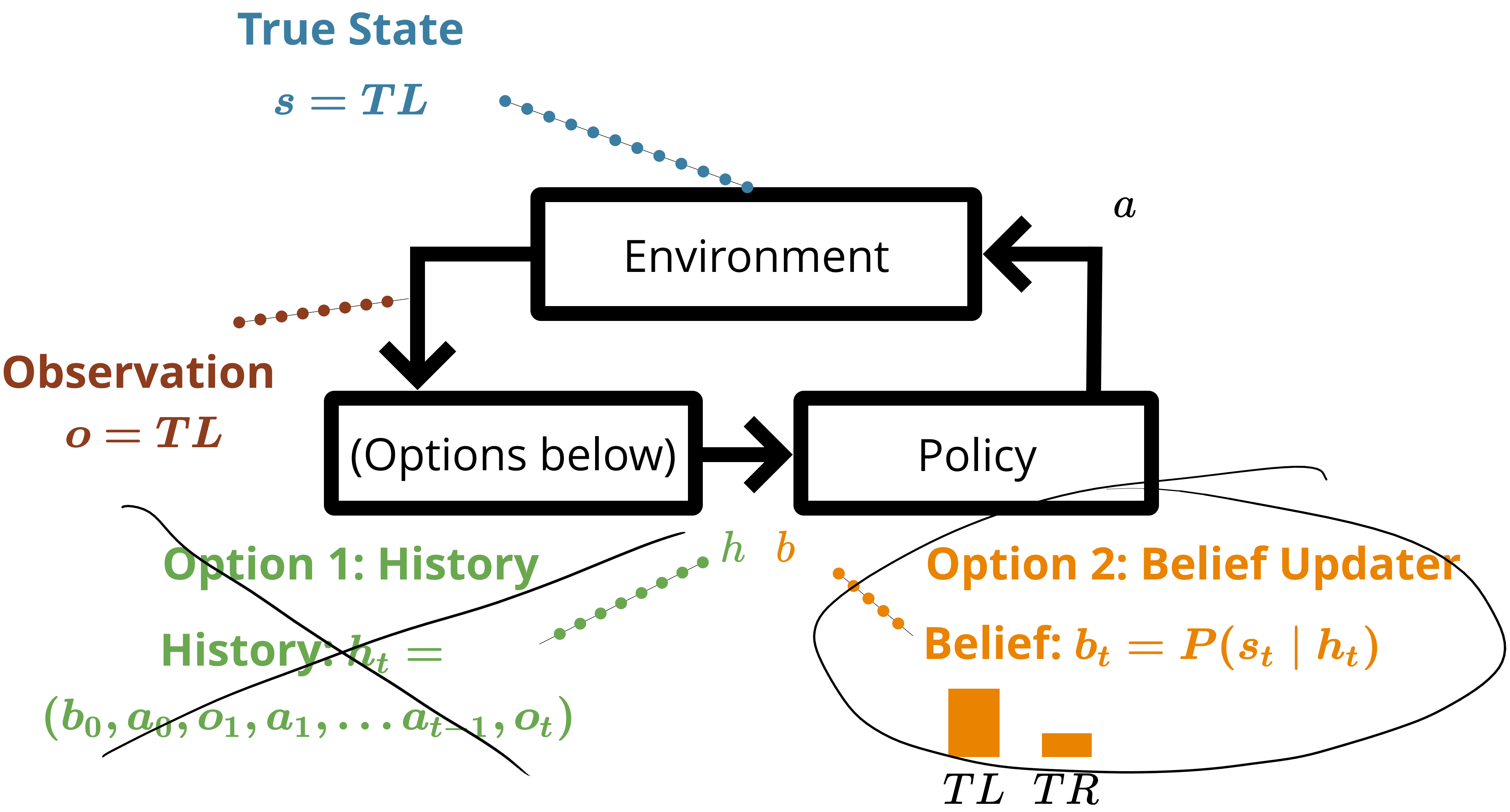
Particle Filters

POMDP Sense-Plan-Act Loop



Review: Bayesian Filter

```julia
function update(b::Vector{Float64}, 𝒫, a, o)
    𝒮, T, O = 𝒫.𝒮, 𝒫.T, 𝒫.O
    b′ = similar(b)
    for (i′, s′) in enumerate(𝒮)
        po = O(a, s′, o)
        b′[i′] = po * sum(T(s, a, s′) * b[i] for (i, s) in enumerate(𝒮))
    end
    if sum(b′) ≈ 0.0
        fill!(b′, 1)
    end
    return normalize!(b′, 1)
