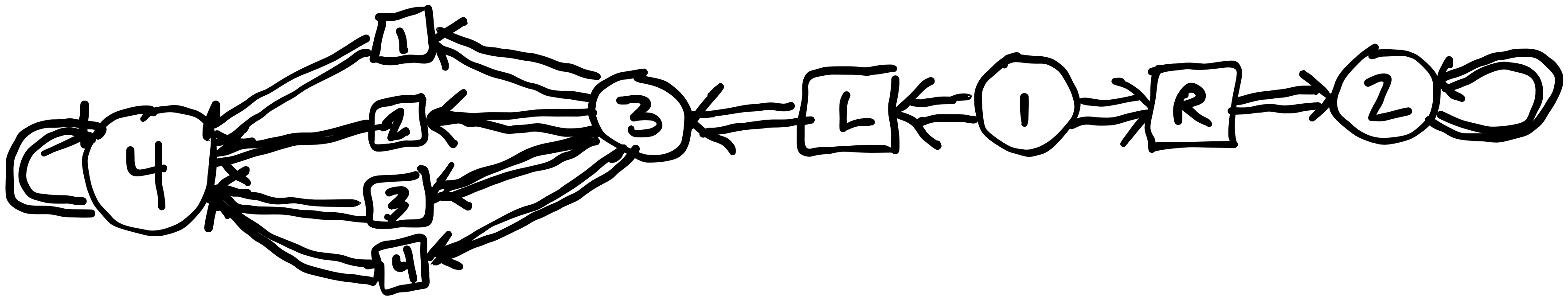

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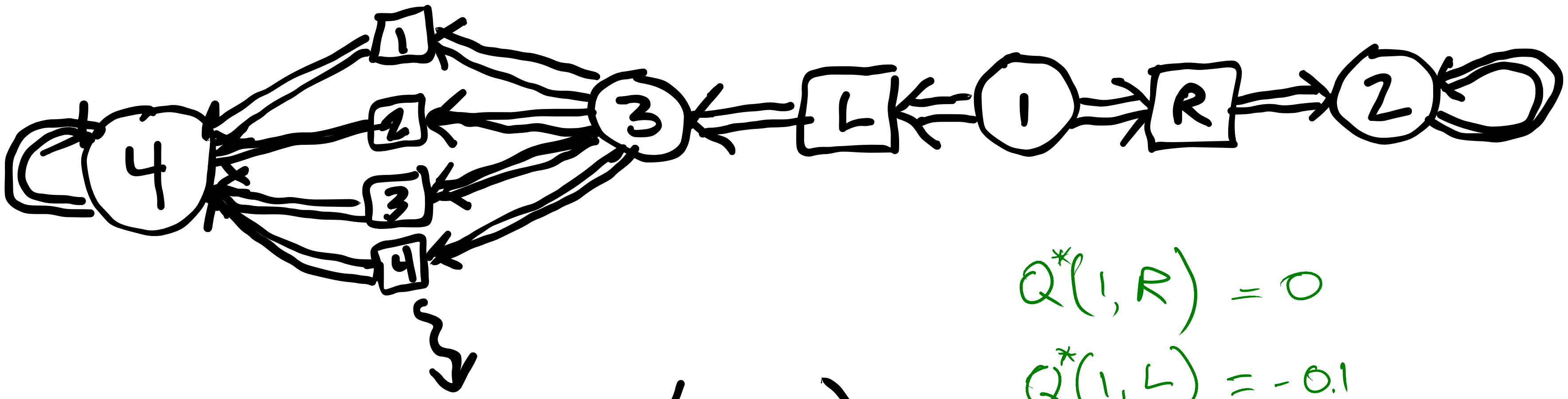
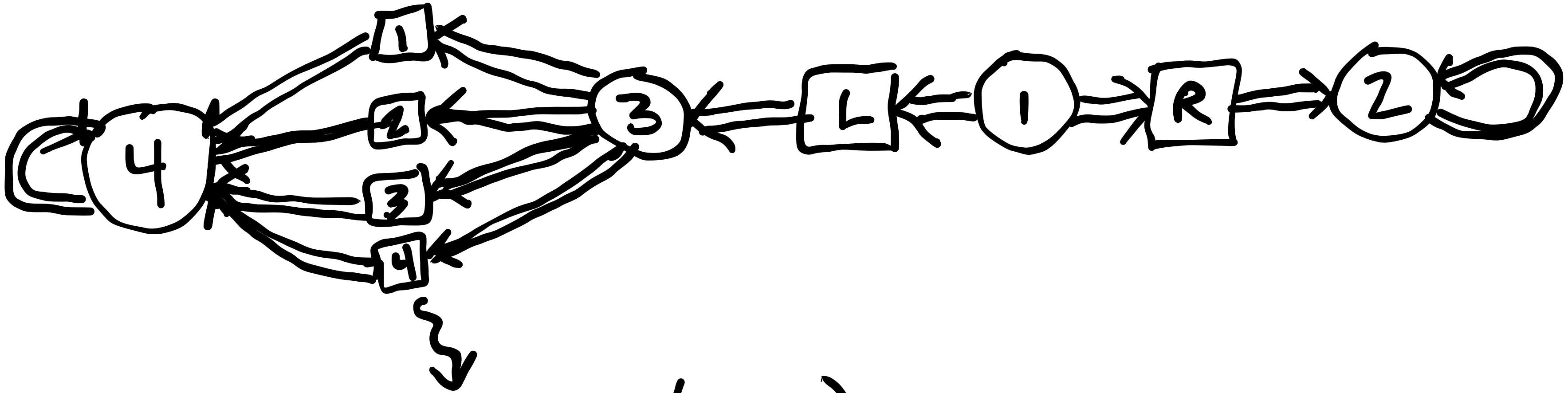


