Last Time

- What does "Markov" mean in "Markov Decision Process"?

$$P(s_{t+1} \mid s_t, \ldots, s_0) = P(s_{t+1} \mid s_t)$$

$$s_{t+1} \perp s_{t-1} \ldots s_0 \mid s_t$$

Guiding Questions

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- What is a **Markov decision process**?

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- What is a **Markov decision process**?
- What is a **policy**?

Guiding Questions

- What is a **Markov decision process**?
- What is a **policy**?
- How do we **evaluate** policies?

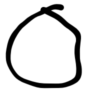
Decision Networks and MDPs

Decision Networks and MDPs

Decision Network

Decision Networks and MDPs

Decision Network



Decision Networks and MDPs

Decision Network

Chance node

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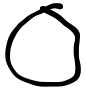
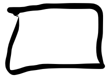
 Chance node



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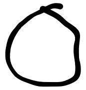
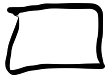
 Chance node

 Decision node

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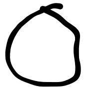
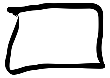
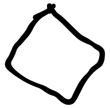
 Chance node

 Decision node



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MDP Dynamic Decision Network

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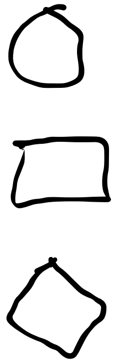

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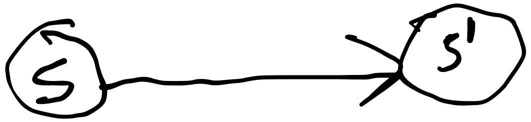

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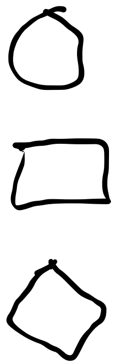

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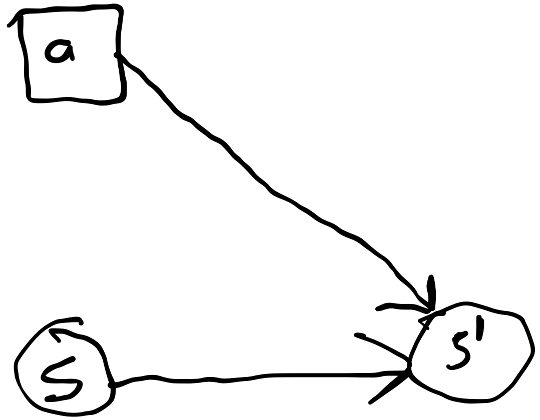

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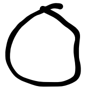

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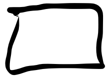

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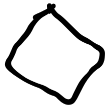

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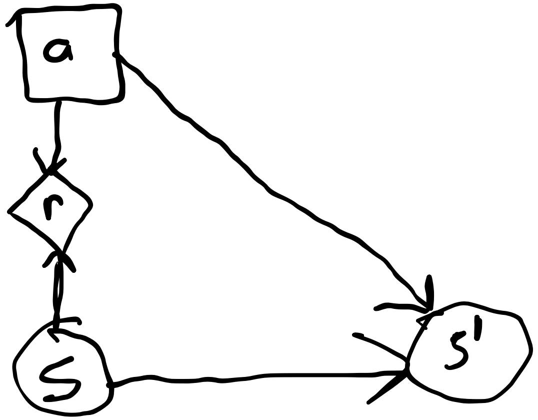

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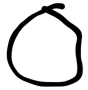
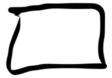
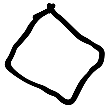
 Chance node

 Decision node

 Utility node

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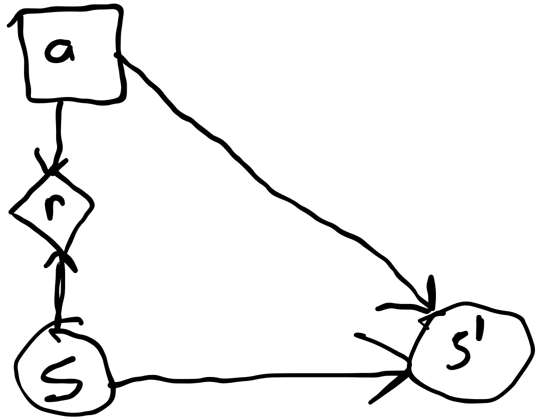

