

# Recap

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- POMDP

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- POMDP  $(S, A, O, R, T, Z, \gamma)$

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- Belief Updates

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$$b_t(s) = P(s_t = s \mid h_t)$$


$$b' = \tau(b, a, o)$$

# Recap

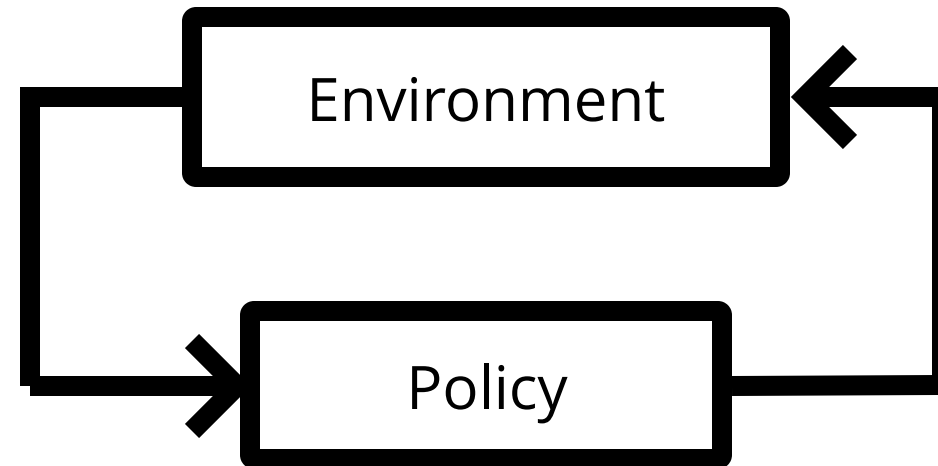
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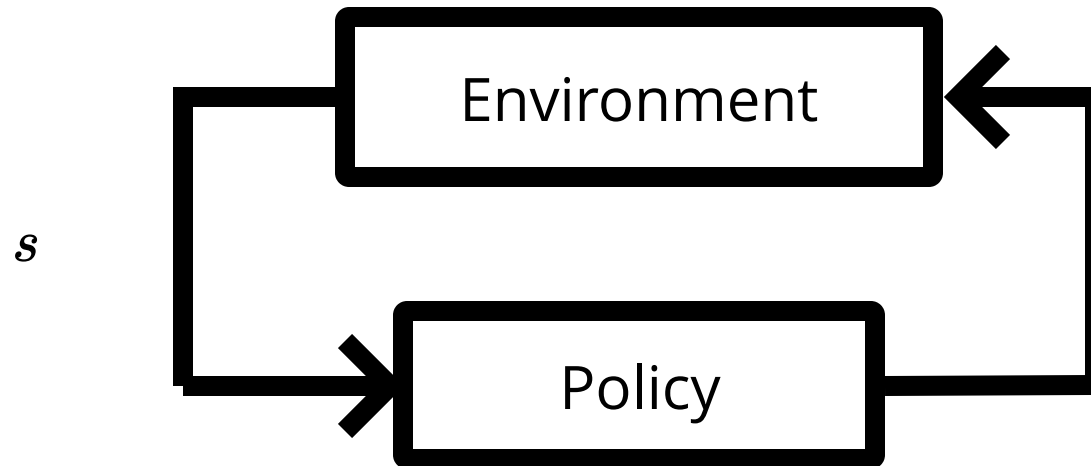
$$b'(s') \propto Z(o \mid a, s') \sum_s T(s' \mid s, a) b(s)$$


# MDP Sense-Plan-Act Loop

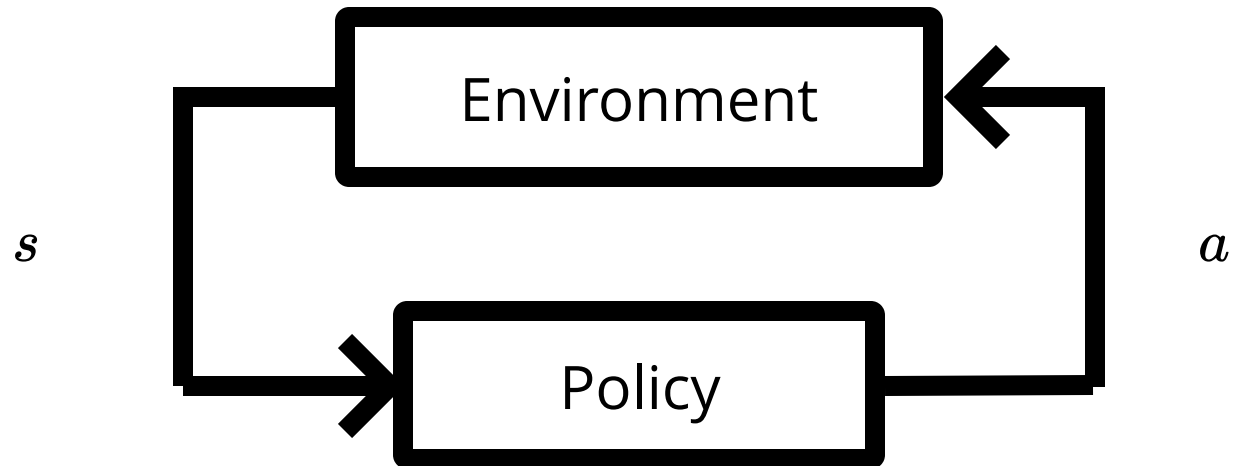




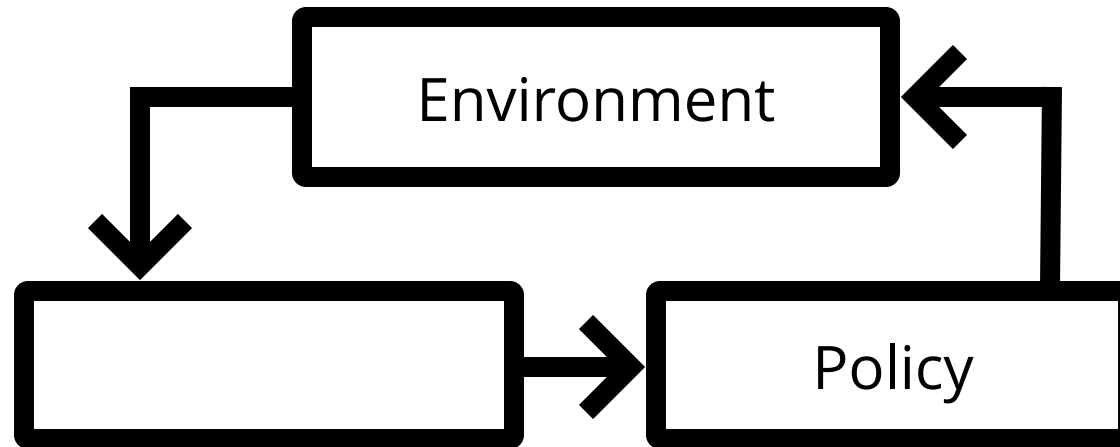
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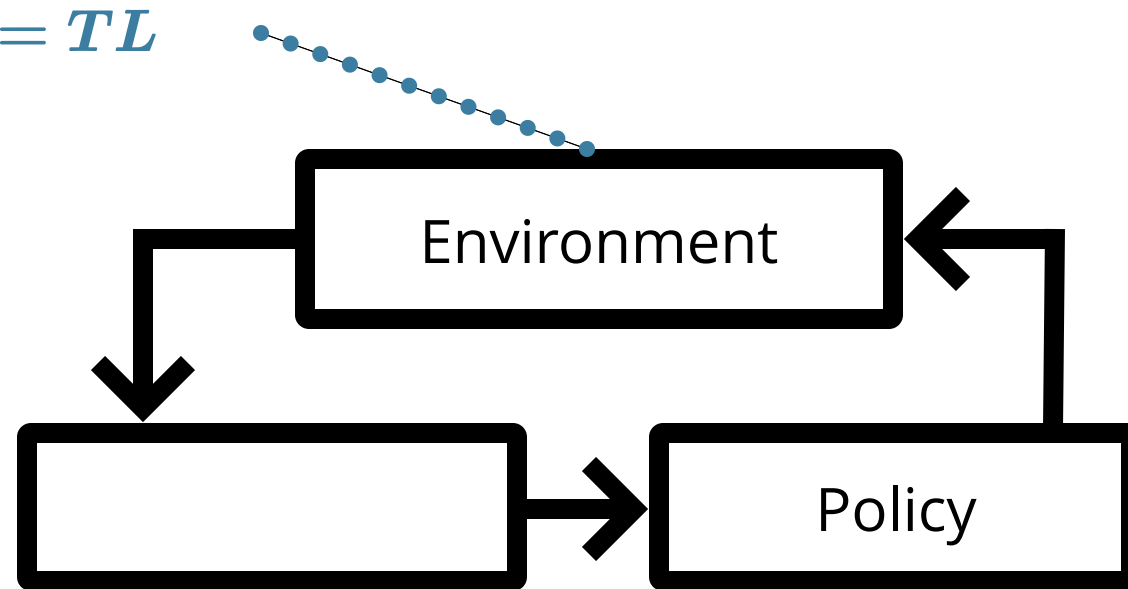
# POMDP Sense-Plan-Act Loop



