Last Time

1. Relationships Between RVs — Joint, Conditional, Marginal
2. Inferrence

Bayes Rule

Today:
What does "Markov" mean.

Stochastic Process

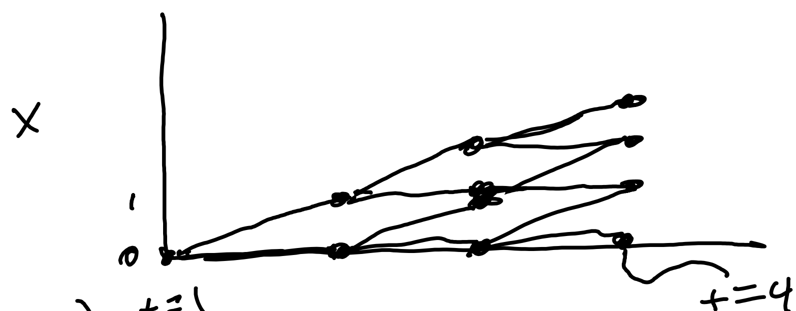
Collection of RVs indexed by time.

$$\{x_t\}_{t=1}^{\infty} = \{x_1, x_2, x_3 \ldots\}$$

Example:

$x_1 = 0$ $\qquad x_{t+1} = x_t + v_t$ $\qquad x' = x + v$

v_t iid $\qquad v_t \sim U(\{0,1\})$



Conditional

$P(x_{t+1} \mid x_t)$

x_{t+1}	
x_t	0.5
x_t+1	0.5

Joint

x_1	x_2	x_3	
0	0	0	0.25
0	0	1	0.25
0	1	1	0.25
0	1	2	0.25

$\overset{1}{P(x_1=0)}\,\overset{0.5}{P(x_2=0 \mid x_1=0)}$
$P(x_3=0 \mid x_2=0)$
0.5

Marginal

x_2	
0	0.25
1	0.2

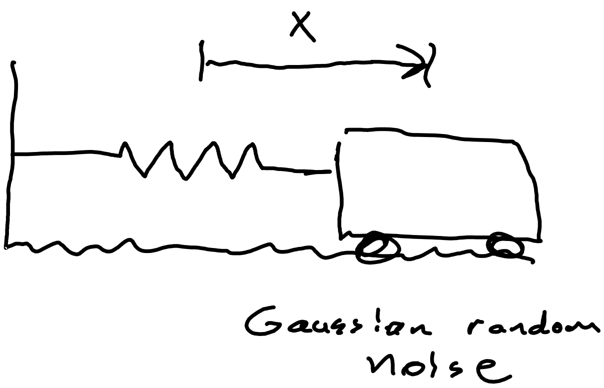
x_3	
0	0.25
1	0.5
2	0.25

Markov Process

Def. A S.P. $\{x_t\}$ is Markov if

$$P(x_t \mid x_{t-1}, x_{t-2} \cdots x_1) = P(x_t \mid x_{t-1})$$

x_t is known as the "state"



Can this be described as a Markov Process

~~$\{x_t\}$ is Markov~~

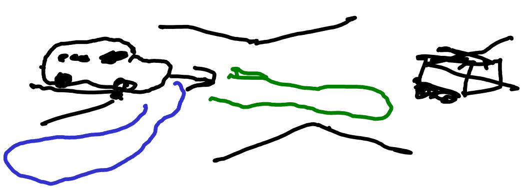
$$\boxed{\begin{bmatrix} x_{t+1} \\ \dot{x}_{t+1} \end{bmatrix}} = \begin{bmatrix} 1 & \Delta t \\ -\frac{k}{m}\Delta t & 1 \end{bmatrix} \boxed{\begin{bmatrix} x_t \\ \dot{x}_t \end{bmatrix}} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_t$$

$$y_t \equiv (x_t, \dot{x}_t)$$

$\{y_t\}$ is Markov!

$$P(y_t \mid y_{t-1} \cdots) = P(y_t \mid y_{t-1})$$

Sometimes you can't measure the whole state



state

(position, velocity, intention)

Hidden Markov Model

Bayesian Network

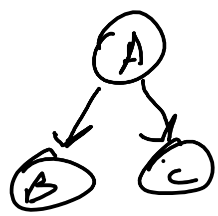
DAG Node: R.V.