Actions 1-4: $r \sim \mathcal{N}(-0.1, 1)$

$Q^*(1, R) = 0$

$Q^*(1, L) = -0.1$

Illustrative Problem

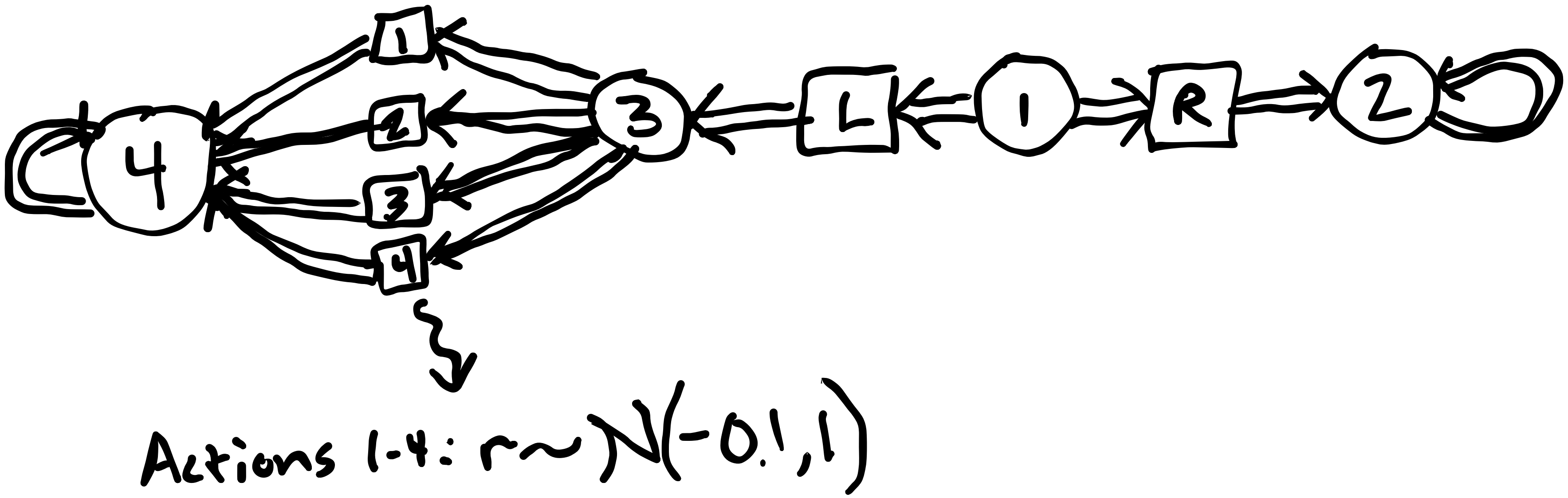


Actions 1-4: $r \sim \mathcal{N}(-0.1, 1)$

1. After a few episodes, what is Q(3, a) for a in 1-4?

Q(3,1)