MDP Optimization problem

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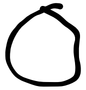
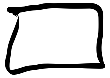
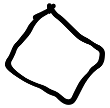
 Chance node

 Decision node

 Utility node

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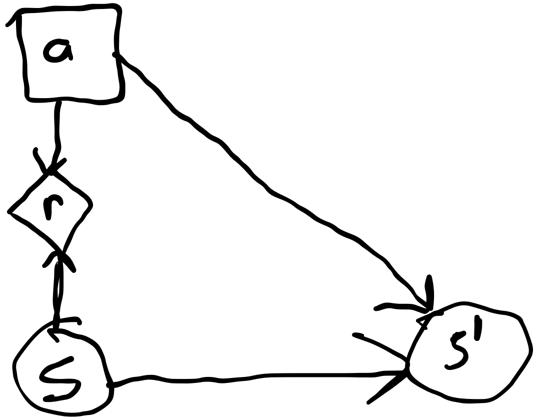

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$$\text{maximize} \quad \mathrm{E}\left[\sum_{t=1}^{\infty} r_t\right]$$

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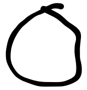
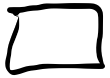
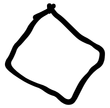
 Chance node

 Decision node

 Utility node

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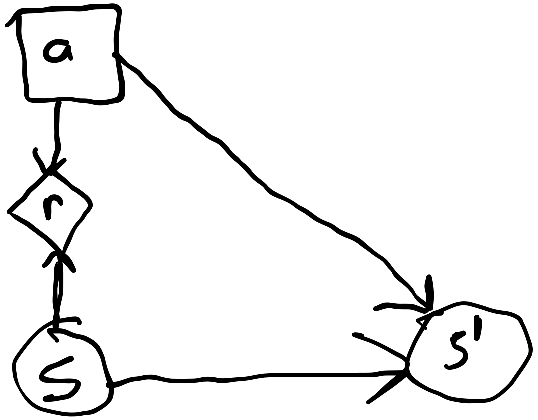

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Not well formulated!

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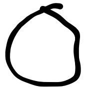
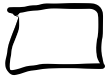
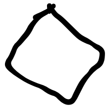
 Chance node

 Decision node

 Utility node

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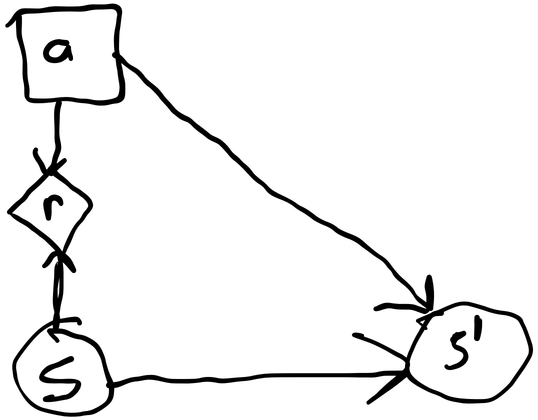

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Infinite

Finite MDP Objectives

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1. Finite time

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4. Terminal States

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Infinite time, but a terminal state (no reward, no leaving) is always reached with probability 1.

