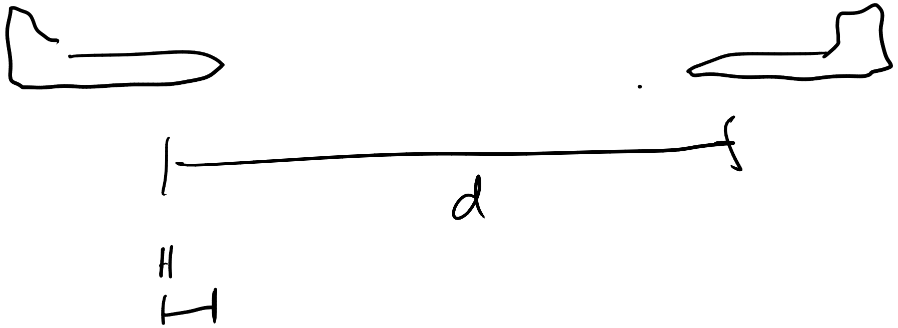
Last Time



$d = 0$, $d = v\Delta t$

Last Time

- What are the differences between online and offline solutions?
- Are there solution techniques that are *independent* of the state space size?

Guiding Questions

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- What tools do we have to solve MDPs with continuous S and A?

Continuous S and A

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e.g. $S \subseteq \mathbb{R}^n$, $A \subseteq \mathbb{R}^m$

Continuous S and A

e.g. $S \subseteq \mathbb{R}^n$, $A \subseteq \mathbb{R}^m$

The old rules still work!

$$V^*(s) = \max_a \left(R(s,a) + \gamma \int_S T(s'|s,a)\, V^*(s') \right)$$

$$V^\pi(s) = R(s, \pi(s)) + \gamma \underset{s' \sim T(s,\pi(s))}{E} \left[V^\pi(s') \right]$$

Linear Dynamics, Quadratic Reward

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$$s' = T_s s + T_a a + w$$

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w is zero-mean, finite variance R.V.

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LQR

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if $w \sim \mathcal{N}(0, \Sigma)$, then $q_{h+1} = \Sigma_{i=1}^h \mathrm{tr}(\Sigma P_i)$

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"Linear Quadratic Regulator" / "LQR"

If not Linear Quadratic…

Offline:

Online:

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Offline:

- Approximate Dynamic Programming (ADP)

Online:

If not Linear Quadratic...

Offline:

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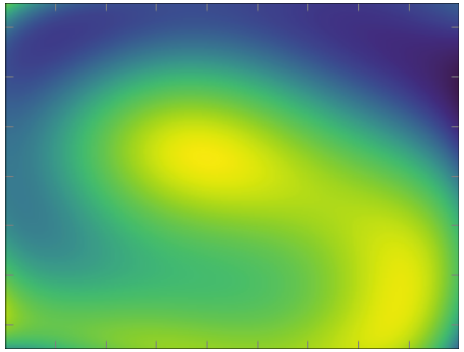
V_θ



<u>Online:</u>

If not Linear Quadratic...

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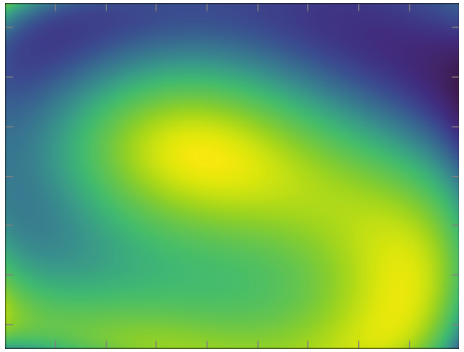

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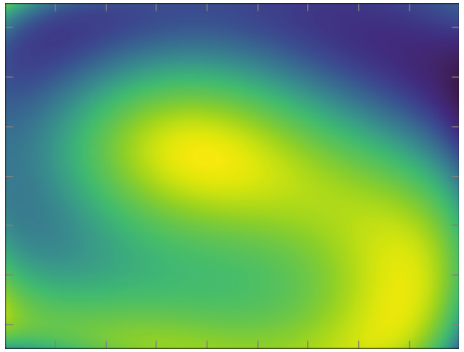

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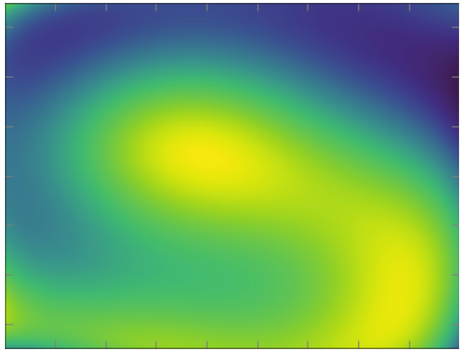

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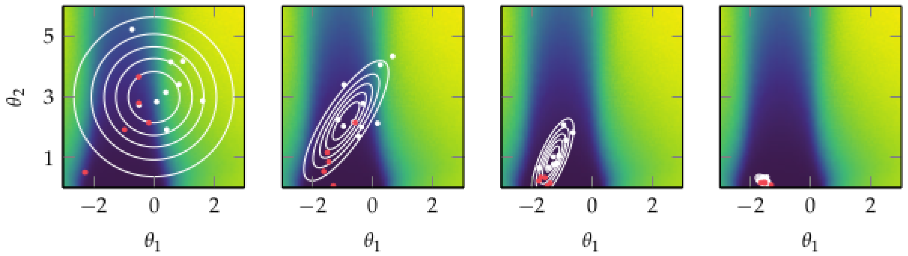