end
```

Review: Bayesian Filter

$$b_t(s) = P(s_t = s \mid h_t)$$

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Review: Bayesian Filter

$$b_t(s) = P(s_t = s \mid h_t)$$

$$b' = \tau(b, a, o)$$

$$b'(s') \propto Z(o \mid a, s') \sum_s T(s' \mid s, a)\, b(s)$$

$|S|^2$

```
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Rejection Particle Filter

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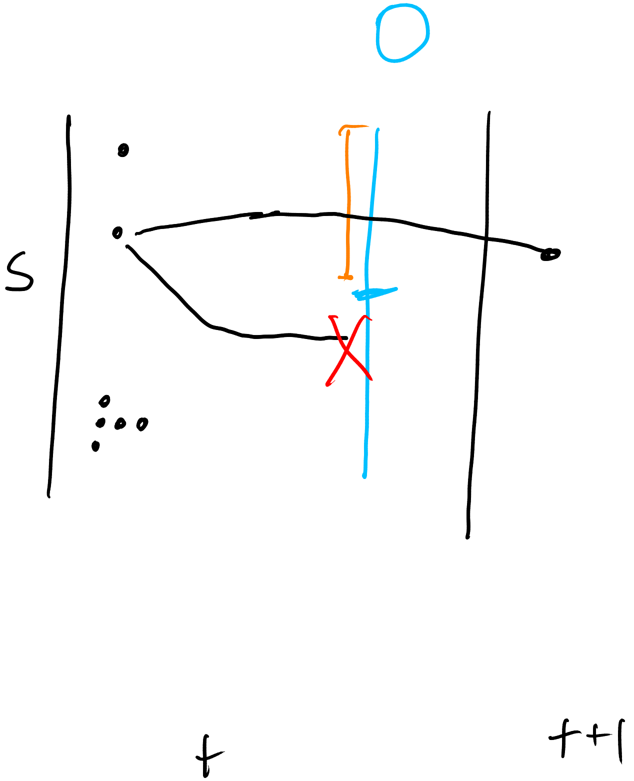
