Guiding Questions:

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1. How do we **encode relationships** between random variables?

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2. How do we **infer** something about one random variable given the value of another related one?

Plausibility

A, B

$A \succ B$

$A \sim B$

$A \prec B$

- Universal Comparibility Exactly one holds
- Transitivity

if $A \succeq B$ and $B \succeq C$ then $A \succeq C$

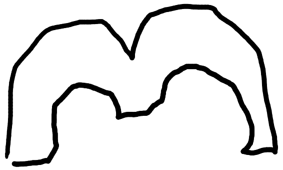
$P(A) > P(B)$ iff $A \succ B$

$P(A) = P(B)$ iff $A \sim B$

What is a Random Variable?

R.V. X ← Today only! Capital Letter = R.V.

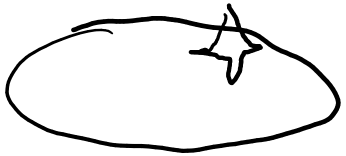
Happy Meal



Variable
- finite set of vals
- Probability for each val

$P(X=1)=0.5$

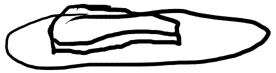
Chipotle



Variable
- continuous/discrete
- related to other R.Vs

$P(X|Y)$

Filet Mignon



$(\Omega, \mathcal{F}, \mathbb{P})$

$X: \Omega \to \mathbb{E}$

Vocabulary/Notation

Vocabulary/Notation

Term	Definition	Coinflip Example	Uniform Example

Vocabulary/Notation

Term	Definition	Coinflip Example $\mathrm{Bernoulli}(0.5)$	Uniform Example

Vocabulary/Notation

Term	**Definition**	$\text{Bernoulli}(0.5)$ **Coinflip Example**	$\mathcal{U}(0, 1)$ **Uniform Example**

Vocabulary/Notation

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support(X)			

Vocabulary/Notation

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Distribution			

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X	**P(X)**
0	0.5
1	0.5

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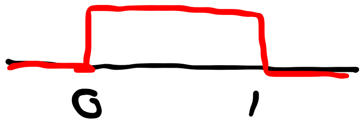
support(X) $x \in X$ $X \in [0,1]$	All the values that X can take	$\{h,t\}$ or $\{0,1\}$	$[0,1]$ 
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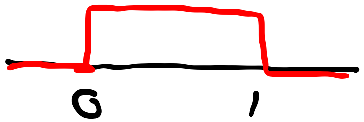
support(X) $x \in X$ $X \in [0,1]$	All the values that X can take	$\{h, t\}$ or $\{0, 1\}$	$[0,1]$ 
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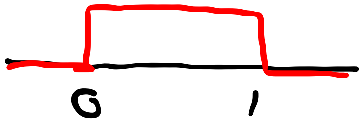

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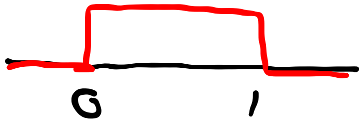
support(X) $x \in X$ $X \in [0,1]$	All the values that X can take	$\{h,t\}$ or $\{0,1\}$	$[0,1]$ 
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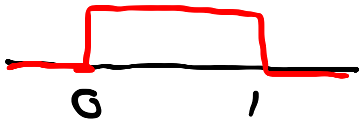 </td></tr>
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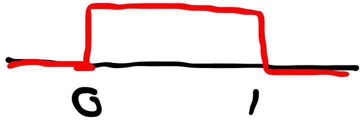 </td></tr>
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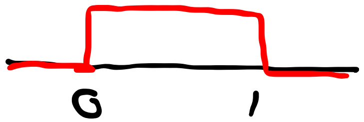
support(X) $x \in X$ $X \in [0,1]$	All the values that X can take	$\{h,t\}$ or $\{0,1\}$	$[0,1]$ 
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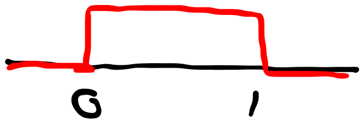 </td></tr>
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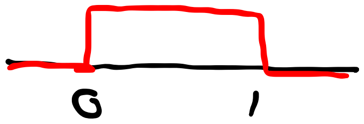 </td></tr>
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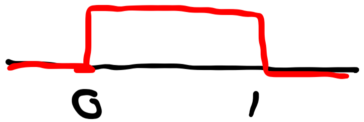 </td></tr>
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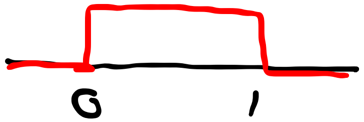
support(X) $x \in X$ $X \in [0,1]$	All the values that X can take	$\{h,t\}$ or $\{0,1\}$	$[0,1]$ 
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1	0.5

