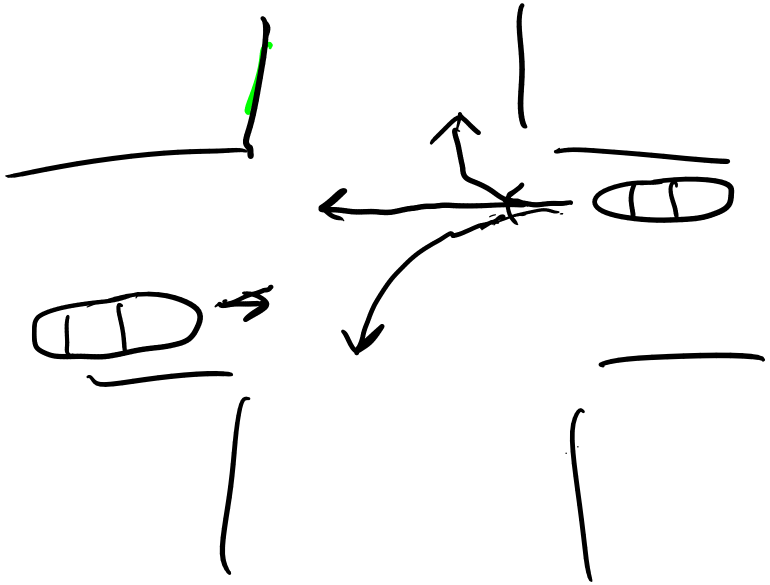
POMDPs

- We've been living a lie:

```
s = observe(env)
```



Types of Uncertainty

Types of Uncertainty

Alleatory

Types of Uncertainty

Alleatory

Types of Uncertainty

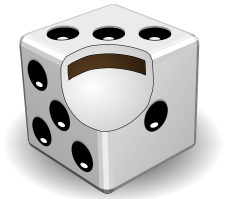
Alleatory

Epistemic (Static)

Types of Uncertainty

Alleatory

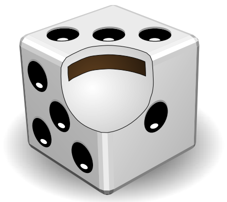
Epistemic (Static)



Types of Uncertainty

Alleatory

Epistemic (Static)



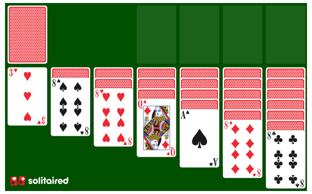
Epistemic (Dynamic)

Types of Uncertainty

Alleatory

Epistemic (Static)

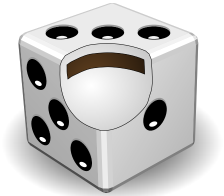
Epistemic (Dynamic)



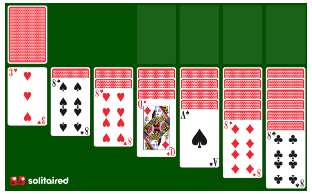
Types of Uncertainty

Alleatory

Epistemic (Static)



Epistemic (Dynamic)

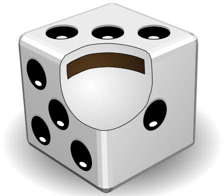


Interaction

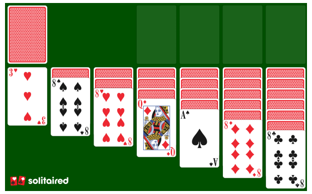
Types of Uncertainty

Alleatory

Epistemic (Static)



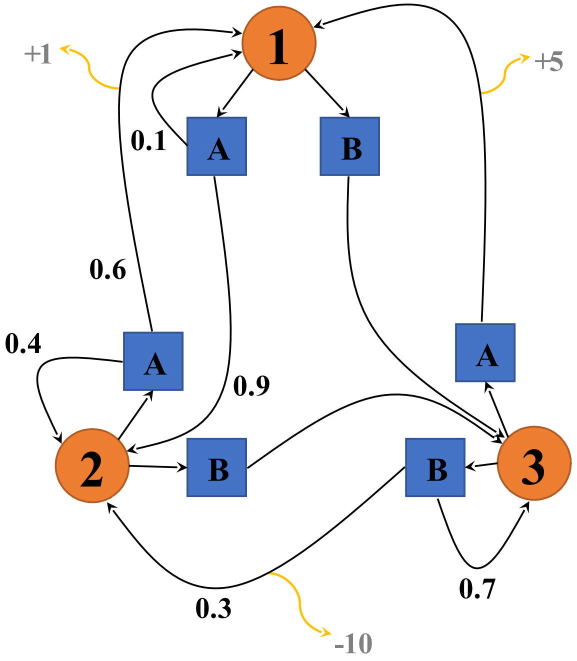
Epistemic (Dynamic)