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Online:

- Model Predictive Control (MPC)

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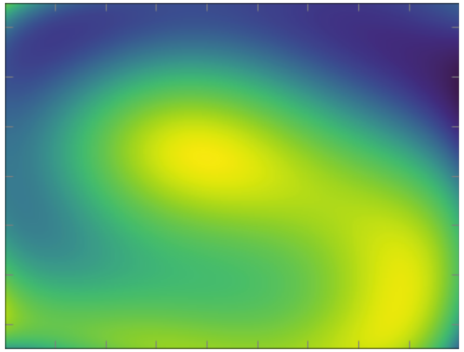

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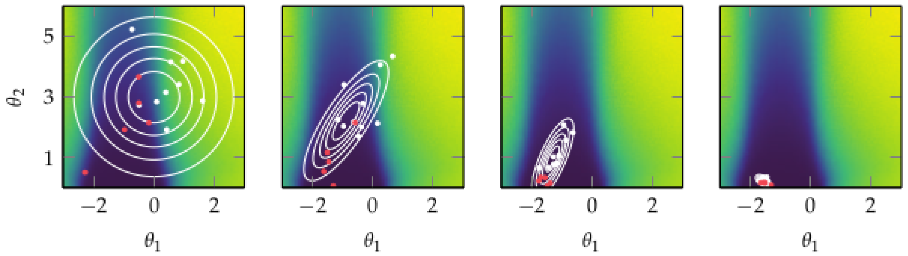

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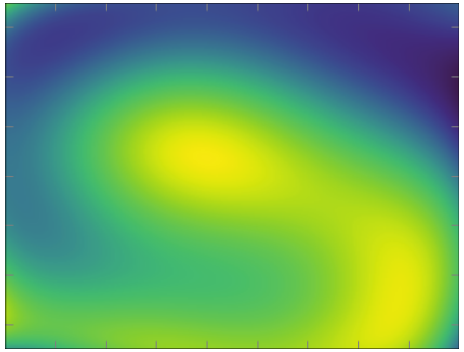

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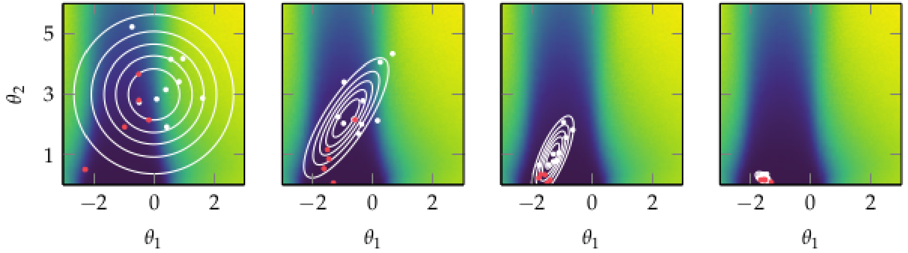

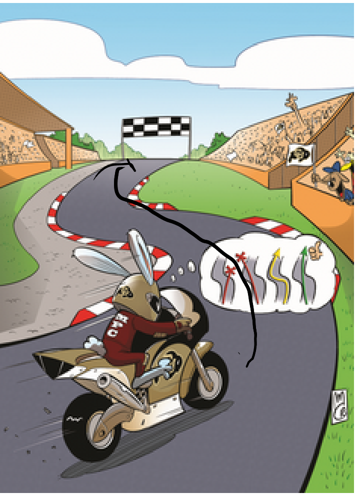

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- Sparse Tree Search/Progressive Widening

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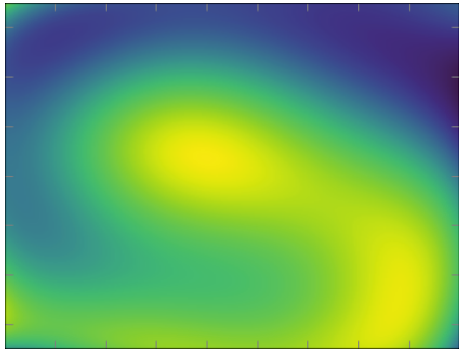

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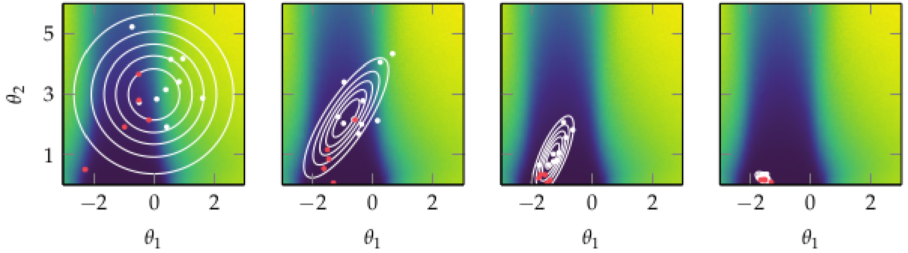