Distributions of related R.V.s

Distributions of related R.V.s

Joint Distribution

Distributions of related R.V.s

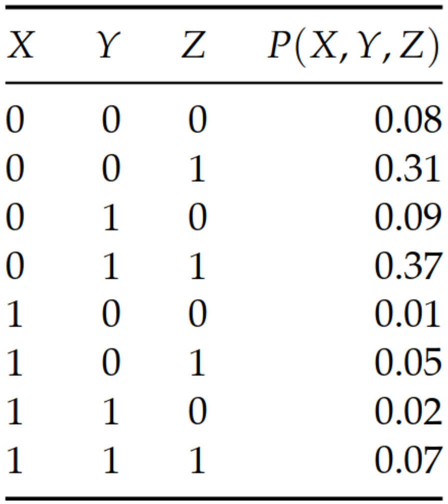
Joint Distribution

$$P(X, Y, Z)$$

Distributions of related R.V.s

Joint Distribution

$$P(X, Y, Z)$$

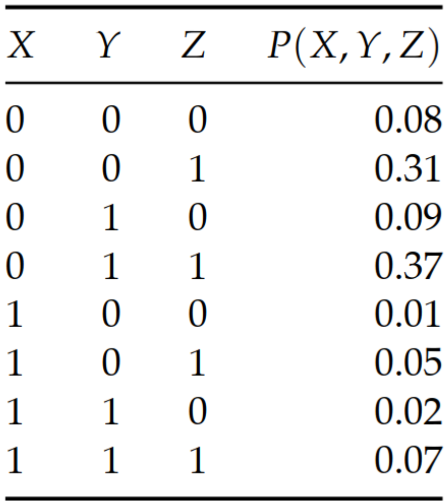


X	Y	Z	$P(X,Y,Z)$
0	0	0	0.08
0	0	1	0.31
0	1	0	0.09
0	1	1	0.37
1	0	0	0.01
1	0	1	0.05
1	1	0	0.02
1	1	1	0.07

Distributions of related R.V.s

Joint Distribution

$$P(X, Y, Z)$$



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Conditional Distribution

Distributions of related R.V.s

Joint Distribution

$$P(X, Y, Z)$$

X	Y	Z	$P(X,Y,Z)$
0	0	0	0.08
0	0	1	0.31
0	1	0	0.09
0	1	1	0.37
1	0	0	0.01
1	0	1	0.05
1	1	0	0.02
1	1	1	0.07

Conditional Distribution

$$P(X \mid Y, Z)$$

Distributions of related R.V.s

Joint Distribution

$$P(X, Y, Z)$$

X	Y	Z	$P(X,Y,Z)$
0	0	0	0.08
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1	0	0	0.01
1	0	1	0.05
1	1	0	0.02
1	1	1	0.07

Conditional Distribution

$$P(X \mid Y, Z)$$

(Distribution - valued function)

Distributions of related R.V.s

Joint Distribution

$$P(X, Y, Z)$$

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1	1	1	0.07

Conditional Distribution

$$P(X \mid Y, Z)$$

(Distribution - valued function)

X	$P(X \mid Y=1, Z=1)$
0	0.888 . . .
1	0.111 . . .

Distributions of related R.V.s

Joint Distribution

$$P(X, Y, Z)$$

X	Y	Z	$P(X,Y,Z)$
0	0	0	0.08
0	0	1	0.31
0	1	0	0.09
0	1	1	0.37
1	0	0	0.01
1	0	1	0.05
1	1	0	0.02
1	1	1	0.07

Conditional Distribution

$$P(X \mid Y, Z)$$

(Distribution - valued function)

X	$P(X \mid Y=1, Z=1)$
0	0.888...
1	0.111...

Marginal Distribution

Distributions of related R.V.s

Joint Distribution

$$P(X, Y, Z)$$

X	Y	Z	$P(X,Y,Z)$
0	0	0	0.08
0	0	1	0.31
0	1	0	0.09
0	1	1	0.37
1	0	0	0.01
1	0	1	0.05
1	1	0	0.02
1	1	1	0.07

Conditional Distribution

$$P(X \mid Y, Z)$$

(Distribution - valued function)

X	$P(X \mid Y=1, Z=1)$
0	0.888...
1	0.111...

