Guiding Question

- What does "Markov" mean in "Markov Decision Process"?

Stochastic Process

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- A stochastic process is a collection of R.V.s indexed by time.

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$x' = x + v$

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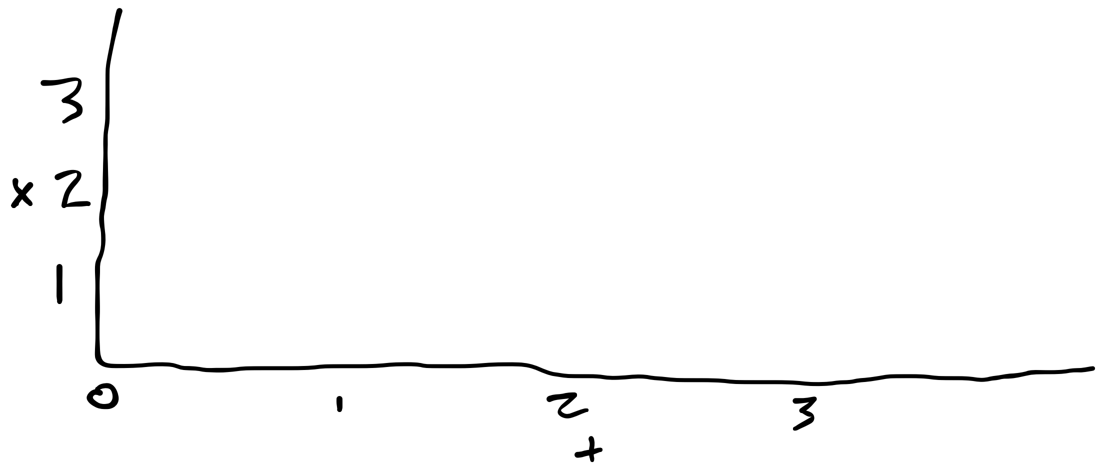

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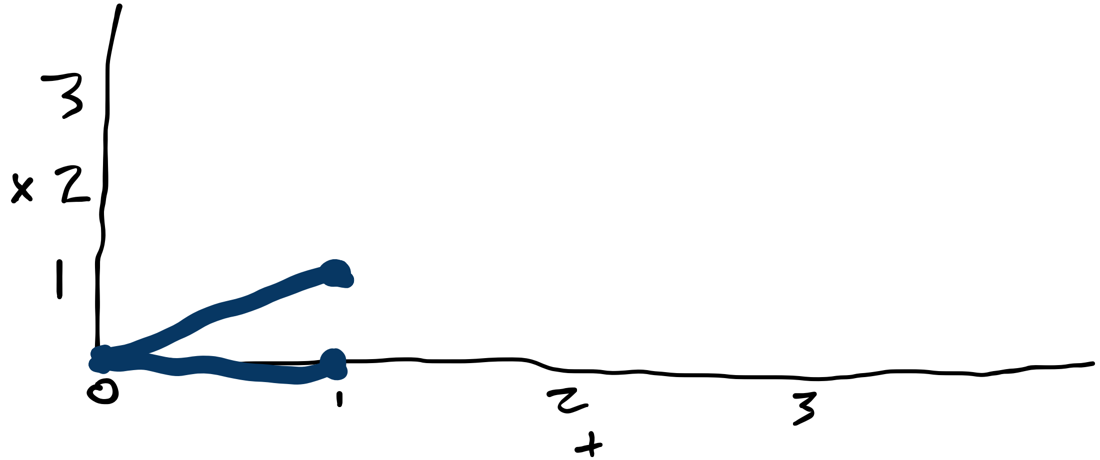

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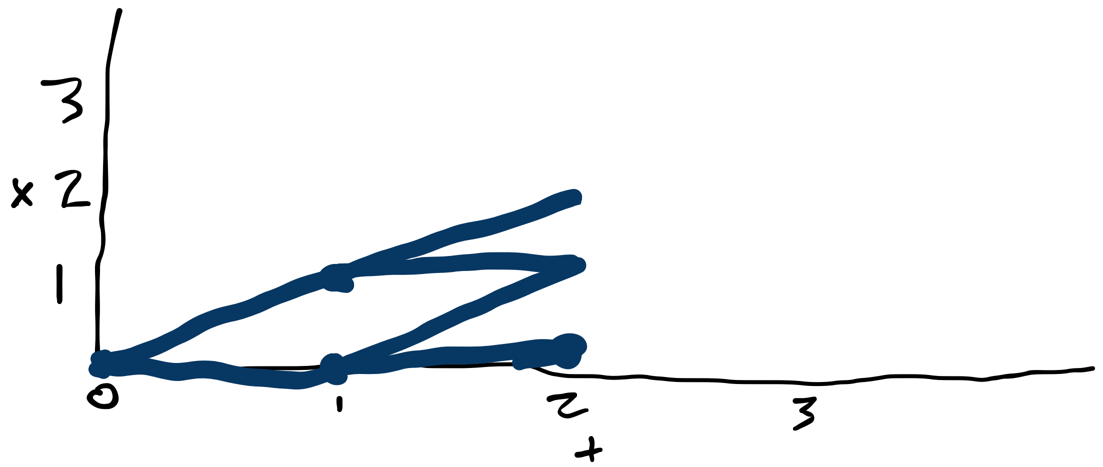

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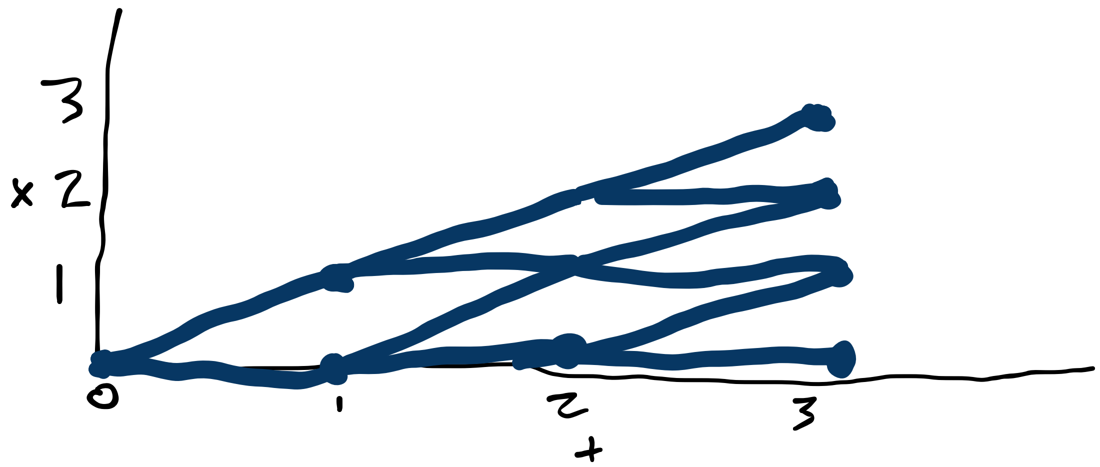

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Example:

Conditional

$x_0 = 0$

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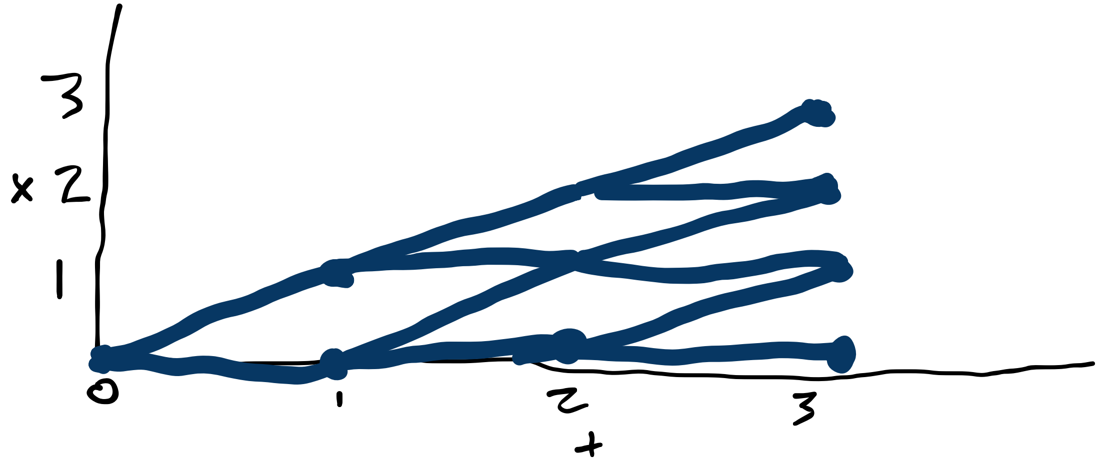

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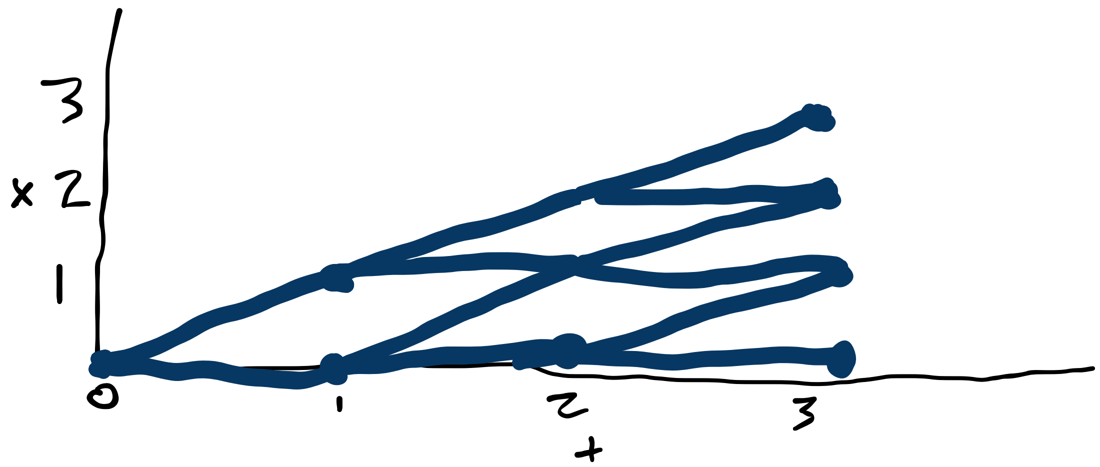
Conditional

x_{t+1}	$P(x_{t+1} \mid x_t)$
x_t	0.5
$x_t + 1$	0.5



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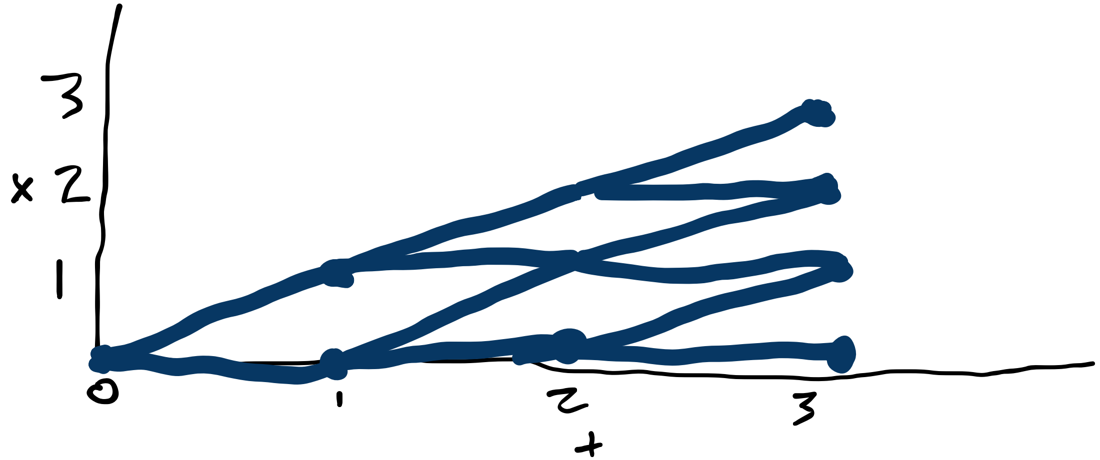

Joint

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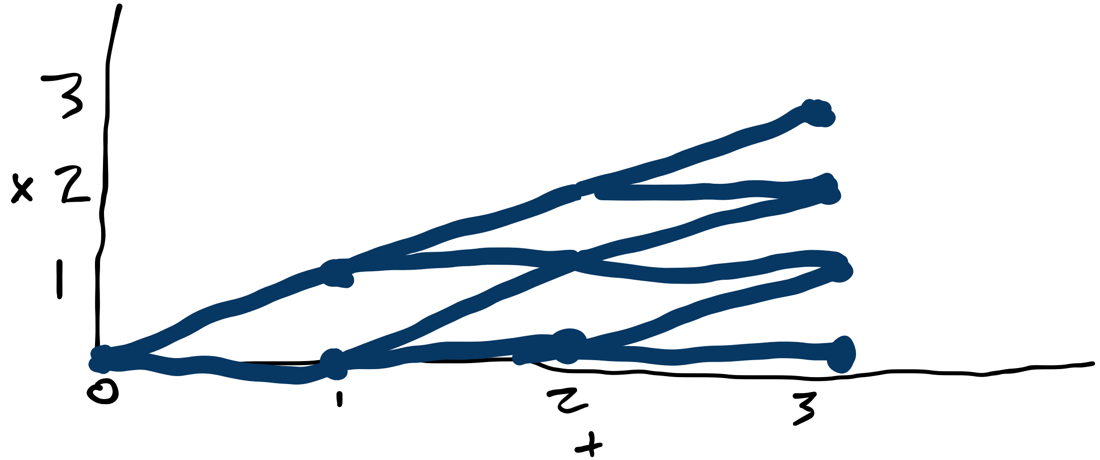

