Guiding Question

- What does "Markov" mean in "Markov Decision Process"?

Stochastic Process

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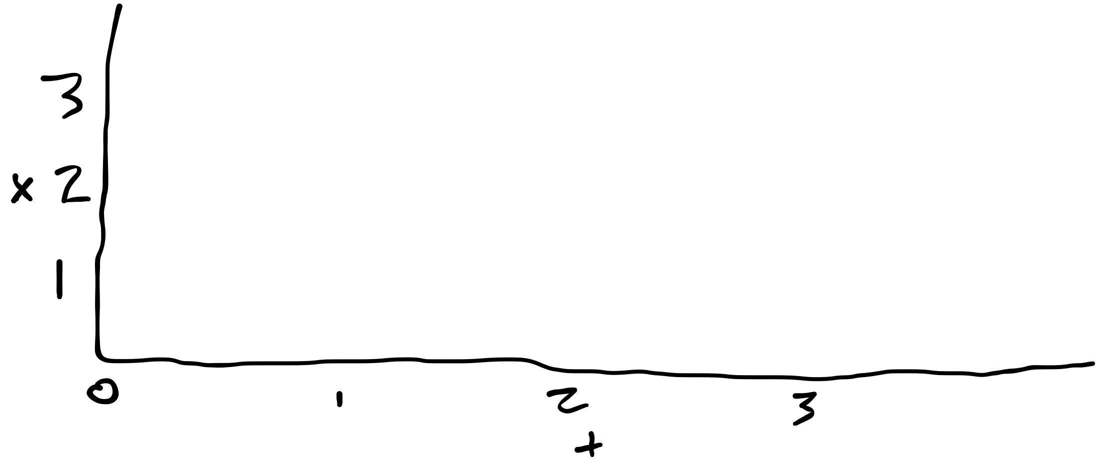

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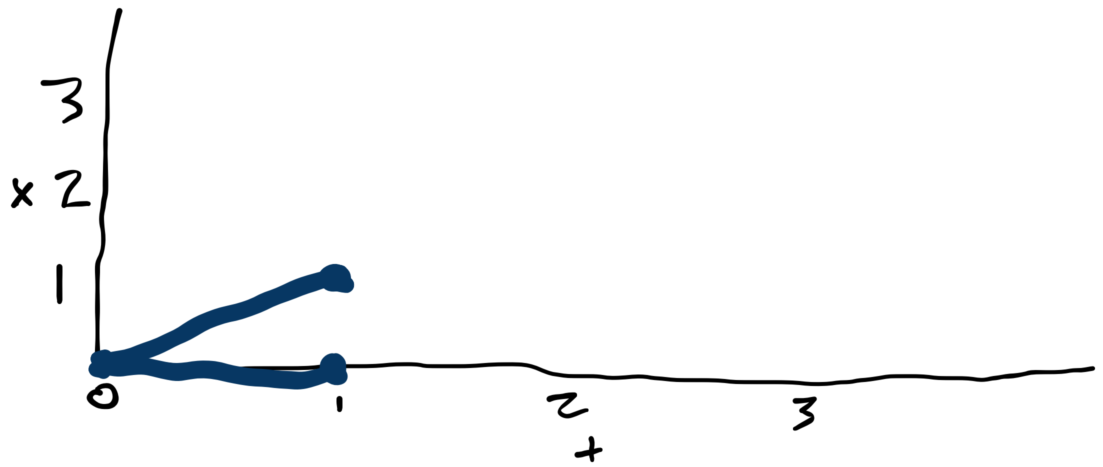

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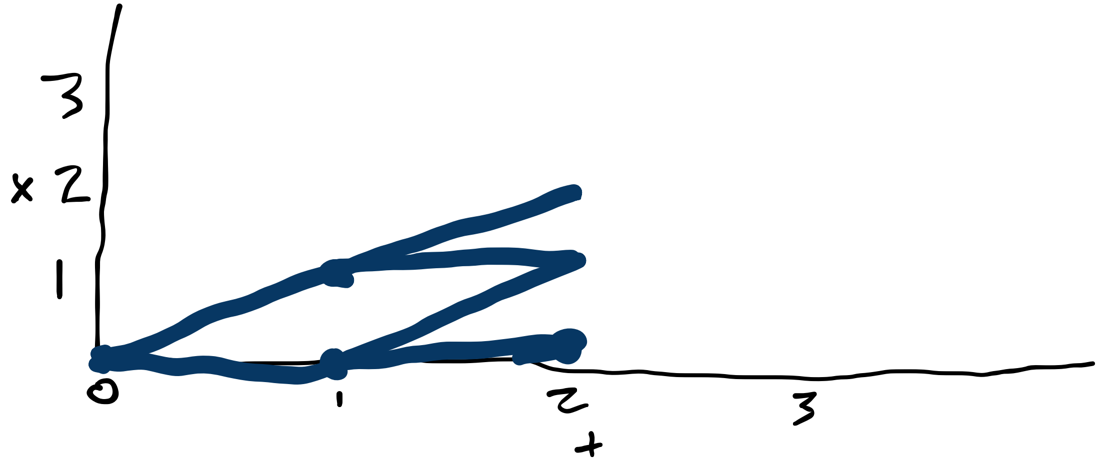