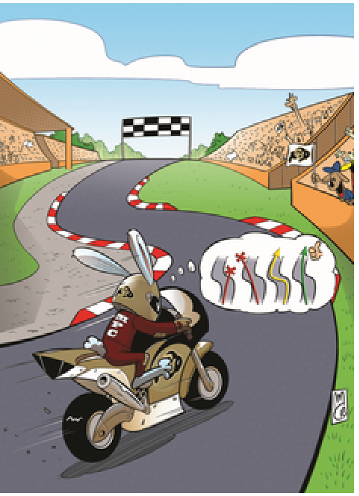

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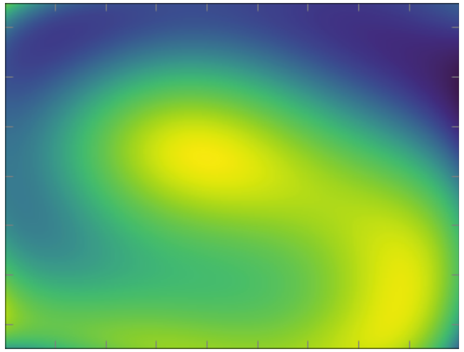

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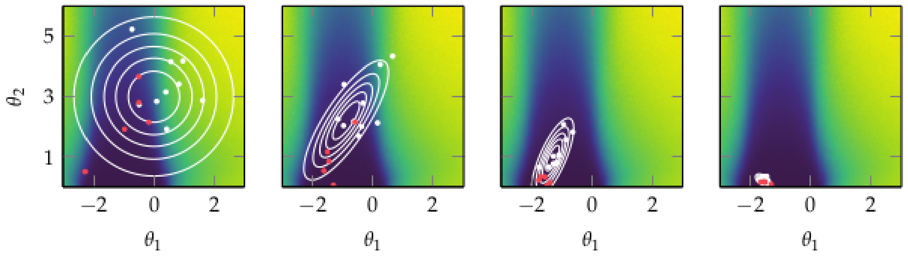

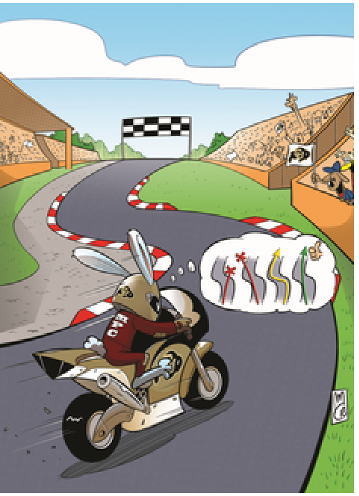

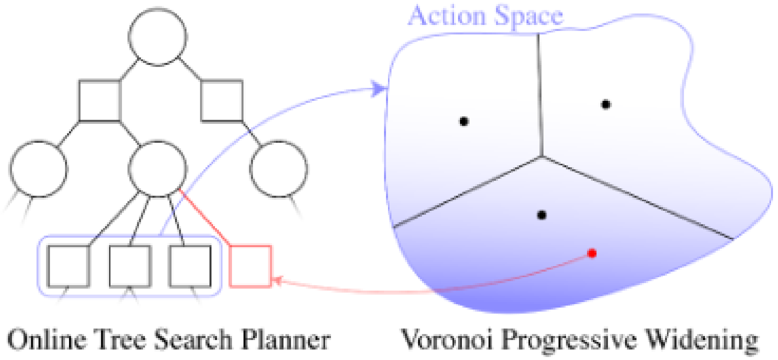

Approximate Dynamic Programming

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$V_\theta(s) = f_\theta(s)$ (e.g. neural network)

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while not converged

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$\hat{V}' \leftarrow B_{\text{approx}}[V_\theta]$