Joint

x0	x1	x2	P(x1, x2, x3)
0	0	0	0.25
0	0	1	0.25
0	1	1	0.25
0	1	2	0.25

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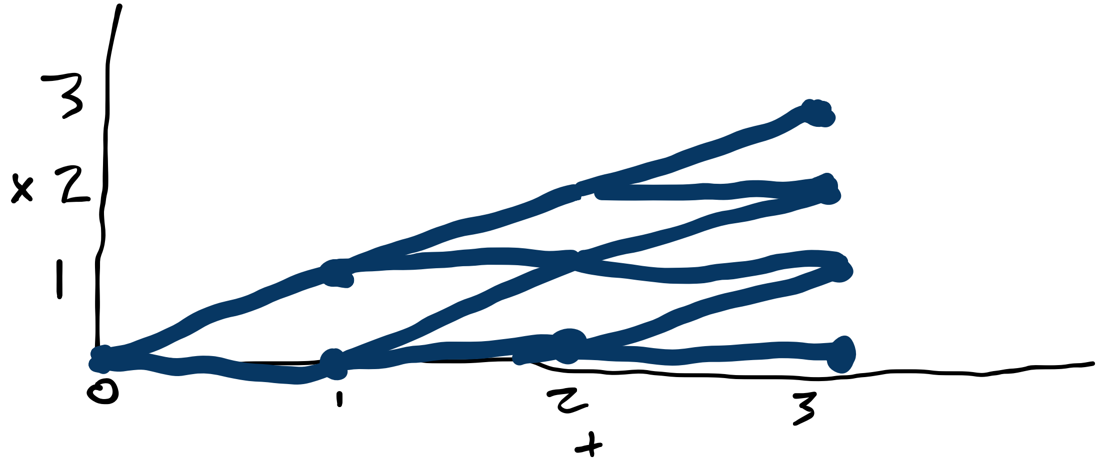

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Marginal

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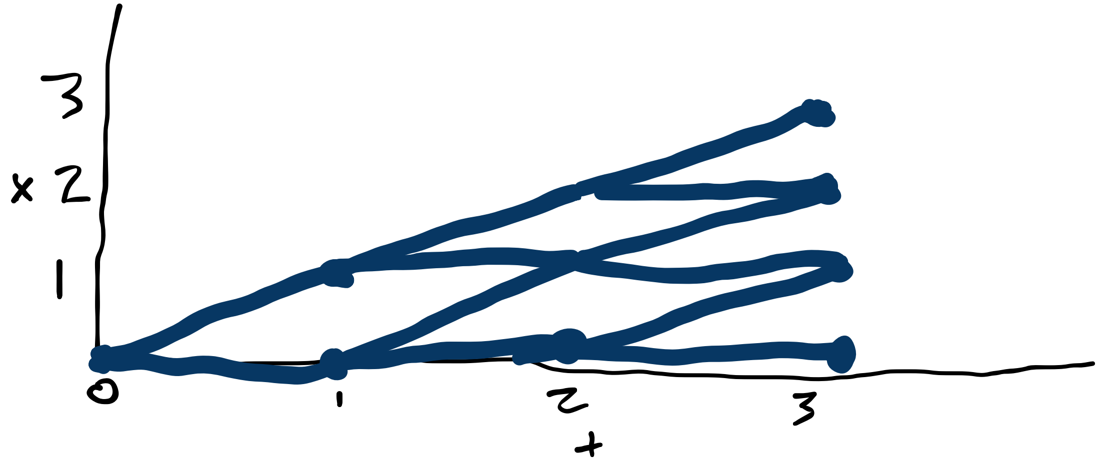

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Marginal

x_2	
0	0.5
1	0.5

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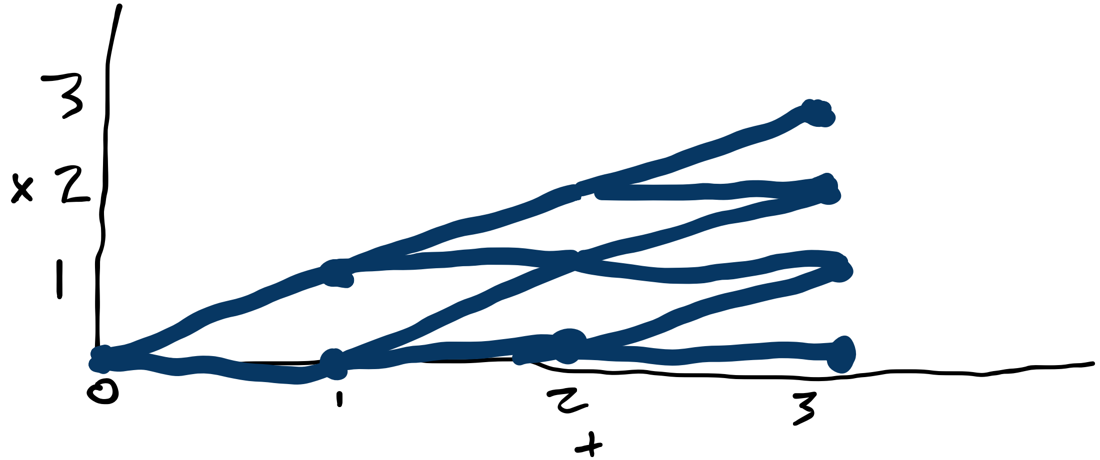

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0	0	0	0.25
0	0	1	0.25
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Marginal

x_2	
0	0.5
1	0.5

x_3	
0	0.25
1	0.5
2	0.25

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$P(x_{t+1} \mid x_t, x_{t-1}, \ldots) = P(x_{t+1} \mid x_t)$

In a *stationary* stochastic process (all in this class), this relationship does not change with time

Example:

$x_0 = 0$

$$\boxed{x_{t+1} = x_t + v_t}$$

$v_t \sim \mathcal{U}(\{0, 1\})$ (i.i.d.)

$U(0,1)$ $U(\{1,2,3\})$

Shorthand:

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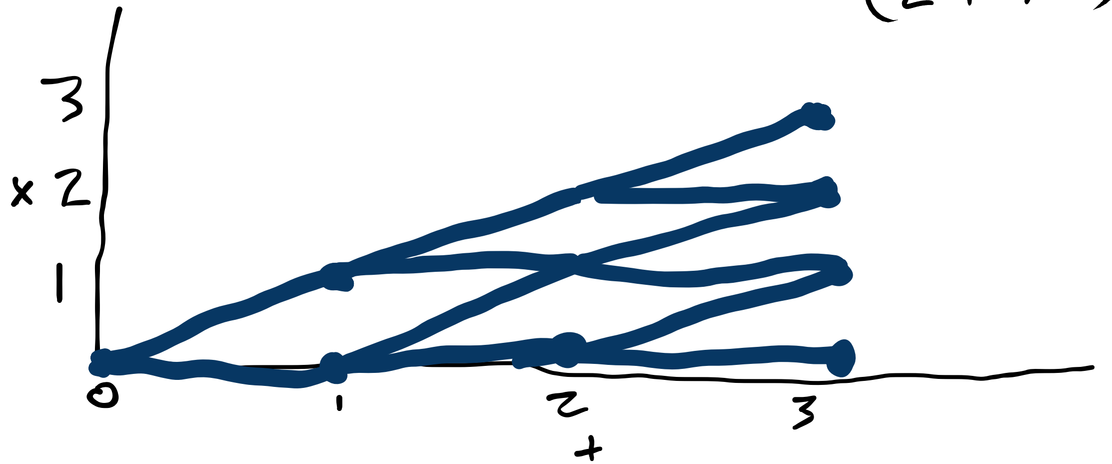