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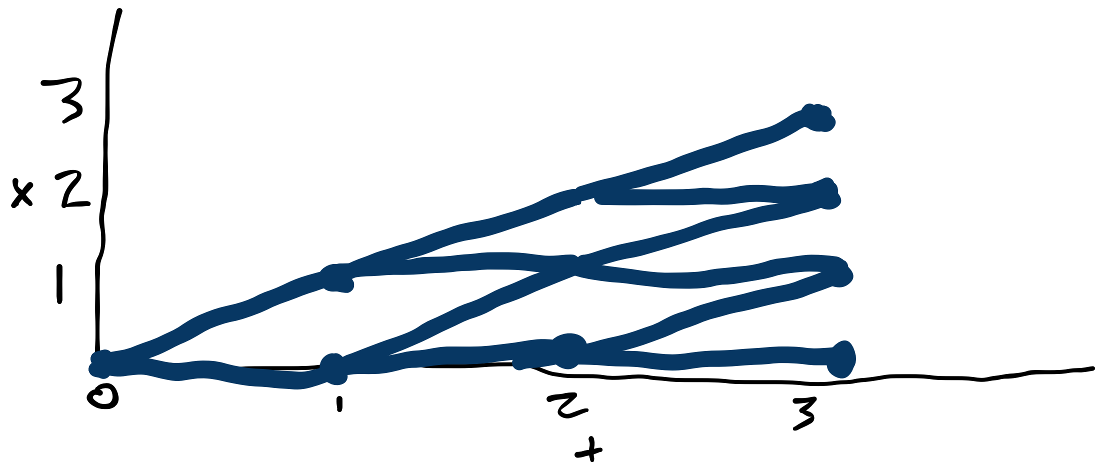

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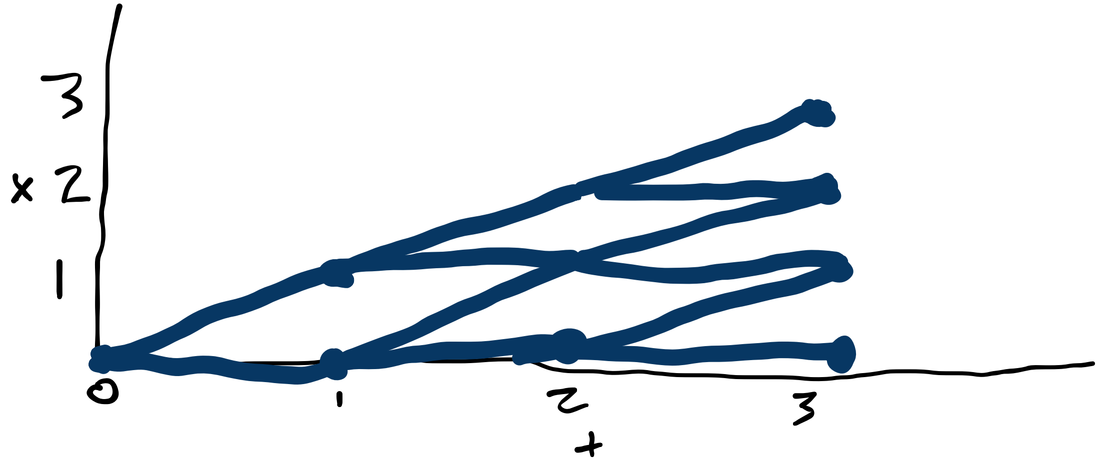