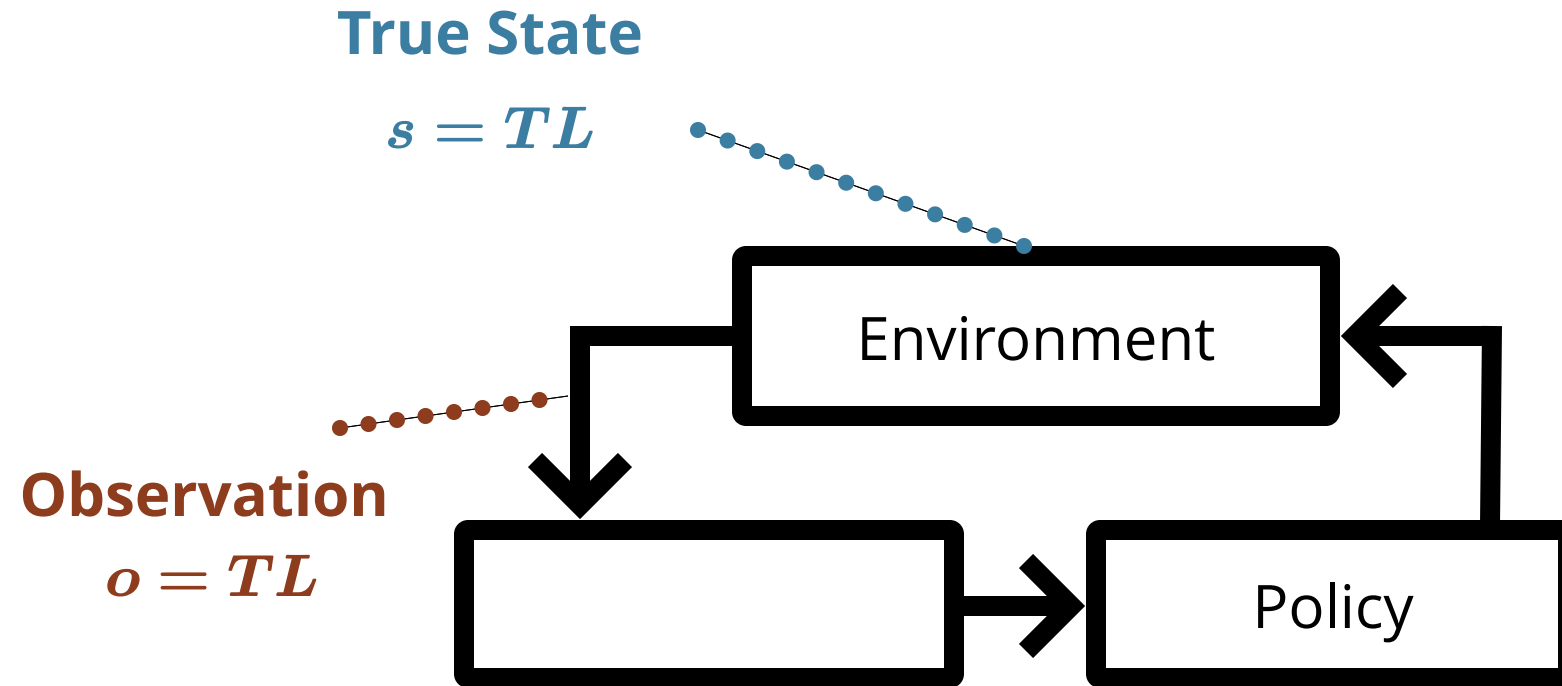
# POMDP Sense-Plan-Act Loop

True State

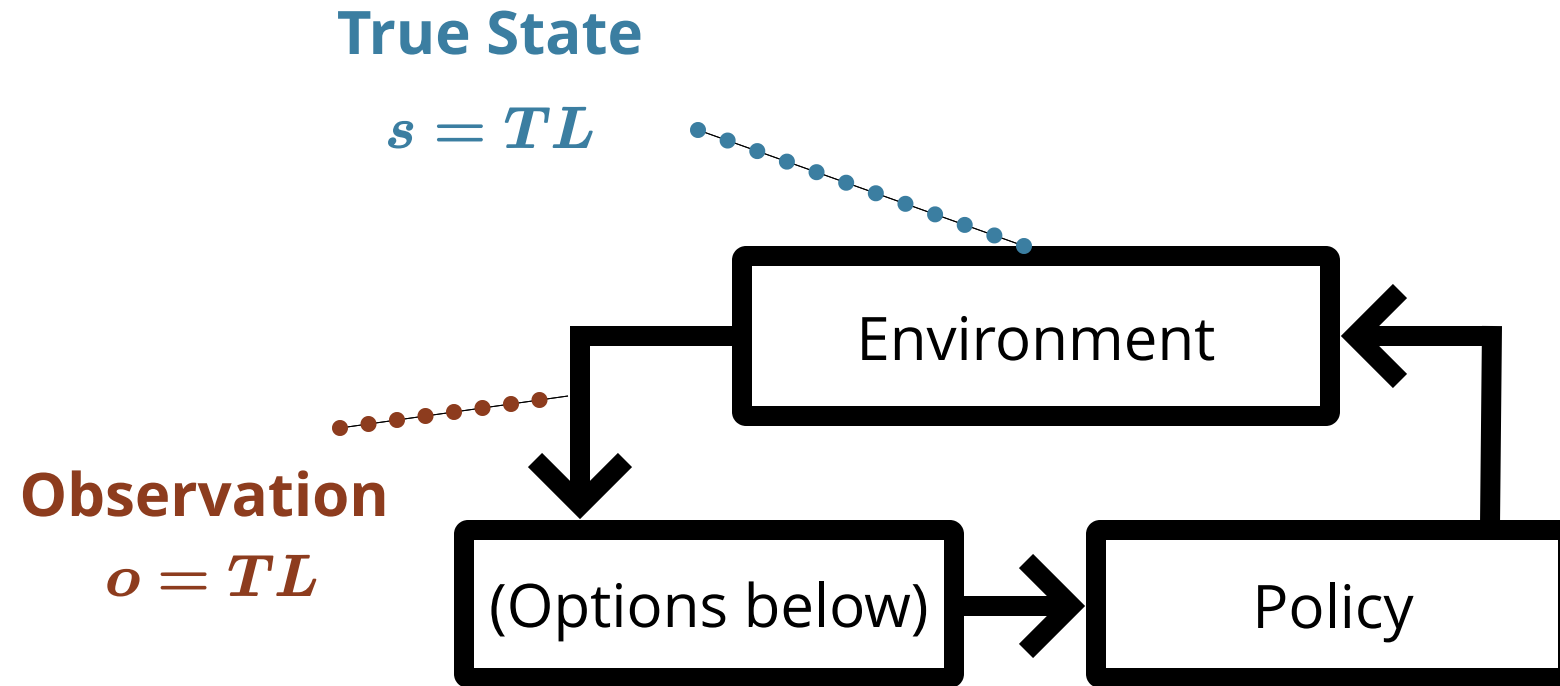
$$s = TL$$



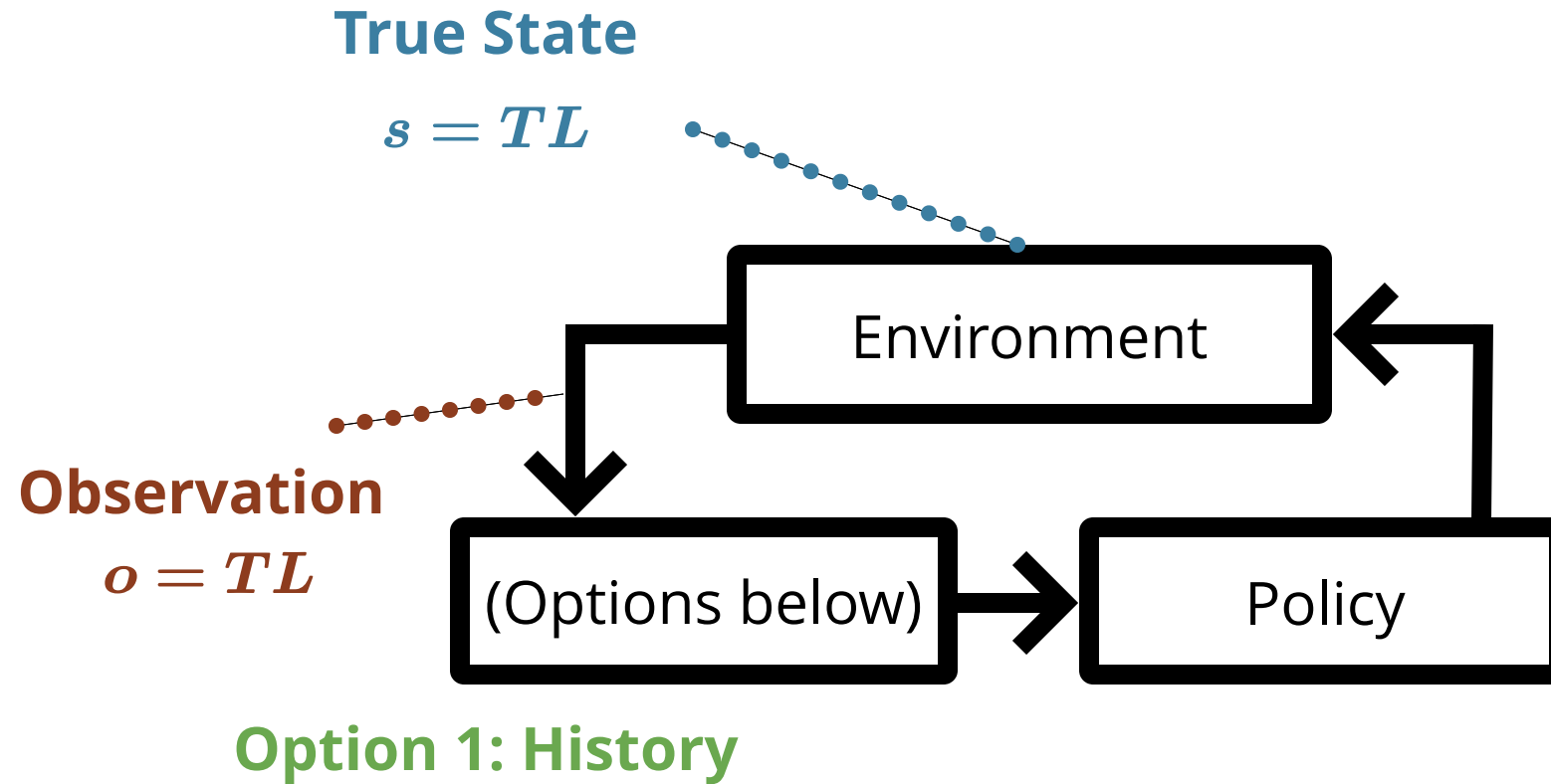
# POMDP Sense-Plan-Act Loop



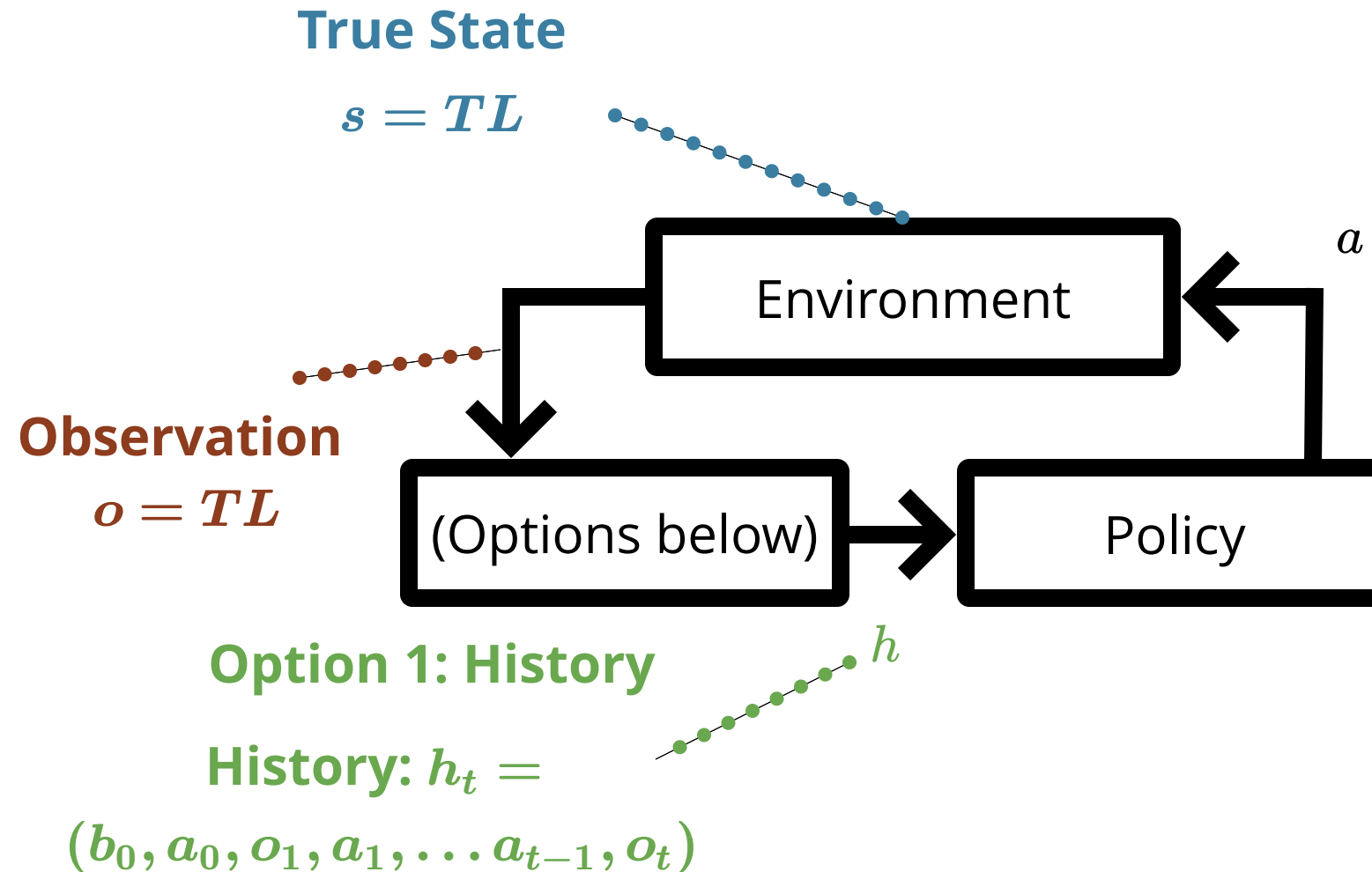
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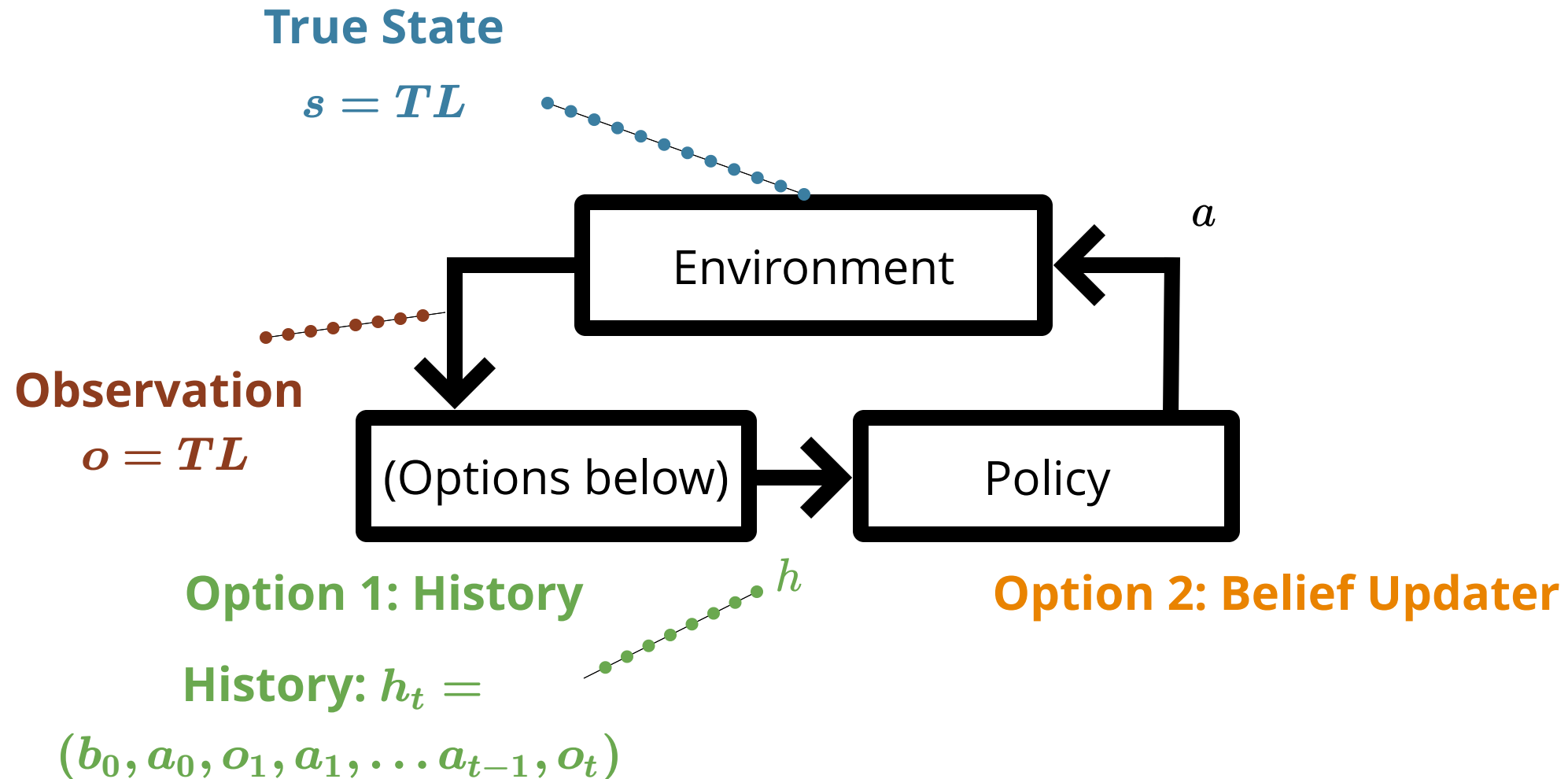