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Simulating a Stochastic Process

030-Stochastic-Processes.ipynb

Markov Process

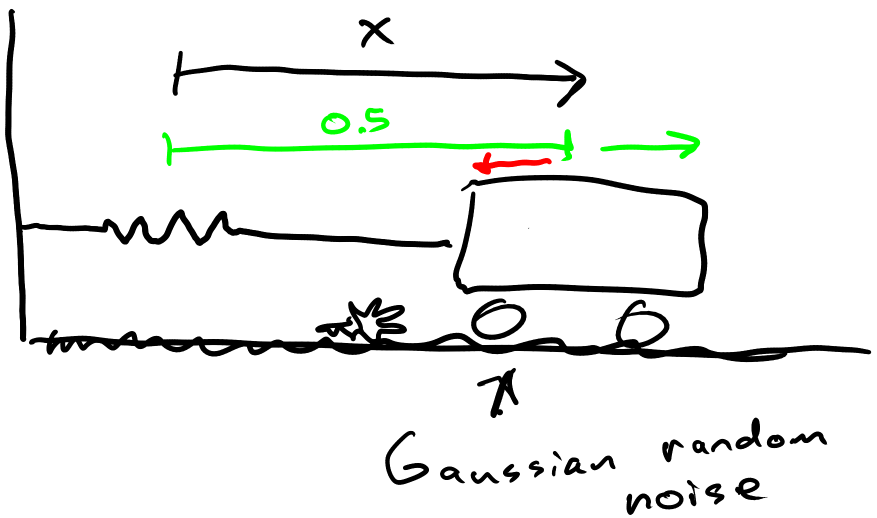
Markov Process

- A stochastic process $\{x_t\}$ is *Markov* if

$$P(x_t \mid x_{t-1}, x_{t-2}, \ldots, x_0) = P(x_t \mid x_{t-1})$$

Markov Process

- A stochastic process $\{x_t\}$ is *Markov* if
  $P(x_t \mid x_{t-1}, x_{t-2}, \ldots, x_0) = P(x_t \mid x_{t-1})$
- x_t is called the "state" of the process



Is $\{x_t\}$ a Markov Process? No

$$\underbrace{\begin{bmatrix} x_{t+1} \\ \dot{x}_{t+1} \end{bmatrix}}_{y_{t+1}} = \begin{bmatrix} 1 & \Delta t \\ -\frac{k}{m}\Delta t & 1 \end{bmatrix} \underbrace{\begin{bmatrix} x_t \\ \dot{x}_t \end{bmatrix}}_{y_t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_t$$

$v_t \sim N(0, \sigma)$

Is $\{y_t\}$ a Markov Process? Yes

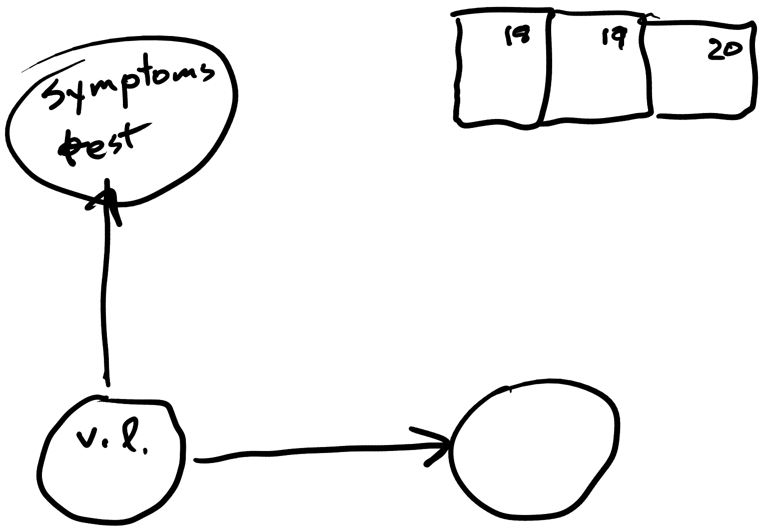
Breakout Rooms

Breakout Rooms

- Name

Breakout Rooms

- Name
- Suppose you want to create a Markov model that describes whether you will test positive for COVID on a given day. What information should be included the state of that model?



state: #cases today
\# exposure in past 5 days
prior test result
days since vaccination
symptoms
observations

Breakout Rooms

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- Suppose you have a factory with an entrance/exit road, and you want to define a Markov process to model when trucks will reach the intersection. What should be in the state?

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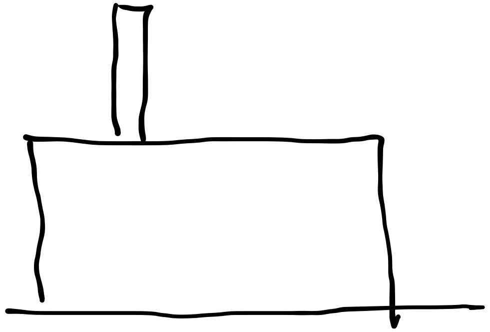

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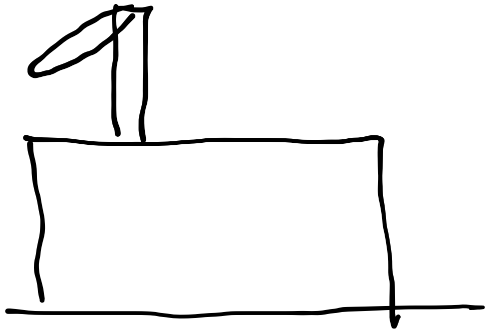

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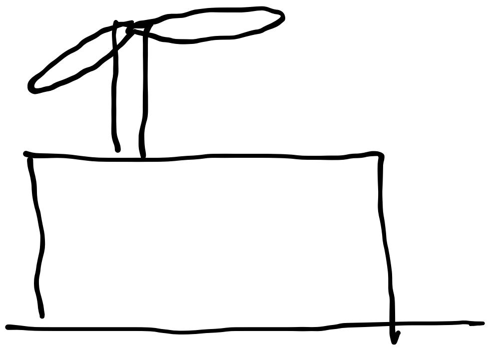

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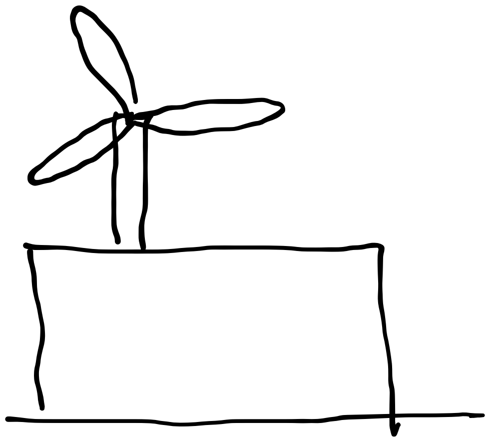