Marginal Distribution

$$P(X)\ P(Y)\ P(Z)$$

Distributions of related R.V.s

Joint Distribution

$$P(X, Y, Z)$$

X	Y	Z	$P(X,Y,Z)$
0	0	0	0.08
0	0	1	0.31
0	1	0	0.09
0	1	1	0.37
1	0	0	0.01
1	0	1	0.05
1	1	0	0.02
1	1	1	0.07

Conditional Distribution

$$P(X \mid Y, Z)$$

(Distribution - valued function)

X	$P(X \mid Y=1, Z=1)$
0	0.888...
1	0.111...

Marginal Distribution

$$P(X)\ P(Y)\ P(Z)$$

X	$P(X)$
0	0.85
1	0.15

Y	$P(Y)$
0	0.45
1	0.55

Z	$P(Z)$
0	0.20
1	0.80

Distributions of related R.V.s

Joint Distribution

$$P(X, Y, Z)$$

Conditional Distribution

$$P(X \mid Y, Z)$$

Marginal Distribution

$$P(X)\ P(Y)\ P(Z)$$

Distributions of related R.V.s

Joint Distribution

$P(X, Y, Z)$

Conditional Distribution

$P(X \mid Y, Z)$

Marginal Distribution

$P(X)\ P(Y)\ P(Z)$

3 Rules

Distributions of related R.V.s

Joint Distribution

$P(X, Y, Z)$

Conditional Distribution

$P(X \mid Y, Z)$

Marginal Distribution

$P(X)\ P(Y)\ P(Z)$

3 Rules

(Burrito-level)

Distributions of related R.V.s

Joint Distribution

$P(X, Y, Z)$

Conditional Distribution

$P(X \mid Y, Z)$

Marginal Distribution

$P(X)\ P(Y)\ P(Z)$

3 Rules (Burrito-level) (Filet Minion Level: Axioms of Probability)

Distributions of related R.V.s

Joint Distribution

$P(X, Y, Z)$

Conditional Distribution

$P(X \mid Y, Z)$

Marginal Distribution

$P(X)\ P(Y)\ P(Z)$

3 Rules

(Burrito-level)

1)

Distributions of related R.V.s

Joint Distribution

$P(X, Y, Z)$

Conditional Distribution

$P(X \mid Y, Z)$

Marginal Distribution

$P(X)\ P(Y)\ P(Z)$

3 Rules (Burrito-level)

1) a) $0 \leq P(X \mid Y) \leq 1$

Distributions of related R.V.s

Joint Distribution	**Conditional Distribution**	**Marginal Distribution**
$P(X, Y, Z)$	$P(X \mid Y, Z)$	$P(X)\ P(Y)\ P(Z)$

3 Rules (Burrito-level)

1) a) $0 \leq P(X \mid Y) \leq 1$
   b) $\sum_{x \in X} P(x \mid Y) = 1$

Distributions of related R.V.s

Joint Distribution

$P(X, Y, Z)$

Conditional Distribution

$P(X \mid Y, Z)$

Marginal Distribution

$P(X)\ P(Y)\ P(Z)$

3 Rules (Burrito-level)

1) a) $0 \leq P(X \mid Y) \leq 1$

   b) $\sum_{x \in X} P(x \mid Y) = 1$

2) "Law of total probability"

$$P(X) = \sum_{y \in Y} P(X, y)$$

Distributions of related R.V.s

Joint Distribution

$P(X, Y, Z)$

Conditional Distribution

$P(X \mid Y, Z)$

Marginal Distribution

$P(X)\; P(Y)\; P(Z)$

3 Rules

(Burrito-level)

1) a) $0 \leq P(X \mid Y) \leq 1$
   b) $\sum_{x \in X} P(x \mid Y) = 1$
2) "Law of total probability"

$$P(X) = \sum_{y \in Y} P(X, y)$$

Joint $\rightarrow$ Marginal

Distributions of related R.V.s

Joint Distribution

$P(X, Y, Z)$

Conditional Distribution

$P(X \mid Y, Z)$

Marginal Distribution

$P(X)\ P(Y)\ P(Z)$

3 Rules

(Burrito-level)

1) a) $0 \leq P(X \mid Y) \leq 1$

   b) $\sum_{x \in X} P(x \mid Y) = 1$

2) "Law of total probability"

$$P(X) = \sum_{y \in Y} P(X, y)$$

Joint → Marginal

3) Definition of Conditional Probability

$$P(X \mid Y) = \frac{P(X, Y)}{P(Y)}$$

Distributions of related R.V.s

Joint Distribution

$P(X, Y, Z)$

Conditional Distribution

$P(X \mid Y, Z)$

Marginal Distribution

$P(X)\ P(Y)\ P(Z)$

3 Rules (Burrito-level)

1) a) $0 \leq P(X \mid Y) \leq 1$

   b) $\sum_{x \in X} P(x \mid Y) = 1$

2) "Law of total probability"

$$P(X) = \sum_{y \in Y} P(X, y)$$

3) Definition of Conditional Probability

$$P(X \mid Y) = \frac{P(X, Y)}{P(Y)}$$

Joint → Marginal

Joint + Marginal → Conditional

Distributions of related R.V.s

Joint Distribution

$P(X, Y, Z)$

Conditional Distribution

$P(X \mid Y, Z)$

Marginal Distribution

$P(X)\ P(Y)\ P(Z)$

3 Rules

(Burrito-level)