# POMDP Sense-Plan-Act Loop

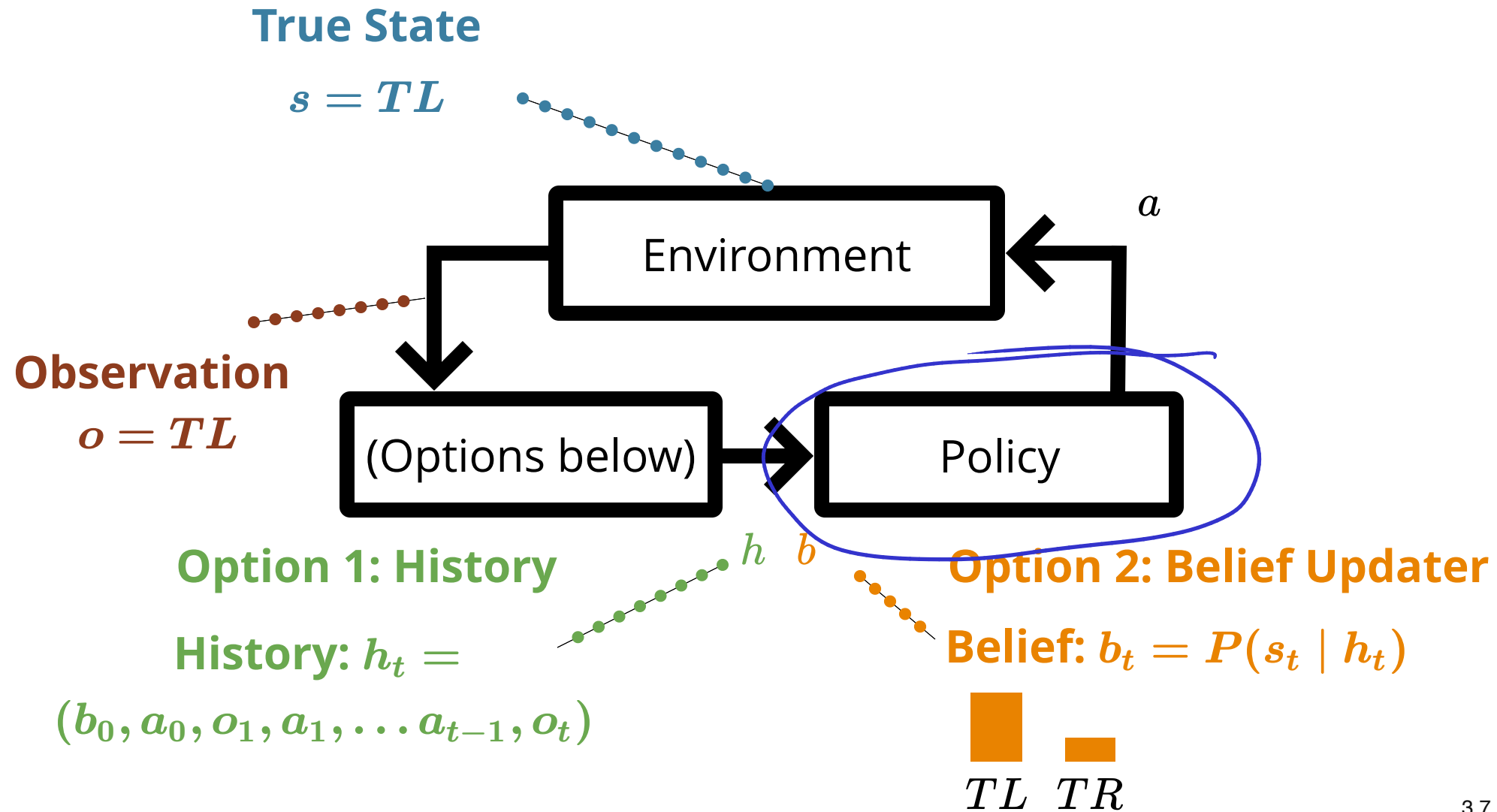




# POMDP Sense-Plan-Act Loop



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# Exercise 1: Crying Baby Belief Update

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$$S = \{h, \neg h\}$$

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# Exercise 1: Crying Baby Belief Update

$$S = \{h, \neg h\} \quad T(h \mid h, \neg f) = 1.0$$

$$A = \{f, \neg f\} \quad T(h \mid \neg h, \neg f) = 0.1$$

$$O = \{c, \neg c\} \quad T(\neg h \mid \cdot, f) = 1.0$$

$$R(s, a) = R(s) + R(a)$$

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$$b'(s') \propto Z(o \mid a, s') \sum_s T(s' \mid s, a) b(s)$$

$$R(s, a) = R(s) + R(a)$$

$$R(s) = \begin{cases} -10 & \text{if } s = h \\ 0 & \text{otherwise} \end{cases}$$

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$$Z(c \mid \cdot, h) = 0.8$$

$$Z(c \mid \cdot, \neg h) = 0.1$$

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$$R(s, a) = R(s) + R(a)$$

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$$Z(c \mid \cdot, h) = 0.8$$

$$Z(c \mid \cdot, \neg h) = 0.1$$

$$\gamma = 0.9$$

# Exercise 1: Crying Baby Belief Update

$$S = \{h, \neg h\} \quad T(h \mid h, \neg f) = 1.0$$

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$$O = \{c, \neg c\} \quad T(\neg h \mid \cdot, f) = 1.0$$

$$b'(s') \propto Z(o \mid a, s') \sum_s T(s' \mid s, a) b(s)$$

Starting at a  $b(h) = 0$ , calculate

$$R(s, a) = R(s) + R(a)$$

$$R(s) = \begin{cases} -10 & \text{if } s = h \\ 0 & \text{otherwise} \end{cases}$$

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$$Z(c \mid \cdot, h) = 0.8$$

$$Z(c \mid \cdot, \neg h) = 0.1$$

$$\gamma = 0.9$$

$b'$  with  $a = \neg f$  and  $o = c$ .

$$b'(h) \propto \overset{Z(c|\neg f,h)}{0.8} \left( \overset{T(h|h,\neg f)}{0.1} \overset{b(h)}{1.0} + \overset{T(h|\neg h,\neg f)}{1.0} \overset{b(h)}{0.0} \right) = 0.08$$

$$b'(h) \propto \underset{0.8}{Z(c|\neg f,h)} \left( \underset{0.1}{\cancel{T(h|\neg h,\neg f)}} \overset{1.0}{b(\neg h)} + \underset{1.0}{T(h|h,\neg f)} \overset{0}{\cancel{b(h)}} \right)$$

$$b'(h) \propto 0.08$$

$$b'(\neg h) \propto \underset{0.1}{Z(c|\neg f,\neg h)} \left( \underset{0.9}{T(\neg h|\neg h,\neg f)} \overset{1.0}{b(\neg h)} + \underset{1.0}{\cancel{T(\neg h|h,\neg f)}} \overset{0}{\cancel{b(h)}} \right)$$

$$= 0.09$$

$b'(h) = 0.08 / (0.08 + 0.09) = 47\%$   
 $b'(\neg h) = 0.09 / (0.08 + 0.09) = 53\%$

$b'(h) =$   
 $b'(\neg h) =$

# Belief Dynamics

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$$b'(s') \propto Z(o \mid a, s') \sum_s T(s' \mid s, a) b(s)$$

# Belief Dynamics

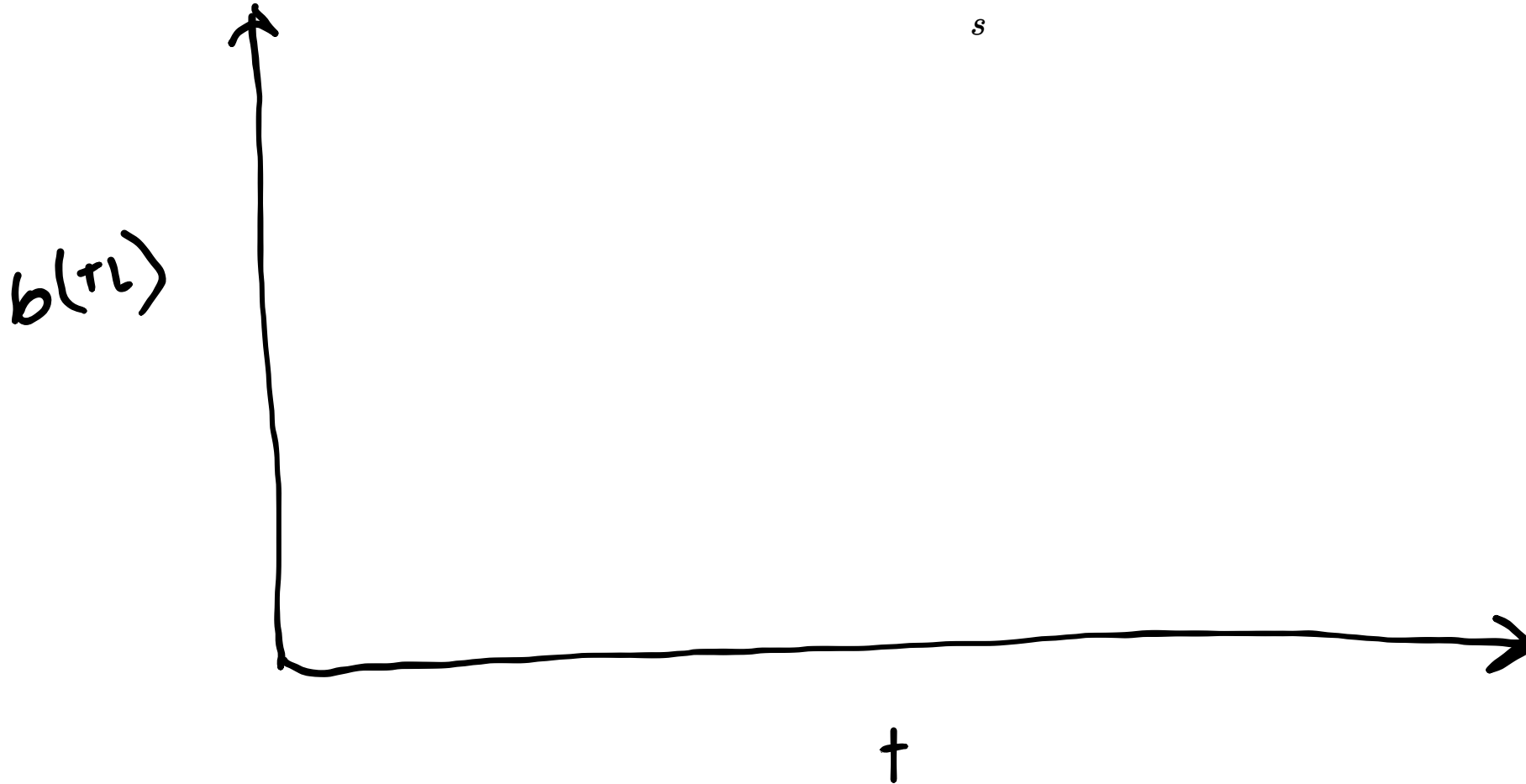
$$b'(s') \propto Z(o \mid a, s') \sum_s T(s' \mid s, a) b(s)$$

