

ASEN 5519-003 Decision Making under Uncertainty

Homework 1: Probabilistic Models

January 15, 2023

1 Conceptual Questions

Question 1. (20 pts) Consider the following joint distribution of three binary-valued random variables, A , B , and C :

A	B	C	$P(A, B, C)$
0	0	1	0.15
0	1	0	0.05
0	1	1	0.01
1	0	0	0.14
1	0	1	0.18
1	1	0	0.29
1	1	1	0.07

- a) What is the probability of the outcome $A = 0, B = 0, C = 0$?
- b) What is the marginal distribution of A ?
- c) What is the conditional distribution of A given $B = 0$ and $C = 1$?

Question 2. (20 pts) Let B be a uniformly-distributed binary random variable and let A be a real-valued random variable with the following conditional distribution:

$$A \mid B = 0 \sim \mathcal{U}(0, 2)$$

$$A \mid B = 1 \sim \mathcal{U}(1, 3)$$

- (a) Plot or draw the probability density functions for the conditional distribution of A .
- (b) Plot or draw the marginal density function of A .
- (c) What is the probability that $A = 1.5$?
- (d) What is the probability that $A \in [1.5, 1.6]$?

Question 3. (20 pts) 2% of women at age forty who participate in routine screening have breast cancer. 86% of those with breast cancer will get positive mammograms. 8% of those without breast cancer will also get positive mammograms. A woman in this age group had a positive mammogram in a routine screening. What is the probability that she actually has breast cancer?

Question 4. (20 pts) Suppose that a stationary stochastic process $\{x_t\}$ is defined by the following equation: $x_{t+1} = 1.5x_t - x_{t-1} + v_t$ where v_t are independent, identically distributed random variables with $v_t \sim \mathcal{N}(0.0, 0.2^2)$.

- (a) Simulate and plot 10 20-step trajectories sampled from this process with $x_0 = x_{-1} = 1$ (as always, submit your code for this).
- (b) Is this process a Markov process if the state is defined as x_t ? Why or why not?
- (c) If you only had access to the trajectories you plotted what evidence could you use to convince someone that this process is or is not Markov?
- (d) What would need to be included in the state at time t to make this a Markov process?

2 Programming

Question 5. (20 pts) In this exercise, you will write a function in Julia that will require you to learn some concepts useful for the rest of the course. Your function should accept four positional arguments:

- **a**: A real number.
- **b**: A square matrix.
- **c**: An array that has the same size and element type as **b**.
- **d**: A Dict with **String** keys and vector values.

The function should return a **NamedTuple** with the following entries:

- **relu**: The maximum of **a** and zero (the “rectified linear unit” function).
- **matrix**: The matrix product of **b** and **c**.
- **elementwise**: The element-wise product of **b** and **c**.
- **max**: The element-wise maximum of all of the vectors in **d** with keys that start with the character '**a**'.

The function must be completely “type-stable” (see the “Performance Tips” section of the Julia manual to learn what this means and use `@code_warntype` to debug) and must return objects with similar types to the input arguments (for instance if **a** is an **Int**, **relu** should also be an **Int**).

Evaluate this function with `DMUStudent.HW1.evaluate` and submit the resulting json file *along with a listing of the code*. A score of 1 will receive full credit.