ASEN 5519-003 Decision Making under Uncertainty
Homework 1: Probabilistic Models

January 17, 2023

1 Questions

Question 1. (20 pts) Consider the following joint distribution of three binary-valued random variables, A, B, and C:

A	B	C	$P(A, B, C)$
0	0	1	0.15
0	1	0	0.05
0	1	1	0.01
1	0	0	0.14
1	0	1	0.18
1	1	0	0.29
1	1	1	0.07

a) What is the probability of the outcome $A = 0$, $B = 0$, $C = 0$?

b) What is the marginal distribution of A?

c) What is the conditional distribution of A given $B = 0$ and $C = 1$?

Question 2. (20 pts) 2% of women at age forty who participate in routine screening have breast cancer. 86% of those with breast cancer will get positive mammograms. 8% of those without breast cancer will also get positive mammograms. A woman in this age group had a positive mammogram in a routine screening. What is the probability that she actually has breast cancer?

Question 3. (40 pts) Suppose that a stationary stochastic process $\{x_t\}$ is defined by the following equation: $x_{t+1} = 1.5\,x_t - x_{t-1} + v_t$ where v_t are independent, identically distributed random variables with $v_t \sim \mathcal{N}(0.0, 0.2^2)$.

(a) Simulate and plot 10 20-step trajectories sampled from this process with $x_0 = x_{-1} = 1$ (You may use any programming language, but as always, submit your code).

(b) Is this process a Markov process if the state is defined as x_t? Why or why not?

(c) If you only had access to the trajectories you plotted what evidence could you use to convince someone that this process is or is not Markov?

(d) What would need to be included in the state at time t to make this a Markov process?

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2 Auto-graded Programming

Question 4. (20 pts) In this exercise, you will write and test a Julia function to ensure that you can get Julia and the course-specific code running and help you learn how to do a task that sometimes trips students up in homework 2. Your function should take two arguments:

- `a`: a matrix, and
- `bs`: a non-empty vector of vectors.

The function should multiply all of the vectors in `bs` by `a` and then return the *elementwise* maximum of the resulting vectors.

In order to get full-credit, the function must be completely "type-stable" (see the "Performance Tips" section of the Julia manual). Your function should always return a vector with the same element type as `a`. You can assume the vectors in `bs` will have the same element type as `a`, but you should be able to handle `a` with any numeric element type.

Evaluate this function with `DMUStudent.HW1.evaluate` and submit the resulting json file *along with a listing of the code*. A score of 1 will receive full credit.