$b(\tau_L)$

†

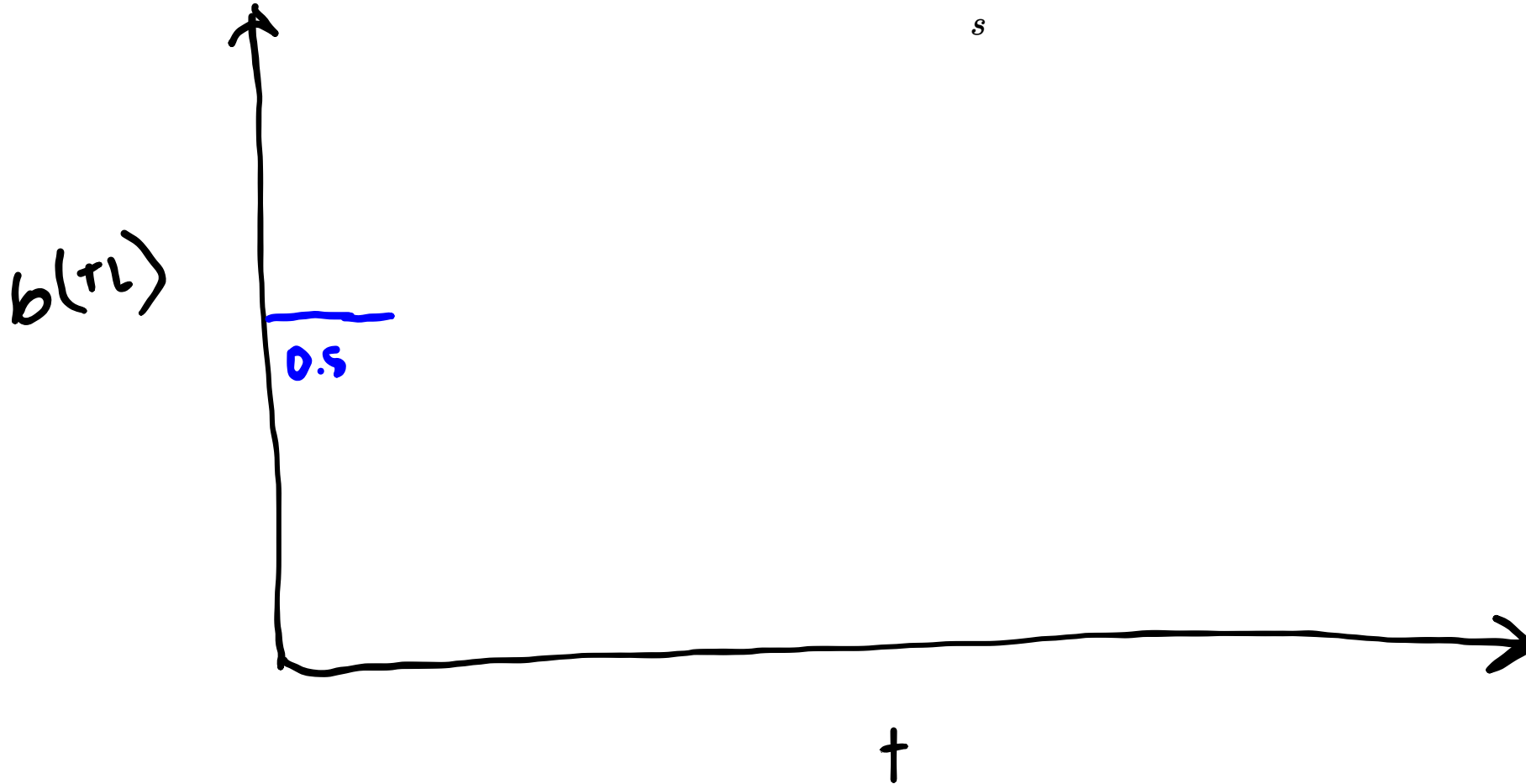
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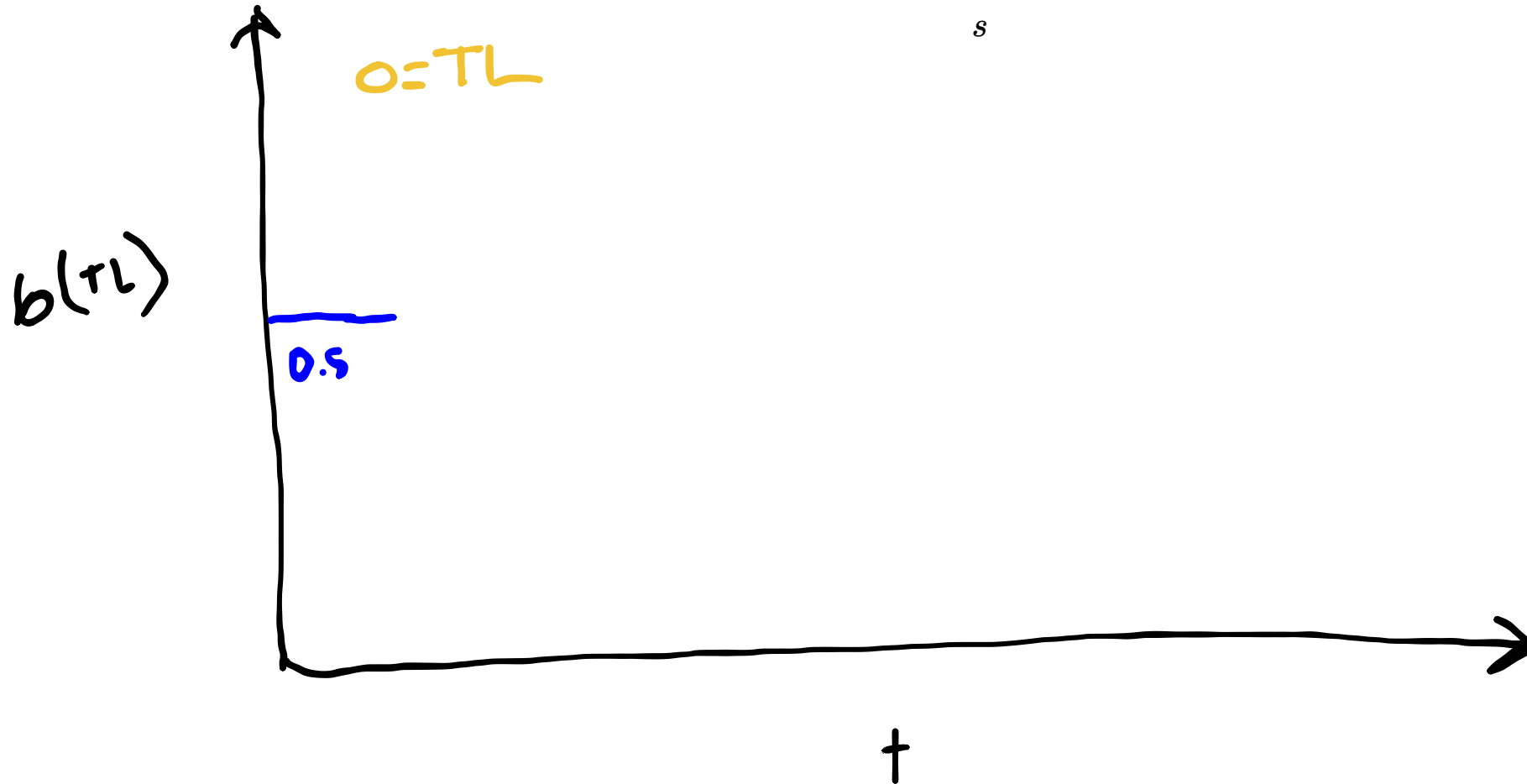
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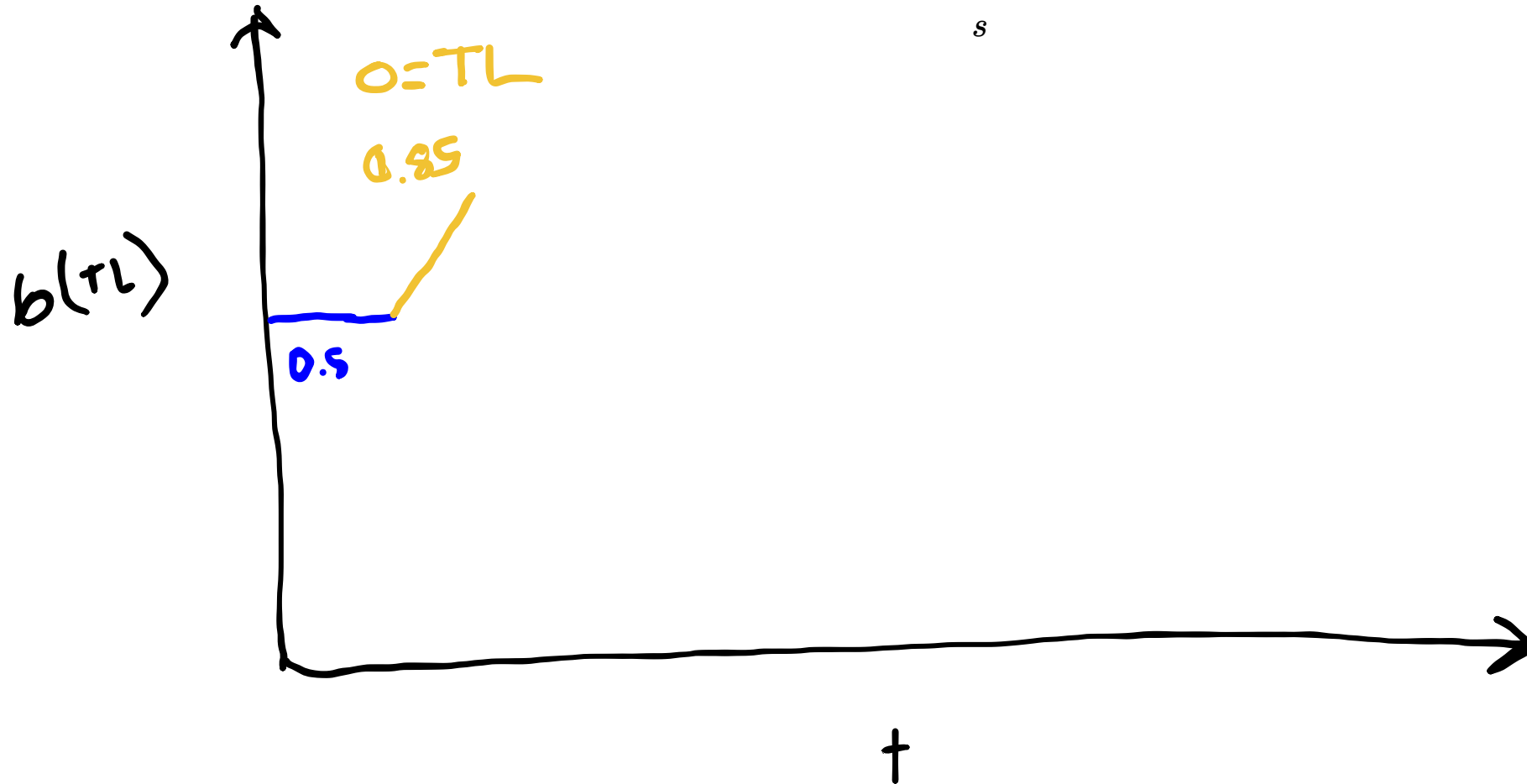
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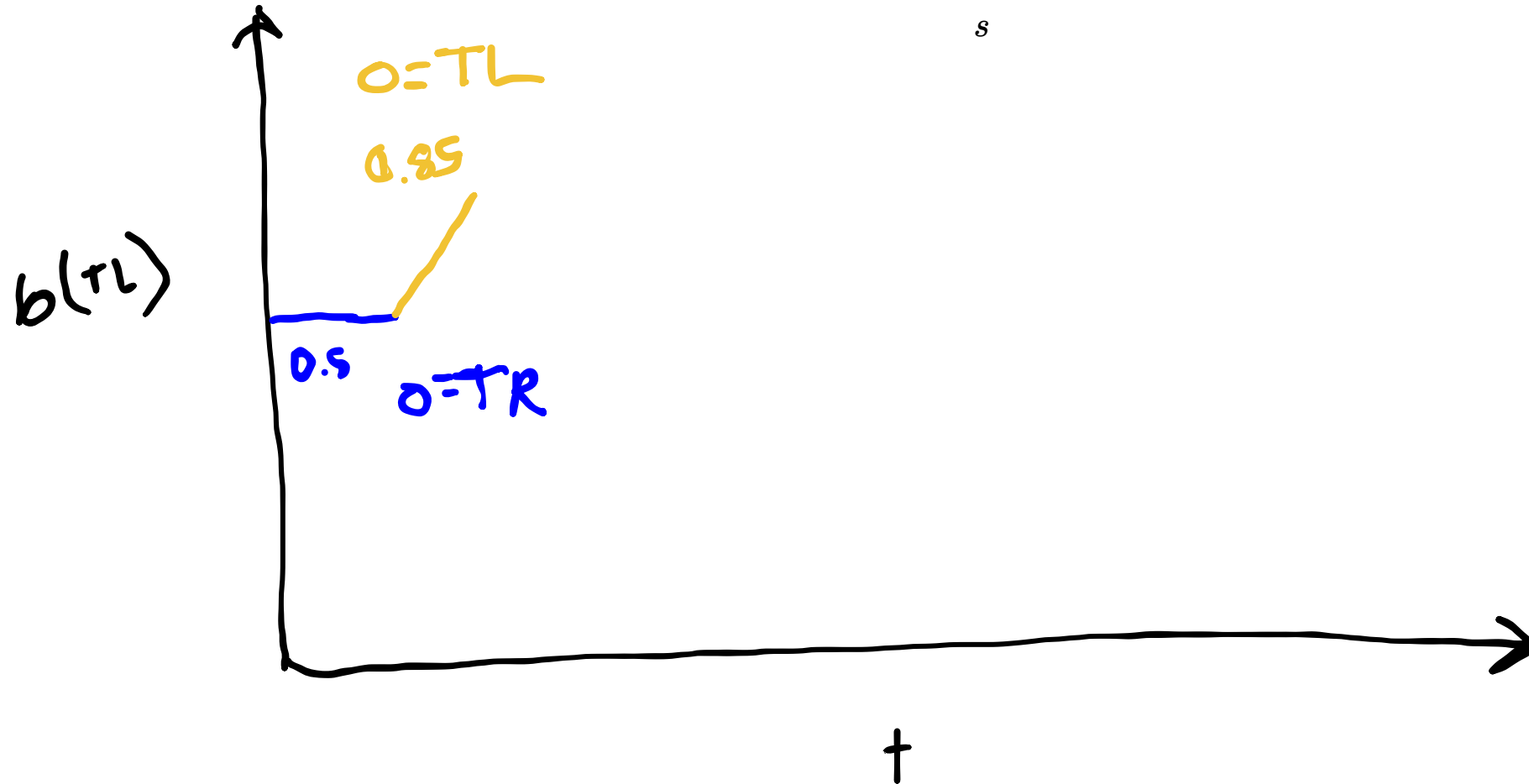