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Monte Carlo

Approximate Dynamic Programming

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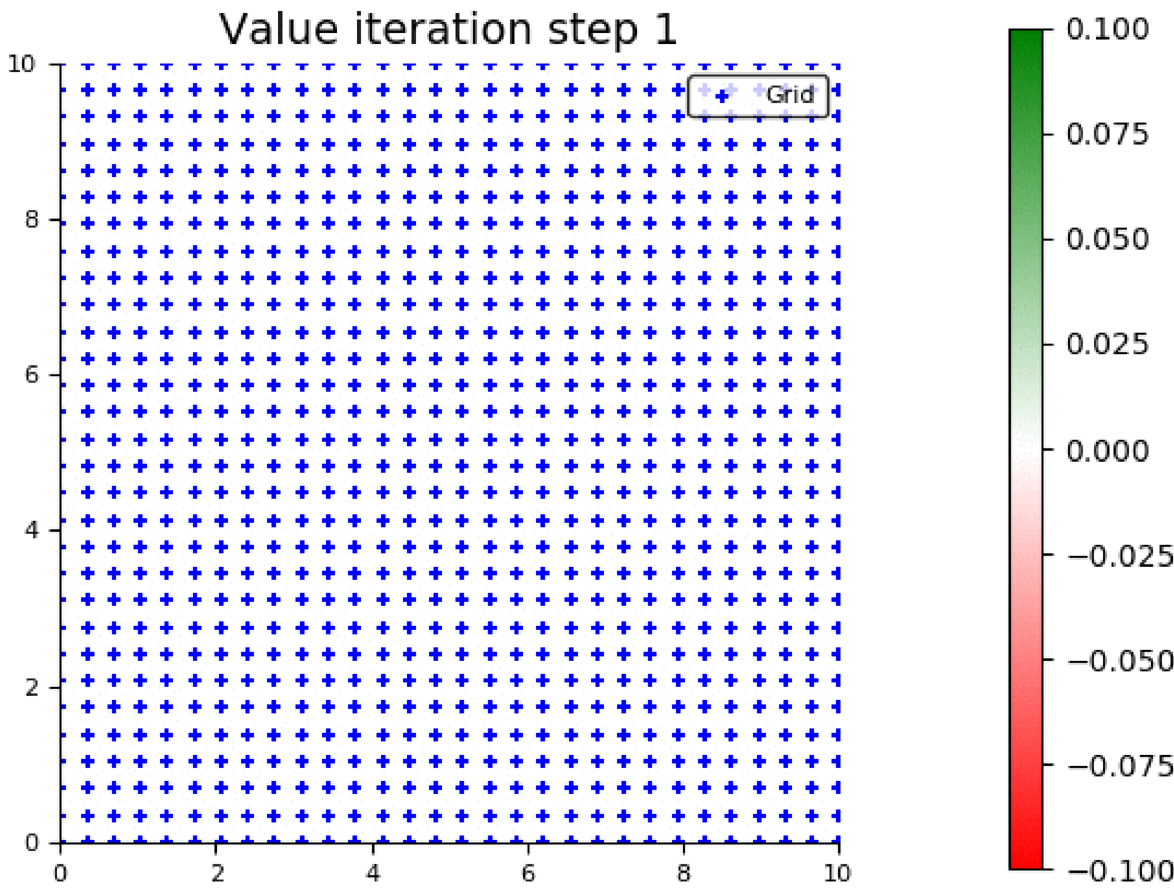
LSVI



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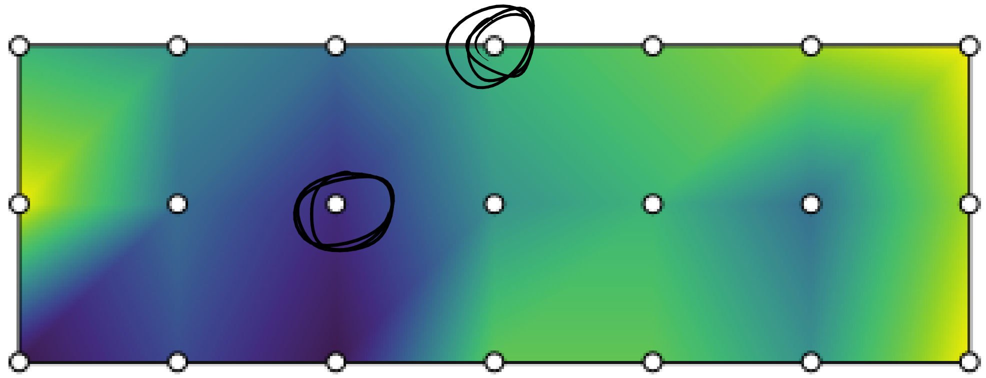

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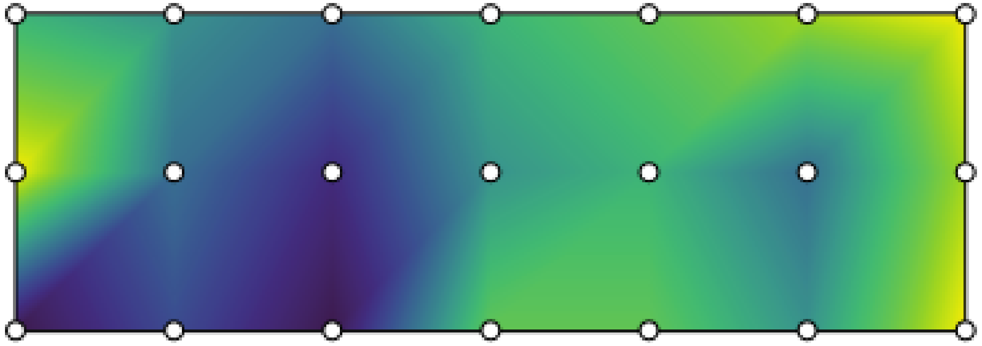
- Local: (e.g. simplex interpolation)
- Global: (e.g. Fourier, neural network)



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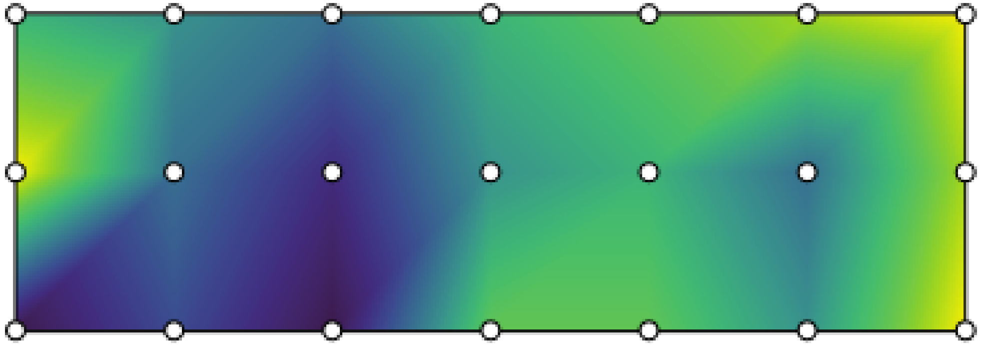