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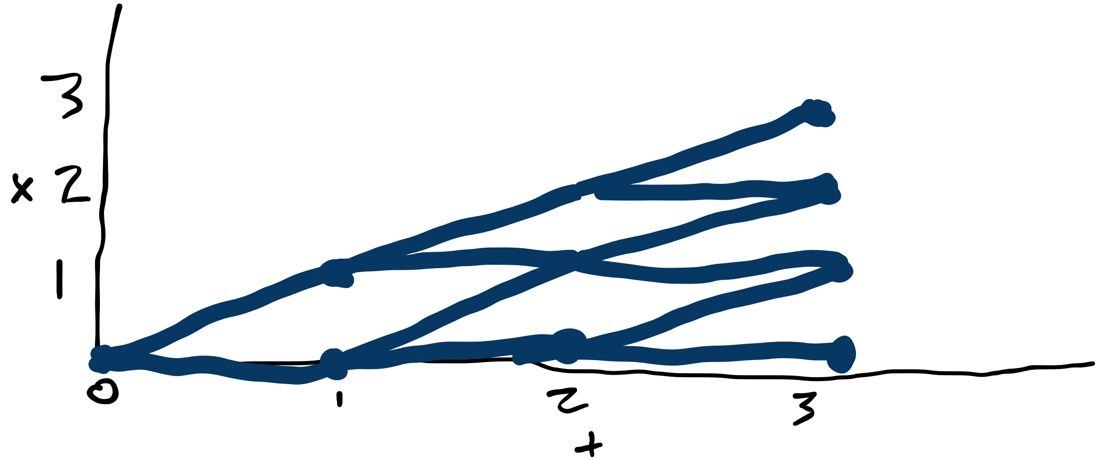

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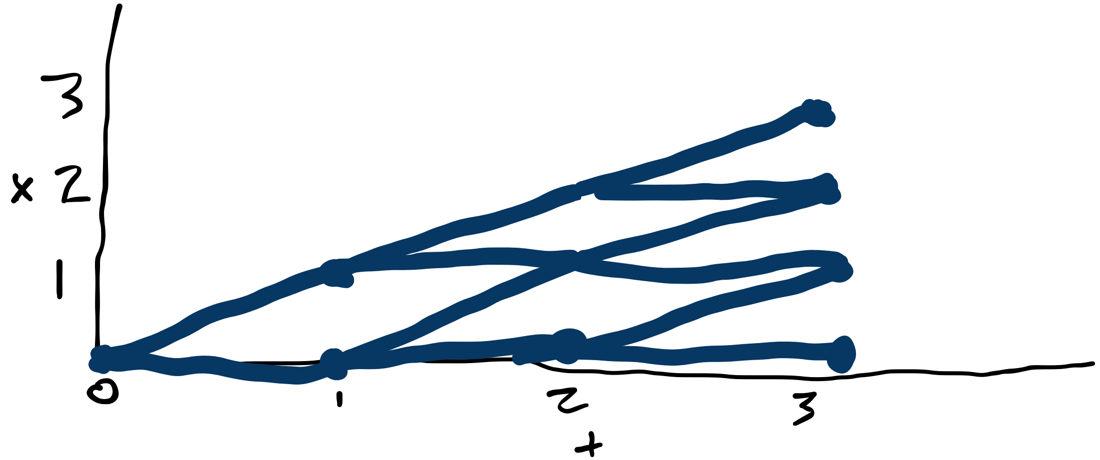

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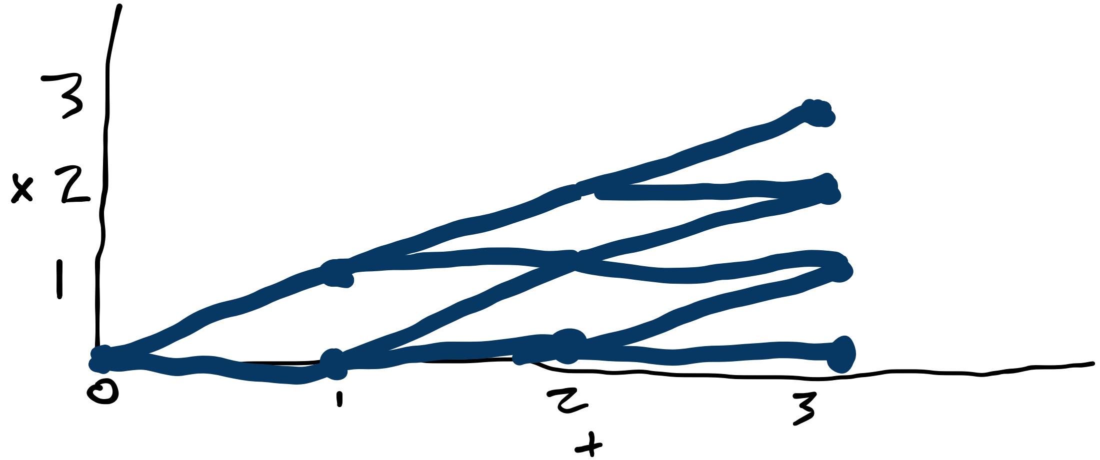