POMDP

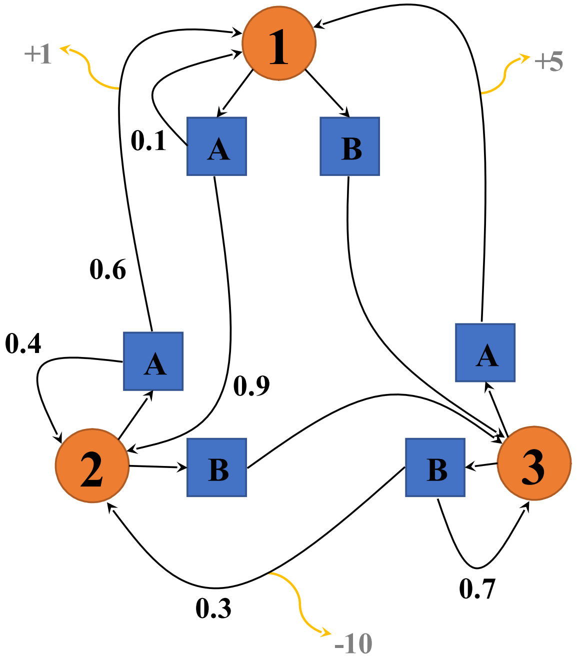
Interaction

Markov Decision Process (MDP)



- $\mathcal{S}$ - State space
- $T : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to \mathbb{R}$ - Transition probability distribution

Markov Decision Process (MDP)



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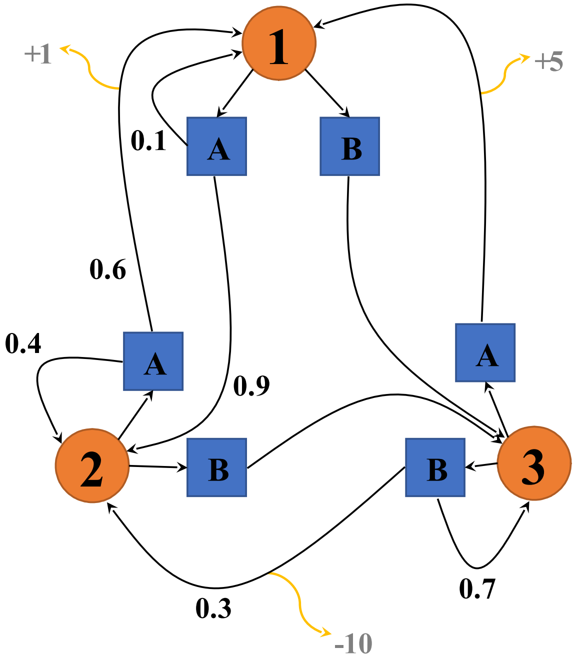
- $\mathcal{A}$ - Action space

Markov Decision Process (MDP)



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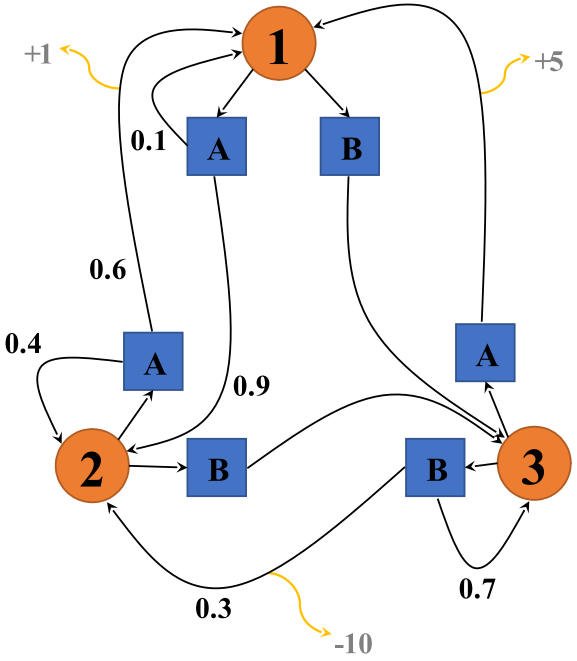
Markov Decision Process (MDP)



