ASEN 5519-003 Decision Making under Uncertainty
Homework 1: Probabilistic Models

January 16, 2023

1 Questions

Question 1. (20 pts) Consider the following joint distribution of three binary-valued random variables, A, B, and C:

A	B	C	$P(A, B, C)$
0	0	1	0.15
0	1	0	0.05
0	1	1	0.01
1	0	0	0.14
1	0	1	0.18
1	1	0	0.29
1	1	1	0.07

a) What is the probability of the outcome $A = 0$, $B = 0$, $C = 0$?

b) What is the marginal distribution of A?

c) What is the conditional distribution of A given $B = 0$ and $C = 1$?

Question 2. (20 pts) 2% of women at age forty who participate in routine screening have breast cancer. 86% of those with breast cancer will get positive mammograms. 8% of those without breast cancer will also get positive mammograms. A woman in this age group had a positive mammogram in a routine screening. What is the probability that she actually has breast cancer?

Question 3. (40 pts) Suppose that a stationary stochastic process $\{x_t\}$ is defined by the following equation: $x_{t+1} = 1.5\,x_t - x_{t-1} + v_t$ where v_t are independent, identically distributed random variables with $v_t \sim \mathcal{N}(0.0, 0.2^2)$.
  (a) Simulate and plot 10 20-step trajectories sampled from this process with $x_0 = x_{-1} = 1$ (as always, submit your code for this).
  (b) Is this process a Markov process if the state is defined as x_t? Why or why not?
  (c) If you only had access to the trajectories you plotted what evidence could you use to convince someone that this process is or is not Markov?
  (d) What would need to be included in the state at time t to make this a Markov process?

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2 Auto-graded Programming

Question 4. (20 pts) In this exercise, you will write a function in Julia that will require you to learn some concepts useful for the rest of the course. Your function should accept four positional arguments:

- `a`: A real number.
- `b`: A square matrix.
- `c`: An array that has the same size and element type as `b`.
- `d`: A `Dict` with `String` keys and vector values.

The function should return a `NamedTuple` with the following entries:

- `relu`: The maximum of `a` and zero (the "rectified linear unit" function).
- `matrix`: The matrix product of `b` and `c`.
- `elementwise`: The element-wise product of `b` and `c`.
- `first`: The vector corresponding to the first key in `d` when the keys are alphabetically sorted.
- `max`: The element-wise maximum over all of the vectors in `d`.

The function must be completely "type-stable" (see the "Performance Tips" section of the Julia manual) and must return objects with similar types to the input arguments (for instance if `a` is an `Int`, `relu` should also be an `Int`.

Evaluate this function with `DMUStudent.HW1.evaluate` and submit the resulting json file *along with a listing of the code.* A score of 1 will receive full credit.