Joint

x0	x1	x2	P(x1, x2, x3)
0	0	0	0.25
0	0	1	0.25
0	1	1	0.25
0	1	2	0.25

Simulating a Stochastic Process

030-Stochastic-Processes.ipynb

Markov Process

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- A stochastic process $\{s_t\}$ is *Markov* if
  $P(s_{t+1} \mid s_t, s_{t-1}, \ldots, s_0) = P(s_{t+1} \mid s_t)$

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  $P(s_{t+1} \mid s_t, s_{t-1}, \ldots, s_0) = P(s_{t+1} \mid s_t)$
- s_t is called the "state" of the process

Break

Break

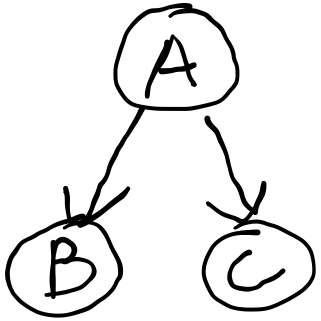
- Suppose you want to create a Markov model that describes how many new COVID cases will be detected on a particular day. What information should be in the state of the model?

Hidden Markov Model

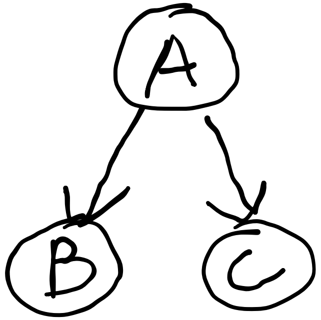
(Often you can't measure the whole state)

Bayesian Networks

Bayesian Networks

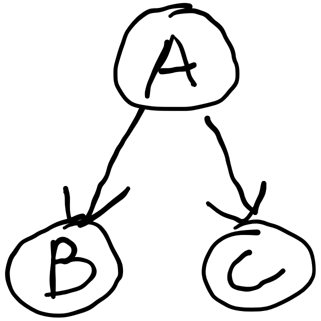


Bayesian Networks



A *Bayesian Network* is a directed acyclic graph (DAG) that encodes probabilistic relationships between R.V.s

Bayesian Networks



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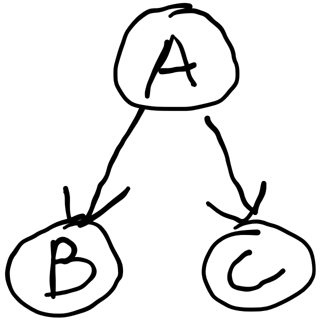

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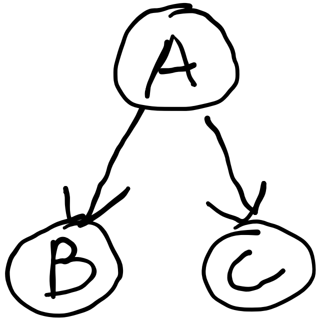

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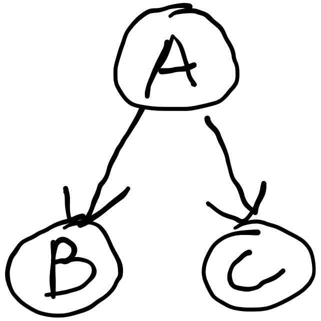
Concretely:

Bayesian Networks



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Concretely: $P(x_{1:n}) = \prod_i P(x_i \mid pa(x_i))$

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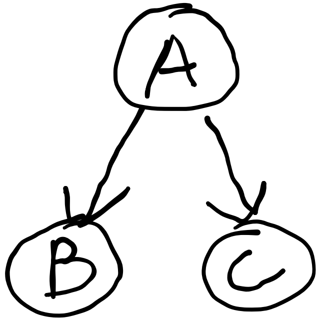

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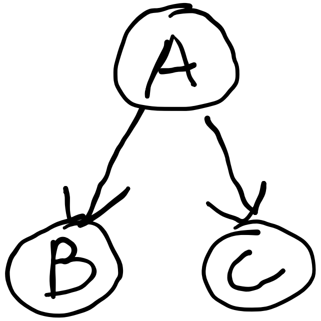
Bayesian Networks



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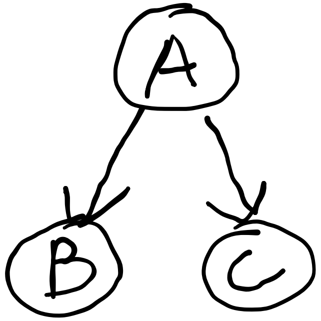