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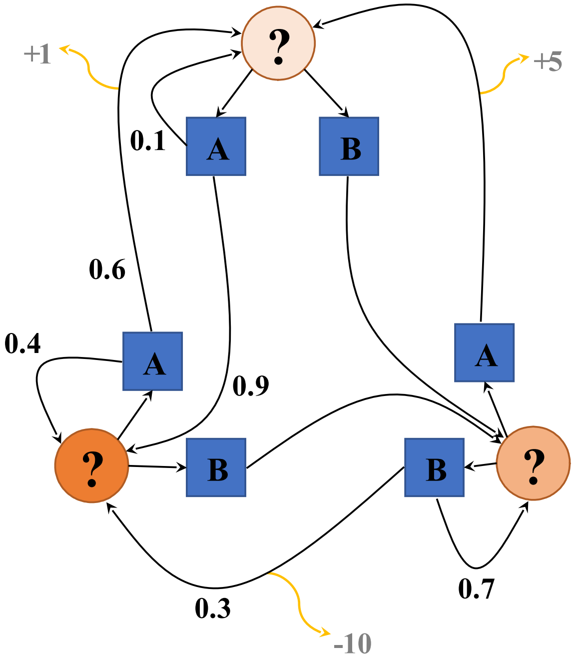
Alleatory

Partially Observable Markov Decision Process (POMDP)



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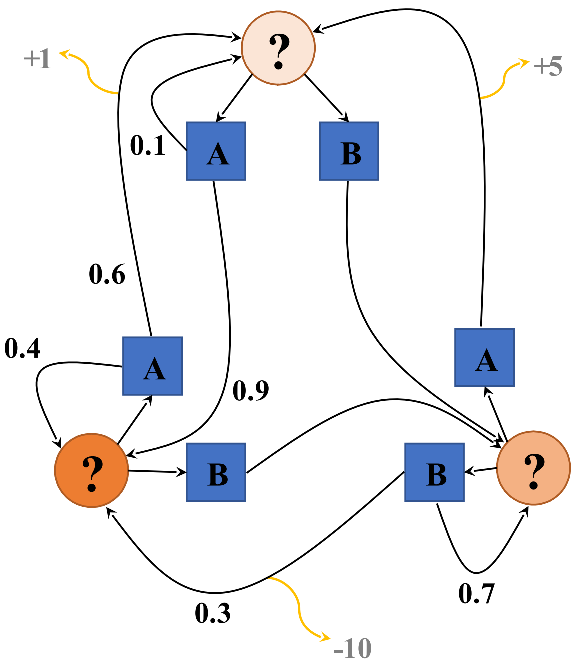
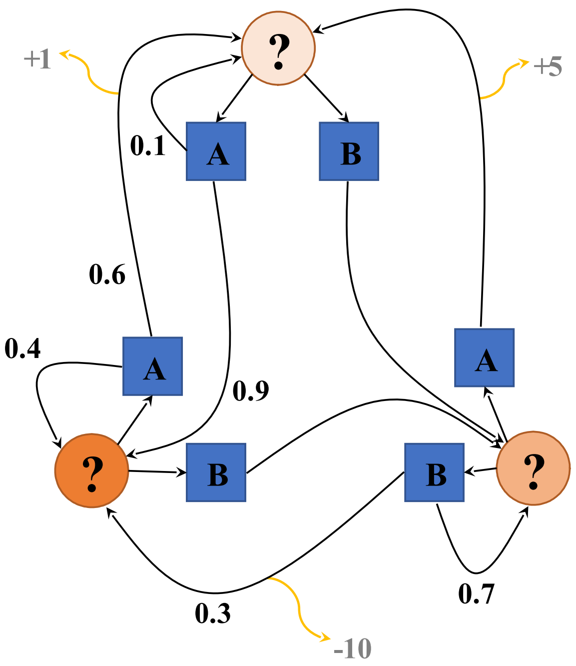
Partially Observable Markov Decision Process (POMDP)