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(S, A, T, R, γ)

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(S, A, T, R, γ) (and b in some contexts)

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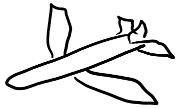
MDP "Tuple Definition"

$s \in S = \mathbb{R}^{12}$

$$\begin{bmatrix} x \\ y \\ z \\ u \\ v \\ w \\ \phi \\ \theta \\ \psi \\ p \\ q \\ r \end{bmatrix}$$



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$$\underbrace{T(s' \mid s, a)}_{\text{Explicit}}$$

$$\downarrow$$

$$s', r = G(s, a)$$

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$T(s' \mid s, a)$

- R (reward function) - maps each state and action to a reward

$s', r = G(s, a)$

MDP "Tuple Definition"

(S, A, T, R, γ) (and b in some contexts)

- S (state space) - set of all possible states

$\{1, 2, 3\}$ $(x, y) \in \mathbb{R}^2$ $\{0, 1\} \times \mathbb{R}^4$

$\{\text{healthy}, \text{pre-cancer}, \text{cancer}\}$ $(s, i, r) \in \mathbb{N}^3$

- A (action space) - set of all possible actions

$\{1, 2, 3\}$ $\mathbb{R}^2$ $\{0, 1\} \times \mathbb{R}^2$

$\{\text{test}, \text{wait}, \text{treat}\}$

- T (transition distribution) - explicit or implicit ("generative") model of how the state changes

$T(s' \mid s, a)$

$s' \sim T(s,a)$

$R(s,a) = \underset{s' \sim T(s,a)}{E}\left[R(s,a,s')\right]$

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- b: initial state distribution

MDP Example

Imagine it's a cold day and you're ready to go to work. You have to decide whether to bike or drive.

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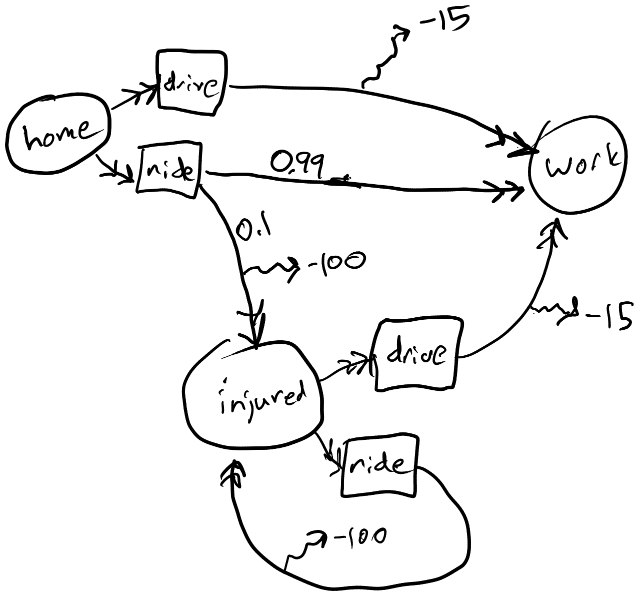
Imagine it's a cold day and you're ready to go to work. You have to decide whether to bike or drive.

- If you drive, you will have to pay $15 for parking; biking is free.
- On 1% of cold days, the ground is covered in ice and you will crash if you bike, but you can't discover this until you start riding. After your crash, you limp home with pain equivalent to losing $100.



$S = \{\text{home}, \text{work}, \text{injured}\}$

$A = \{\text{drive}, \text{ride}\}$

$$T^{\text{drive}} = \begin{array}{c|c} & s' \\ \hline s & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array} \qquad T^{\text{ride}} = \begin{bmatrix} 0 & 0.99 & 0.01 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(s,a,s') = \begin{cases} -15 & \text{if } a = \text{drive} \\ -100 & \text{if } s' = \text{injured} \\ 0 & \text{o.w.} \end{cases} \qquad \gamma = 0.99$$

Policies and Simulation

Policies and Simulation

- A *policy*, denoted with π, as in $a_t = \pi(s_t)$ is a function mapping every state to an action.
- When a policy is combined with a Markov decision process, it becomes a Markov stochastic process with

$$P(s' \mid s) = T(s' \mid s, \pi(s))$$

Simulate

Input: (S, A, T, R, γ, b)

Output: $\hat{u}$ (accumulated reward)

$s \leftarrow$ sample(b)