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Markov Process

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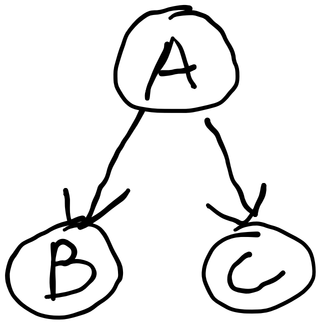

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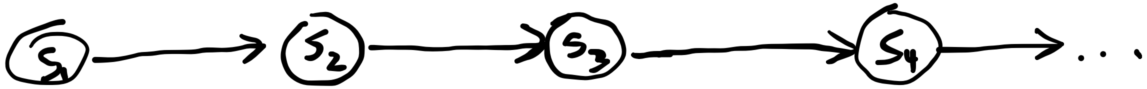

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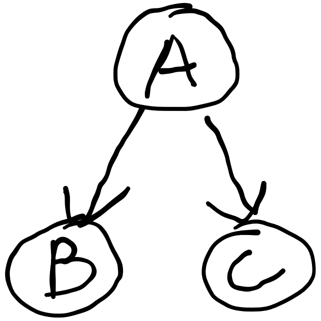

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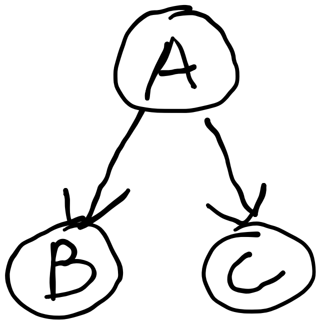
Parents

Markov Process

$s_1 \to s_2 \to s_3 \to s_4 \to \cdots$

Hidden Markov Model

Bayesian Networks



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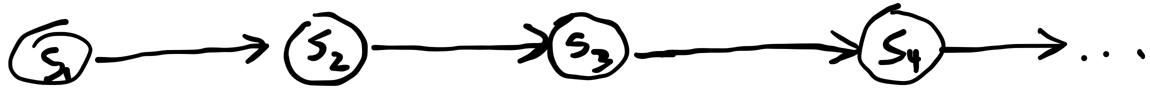

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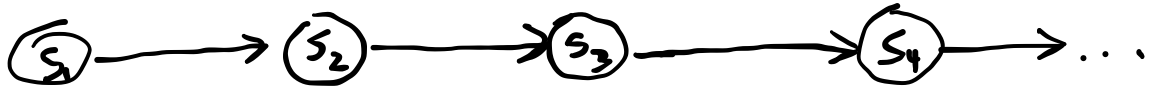

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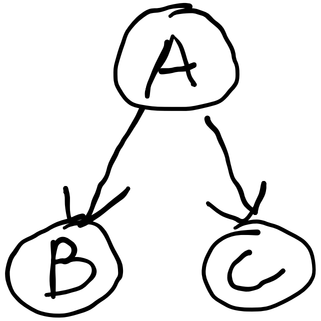

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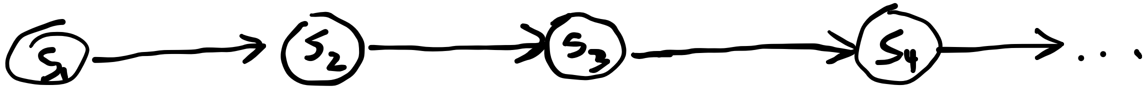

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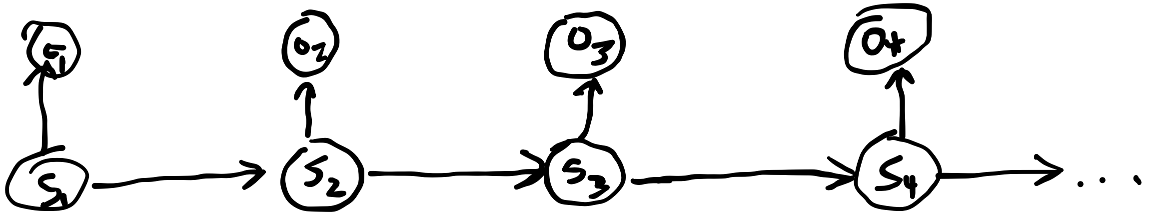

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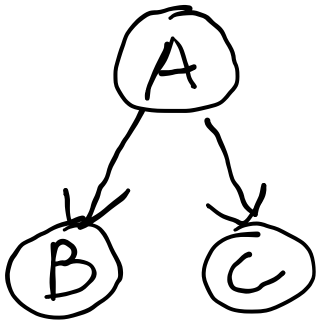