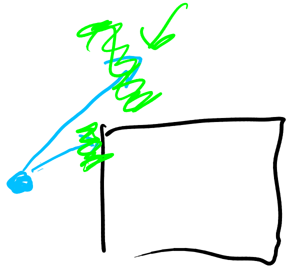




```
function update(b::RejectionParticleFilter, 𝒫, a, o)
    T, O = 𝒫.T, 𝒫.O
    states = similar(b.states)
    i = 1
    while i ≤ length(states)
        s = rand(b.states)
        s′ = rand(T(s,a))
        if rand(O(a,s′)) == o
            states[i] = s′
            i += 1
        end
    end
    return RejectionParticleFilter(states)
end
```

Simulates

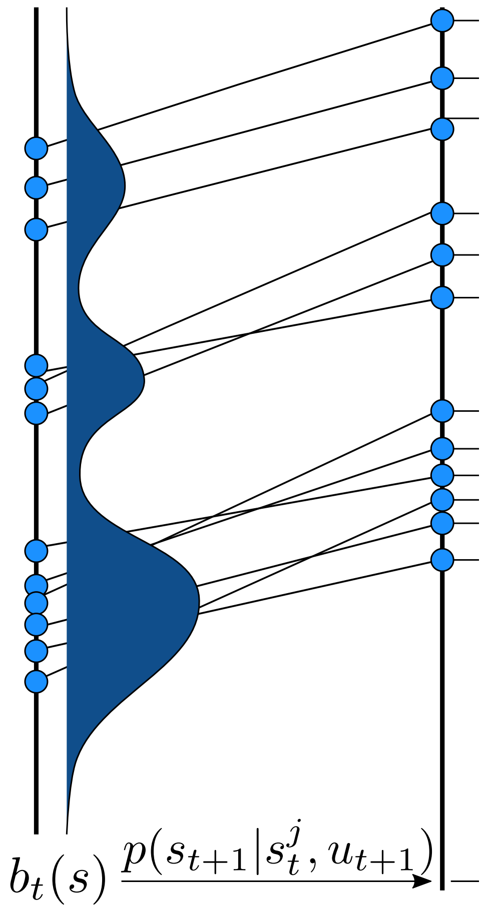
Only requires a Generative Model

$Z \quad T \qquad s', o \leftarrow G(s,a)$

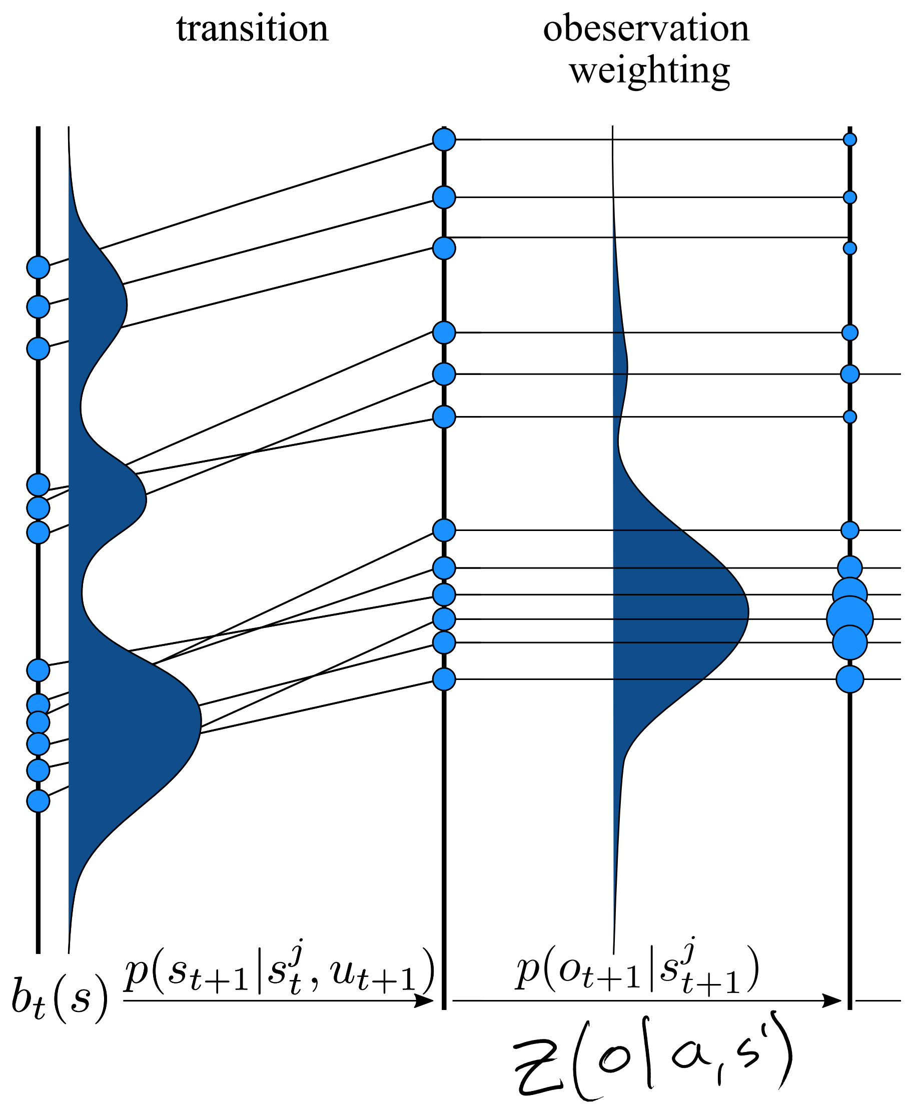
Weighted Particle Filtering

Weighted Particle Filtering

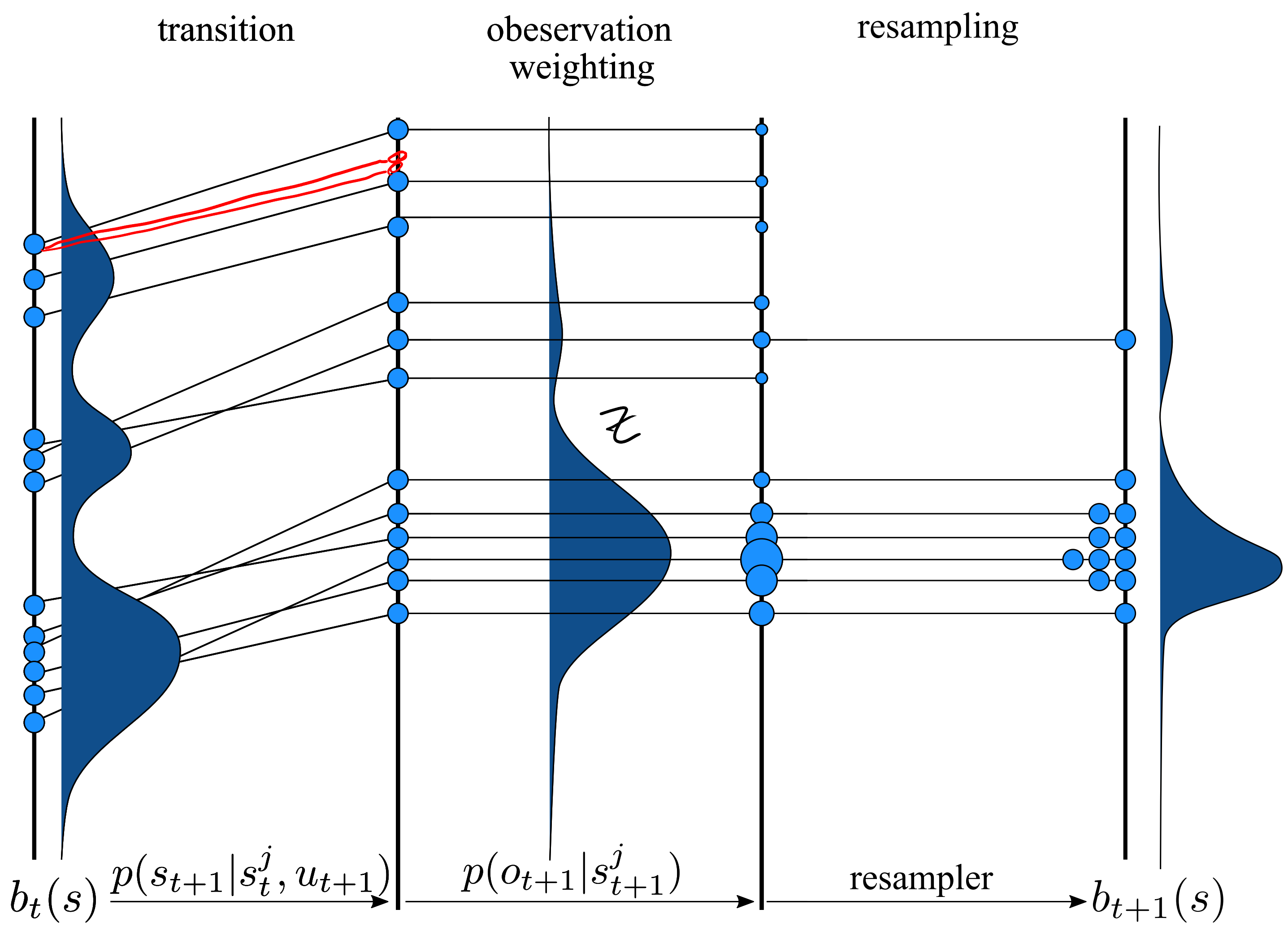
transition



Weighted Particle Filtering



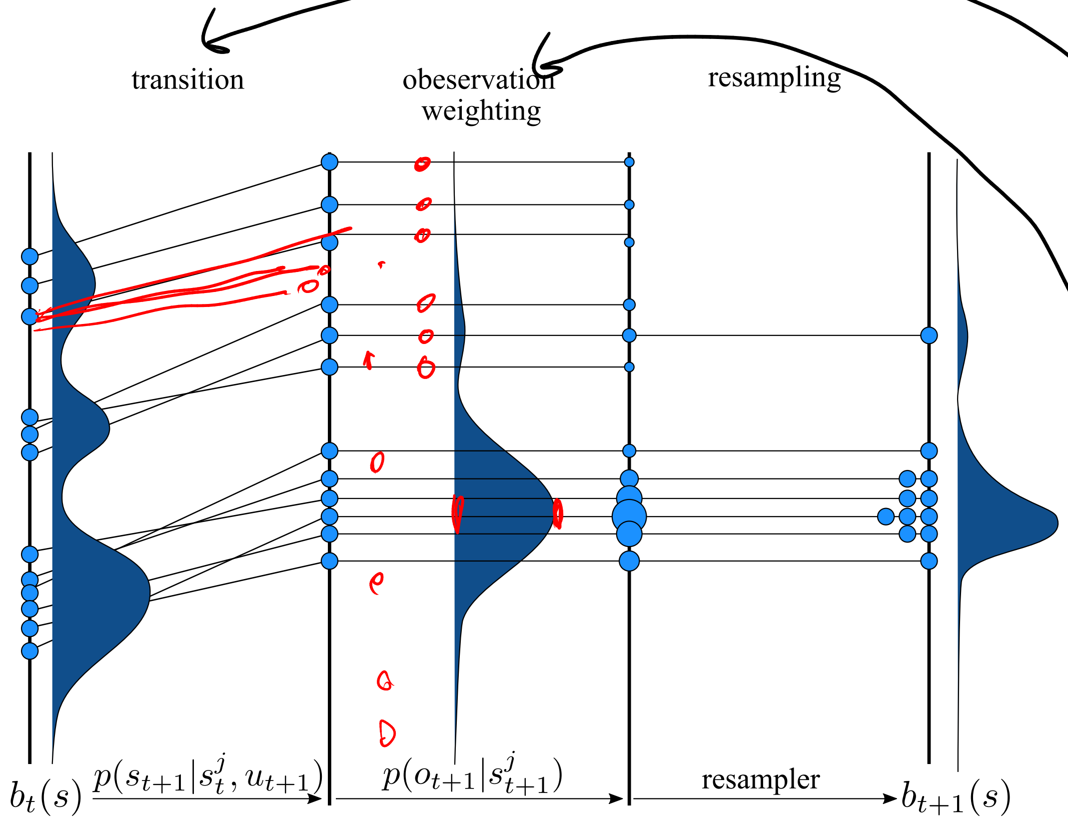
Weighted Particle Filtering



Weighted Particle Filtering

```