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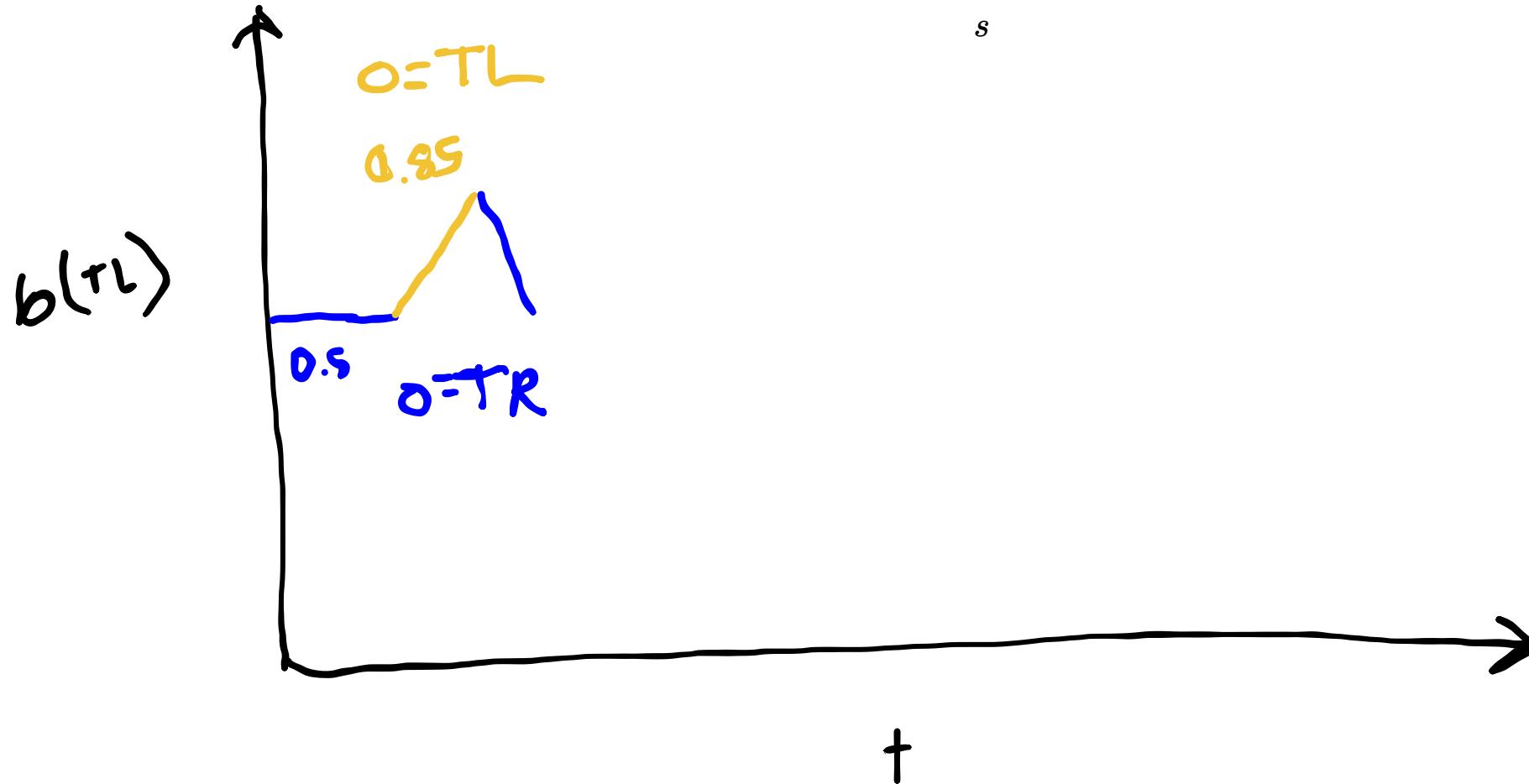
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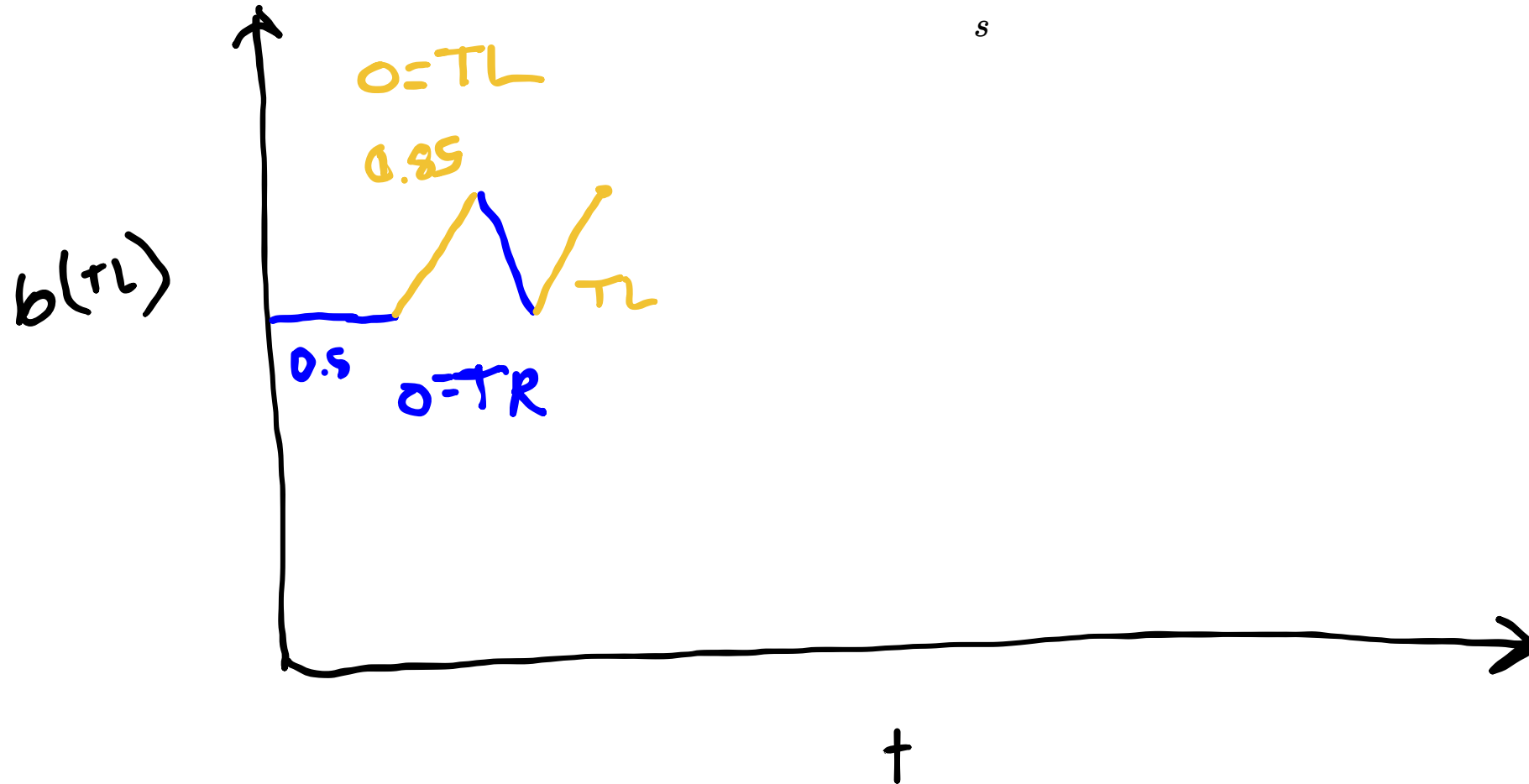
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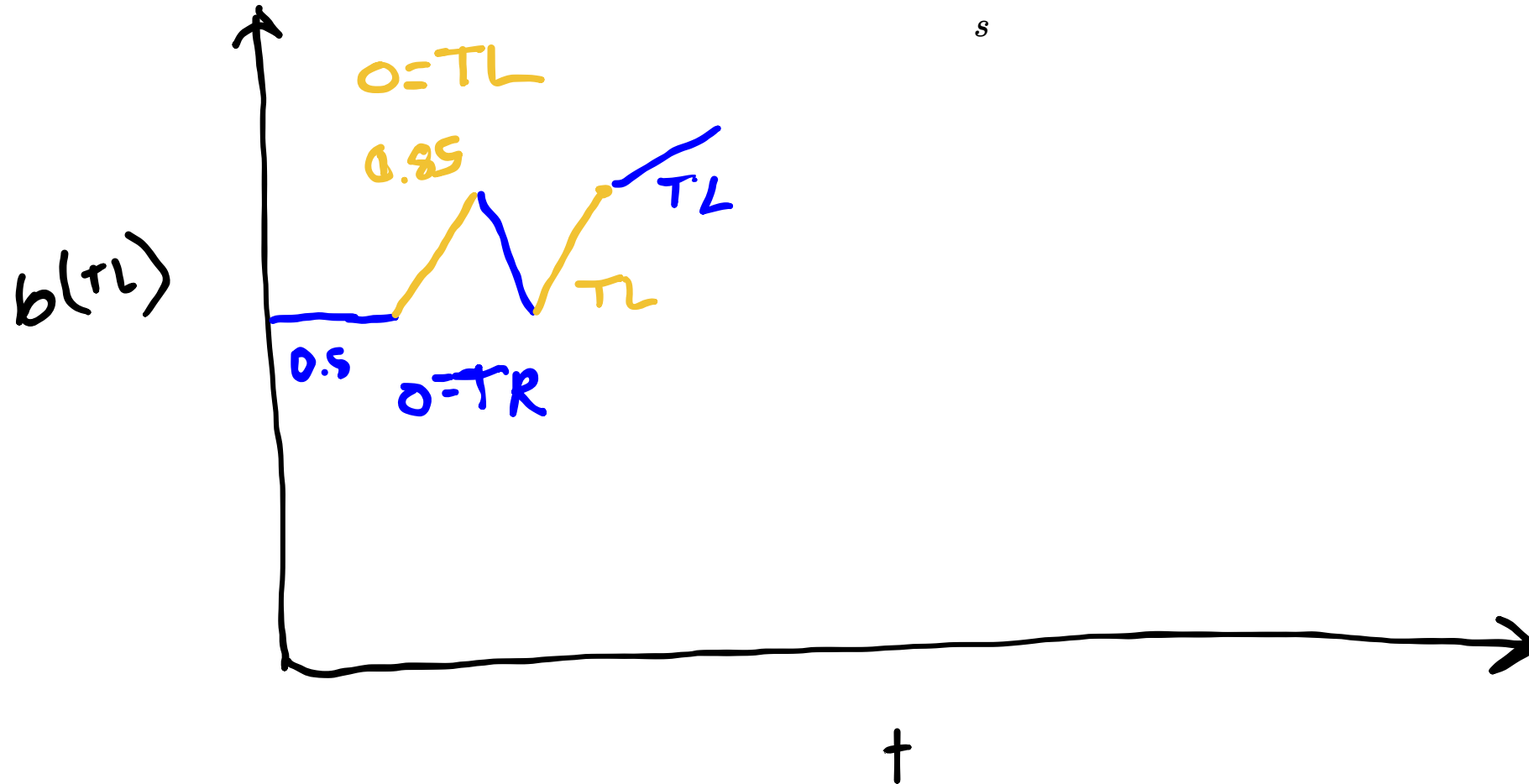