- $\mathcal{S}$ - State space
- $T : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow \mathbb{R}$ - Transition probability distribution
- $\mathcal{A}$ - Action space
- $R : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$ - Reward
- $\mathcal{O}$ - Observation space

Partially Observable Markov Decision Process (POMDP)



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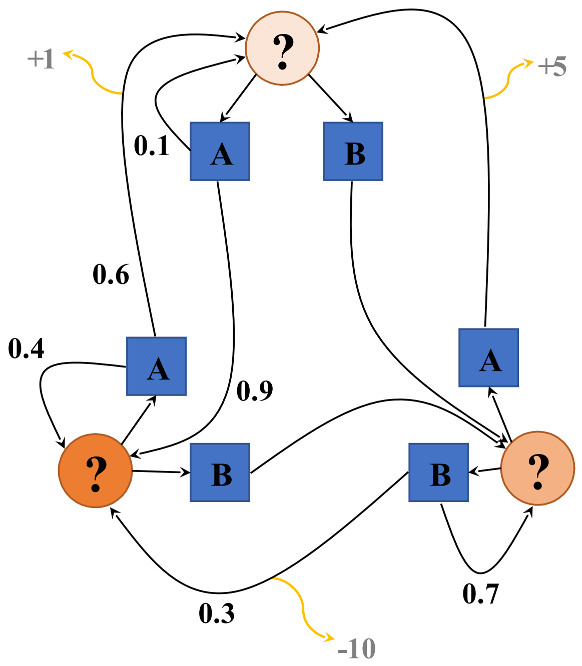
- $R : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$ - Reward
- $\mathcal{O}$ - Observation space
- $Z : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \times \mathcal{O} \to \mathbb{R}$ - Observation probability distribution

Partially Observable Markov Decision Process (POMDP)



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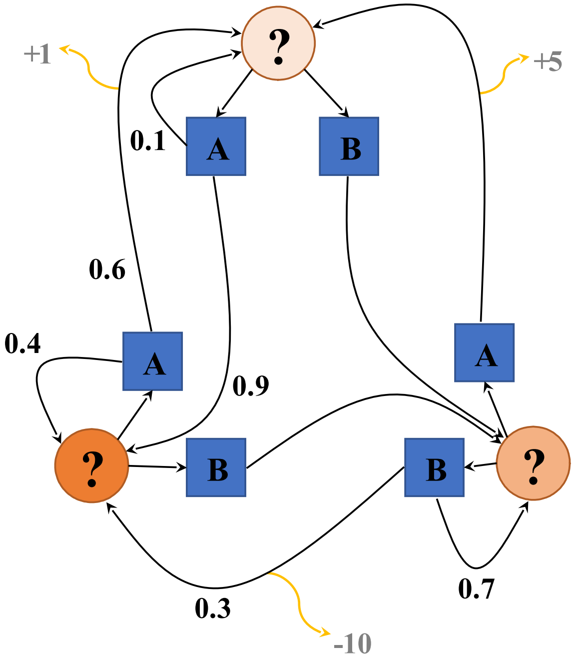
Alleatory

Partially Observable Markov Decision Process (POMDP)



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$Z(o \mid a, s')$

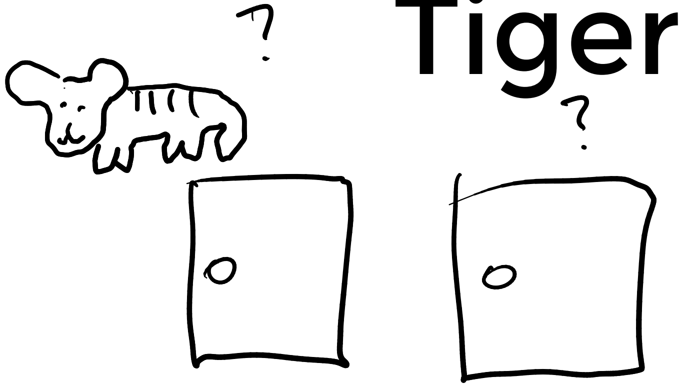
$Z(o \mid s, a, s')$

Alleatory

Epistemic (Static)

Epistemic (Dynamic)