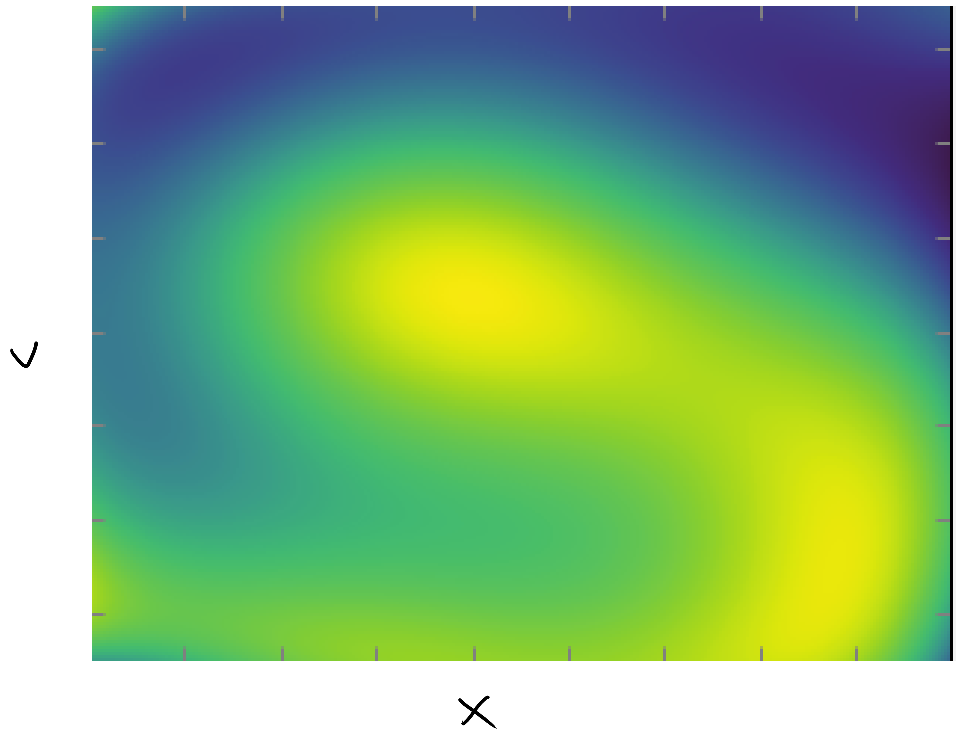

$s = (x, v)$

$V_\theta(s) = \theta^T \beta(s)$

$$\beta(s) = \begin{array}{l} [1, \\ x, \quad v, \\ x^2, \quad xv, \quad v^2, \\ x^3, \quad x^2v, \quad xv^2, \quad v^3, \\ x^4, \quad x^3v, \quad x^2v^2, \quad xv^3, \quad v^4, \\ x^5, \quad x^4v, \quad x^3v^2, \quad x^2v^3, \quad xv^4, \quad v^5, \\ x^6, \quad x^5v, \quad x^4v^2, \quad x^3v^3, \quad x^2v^4, \quad xv^5, \quad v^6] \end{array}$$

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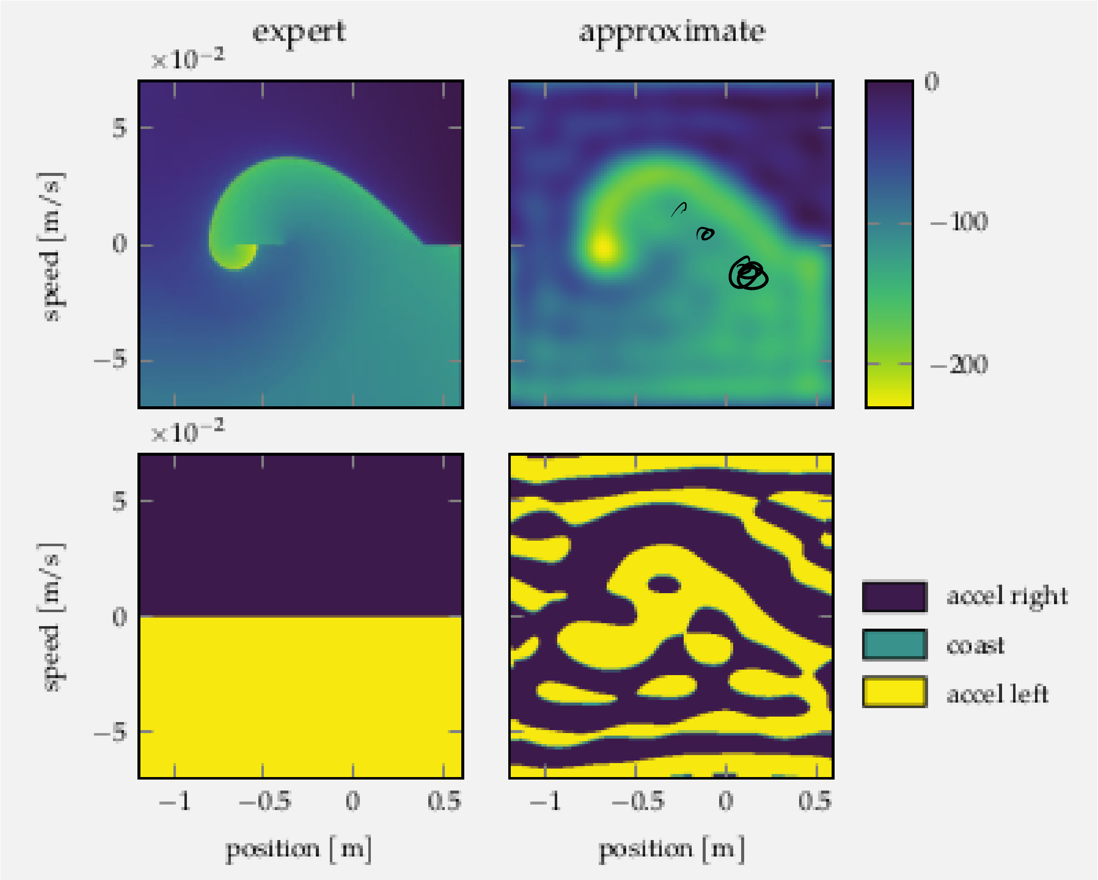