Illustrative Problem



1. After a few episodes, what is Q(3, a) for a in 1-4?
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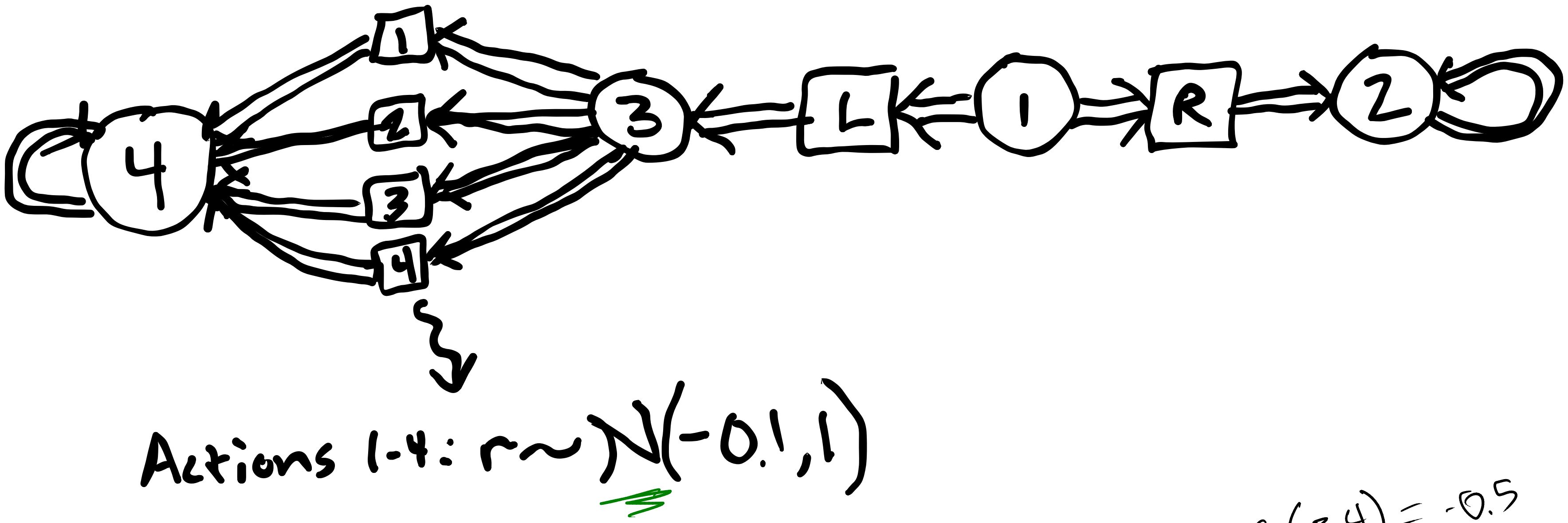
Illustrative Problem



1. After a few episodes, what is Q(3, a) for a in 1-4?
2. After a few episodes, what is Q(1, L)?
3. Why is this a problem and what are some possible solutions?

$Q(3,4) = -0.5$
$Q(3,1) = 0.1$
$Q(3,2) = -0.3$

$Q(1,L) = 0.1$

Big Problem: Maximization Bias

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Even if all $Q(s', a')$ unbiased, $\max_{a'} Q(s', a')$ is biased!

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Solution: Double Q Learning

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Solution: Double Q Learning Q_1, Q_2

Big Problem: Maximization Bias

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Solution: Double Q Learning Q_1, Q_2

$$Q_1(s,a) \leftarrow Q_1(s,a) + \alpha \ (r + \gamma Q_2(s', \text{argmax}_{a'} Q_1(s', a')) - Q_1(s,a))$$

$r + \gamma \max_{a'} Q(s', a')$

Big Problem: Maximization Bias

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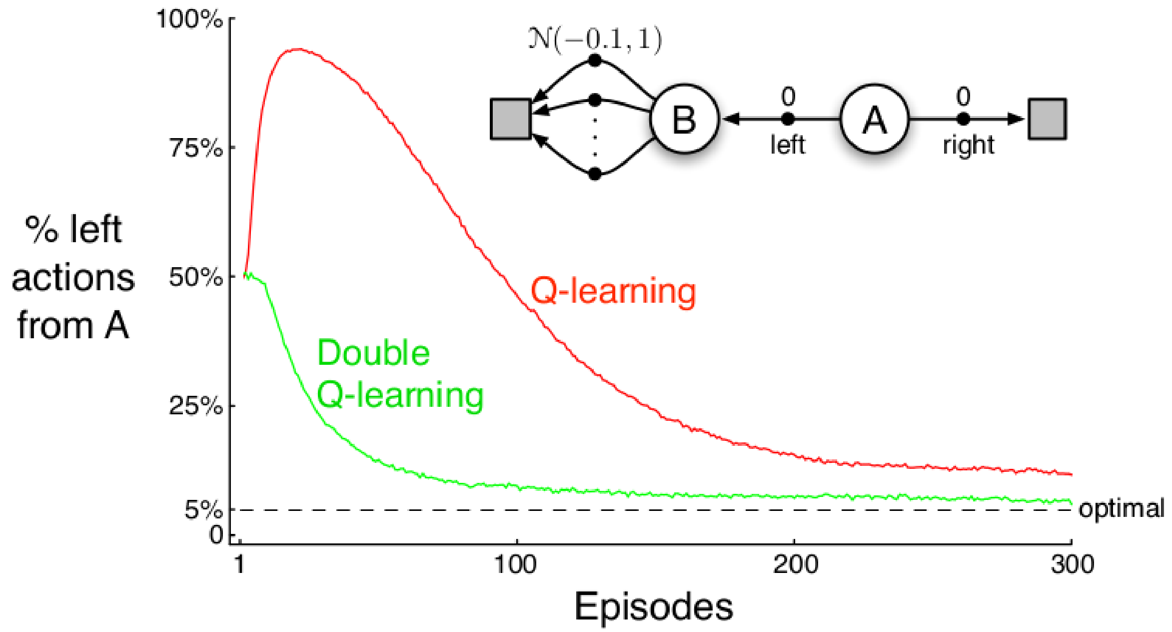
Solution: Double Q Learning Q_1, Q_2

$$Q_1(s,a) \leftarrow Q_1(s,a) + \alpha \left(r + \gamma Q_2(s', \text{argmax}_{a'} Q_1(s', a')) - Q_1(s,a)\right)$$