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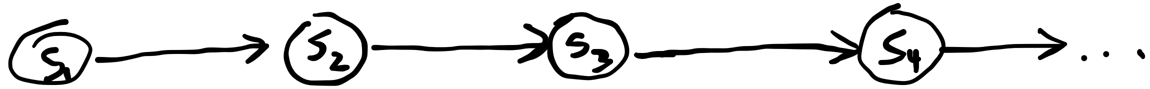

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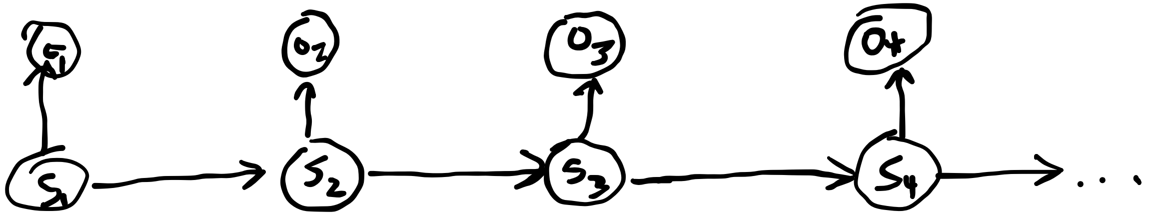

Dynamic Bayesian Network

Bayesian Networks

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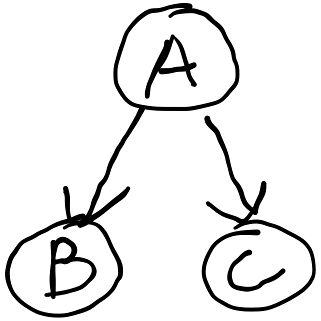

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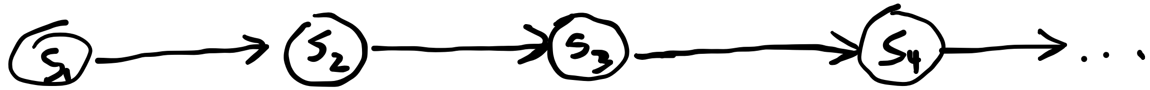

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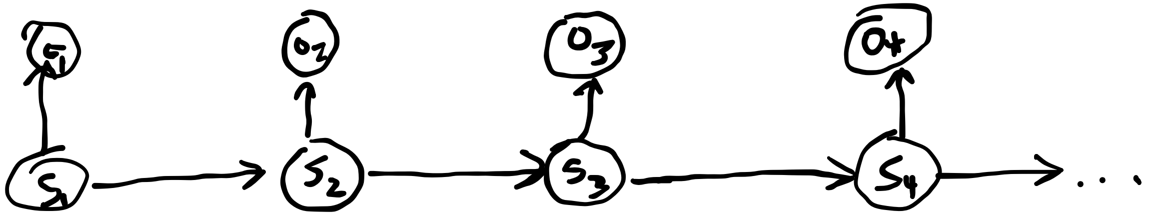
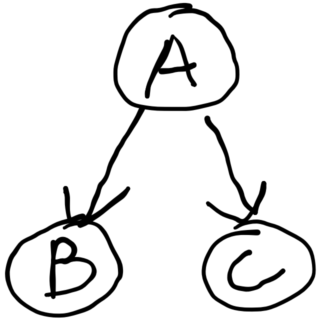


Dynamic Bayesian Network (One step)

Bayesian Networks



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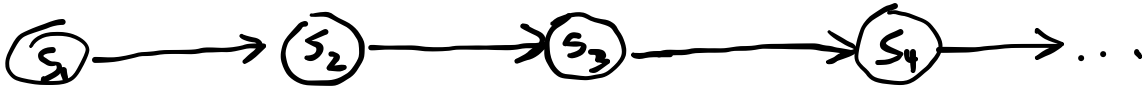

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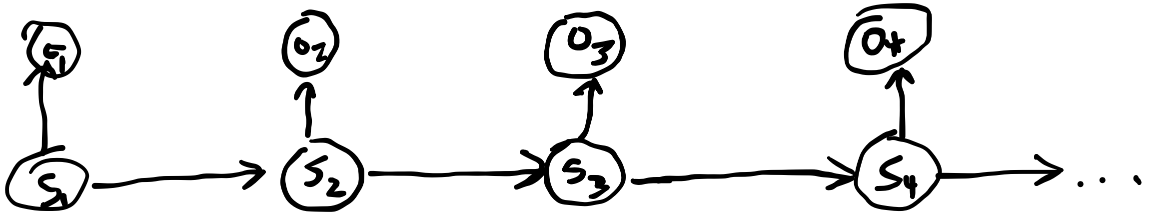