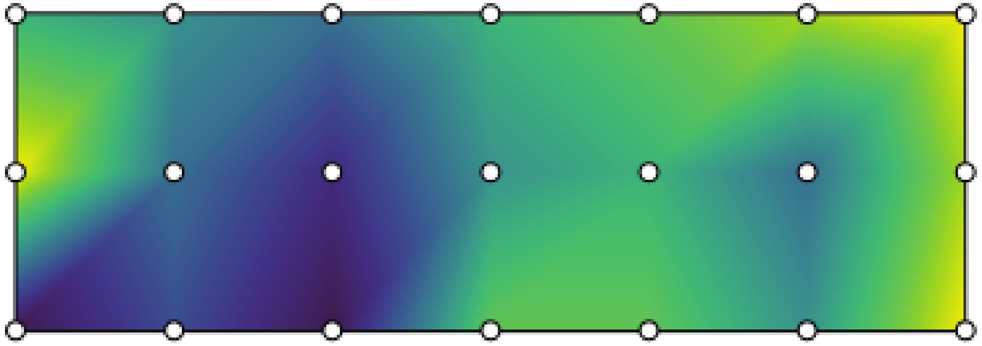

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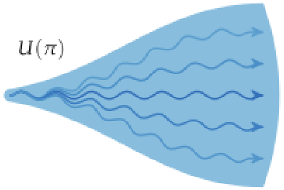

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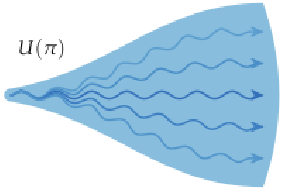

Common Approaches

Policy Search

$$\underset{\theta}{\text{maximize}} \quad U(\pi_\theta)$$

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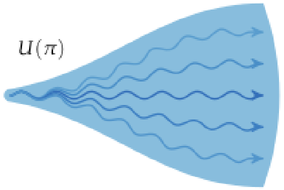

<u>Common Approaches</u>

- Evolutionary Algorithms

Policy Search

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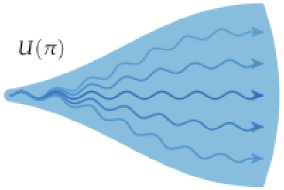

Common Approaches

- Evolutionary Algorithms
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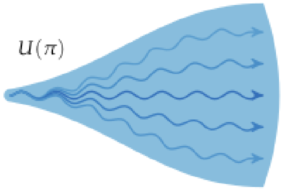

<u>Common Approaches</u>

- Evolutionary Algorithms
- Cross Entropy Method
- Policy Gradient (will cover in RL section)

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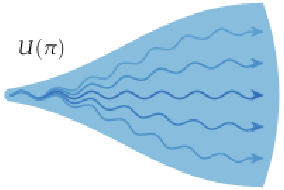

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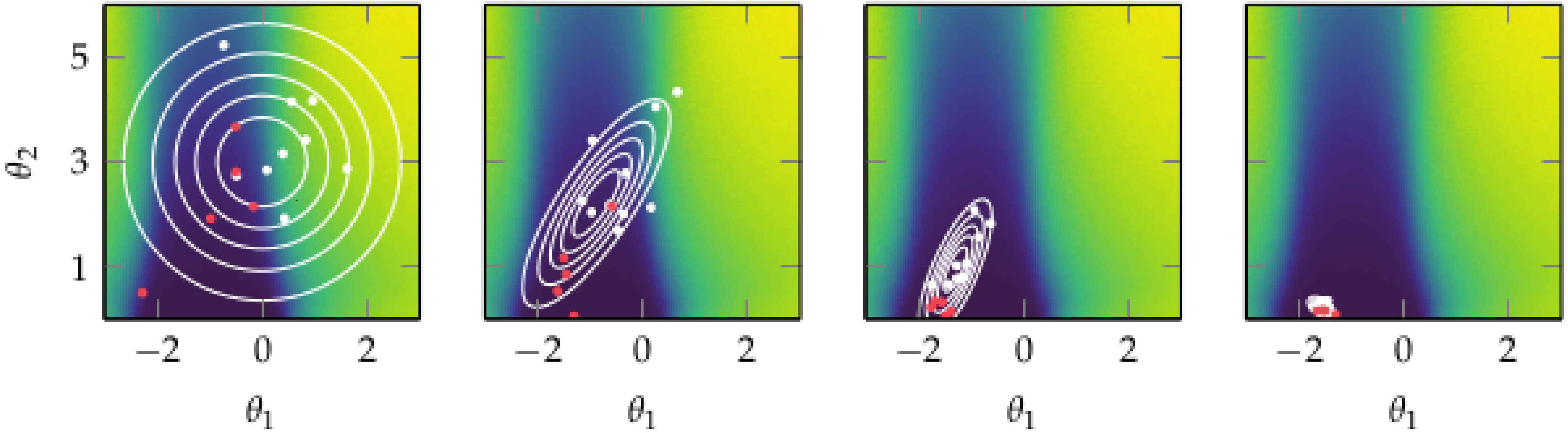