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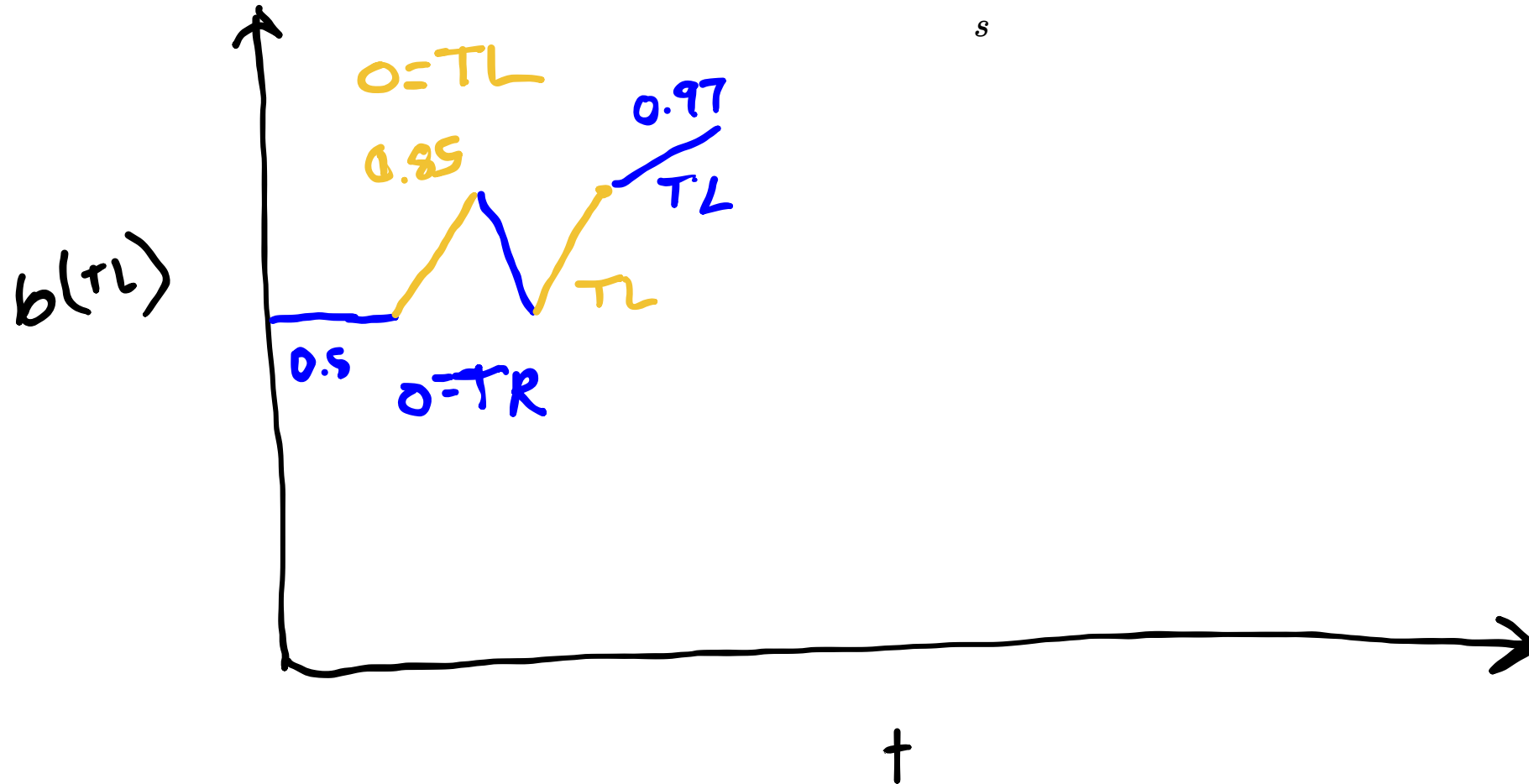
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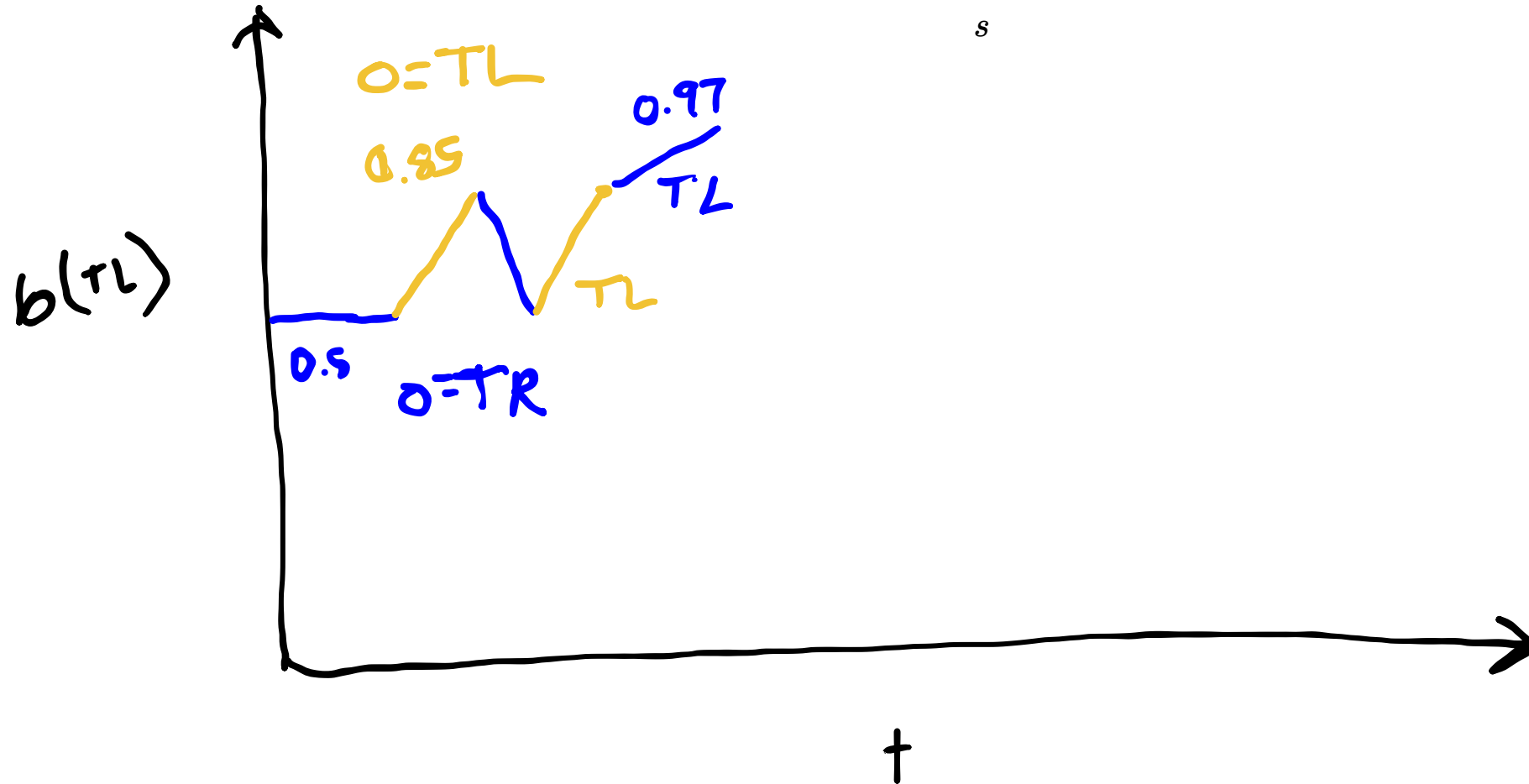
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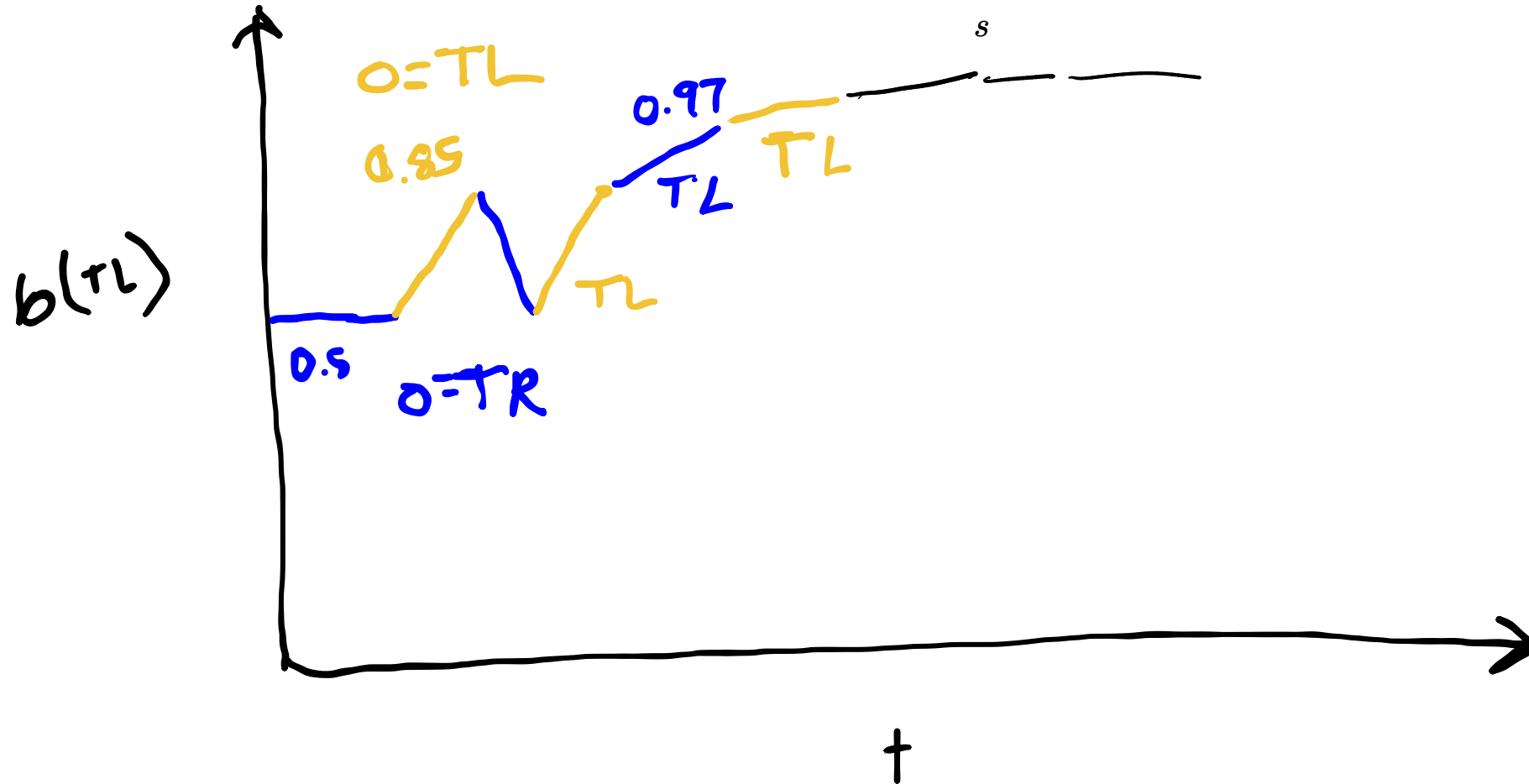
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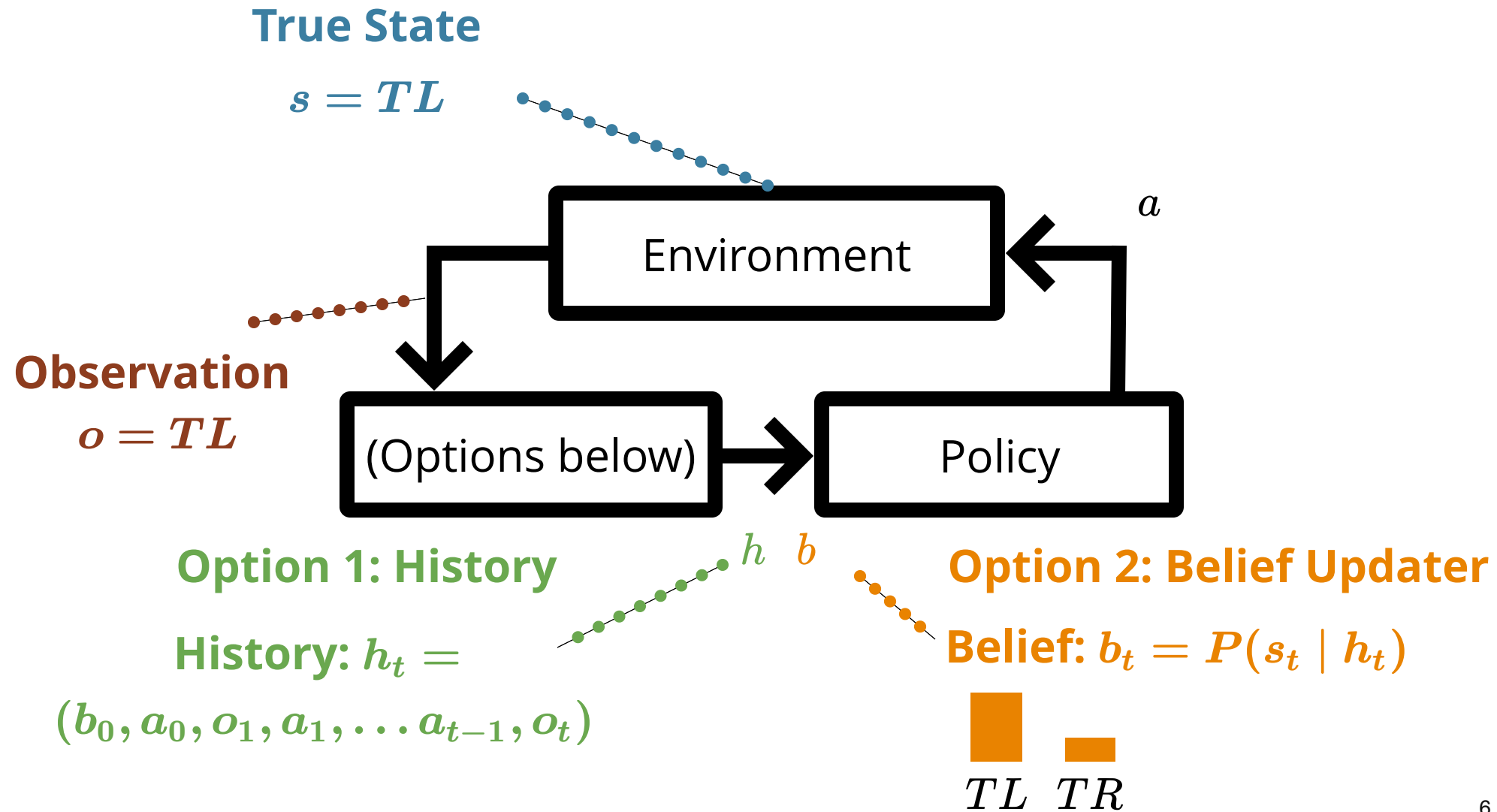