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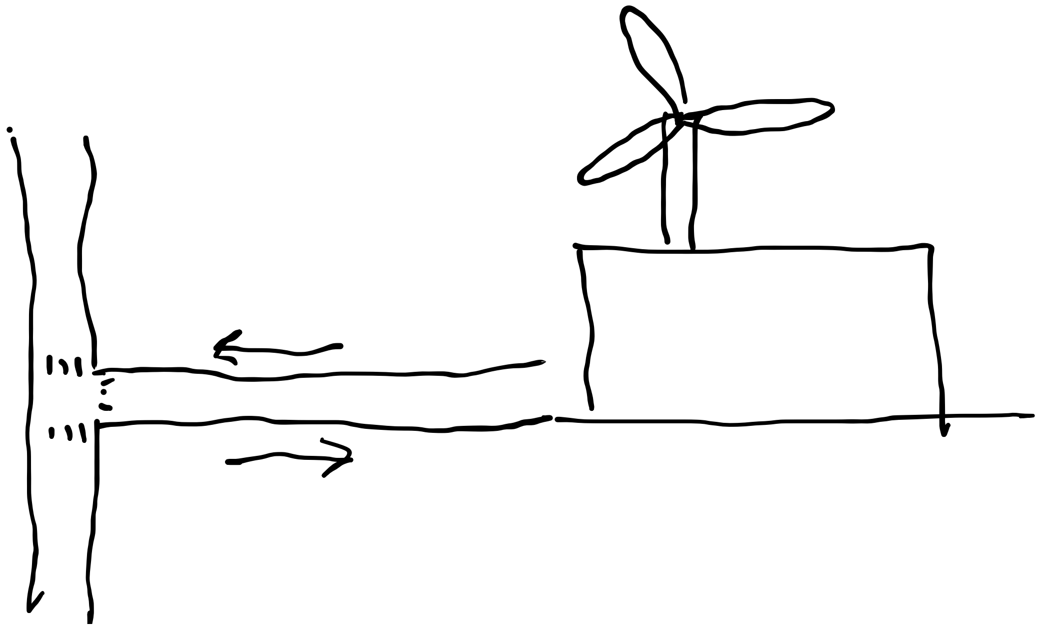
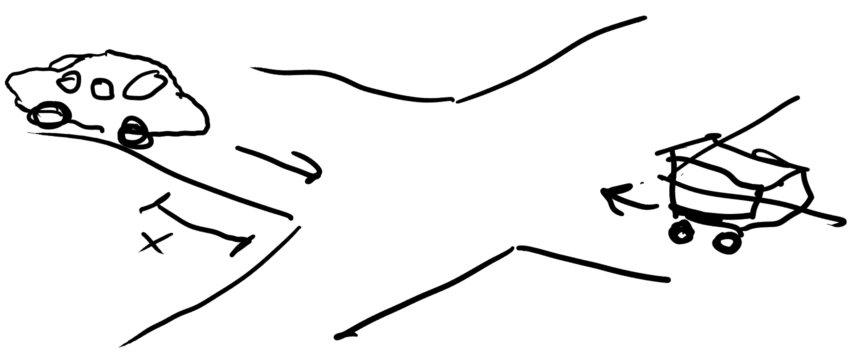


Hidden Markov Model

(Often you can't measure the whole state)

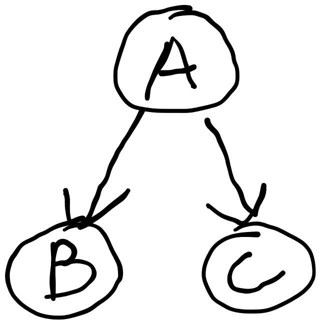


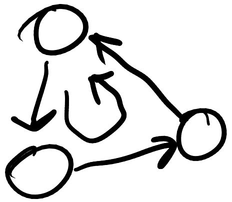
state: $(x, \dot{x}, \text{intention})$

observation: $(x, \dot{x})$

Bayesian Networks

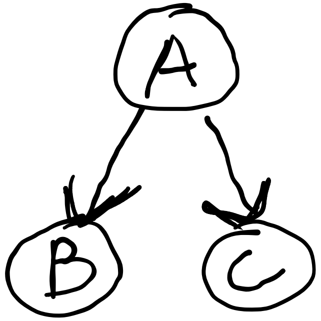
Bayesian Networks



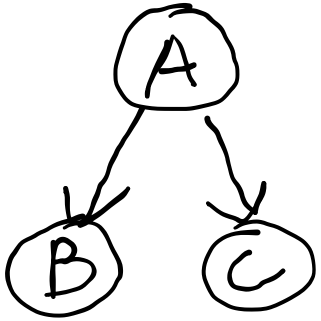


Bayesian Networks

A *Bayesian Network* is a directed acyclic graph (DAG) that encodes probabilistic relationships between R.V.s



Bayesian Networks



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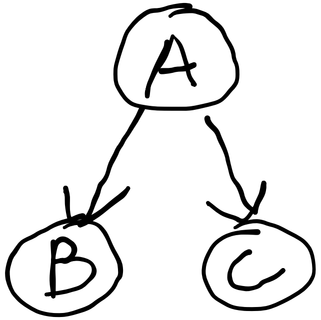

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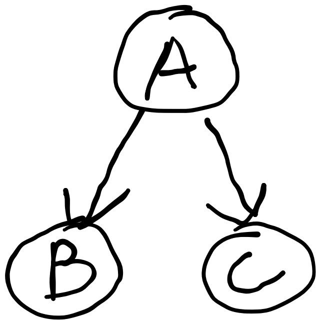

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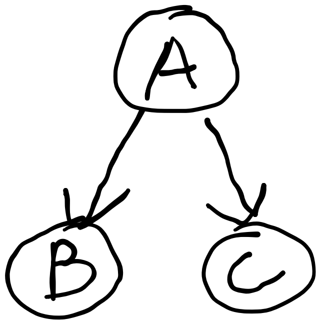
Concretely:

Bayesian Networks



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x_1
x_2
$\vdots$
x_n

- Nodes: R.V.s
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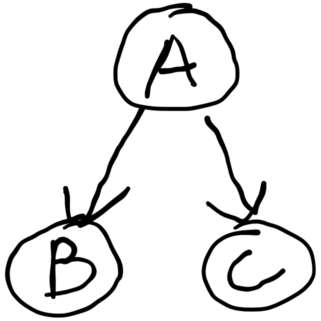
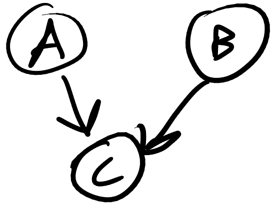
Concretely: $$P(x_i \mid \underline{x_{1:n \setminus i}}) = P(x_i \mid Pa(x_i))$$

Bayesian Networks





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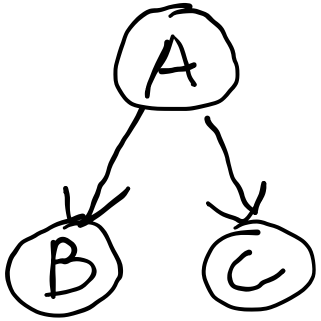
Parents