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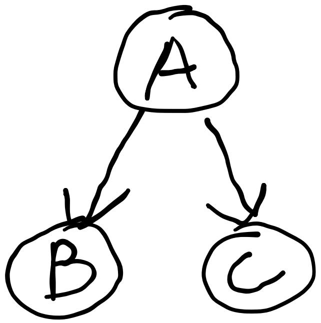

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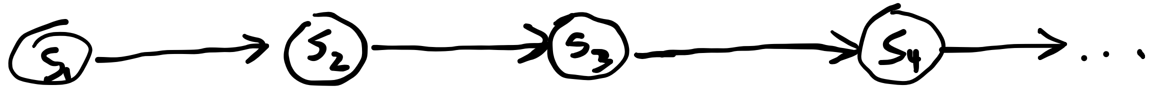

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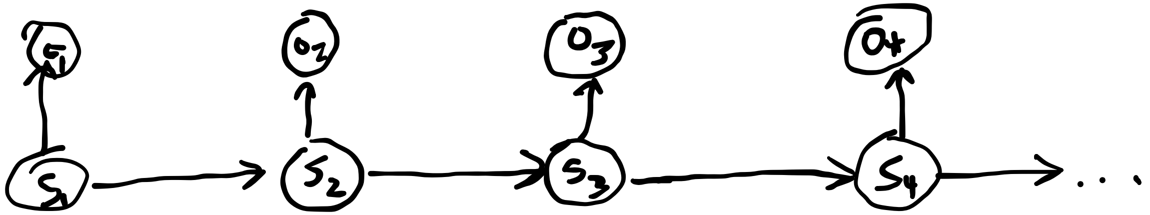

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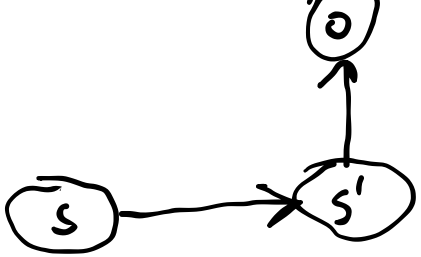

Decision Networks and MDPs

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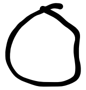

Decision Networks and MDPs

Decision Network

Chance node

Decision Networks and MDPs

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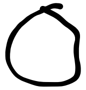
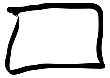
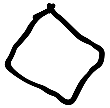
Decision Networks and MDPs

Decision Network

 Chance node

 Decision node



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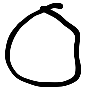
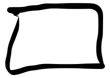
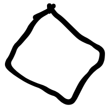
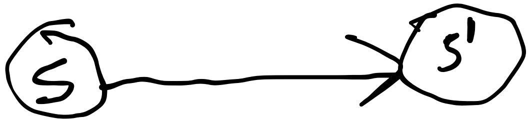
 Chance node

 Decision node

 Utility node

MDP Dynamic Decision Network



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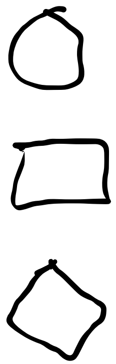

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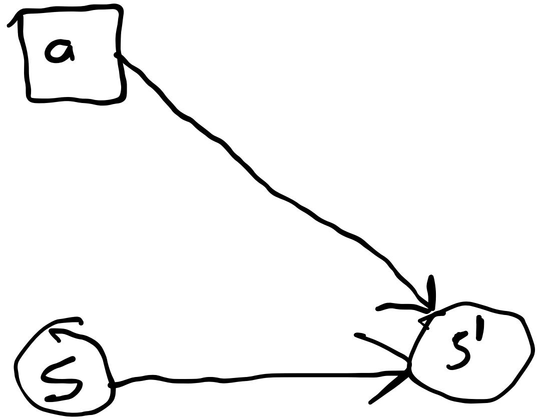

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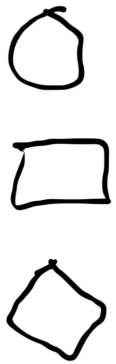