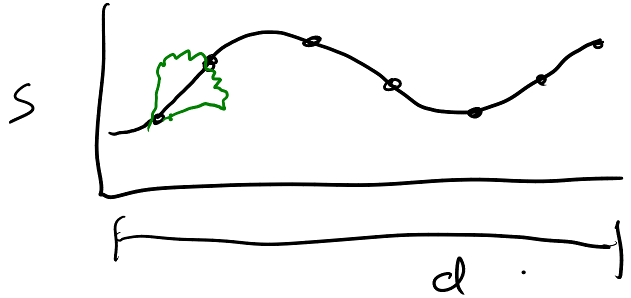
Model Predictive Control

Model Predictive Control

$$\underset{a_{1:d}, s_{1:d}}{\text{maximize}} \quad \sum_{t=1}^{d} \gamma^t R(s_t, a_t)$$

$$\text{subject to} \quad s_{t+1} = \mathrm{E}[T(s_t, a_t)] \quad \forall t$$

mean



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Optimistic

$$\underset{a_{1:d}, s_{1:d}^{(1:m)}}{\text{maximize}} \quad \frac{1}{m} \sum_{i=1}^{m} \sum_{t=1}^{d} \gamma^t R(s_t^{(i)}, a_t)$$

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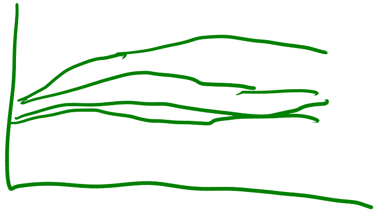
Open Loop



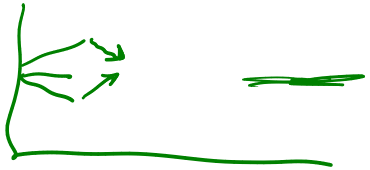
Pessimistic /conservative

$$\underset{a_{1:d}^{(1:m)}, s_{1:d}^{(1:m)}}{\text{maximize}} \quad \frac{1}{m} \sum_{i=1}^{m} \sum_{t=1}^{d} \gamma^t R(s_t^{(i)}, a_t^{(i)})$$

$$\text{subject to} \quad s_{t+1} = G(s_t^{(i)}, a_t^{(i)}, w_t^{(i)}) \quad \forall t, i$$

$$a_1^{(i)} = a_1^{(j)} \quad \forall i, j$$

Hindsight Opt



Optimistic

Sparse Tree Search/Progressive Widening

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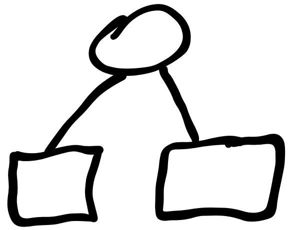

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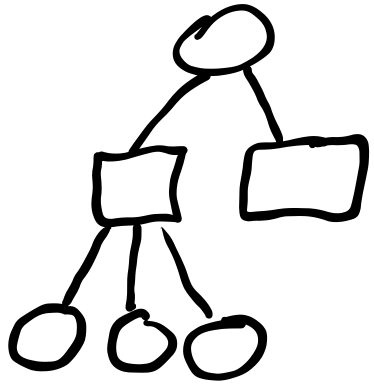