Edge: Direct Probabilistic Relationship

Concretely $\quad P(x_i \mid x_{1:n \setminus i}) = P(x_i \mid Pa(x_i))$

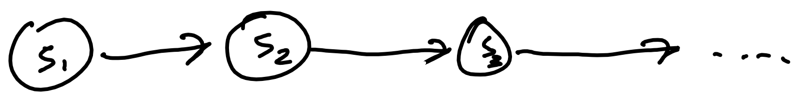
$x_{1:n}$

$P(B \mid A, C) = P(B \mid A)$

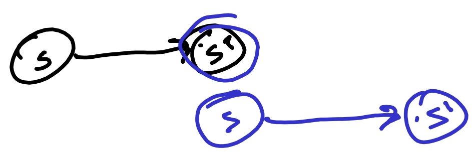


B and C independent?
$B \perp C$ Not necesarily
$B \perp C \mid A$

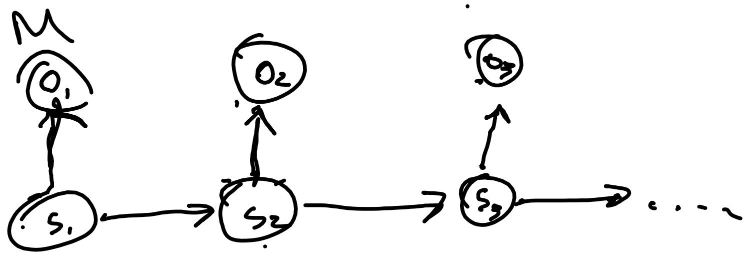
Markov Process

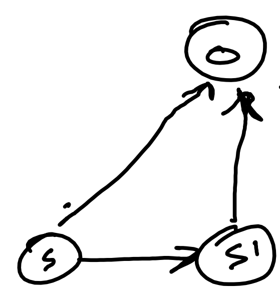


Dynamic Bayesian Networks



HMM

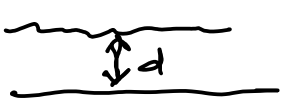




Breakout Rooms Favorite Movie

Boulder Creek.

- Character has to make decision under uncertainty

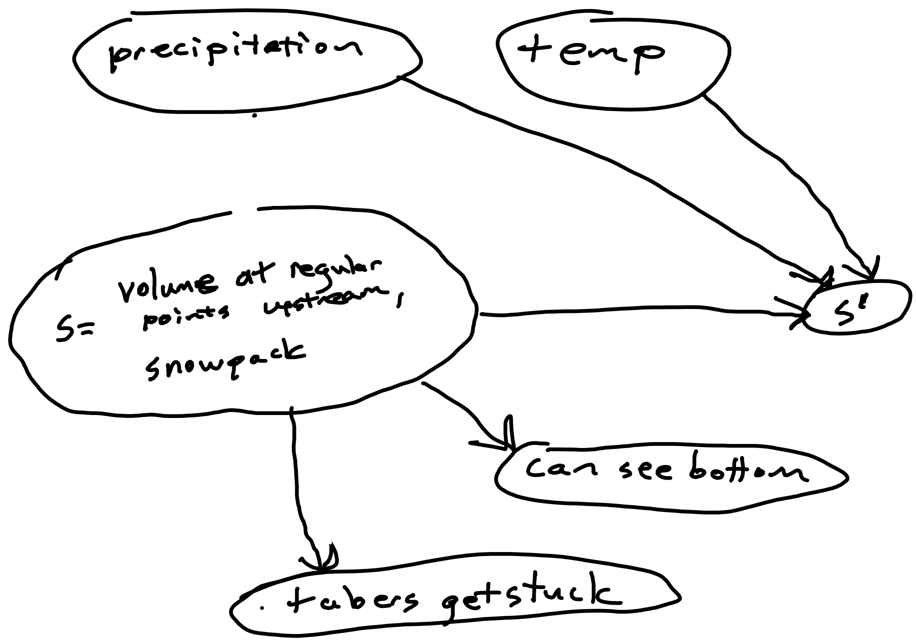


state?

tubers get stuck
see bottom
} observation

physical width, length of river

flow rate, precipitation, temp
width river, volumetric flow rate
source of wate, snow, width, flow rate

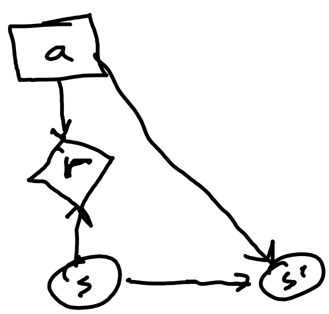


Markov Decision Process

Decision Network

- chance nodes
- decision node
- utility node

MDP Dynamic Decision Network



Optimization

$$\text{maximize } E\left[\sum_{t=1}^{\infty} r_t\right]$$

"return"

Finite Rewards

1) Finite time

$$\sum_{t=1}^{T} r_t$$

2) Average reward

$$\lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} r_t$$

3) Discount

$$\sum_{t=1}^{\infty} \gamma^t r_t \leq \frac{\bar{r}}{1-\gamma}$$ discount $\gamma \in [0,1)$

0.9, 0.95, 0.99

4) Terminal states

problem reaches terminal state w.p. 1