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$$b'(s') \propto Z(o | a, s') \sum_s T(s' | s, a) b(s)$$

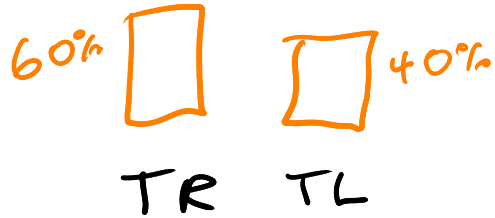


# POMDP Sense-Plan-Act Loop



# Guiding Quesiton

How do we calculate the optimal action in a POMDP?



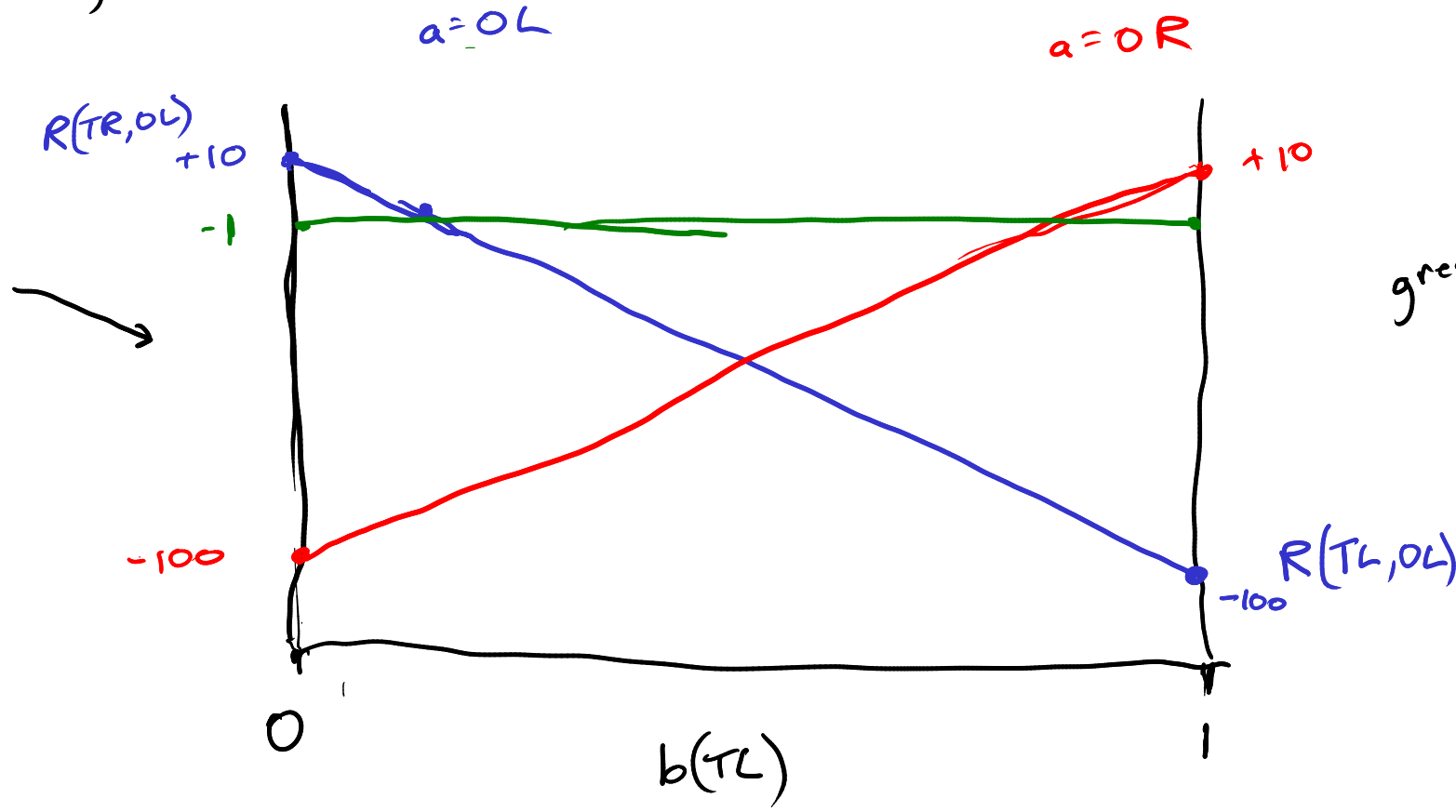
# One-step utility

Reward    +10    empty  
              -1    Listen  
              -100   tiger

$$R(b, a)$$

$$= E[R(s, a)]$$

$s \sim b$



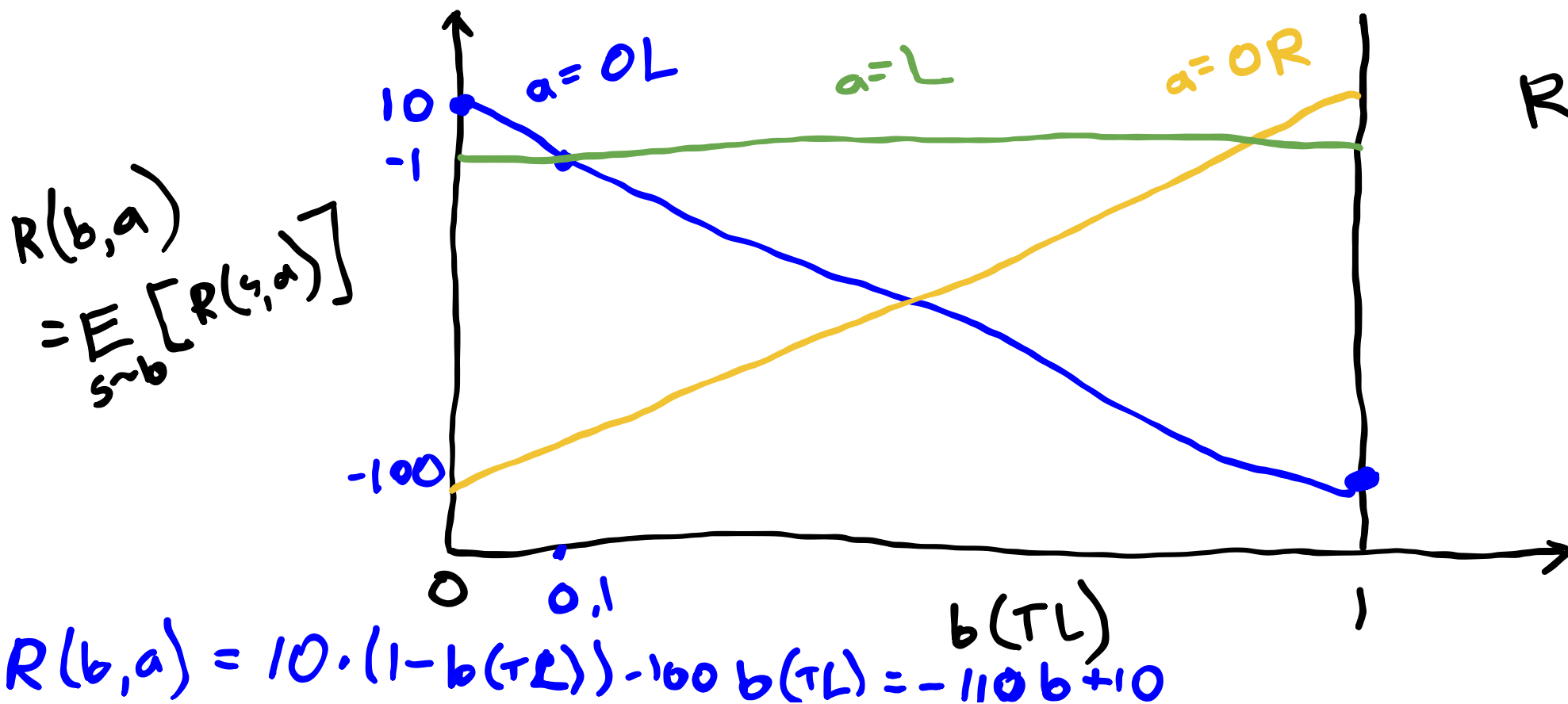
$R(b, a) = \bar{r}^a \cdot b$   
 greedy action  
 $\arg \max_a \bar{r}^a \cdot b$   
 1-step  
 $\alpha$ -vector

$$R(b, OL) = +10(1 - b(TL)) + (-100)b(TL)$$

$$b(TL) = 0.1 : 10 \cdot 0.9 + (-100) \cdot 0.1$$

# One-step utility

Reward: +10 empty door  
-1 Listen  
-100 Tiger



$$R(b,a) = \bar{r}_a \cdot b$$

↑  
 $\alpha$ -vector

# Exercise 2: Crying Baby 1-Step Utility

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$$\gamma = 0.9$$

# Exercise 2: Crying Baby 1-Step Utility

$$\begin{aligned} S &= \{h, \neg h\} & T(h \mid h, \neg f) &= 1.0 \\ A &= \{f, \neg f\} & T(h \mid \neg h, \neg f) &= 0.1 \\ O &= \{c, \neg c\} & T(\neg h \mid \cdot, f) &= 1.0 \end{aligned}$$

$$R(s, a) = R(s) + R(a)$$

$$R(s) = \begin{cases} -10 & \text{if } s = h \\ 0 & \text{otherwise} \end{cases}$$

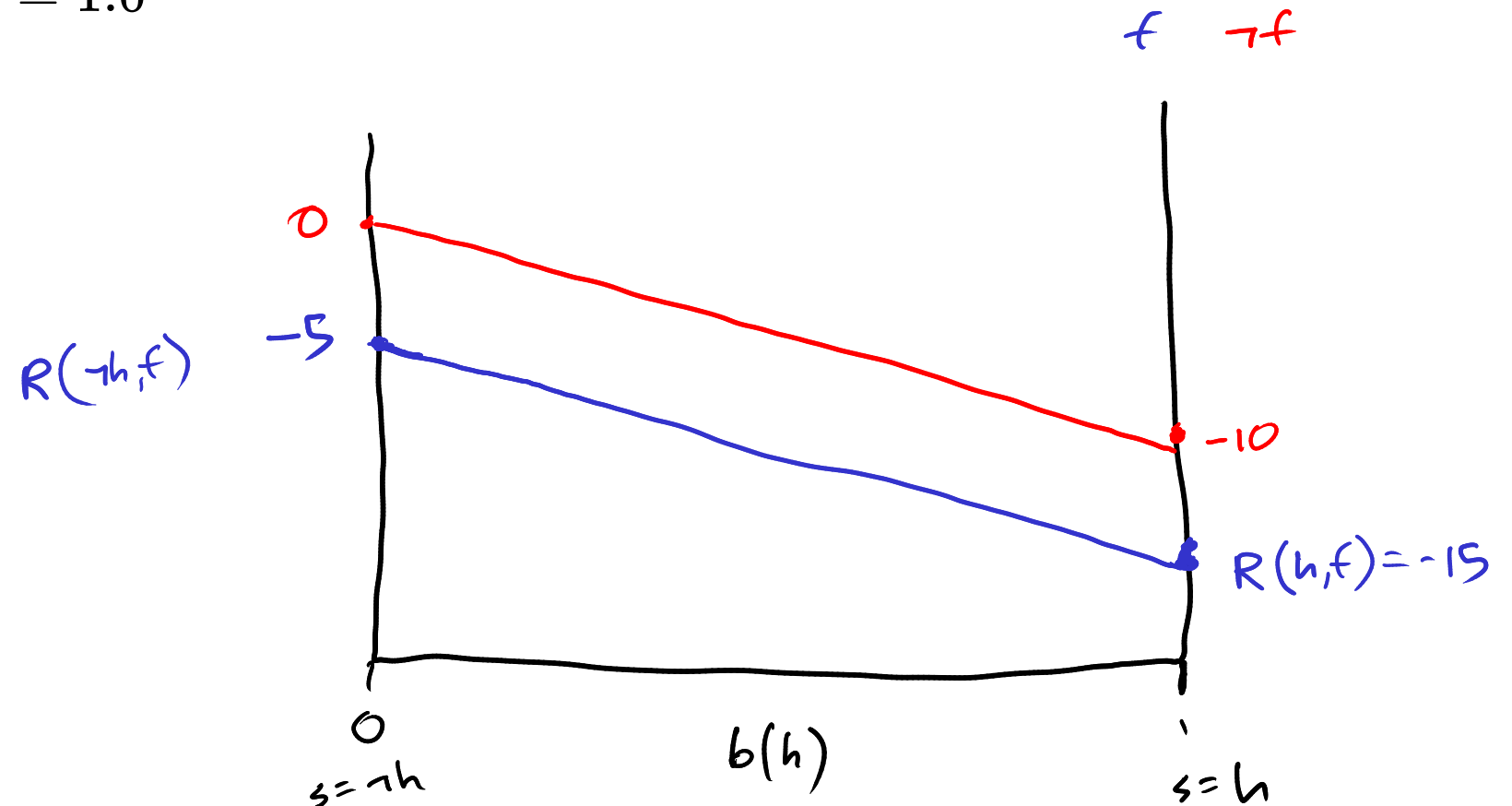
$$R(a) = \begin{cases} -5 & \text{if } a = f \\ 0 & \text{otherwise} \end{cases}$$

$$Z(c \mid \cdot, h) = 0.8$$

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Draw the 1-step utility  $\alpha$ -vectors for the Crying Baby problem.



# Alpha Vectors for Conditional Plans

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Conditional Plans: fixed-depth history-based policies



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1 Step:

# Alpha Vectors for Conditional Plans

Conditional Plans: fixed-depth history-based policies

1 Step:            

# Alpha Vectors for Conditional Plans

Conditional Plans: fixed-depth history-based policies

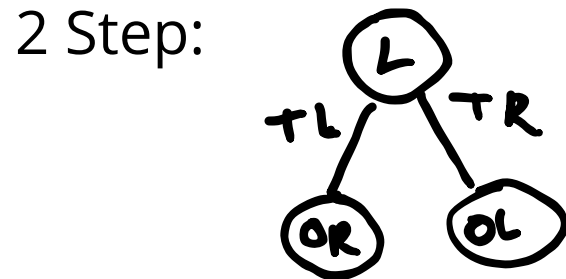
1 Step:    (L)    (OL)    (OR)

2 Step:

# Alpha Vectors for Conditional Plans

Conditional Plans: fixed-depth history-based policies

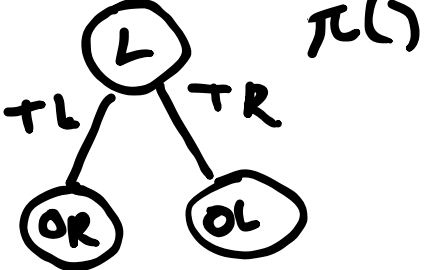
1 Step:     (L)     (OL)     (OR)



# Alpha Vectors for Conditional Plans

Conditional Plans: fixed-depth history-based policies

1 Step: 

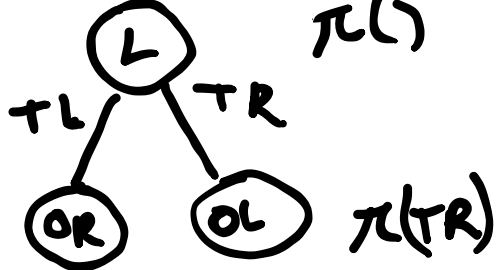
2 Step: 

# Alpha Vectors for Conditional Plans

Conditional Plans: fixed-depth history-based policies

1 Step:  $(L)$   $(OL)$   $(OR)$

2 Step:



```
graph TD; L((L)) -- TL --> OR1((OR)); L -- TR --> OL((OL));
```

$\pi(L)$

$\pi(TR)$

# Alpha Vectors for Conditional Plans

Conditional Plans: fixed-depth history-based policies

1 Step:  $(L)$   $(OL)$   $(OR)$

2 Step:

The first 2-step plan is a tree with root node  $(L)$ . It has two children:  $(OR)$  reached via edge  $TL$ , and  $(OL)$  reached via edge  $TR$ . The label  $\pi(L)$  is written next to the  $(OL)$  node. The second 2-step plan is a tree with root node  $(L)$ . It has two children:  $(OR)$  reached via edge  $TL$ , and  $(L)$  reached via edge  $TR$ . The label  $\pi(TR)$  is written next to the  $(L)$  node.

# Alpha Vectors for Conditional Plans

Conditional Plans: fixed-depth history-based policies

1 Step:  $(L)$   $(OL)$   $(OR)$

2 Step:

The diagrams show three conditional plans for 2 steps:

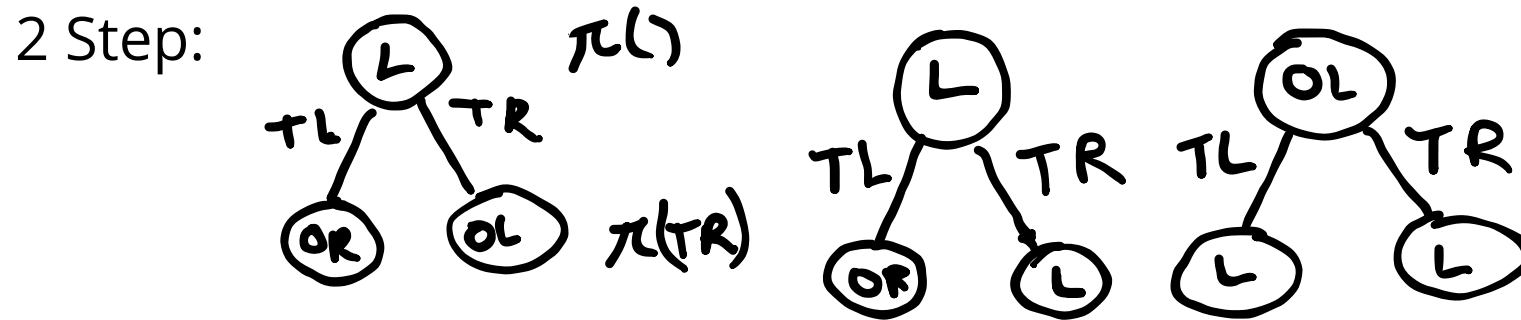
- Plan 1: Root node  $(L)$  with children  $(OR)$  (labeled  $TL$ ) and  $(OL)$  (labeled  $TR$ ). The label  $\pi(L)$  is next to the root, and  $\pi(TR)$  is next to the  $(OL)$  node.
- Plan 2: Root node  $(L)$  with children  $(OR)$  (labeled  $TL$ ) and  $(L)$  (labeled  $TR$ ).
- Plan 3: Root node  $(OL)$  with children  $(L)$  (labeled  $TL$ ) and  $(L)$  (labeled  $TR$ ).



