$$P(A) = \sum_c P(A|C=c)P(C=c)$$

QUIZ 1 SOLUTIONS

1) a) $P(A=1) = P(A=1|C=0)P(C=0) + P(A=1|C=1)P(C=1)$

$= \quad 0.7 \quad 0.4 \quad 0.1 \quad 0.6$

$= \boxed{0.34}$

b)

A	B	P(A,B)
1	1	0.204
1	0	0.136
0	0	0.33
0	1	0.33

$P(A=1)\cdot P(B=1|A=1) = 0.34\cdot 0.6$

$P(A=1)\ P(B=0|A=1) = 0.34\cdot 0.4$

$P(A=0)\ P(B=0|A=0) = 0.66\cdot 0.5$

$P(A=0)\ P(B=1|A=0) = 0.66\cdot 0.5$

c) $P(B=1, C=1) = P(C=1)\big(P(A=1|C=1)P(B=1|A=1) + P(A=0|C=1)P(B=1|A=0)\big)$

$= 0.6\cdot(0.1\cdot 0.6 + 0.9\cdot 0.5) =$

$= 0.306$

$P(B=1) = P(A=1, B=1) + P(A=0, B=1) = 0.204 + 0.33 = 0.534$

$P(C=1) = 0.6$

$P(B=1)P(C=1) = 0.3204 \neq 0.306 = P(B=1, C=1)$

$\therefore$ B and C are not independent

2) $N \equiv$ neice location $\in \{t, c, d\}$ (t: table, c: closet, d: door)

$L \equiv$ leg sticking out $\in \{0,1\}$

Bayes Rule

$$P(N|L) = \frac{P(L|N)\,P(N)}{P(L)}$$

$$P(N=t|L=0) = \frac{\overset{1}{P(L=0|N=t)}\ \overset{1/3}{P(N=t)}}{\underset{5/6}{P(L=0)}}$$

$$P(L) = \sum_n P(L=0|N=n)P(N=n)$$

$$= 0.5\cdot\tfrac{1}{3} + \tfrac{1}{3} + 1\cdot\tfrac{1}{3}$$

$$= \tfrac{5}{6}$$

$$= \frac{\frac{1}{3}}{\frac{5}{6}}$$

$$= \tfrac{2}{5} = \boxed{40\%}$$

3]
$$\pi'(s) = \arg\max_a \{R(s,a) + \gamma V^{\pi}(s')\}$$
$$= \arg\max_a \{ s - a + \gamma V^{\pi}(\text{clamp}(s+a, 1, 3))\}$$

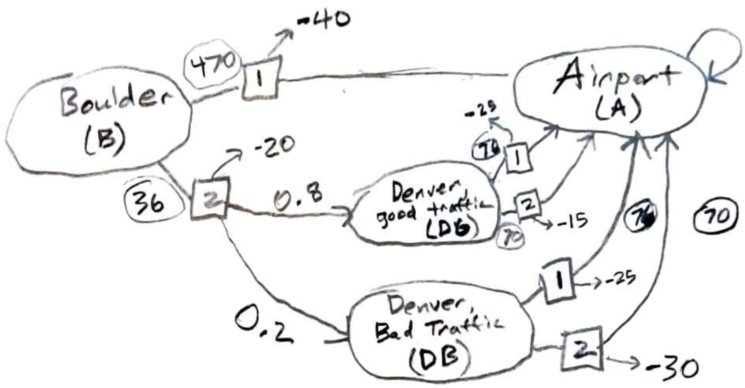
s	a	s'	$R(s,a) + \gamma V^{\pi}(s')$	$Q^{\pi}(s,a)$
1	1	2	$0 + 0.95 \cdot 41$	38.95
1	-1	1	(by definition from V^{π})	40
2	-1	1	"	41
2	1	3	$1 + 0.95 \cdot 42.95$	41.8025
3	-1	2	(By definition)	42.95
3	1	3	$2 + 0.95 \cdot 42.95$	42.8025

$\pi'(1) = -1$

$\pi'(2) = 1$

$\pi'(3) = -1$

4] a)



$S = \{B, A, DG, DB\}$

$A = \{1, 2\}$

$$R(s,a) = \begin{cases} -40 & \text{if } s = B, a = 1 \\ -20 & \text{if } s = B, a = 2 \\ -25 & \text{if } s \in \{DG, DB\}, a = 1 \\ -15 & \text{if } s = DG, a = 2 \\ -30 & \text{if } s = DB, a = 2 \\ 0 & \text{otherwise} \end{cases}$$

$$T(s'|s,a) = \begin{cases} 1 & \text{if } s = B, a = 1, s' = A \\ 1 & \text{if } s \in \{DG, DB\}, s' = A \\ 0.8 & \text{if } s = B, a = 2, s' = DG \\ 0.2 & \text{if } s = B, a = 2, s' = DB \\ 1 & \text{if } s = A, s' = A \\ 0 & \text{o.w.} \end{cases}$$

$\gamma = 1$

b)

$$Q^*(s,a) = R(s,a) + \gamma \sum_{s'} T(s'|s,a)\, V^*(s')$$

$$V^*(s) = \max_a Q^*(s,a)$$

$V^*(A) = 0$ since this is an absorbing state with no reward.

$Q^*(DG,1) = -25 + 0 = -25$

$Q^*(DG,2) = -15 + 0 = -15$

$V^*(DG) = -15$

$Q^*(DB,1) = -25 + 0 = -25$

$Q^*(DB,2) = -30 + 0 = -30$

$V^*(DB) = -25$

$Q^*(B,1) = -40 + 0 = -40$

$Q^*(B,2) = -20 + 0.8\, V^*(DG) + 0.2\, V^*(DB) = -37$

since $Q^*(B,2) > Q^*(B,1)$

taking highway 36 is best