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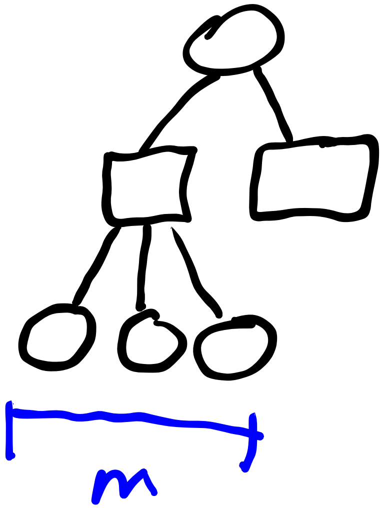
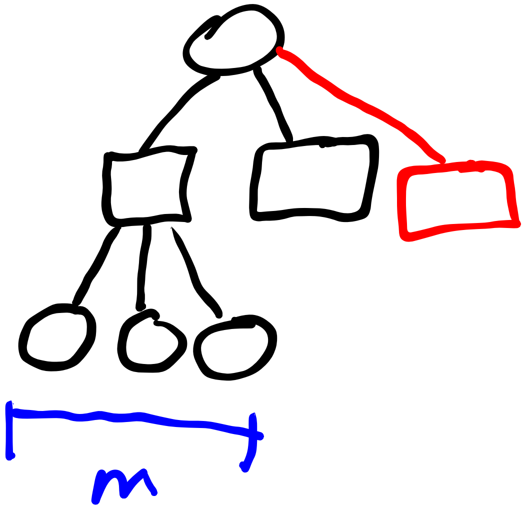
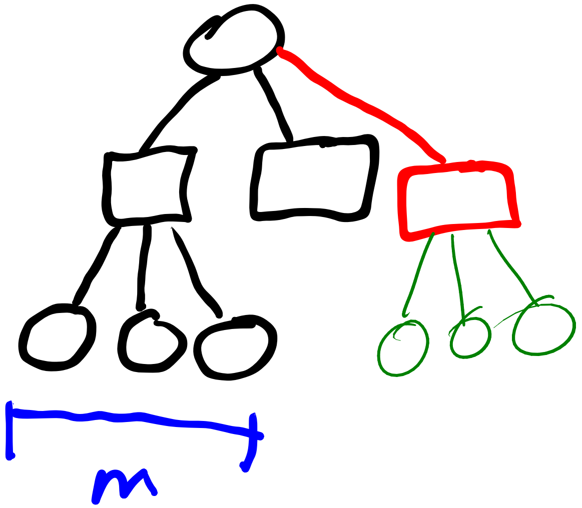


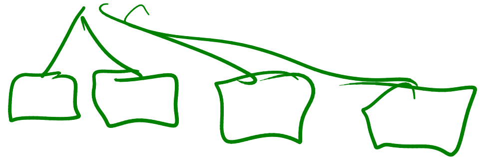
Sparse Tree Search/Progressive Widening

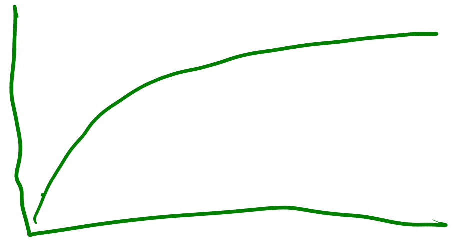


Sparse Tree Search/Progressive Widening

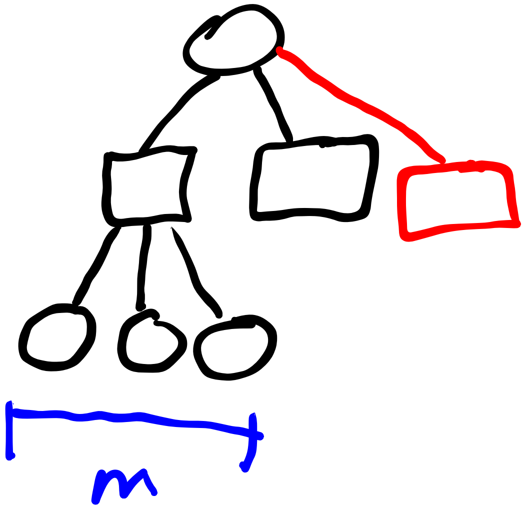


add new branch if $C < kN^{\alpha}$

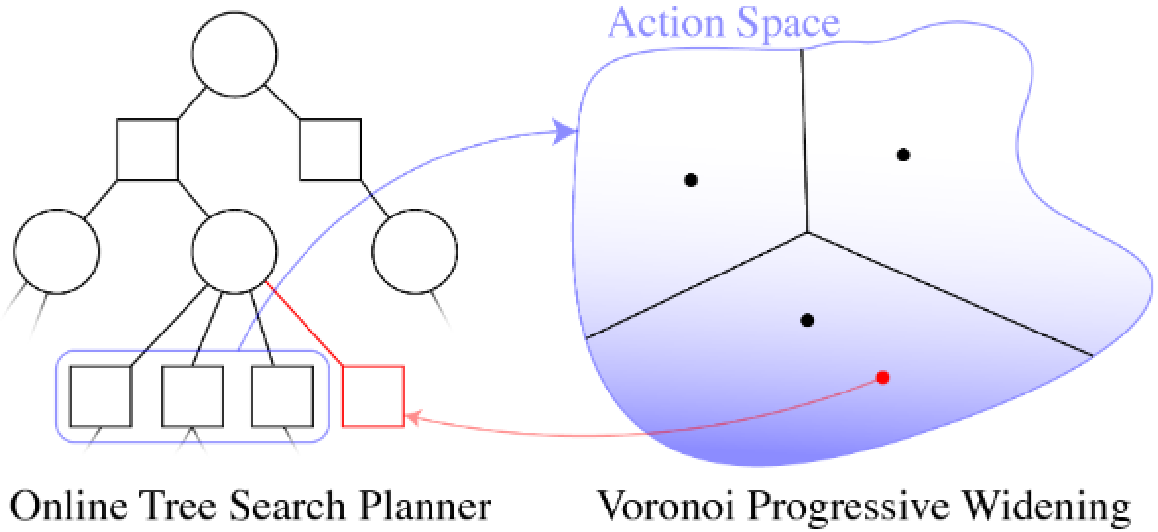




Sparse Tree Search/Progressive Widening



add new branch if $C < kN^{\alpha}$



Guiding Questions

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- What tools do we have to solve MDPs with continuous S and A?