1) a) $0 \leq P(X \mid Y) \leq 1$

   b) $\sum_{x \in X} P(x \mid Y) = 1$

2) "Law of total probability"

$$P(X) = \sum_{y \in Y} P(X, y)$$

3) Definition of Conditional Probability

$$P(X \mid Y) = \frac{P(X, Y)}{P(Y)}$$

Joint → Marginal

Joint + Marginal → Conditional

Marginal + Conditional → Joint

$$P(X, Y) = P(X|Y)\,P(Y)$$

Breakout Rooms

~~First, Introduce Yourself~~

Next, Answer Question: $\sum_a P(a|B)=1$

- $P \in \{0,1\}$: Powder Day
- $C \in \{0,1\}$: Pass Clear
- 1 in 5 days is a powder day
- The pass is clear 8 in 10 days
- If it is a powder day, there is a 50% chance the pass is blocked

- What is the probability that there is a powder day and the pass is clear?
- What is the probability that the pass is blocked on a non-powder day

$P(P=1) = 0.2$ $\quad$ $P(P=0) = 0.8$

$P(C=1) = 0.8$ $\quad$ $P(C=0) = 0.2$

$\rightarrow P(C=0|P=1) = 0.5$ $\quad$ $P(C=1|P=1) = 1 - P(C=0|P=1) = 0.5$

$$P(C=1, P=1) = \underset{0.5}{P(C=1|P=1)}\ \underset{0.2}{P(P=1)}$$

$$\boxed{=0.1}$$

$$P(C=0|P=0) = \frac{P(C=0, P=0)}{P(P=0)}$$

$$\boxed{=0.125}$$

C	P	
0	0	0.1
1	0	0.7
0	1	0.1
1	1	0.1

$$\overset{0.5}{P(C=0|P=1)}\ \overset{0.2}{P(P=1)}$$

$$P(C=1) = P(C=1, P=0) + P(C=1, P=1)$$

$$\underset{0.7}{P(C=1, P=0)} = \underset{0.8}{P(C=1)} - \underset{0.1}{P(C=1, P=1)}$$

Bayes Rule

- Know: $P(\overset{\text{measurement}}{B} \mid A)$
- Want: $P(A \mid B)$

$$P(A|B) = \frac{P(A,B)}{P(B)} \qquad P(B|A) = \frac{P(A,B)}{P(A)}$$

$$P(A|B)P(B) = P(A,B) = P(B|A)P(A)$$

$$\boxed{P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}}$$

$$P(A|B,C) = \frac{P(B|A,C)\,P(A|C)}{P(B|C)}$$

Independence

Independence

Definition: X and Y are *independent* iff $\underline{P(X, Y)} = \underline{P(X)\, P(Y)}$

Independence

Definition: X and Y are *independent* iff $P(X, Y) = P(X)\,P(Y)$

$X \perp Y$

Independence

Definition: X and Y are *independent* iff $P(X, Y) = P(X)\,P(Y)$

$X \perp Y$

$P(X|Y) = P(X)$

Independence

Definition: X and Y are *independent* iff $P(X, Y) = P(X)\,P(Y)$

$X \perp Y$

$P(X|Y) = P(X)$

Definition: X and Y are *conditionally independent* given Z iff
$P(\underline{X, Y \mid} Z) = P(\underline{X \mid Z})\,P(\underline{Y \mid Z})$

Independence

Definition: X and Y are *independent* iff $P(X, Y) = P(X)\,P(Y)$

$X \perp Y$

$P(X \mid Y) = P(X)$

Definition: X and Y are *conditionally independent* given Z iff $P(X, Y \mid Z) = P(X \mid Z)\,P(Y \mid Z)$

$X \perp Y \mid Z$ ←

Guiding Questions:

Guiding Questions:

1. How do we **encode relationships** between random variables?

- Joint
- Conditional
  Marginal

Guiding Questions:

1. How do we **encode relationships** between random variables?
2. How do we **infer** something about one random variable given the value of another related one?

Bayes
Rule