$\hat{u} \leftarrow 0$

for t in $0..T-1$ until $\gamma^t < \varepsilon$

    $a \leftarrow \pi(s)$

    $s', r \leftarrow G(s,a)$ samples from T, R

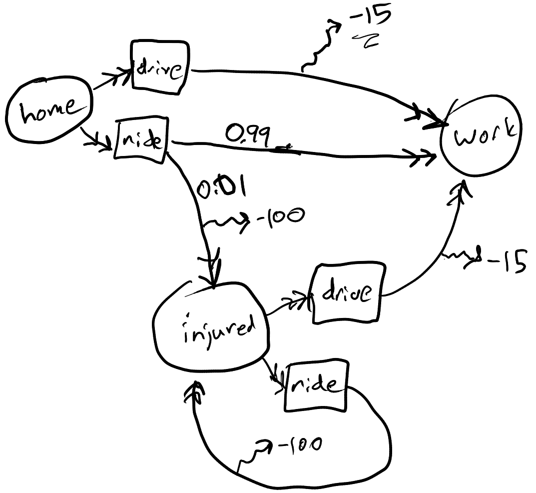
    $\hat{u} \leftarrow \hat{u} + \gamma^t r$

    $s \leftarrow s'$

return $\hat{u}$

Break

- Suggest a policy that you think is optimal for the icy day problem



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$$0.99 \cdot 0 + 0.01(-100 - 15) = -1.15$$

$$\pi(s) = \begin{cases} & \text{if } s = \text{home} \\ \text{drive} & \text{if } s = \text{injured} \\ & \end{cases}$$

P

U

Utility

$A \succ B$: prefer A to B

$A \sim B$: indifferent

$A \succeq B$: prefer A or indifferent

Lottery: $[S_1 : p_1 ; S_2 : p_2 ; \ldots S_n : p_n]$

Completeness: Exactly 1 holds:

$A \succ B \quad B \succ A \quad A \sim B$

Transitivity: If $A \succeq B$ and $B \succeq C$ the $A \succeq C$

Continuity: If $A \succeq C \succeq B$ then $\exists p$ s.t.

$[A : p ; B : 1-p] \sim C$

Independence: If $A \succ B$ then

$[A : p ; C : 1-p] \succeq [B : p ; C : 1-p]$

$\exists$ U s.t.

$U(A) > U(B)$ iff $A \succ B$

$U(A) = U(B)$ iff $A \sim B$

$\underbrace{U([S_1 : p_1 ; \ldots S_n : p_n])}$ $= \sum_i p_i S_i$

Slide not on Exam

Policy Evaluation

$$U(\pi) = E\left[\sum_{t=0}^{\infty} \gamma^t R(s_t, a_t) \mid a_t = \pi(s_t)\right]$$

Naive:

marginal of s_t given π is being executed

$$U(\pi) = \sum_{t=0}^{\infty} \gamma^t P^{\pi}(s_t) R(s_t, \pi(s_t))$$

$$P^{\pi}(s_t) = \sum_{s_{t-1}} T(s_t \mid s_{t-1}, \pi(s_{t-1})) P^{\pi}(s_{t-1})$$

Naive Policy Evaluation not on Exam

Value Function-Based Policy Evaluation

$$U(\pi) = \underset{\tau}{E}\left[\sum \gamma^t R(s_t, a_t) \right|$$

Monte Carlo Policy Evaluation

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- Running a large number of simulations and averaging the accumulated reward is called *Monte Carlo Evaluation*

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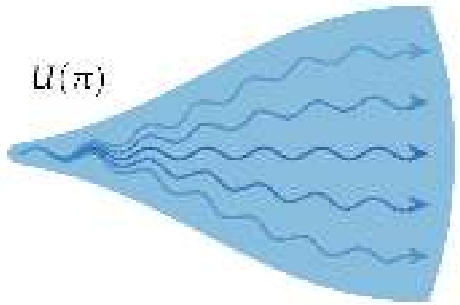