function update(b::ParticleFilter, 𝒫, a, o)
    T, O = 𝒫.T, 𝒫.O
    states = [rand(T(s, a)) for s in b.states]
    weights = [O(a, s′, o) for s′ in states]
    D = SetCategorical(states, weights)
    return ParticleFilter(rand(D, length(states)))
end
```

Weighted Particle Filtering

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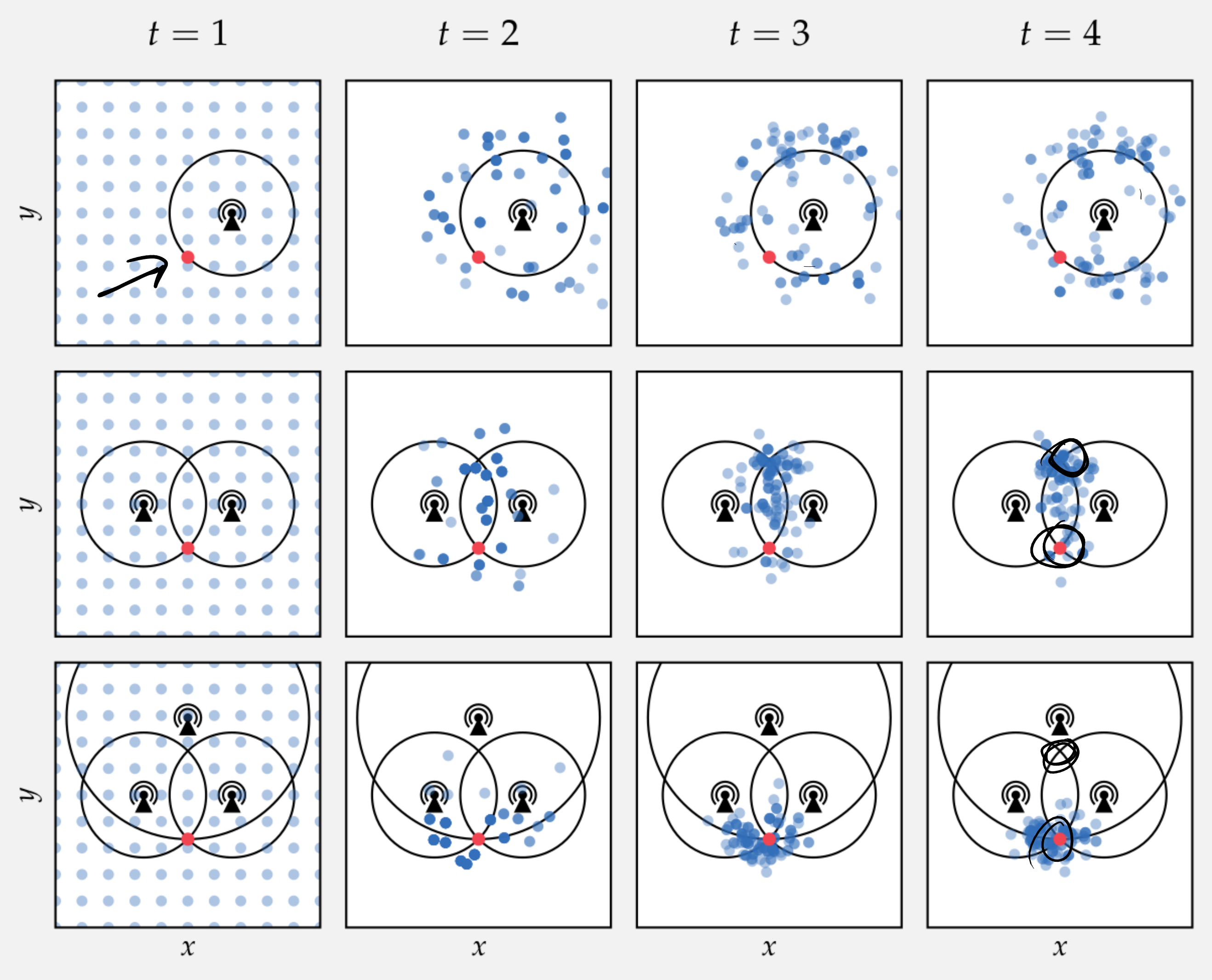
Weighted Particle Filtering



Particle Depletion