Bayesian Networks



$$P(B \mid A, C) = P(B \mid A)$$

$B \perp C$? No
$B \perp C \mid A$ Yes

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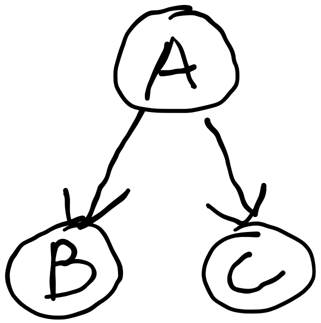
Parents

Bayesian Networks



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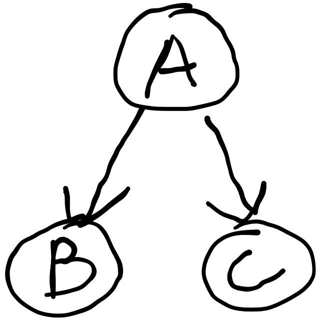
Markov Process

Bayesian Networks



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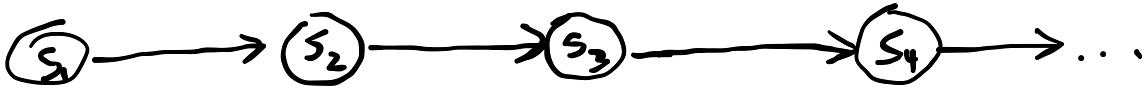
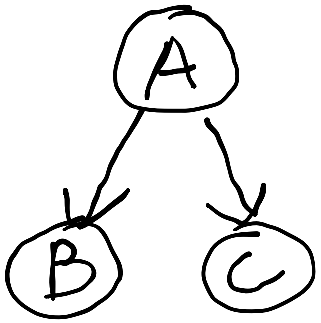
Parents

$P(s_3 \mid s_1, s_2) = P(s_3 \mid s_2)$

Markov Process



Bayesian Networks



$$P(B \mid A, C) = P(B \mid A)$$

A *Bayesian Network* is a directed acyclic graph (DAG) that encodes probabilistic relationships between R.V.s

- Nodes: R.V.s
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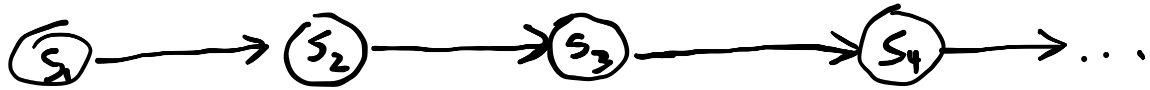

Hidden Markov Model

Bayesian Networks

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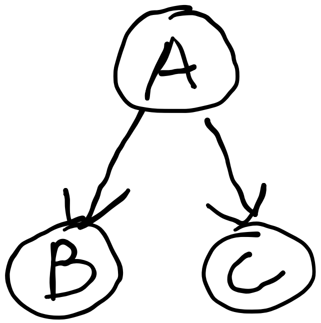

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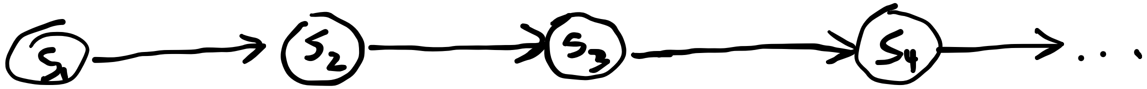

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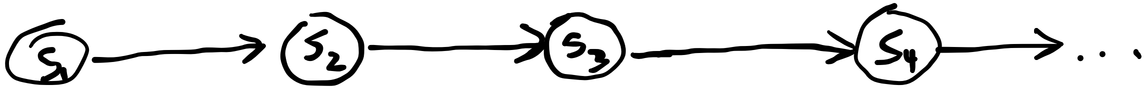

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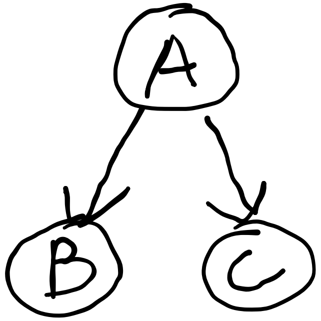

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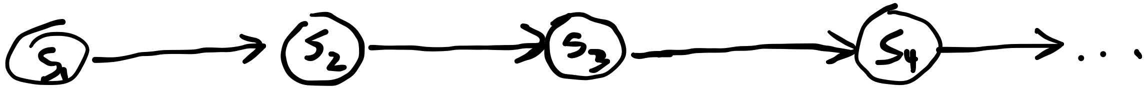

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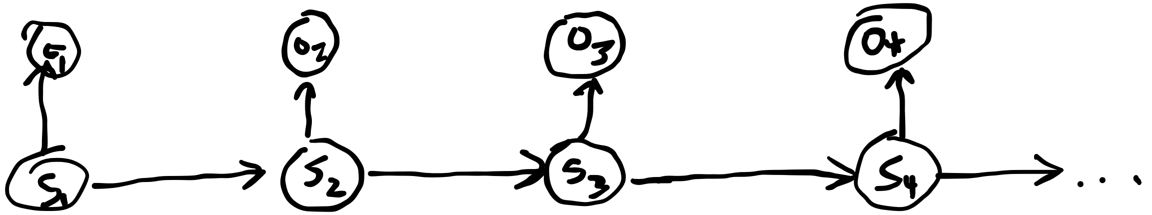

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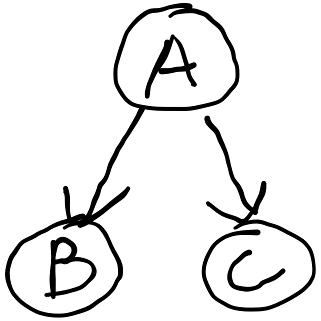

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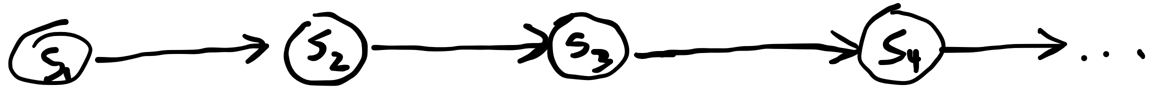

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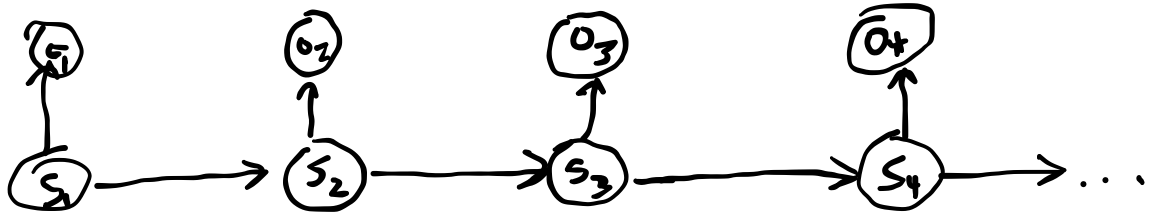
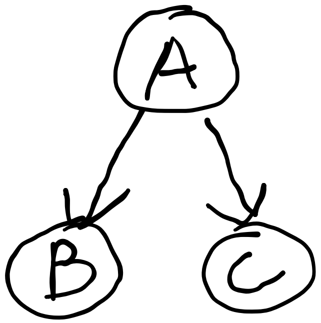