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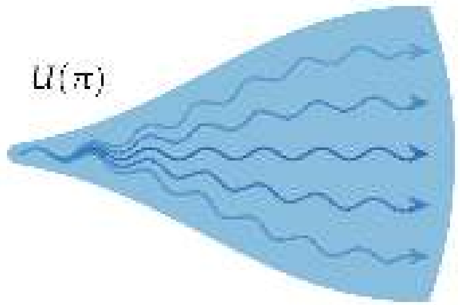
Let $\tau = (s_0, a_0, r_0, s_1, \ldots, s_T)$ be a *trajectory* of the MDP



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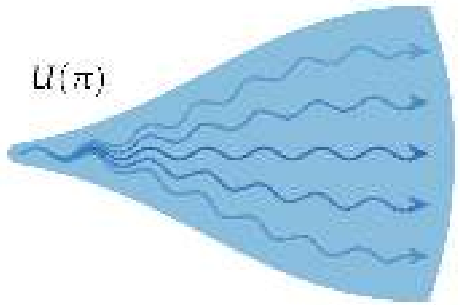

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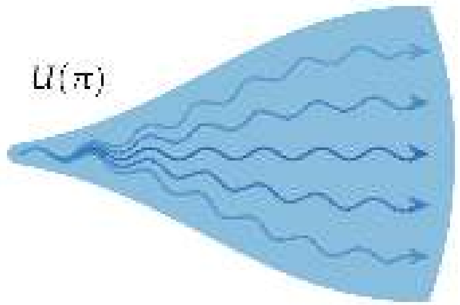

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random variable

$$U(\pi) \approx \bar{u}_m = \frac{1}{m} \sum_{i=1}^{m} \hat{u}^{(i)}$$

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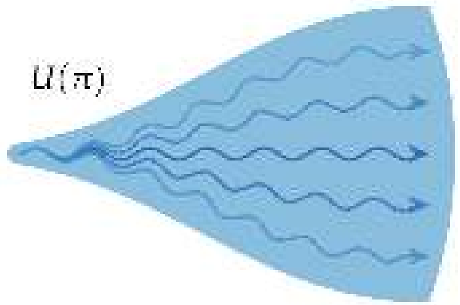

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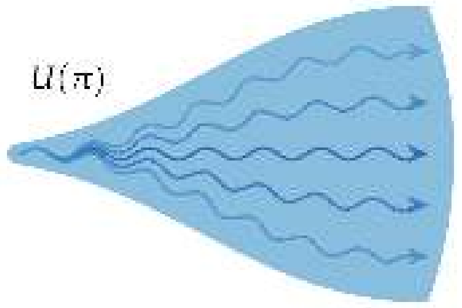

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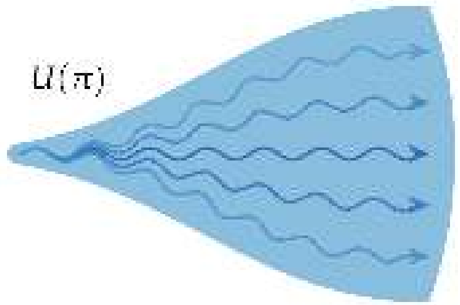

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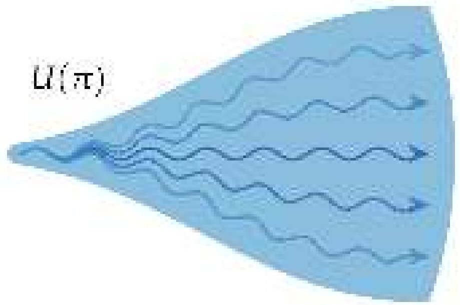

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CLT not on exam

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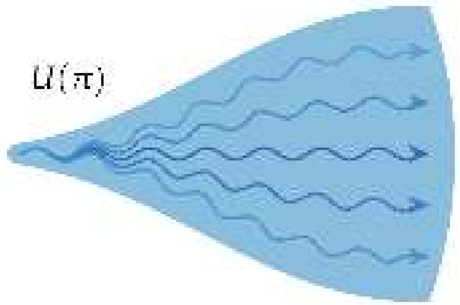

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CLT not on exam

$$\text{s.e.m.} = \frac{\text{std}(\hat{u})}{\sqrt{m}}$$

Standard Error of the Mean

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