Tiger POMDP Definition



-100

+10

Listen: 85% correct observation

$S = \{L, R\}$

$A = \{L, R, Listen\}$

$O = \{L, R\}$

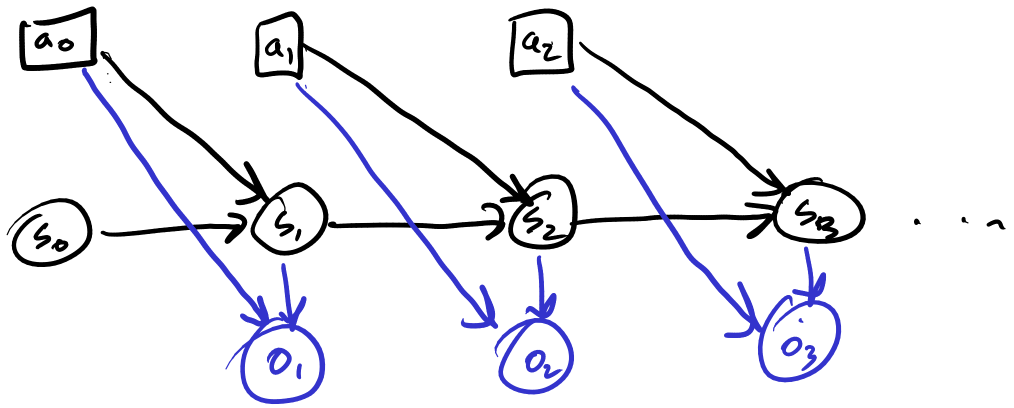
$T^{listen} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $T^{L} = T^{R} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$

$$Z(o \mid s, a) = \begin{cases} 0.85 & \text{if } a = \text{listen and } s = o \\ 0.15 & \text{if } a = \text{listen and } s \neq o \\ 0.5 & \text{if } a \neq \text{listen} \end{cases}$$

$$R(s, a) = \begin{cases} -100 & \text{if } a = s \\ -1 & \text{if } a = \text{listen} \\ 10 & \text{o.w.} \end{cases}$$

$\gamma = 0.95$

Hidden Markov Models and Beliefs



$$P(s_{t+1} \mid s_0, a_0 \ldots s_t, a_t) = T(s_{t+1} \mid s_t, a_t)$$

$$\cancel{P(o_{t+1} \mid o_0, a_0 \ldots o_t, a_t) = P(o_{t+1} \mid a_t, o_{t+1})}$$

L L L L

R R R

L L L

Let $b_0(s) \equiv P(s_0 = s)$

$h_t \equiv (b_0, a_0, o_1, a_1 \;\ldots\; a_{t-1}, o_t)$

$b_t(s) \equiv P(s_t = s \mid h_t)$

$$P(b_{t+1} \mid b_0, a_0, \ldots b_t, a_t) = P(b_{t+1} \mid b_t, a_t)$$

Bayesian Belief Updates

$h_t = (b_0\, a_0, o_1, \ldots\ a_{t-1}, o_t)$

$Z(o \mid a, s')$

$$b_t = P(s_t \mid h_t) = P(s_t \mid h_{t-1}, a_{t-1}, o_t)$$

$$= \frac{P(o_t \mid s_t, h_{t-1}, a_{t-1})\, P(s_t \mid h_{t-1}, a_{t-1})}{P(o_t \mid h_{t-1}, a_{t-1})}$$

$$\sum_{s_t} P(s_t \mid h_{t-1}, a_{t-1})\, P(o_t \mid s_t, h_{t-1}, a_{t-1})$$

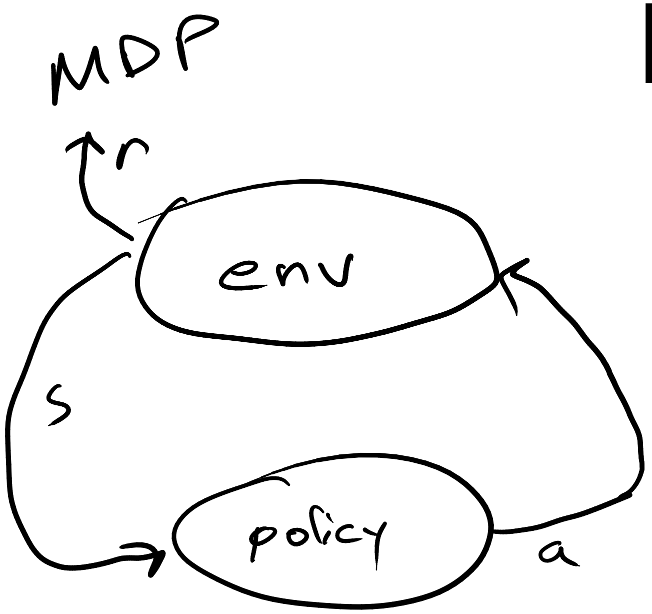
$$\propto P(o_t \mid s_t, h_{t-1}, a_{t-1})\, P(s_t \mid h_{t-1}, a_{t-1})$$