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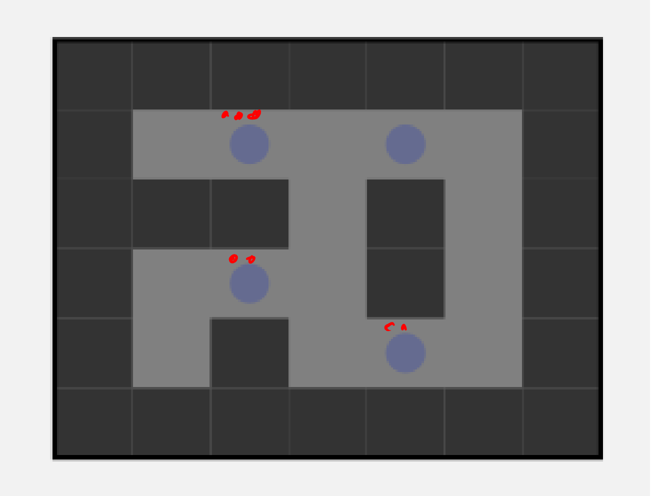

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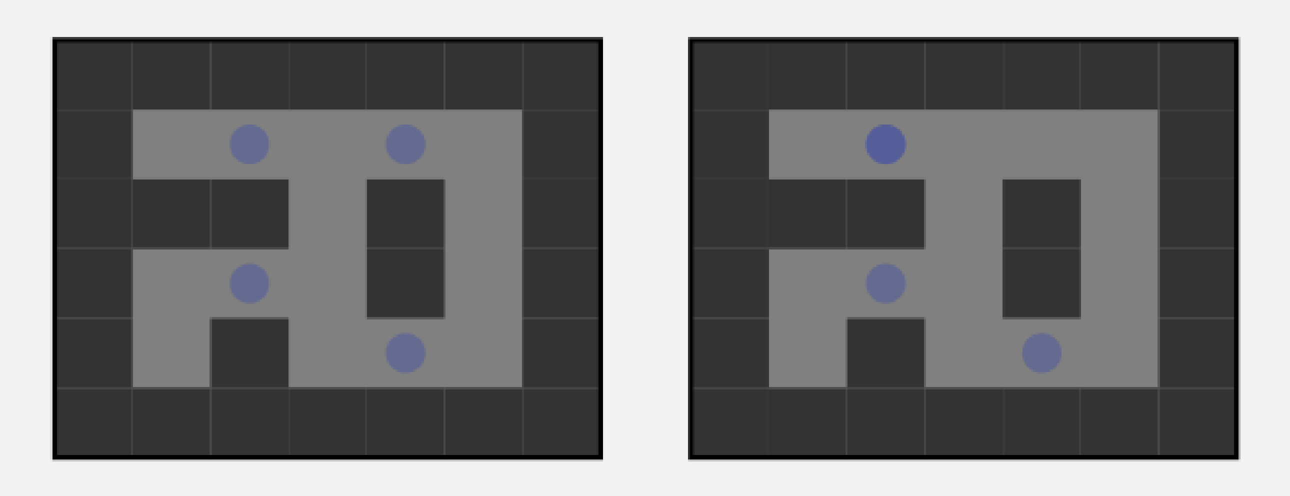

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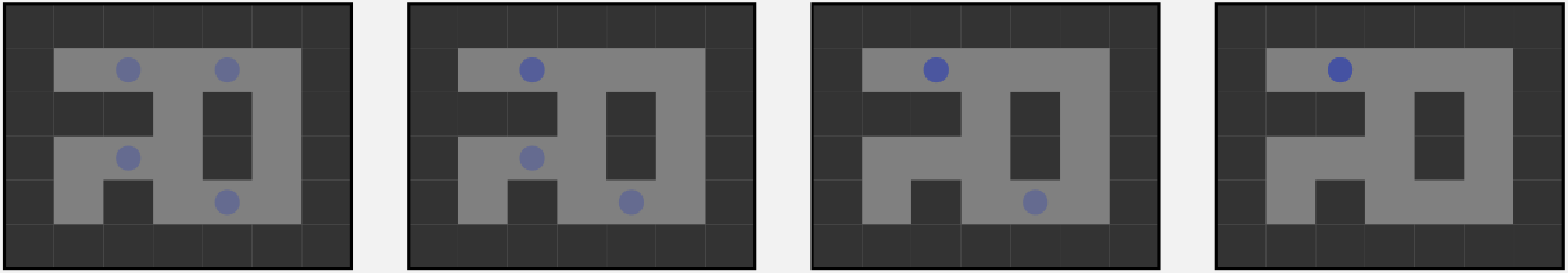

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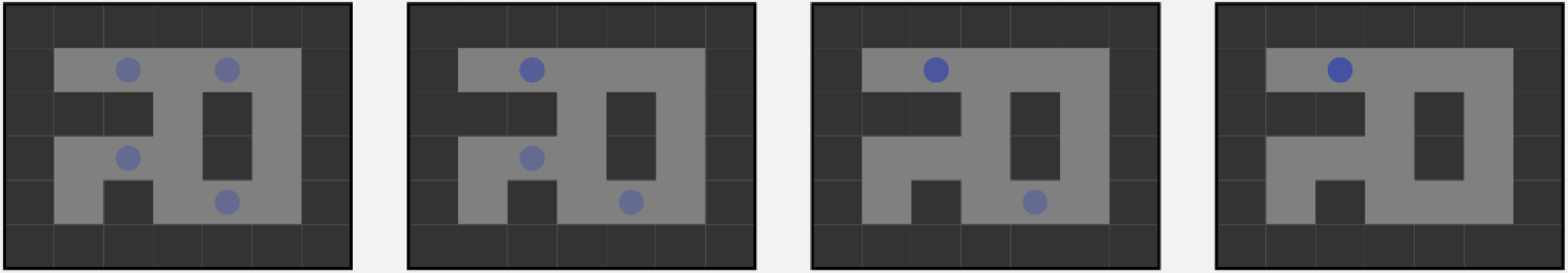
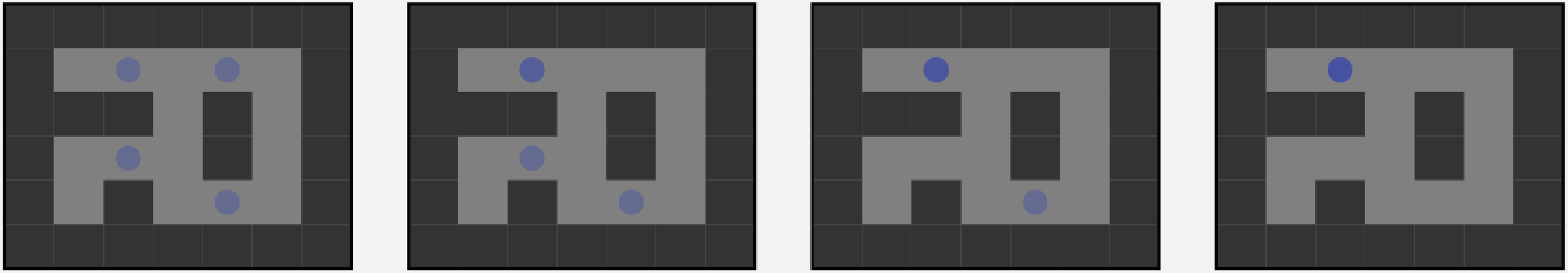


Solution: Domain specific particle injection based on:

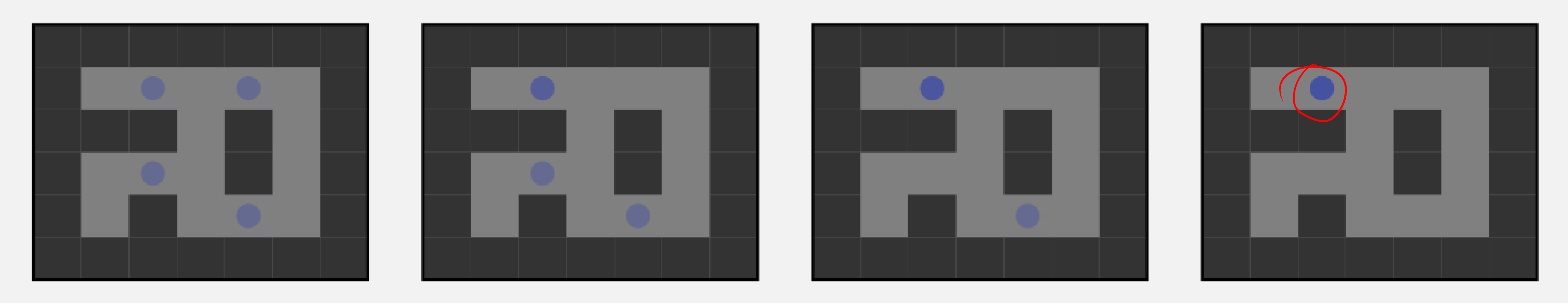
Particle Depletion



Solution: Domain specific particle injection based on:

- Weights

Particle Depletion



Solution: Domain specific particle injection based on:

- Weights
- Particle Diversity

Important Particle Filter Properties

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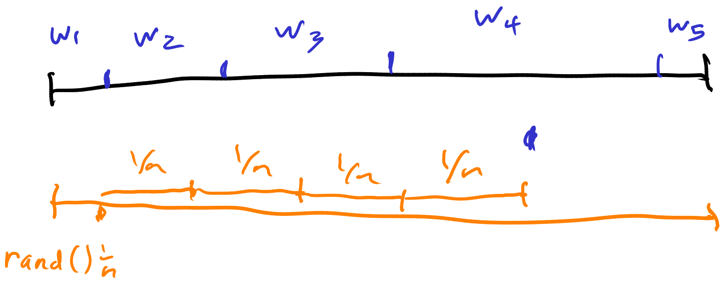
- Often the number of particles does **NOT** need to scale exponentially with the dimension (i.e. $n \neq k^d$)

$\text{error} \propto \frac{1}{\sqrt{n}}$

Important Particle Filter Properties

- Often the number of particles does **NOT** need to scale exponentially with the dimension (i.e. $n \neq k^d$)
- Implementation should have $O(n)$ complexity.



for i in 1:n
rand($\bar{w}$)
end

$O(n)$ $\Big\}$ $O(n^2)$

rand($\bar{w}$, n) $O(n)$

Low variance resampling