Dynamic Bayesian Network

Bayesian Networks



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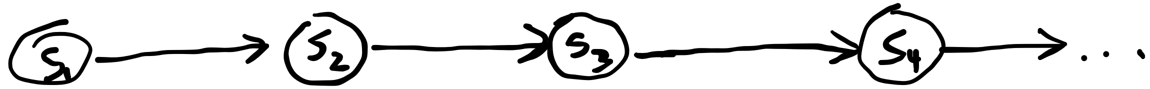

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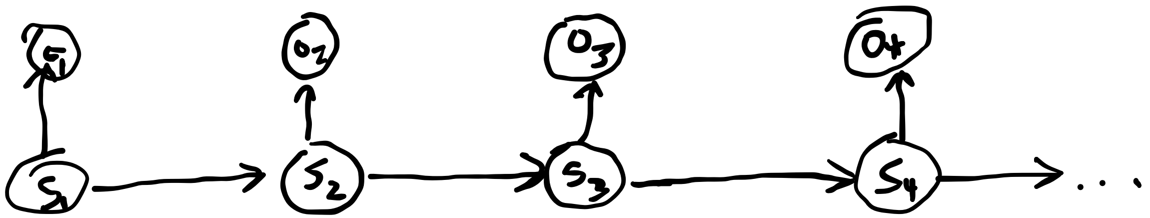
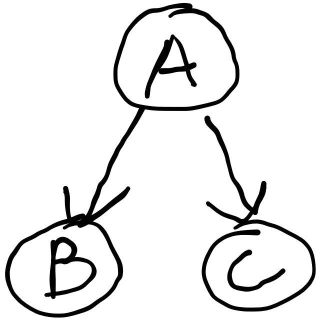


Dynamic Bayesian Network (One step)

Bayesian Networks



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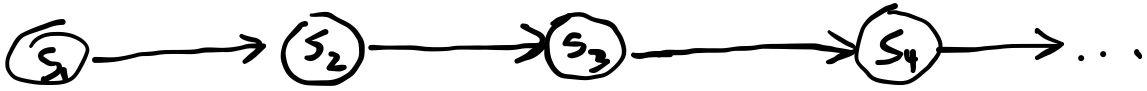

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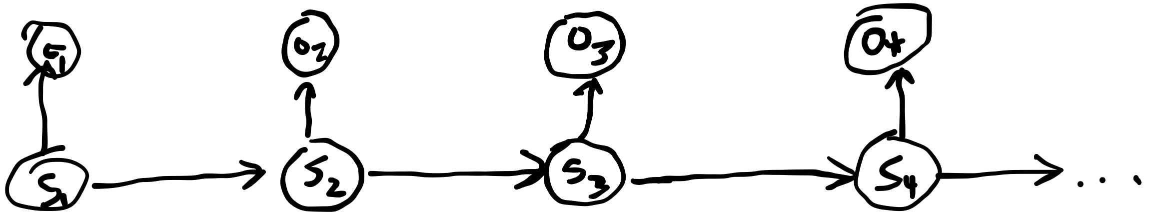

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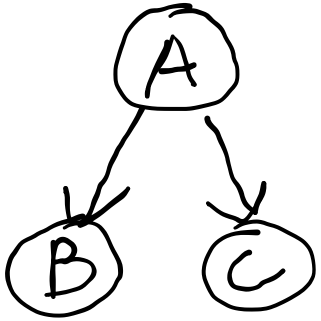

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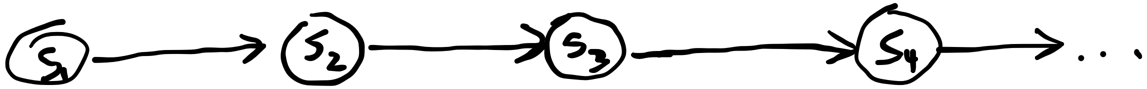

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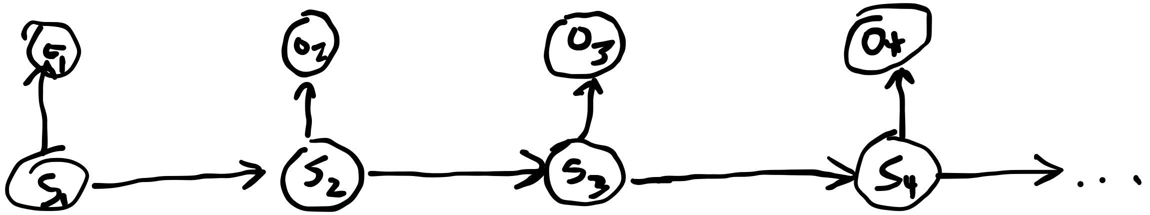

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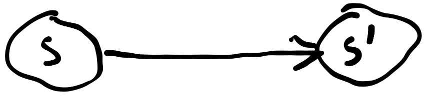

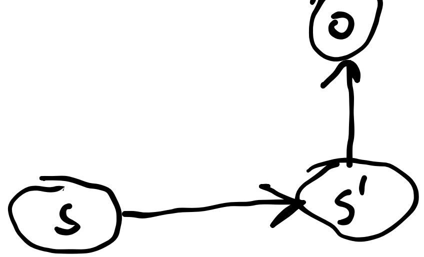

Decision Networks and MDPs

Decision Networks and MDPs

Decision Network

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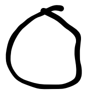

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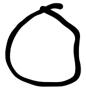 Chance node

Decision Networks and MDPs

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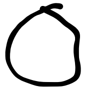
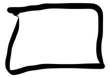
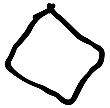
 Chance node

 Decision node



Decision Networks and MDPs

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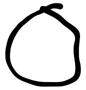
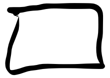
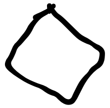
 Chance node

 Decision node

 Utility node

Decision Networks and MDPs

Decision Network

Chance node

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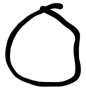
Utility node

MDP Dynamic Decision Network

Decision Networks and MDPs

Decision Network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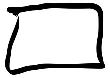
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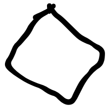
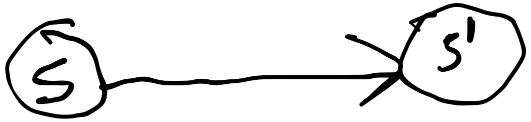
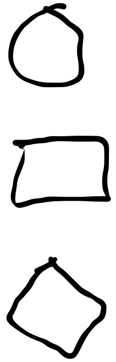

Utility node

MDP Dynamic Decision Network



Decision Networks and MDPs

Decision Network



Chance node

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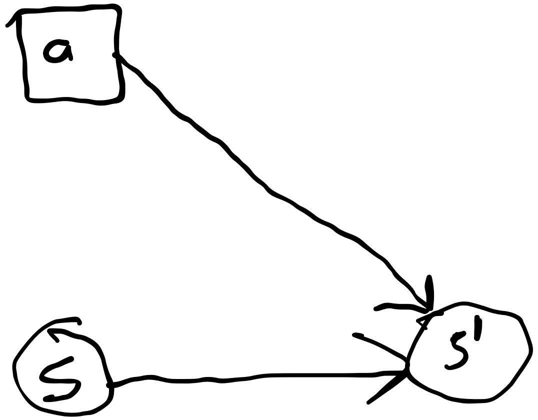

