1. Relationships between R.V.s
2. Infer info about 1 RV given another

Events A + B

1 holds
- $A \succ B$ — more plausible
- $A \sim B$ — as plausible
- $A \prec B$

Universal Comparability

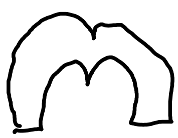
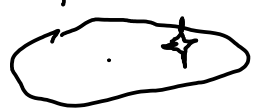
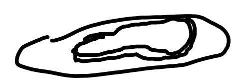
Transitivity

if $A \succeq B$ and $B \succeq C \Rightarrow A \succeq C$

$P(A) > P(B)$ iff $A \succ B$

$P(A) = P(B)$ iff $A \sim B$

Random Variable — Today only? R.V. capital

Happy Meal	Chipotle	Filet Minion
		

Variable
- take on finite set of values
- each has probability

$P(X=1)=0.5$

Variable
- Continuous / Discrete
- Related to other RVs

$P(X \mid Y)$

↑

DMU

(Ω, F, P)

$X: \Omega \to E$

Vocab

support(X) — set of all values X can take

Distribution — function that matches each value in support to a real number

discrete: PMF
continuous: PDF

CDF — $\mathrm{cdf}(x) = P(X \le x)$

Coinflip

$\{h, t\}$
$\{1, 0\}$

$P(X=1) = 0.5$

P 

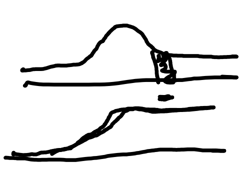
h, t

cdf 

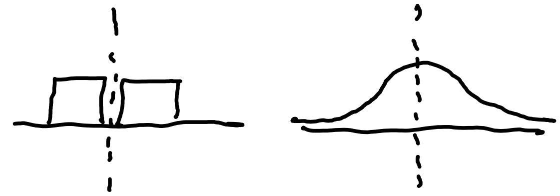
$N(\mu, \sigma)$

$\mathbb{R}$

$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$



Expectation $E[X]$ $= \sum_{i \in X} i\, P(X=i)$ $\int_{-\infty}^{\infty} x\, p(x)\, dx$



Multiple R.Vs X, Y

Joint $P(X,Y,Z)$

Conditional Dist $P(X \mid Y, Z)$

Marginal Dist $P(X), P(Y), P(Z)$

X	Y	Z	P(X,Y,Z)
0	0	0	0.08
0	0	1	0.31
0	1	0	0.09
0	1	1	0.37
1	0	0	0.01
1	0	1	0.05
1	1	0	0.02
1	1	1	0.07

Dist-valued function

$Y, Z \rightarrow$ dist of X

X	P(X | Y=0, Z=0)
0	0.888....
1	0.111....

$P(X \mid Y, Z)$

X	P(X)
0	0.85
1	0.15

Y	P(Y)
0	0.45
1	0.55

Z	P(Z)
0	0.20
1	0.80

Chipotle

Filet Mignon

Axiomization

3 Rules

1) a) $0 \leq P(X \mid Y) \leq 1$

b) $\sum_{x \in X} P(x \mid Y) = 1$

2) "Law of total Probability"

$$P(x) = \sum_{y \in Y} P(x, y)$$

3) Def. of Conditional Prob

$$P(X \mid Y) = \frac{P(X,Y)}{P(Y)}$$

joint $\rightarrow$ marginal

joint $\rightarrow$ conditional

Marginal + Cond. $\rightarrow$ Joint

$$P(X,Y) = P(X,Y)\, P(Y)$$

Often:

A : hidden   know $\underline{P(B|A)}$

B : measure

Bayes Rule:

$$P(A|B) = \frac{P(A,B)}{P(B)} \qquad P(B|A) = \frac{P(B,A)}{P(A)}$$

$$P(A|B)P(B) = P(A,B) = P(B|A)P(A)$$

$$\boxed{P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}}$$

$$P(A|B,C) = \frac{P(B|A,C)\,P(A|C)}{P(B|C)}$$

Breakout Rooms

Introduce Yourself:
Favorite Prog. Lang.

S = scam $P(S=1) = 0.1$

C_t = clue $P(C_t=1 \mid S=1) = 0.9$

$P(C_t=1 \mid S=0) = 0.2$

How many clues $(C_1=1, C_2=1, C_3=1 \ldots)$ do you need to get in a row to be 90% sure that there is a scam?

$$P(S=1 \mid C_1=1 \ldots C_n=1) = \frac{P(C_1=1 \ldots C_n=1 \mid S=1)\,P(S=1)}{P(C_1=1 \ldots C_n=1)}$$

$$P(C_{1:t}=1 \mid S=1) = 0.9^t$$

$$P(S=1) = 0.1$$

$$P(C_1=1 \ldots C_n=1) = P(C_1=1 \ldots C_n=1 \mid S=0)P(S=0) + P(C_{1:t} \mid S=1)P(S=1)$$

3 clues!

$$0.2^t \cdot 0.9 + 0.9^t \cdot 0.1$$

Independence

Def X and Y are Independent iff $P(X,Y) = P(X)\,P(Y)$

$$X \perp Y$$

$$P(X|Y) = P(X)$$

Def X and Y are conditionally Independent given Z

iff $P(X,Y|Z) = P(X|Z)\,P(Y|Z)$

$$X \perp Y \mid Z$$

$$P(X|Y,Z) = P(X|Z)$$

$$P(C_1, C_2 | S) = P(C_1|S)\,P(C_2|S)$$