Question 1

19:27

2022 Quiz 1 Solutions

a) $P(A=1, B=1) = P(A=1 \mid B=1)\, P(B=1) = 0.7 \cdot 0.25 = \boxed{0.175}$

b)

C	$P(C)$
0	0.5
1	0.5

$$\sum_b P(C=0, B=b) = \sum_b P(C=0 \mid B=b)\, P(B=b)$$

$$= P(C=0 \mid B=0)\, P(B=0) + P(C=0 \mid B=1)\, P(B=1)$$

$$= 0.6 \cdot 0.75 + 0.2 \cdot 0.25$$

$$= 0.5$$

$$P(C=1) = 1 - P(C=0) = 0.5$$

c) Want $P(C=1 \mid A=1)$

$$P(C=1 \mid A=1) = \frac{P(C=1, A=1)}{P(A=1)} = \frac{\sum_b P(C=1, A=1 \mid B=b)\, P(B=b)}{\sum_b P(A=1 \mid B=b)\, P(B=b)}$$

(conditional independence)

$$= \frac{\sum_b P(C=1 \mid B=b)\, P(A=1 \mid B=b)\, P(B=b)}{\sum_b P(A=1 \mid B=b)\, P(B=b)}$$

$$= \frac{0.4 \cdot 0.4 \cdot 0.75 + 0.7 \cdot 0.8 \cdot 0.25}{0.4 \cdot 0.75 + 0.7 \cdot 0.25}$$

$$= 0.55$$

greater than 50%

Question 2 | 5:57

T = temp increase $\quad T \in \{0, 1\}$

F = pump failure $\quad F \in \{0, 1\}$

$$P(F|T) = \frac{P(T|F)\,P(F)}{P(T)}$$

$$= \frac{1.0 \cdot 0.01}{P(T|F=1)P(F=1) + P(T|F=0)\,P(F=0)}$$

$$= \frac{0.01}{1.0 \cdot 0.01 + 0.1 \cdot 0.99}$$

$$= 0.0917$$

$$= 9.17\,\%$$

Question 3 | 11:25

$S = \{Kf, H, Hf, B, Bf, +\}$

$A = \{L, R\}$

$R(s,a) = \begin{cases} 1 \text{ if } s = Bf \\ 0 \text{ otherwise} \end{cases}$

f indicates that the robot is carrying food, + is a terminal state

move left or right

s	a	s'	T(s' | s,a)
Kf	L	Kf	1
Kf	R	Kf	0.1
Kf	R	Hf	0.9
Hf	L	Kf	0.9
Hf	L	Hf	0.1
Hf	R	Bf	0.9
Hf	R	Hf	0.1
Bf	any	+	1.0
+	any	+	1.0
H	L	H	0.1
H	L	Kf	0.9
H	R	B	0.9
H	R	H	0.1
B	L	B	0.1
B	L	H	0.9
B	R	B	1.0
any other combination			0

$\gamma = 0.9$

Question 4 | 13:06

s	π^*	V^*	Q^* antibiotic	Q^* wait	$R + \gamma E[V^*]$
recovered	wait	0			(terminal if wait action taken)
ICU	wait	0			(terminal if wait action taken)
severe	wait	−10	−11		−11 + 0
				−10	−10 + 0
minor	either	−5	−5		−1 + (0.6 · V*(recovered) + 0.4 · V*(severe))
				−5	−1 + 0 − 4
					0 + (0.5 · V*(recovered) + 0.5 · V*(severe))
					0 + 0 − 5

Two optimal policies are

$$\pi_1^*(s) = \text{wait}$$

$$\pi_2^*(s) = \begin{cases} \text{antibiotic} & \text{if } s = \text{minor} \\ \text{wait} & \text{otherwise} \end{cases}$$

Both waiting and giving the antibiotic have the same value, so the doctors would be indifferent.