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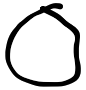
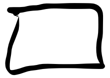
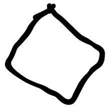
 Chance node

 Decision node

 Utility node

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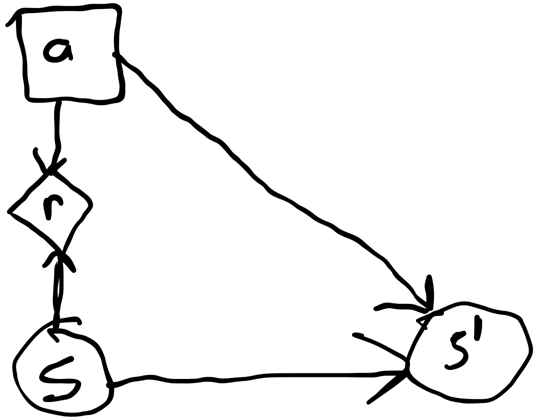

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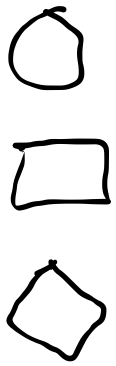

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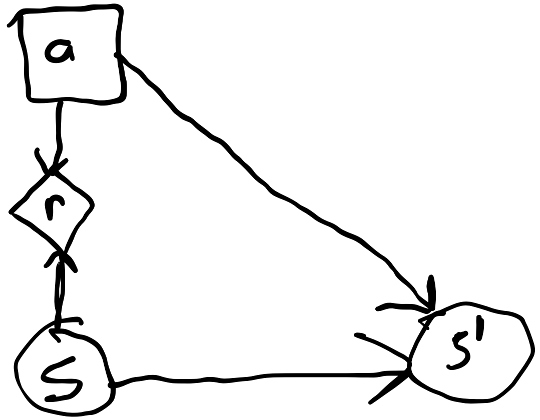
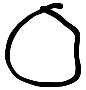
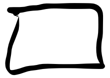


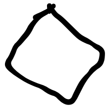
MDP Optimization problem

Decision Networks and MDPs

Decision Network

 Chance node

 Decision node

 Utility node

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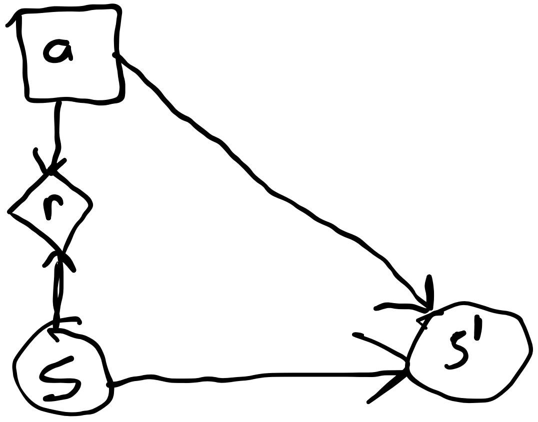
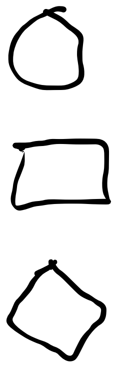


MDP Optimization problem

$$\text{maximize} \quad \mathrm{E}\left[\sum_{t=1}^{\infty} r_t\right]$$

Decision Networks and MDPs

Decision Network



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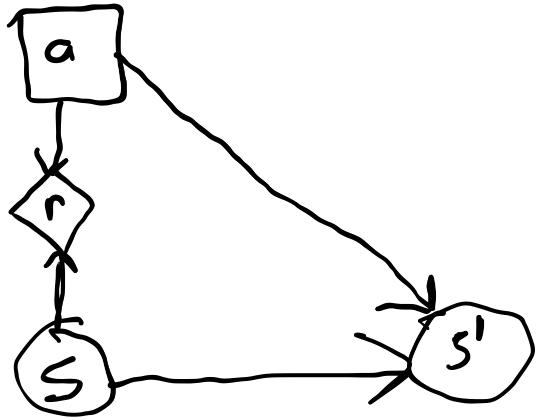

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Not well formulated!

Decision Networks and MDPs

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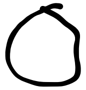

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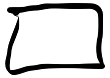

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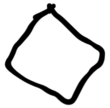

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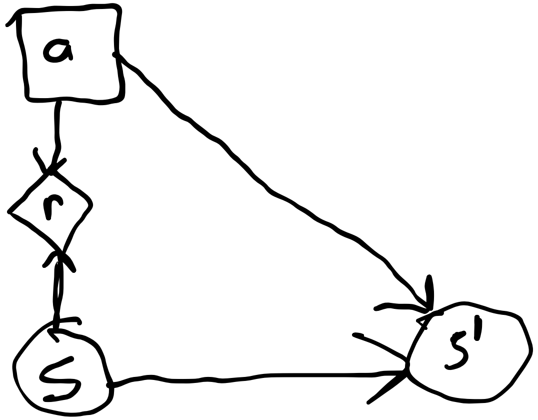

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Infinite

Finite MDP Objectives

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1. Finite time

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Finite MDP Objectives

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$$\mathbf{E}\left[\sum_{t=0}^{T} r_t\right]$$

2. Average reward

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$$\lim_{n\to\infty} \mathbf{E}\left[\sum_{t=0}^{n} r_t\right]$$

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3. Discounting

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discount $\gamma \in [0, 1)$

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then

$$\frac{\underline{r}}{1-\gamma} \leq \sum_{t=0}^{\infty} \gamma^t r_t \leq \frac{\bar{r}}{1-\gamma}$$

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if $\underline{r} \le r_t \le \bar{r}$

then

$$\frac{\underline{r}}{1-\gamma} \le \sum_{t=0}^{\infty} \gamma^t r_t \le \frac{\bar{r}}{1-\gamma}$$

4. Terminal States

Finite MDP Objectives

1. Finite time

$$\mathbf{E}\left[\sum_{t=0}^{T} r_t\right]$$

2. Average reward

$$\lim_{n\to\infty}\mathbf{E}\left[\sum_{t=0}^{n} r_t\right]$$

3. Discounting

$$\mathbf{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t\right]$$

discount $\gamma \in [0,1)$

typically 0.9, 0.95, 0.99

if $\underline{r} \leq r_t \leq \bar{r}$

then

$$\frac{\underline{r}}{1-\gamma} \leq \sum_{t=0}^{\infty} \gamma^t r_t \leq \frac{\bar{r}}{1-\gamma}$$

4. Terminal States

Infinite time, but a terminal state (no reward, no leaving) is always reached with probability 1.

Guiding Question

- What does "Markov" mean in "Markov Decision Process"?

$$P(x_t \mid x_{t-1} \ldots x_0) = P(x_t \mid x_{t-1})$$