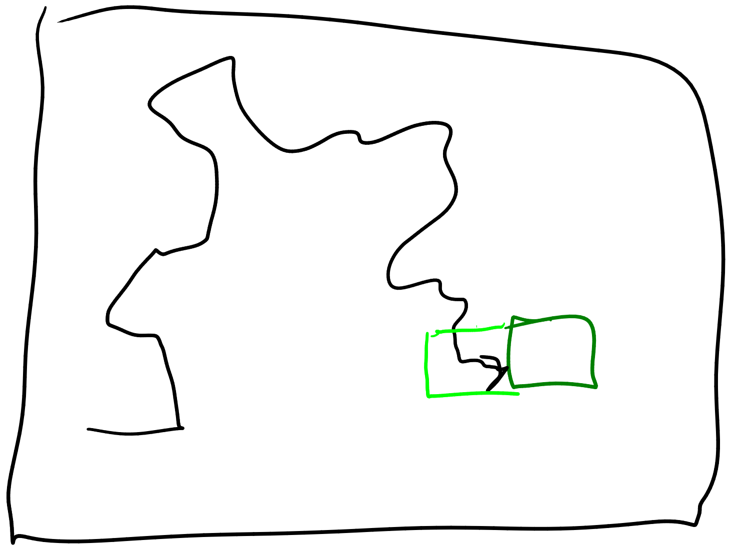
SARSA

Q-learning: $Q(s,a) \leftarrow Q(s,a) + \alpha\ (r + \gamma \max_{a'} Q(s',a') - Q(s,a))$

SARSA: $Q(s,a) \leftarrow Q(s,a) + \alpha\ (r + \gamma Q(s',a') - Q(s,a))$

(s, a, r, s', a')

Eligibility Traces



SARSA-λ

SARSA-λ

Games

Half-Life at 20: why it is the most important shooter ever made

From its opening scenes, Valve's pioneering sci-fi horror game reinvented storytelling and universe building - what made it such a terrifying success?

'It taught a whole generation of big-budget game developers how to tell stories' … the Half-Life box art. Illustration: Valve

SARSA-λ

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$$Q(s,a), N(s,a) \leftarrow 0$$

initialize s, a, r, s'

loop

$$a' \leftarrow \operatorname{argmax} Q(s', a) \text{ w.p. } 1-\epsilon, \quad \text{rand}(A) \text{ o.w.}$$

$$N(s,a) \leftarrow N(s,a) + 1$$

$$\delta \leftarrow r + \gamma Q(s', a') - Q(s,a)$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha\delta N(s,a) \quad \forall s, a$$

$$N(s,a) \leftarrow \gamma\lambda N(s,a) \quad \forall s, a$$

$\lambda \in [0, 1)$

$$s \leftarrow s', \quad a \leftarrow a'$$

$$r \leftarrow \text{act!}(\text{env}, a)$$

$$s' \leftarrow \text{observe}(\text{env})$$

Convergence

Convergence

- Q learning converges to optimal Q-values w.p. 1 (Sutton and Barto, p. 131)

Convergence

- Q learning converges to optimal Q-values w.p. 1
  (Sutton and Barto, p. 131)
- SARSA converges to optimal Q-values w.p. 1 ***provided that***

  $\pi \rightarrow \mathrm{greedy}$

  (Sutton and Barto, p. 129)

On vs Off-Policy

On vs Off-Policy

On Policy

On vs Off-Policy

On Policy

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On vs Off-Policy

On Policy

SARSA:

$$Q(s,a) \leftarrow Q(s,a) + \alpha \ (r + \gamma Q(s',a') - Q(s,a))$$

Off Policy

On vs Off-Policy

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On vs Off-Policy

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Will eligibility traces work with Q-learning?

On vs Off-Policy

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Not easily

Policy Gradient:

$$\theta \leftarrow \theta + \alpha \sum_{k=0}^{d} \nabla_\theta \log \pi_\theta(a_k \mid s_k) R(\tau)$$

Today

Double Q

SARSA-λ

- Basic On- and Off-Policy **value based** model free RL algorithms
- Tricks for tabular value based RL algorithms
- What makes it hard for some approaches to use Off-Policy data?