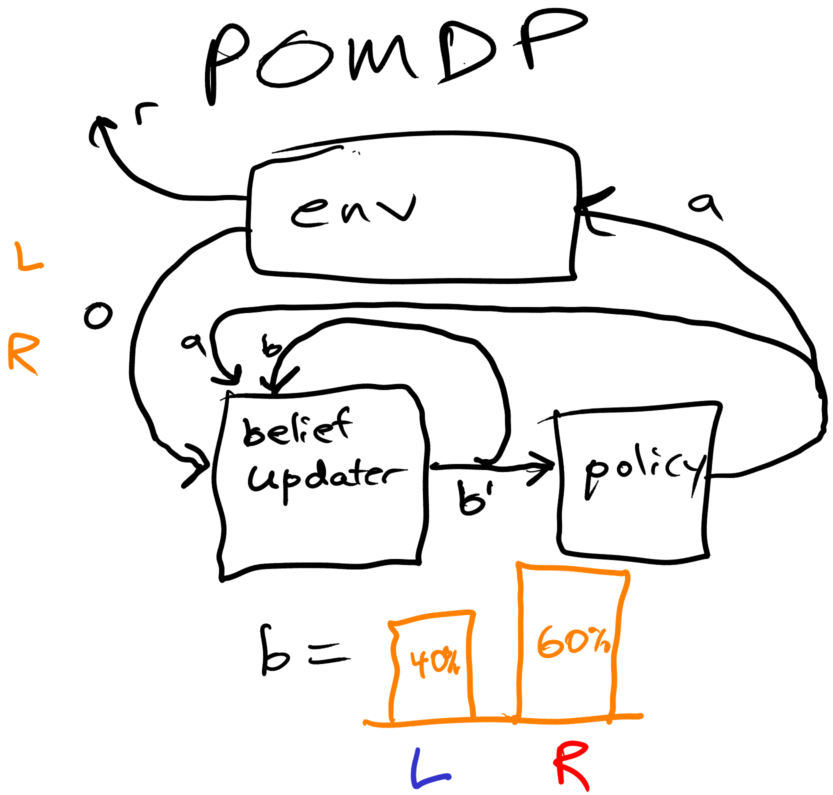
$$= P(o_t \mid s_t, a_{t-1}) \sum_{s_{t-1}} P(s_t \mid s_{t-1}, a_{t-1})\, P(s_{t-1} \mid h_{t-1})$$

$$= Z(o_t \mid a_{t-1}, s_t) \sum_{s_{t-1}} T(s_t \mid s_{t-1}, a_{t-1})\, b_{t-1}(s_{t-1})$$

$$\boxed{b'(s') \propto Z(o \mid a, s') \sum_{s} T(s' \mid s, a)\, b(s)}$$

Filtering Loop





Tiger Example

$$b_0(s) = \begin{cases} 0.5 \text{ if } s = L \\ 0.5 \text{ if } s = R \end{cases}$$

take listen action

receive a L observation

what is b_1?

$$b'(s') \propto Z(o \mid a, s') \sum_s T(s' \mid s, a)\, b(s)$$

$$b_1(L) \propto Z(L \mid \text{Listen}, L) \sum_s T(L \mid s, \text{Listen})\, b(s)$$

$$= 0.85 * \left(\overset{s=L}{1.0 \cdot 0.5} + \overset{s=R}{0.0 \cdot 0.5} \right)$$

$$= 0.425$$

$$b_1(R) \propto Z(L \mid \text{Listen}, R) \sum_s T(R \mid s, \text{Listen})\, b(s)$$

$$= 0.15 \left(0.0 \cdot 0.5 + 1.0 \cdot 0.5 \right)$$

$$= 0.075$$

$$b_1(L) = 0.425 / (0.425 + 0.075) = 0.85$$

$$b_1(R) = 0.075 / (0.425 + 0.075) = 0.15$$

Recap