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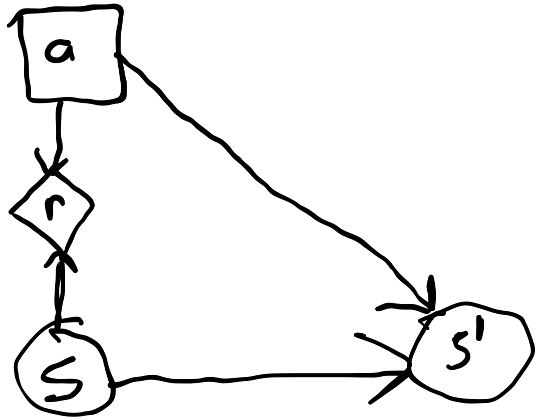

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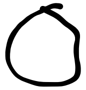
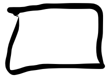
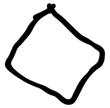
 Chance node

 Decision node

 Utility node

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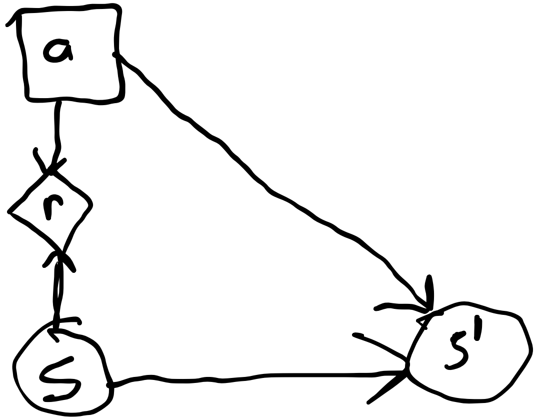

MDP Optimization problem

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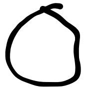
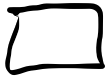
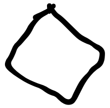
 Chance node

 Decision node

 Utility node

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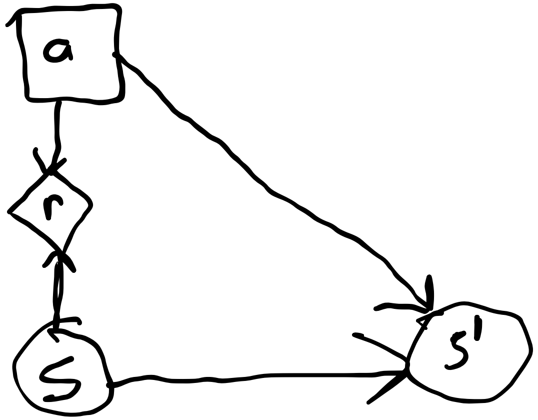

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$$\text{maximize} \quad \mathrm{E}\left[\sum_{t=1}^{\infty} r_t\right]$$

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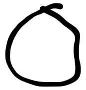
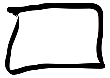
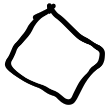
 Chance node

 Decision node

 Utility node

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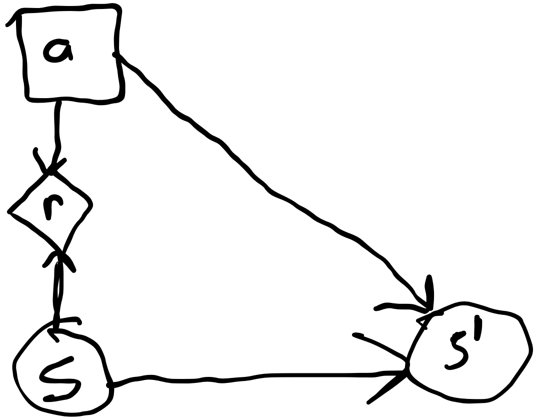

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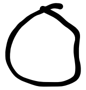
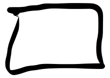
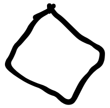
 Chance node

 Decision node

 Utility node

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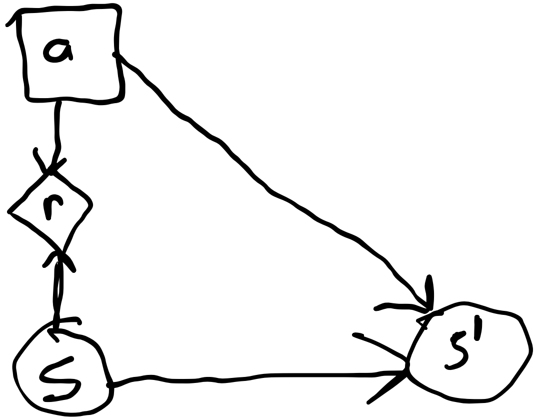

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Finite MDP Objectives

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4. Terminal States

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4. Terminal States

Infinite time, but a terminal state (no reward, no leaving) is always reached with probability 1.

Guiding Question

- What does "Markov" mean in "Markov Decision Process"?