# Alpha Vectors for Conditional Plans

Conditional Plans: fixed-depth history-based policies

1 Step: (L) (OL) (OR)

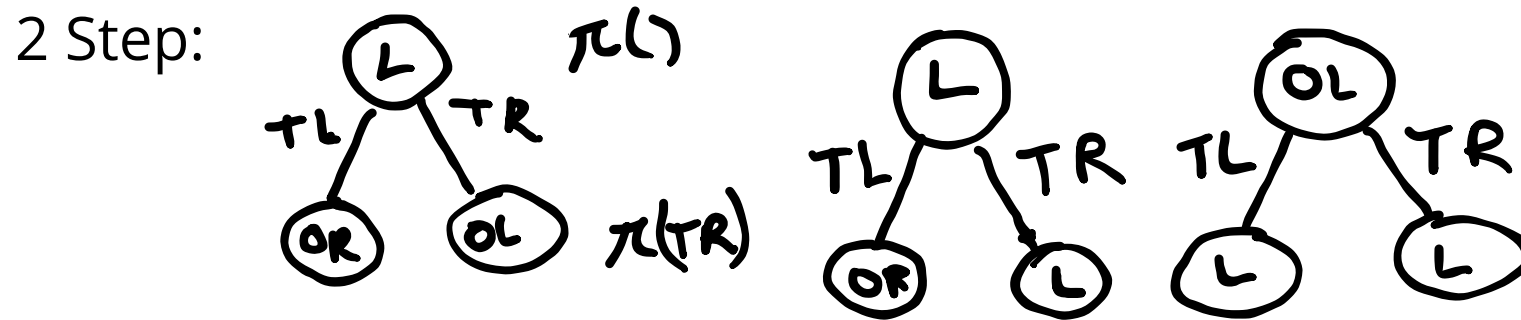


$$|A| \frac{(|O|^h - 1)}{(|O| - 1)}$$

# Alpha Vectors for Conditional Plans

Conditional Plans: fixed-depth history-based policies

1 Step: (L) (OL) (OR)



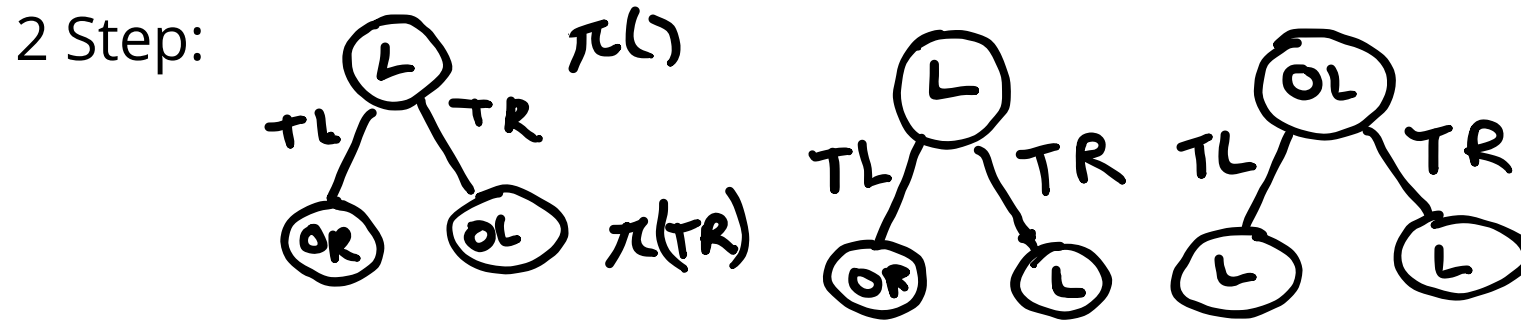
$$|A| \frac{(|O|^h - 1)}{(|O| - 1)}$$

27 two step plans!

# Alpha Vectors for Conditional Plans

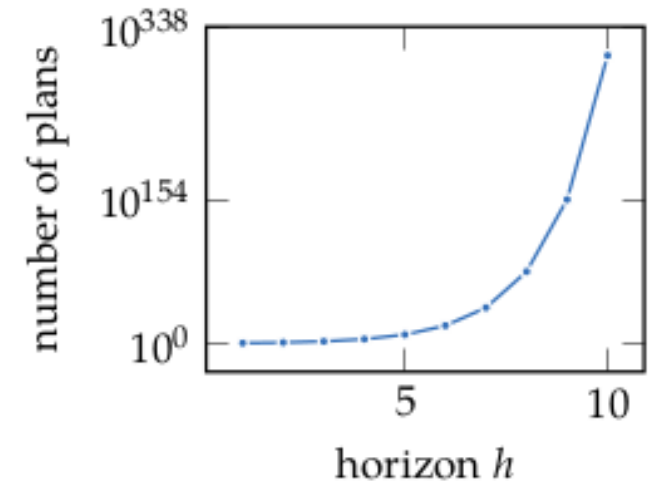
Conditional Plans: fixed-depth history-based policies

1 Step: (L) (OL) (OR)



$$|A| \frac{(|O|^h - 1)}{(|O| - 1)}$$

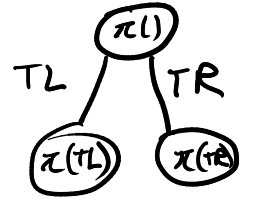
27 two step plans!



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$$U^\pi(s) = R(s, \pi()) + \gamma \left[ \sum_{s'} T(s' | s, \pi()) \sum_o O(o | \pi(), s') U^{\pi(o)}(s') \right]$$

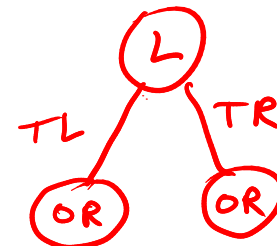
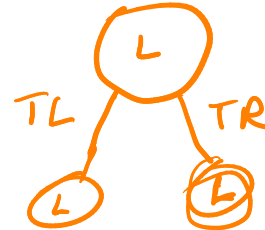
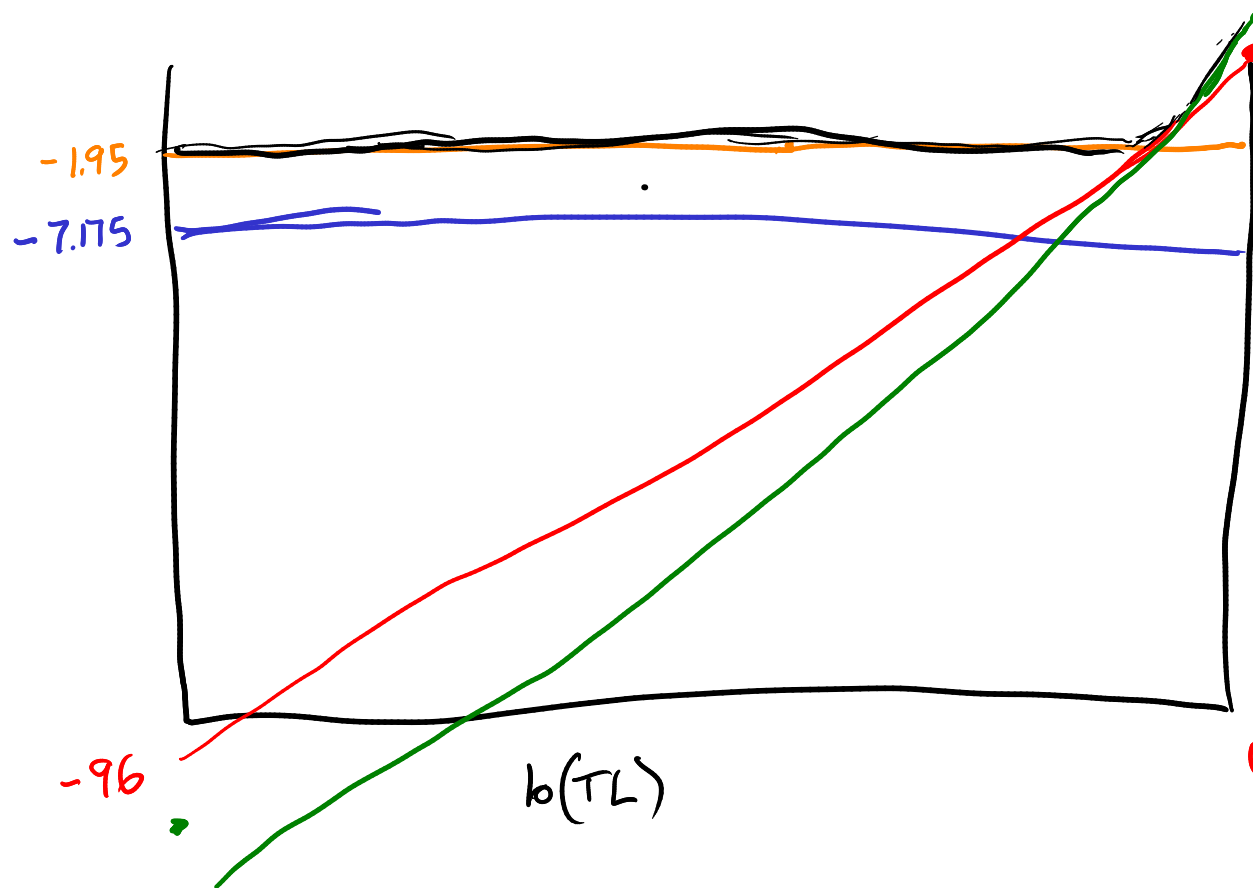


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$$U^\pi(s) = R(s, \pi()) + \gamma \left[ \sum_{s'} T(s' | s, \pi()) \sum_o \cancel{P}(o | \pi(), s') \underline{U^{\pi(o)}(s')} \right]$$

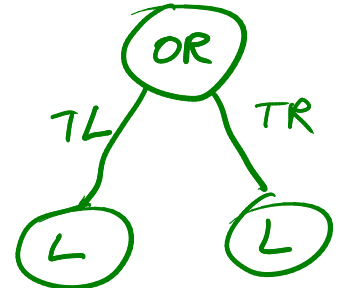
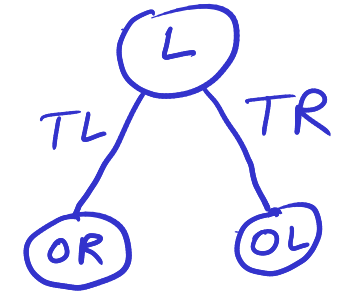
$$U^*(b) = \max_{\pi} \alpha^\pi \cdot b$$

For 1-step:  $U^\pi(s) = R(s, \pi())$



$$U^\pi(TL) = -1 + \gamma 10 = 8.5$$

$$U^\pi(TR) = -1 + \gamma -100 = -96$$



$$U^\pi(TL) = 10 + \gamma(-1) = 9.5$$

$$U^\pi(TR) = -100 + \gamma(-1) = -100.95$$

# POMDP Value Functions

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$$\underline{V^*(b) = \max_{\alpha \in \Gamma} \alpha^\top b}$$



# Exercise: 2-Step Crying Baby $\alpha$ Vectors

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$$S = \{h, \neg h\} \quad T(h \mid h, \neg f) = 1.0$$

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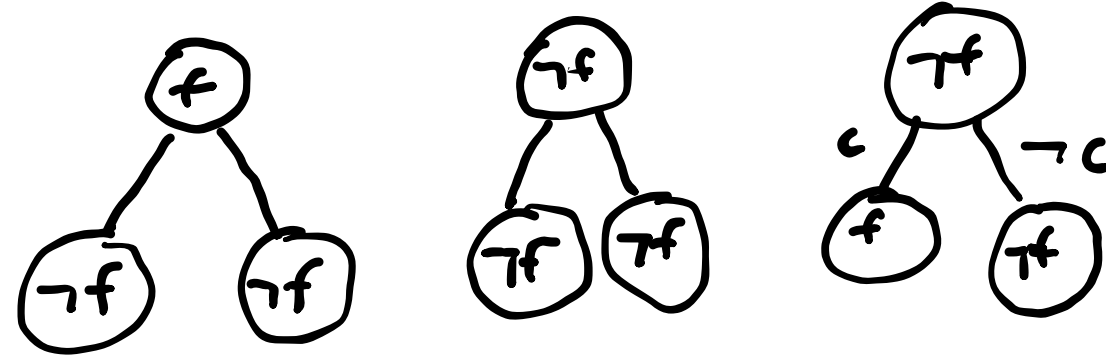
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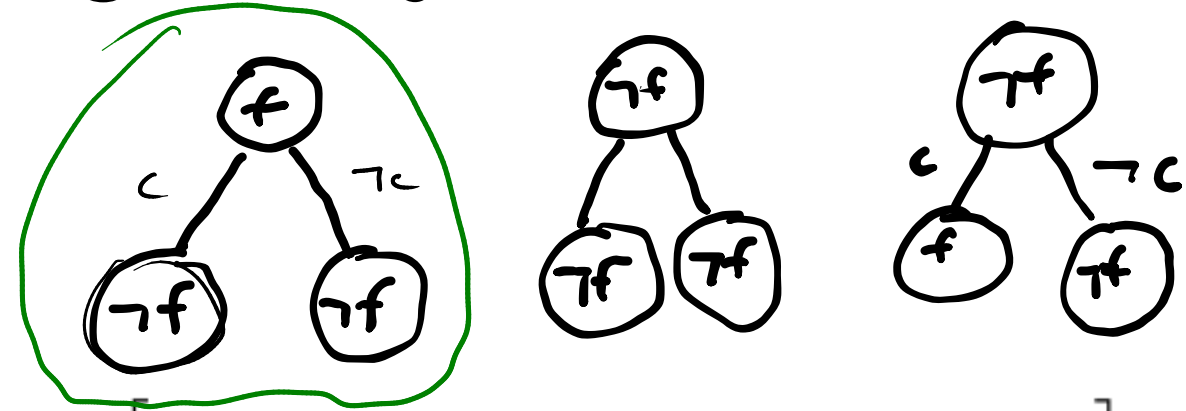
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$$U^\pi(s) = R(s, \pi()) + \gamma \left[ \sum_{s'} T(s' \mid s, \pi()) \sum_o O(o \mid \pi(), s') \underbrace{U^{\pi(o)}(s')}_{U^{\pi(c)}(h), U^{\pi(\neg c)}(h)} \right]$$

$$U^\pi(h) = R(h, f) + \gamma \left( T(h \mid h, f) (Z(c \mid f, h) U^{\pi(c)}(h) + Z(\neg c \mid f, h) U^{\pi(\neg c)}(h)) + T(\neg h \mid h, f) (Z(c \mid f, h) U^{\pi(c)}(\neg h) + Z(\neg c \mid f, h) U^{\pi(\neg c)}(\neg h)) \right)$$

$$-15 + 0.9 (1.0 (0.8 \cdot 0 + 0.9 \cdot 0)) = -15$$

# $\alpha$ -Vector Pruning

# Alpha Vector Expansion

# POMDP Value Iteration (horizon $d$ )

$\Gamma^0 \leftarrow \emptyset$

for  $n \in 1 \dots d$

    Construct  $\Gamma^n$  by expanding with  $\Gamma^{n-1}$

    Prune  $\Gamma^n$

# Recap

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- A POMDP is an MDP on the \_\_\_\_\_

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- Each  $\alpha$  vector corresponds to a conditional plan