HW 1 due tonight

Part way through!

Last Time

- What does "Markov" mean in "~~Markov Decision Process~~"?

Stochastic Process $\{x_t\}$ is Markov if

$$P(x_{t+1} \mid x_t, x_{t-1} \ldots x_0) = P(x_{t+1} \mid x_t)$$

Guiding Questions

Guiding Questions

- What is a **Markov decision process**?

Guiding Questions

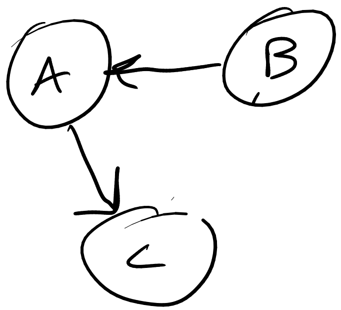
- What is a **Markov decision process**?
- What is a **policy**?

Guiding Questions

- What is a **Markov decision process**?
- What is a **policy**?
- How do we **evaluate** policies?

Decision Networks and MDPs

Bayes Net



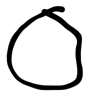
$P(C \mid A, B) = P(C \mid A)$

Decision Networks and MDPs

Decision Network

Decision Networks and MDPs

Decision Network



Decision Networks and MDPs

Decision Network

Chance node

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Decision Networks and MDPs

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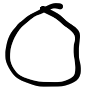
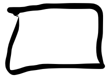
 Chance node

 Decision node

Decision Networks and MDPs

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Chance node

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Decision Networks and MDPs

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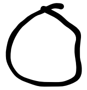
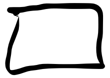
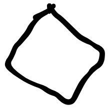
Utility node

Decision Networks and MDPs

Decision Network

 Chance node

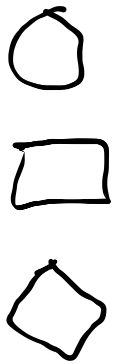
 Decision node

 Utility node

MDP Dynamic Decision Network

Decision Networks and MDPs

Decision Network



Chance node

Decision node

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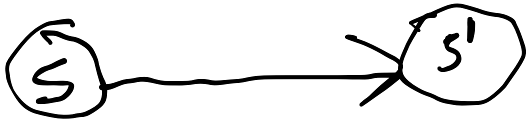
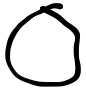
MDP Dynamic Decision Network

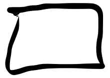


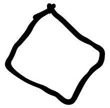
DBN for
a Markov
Stoch. Process

Decision Networks and MDPs

Decision Network

 Chance node

 Decision node

 Utility node

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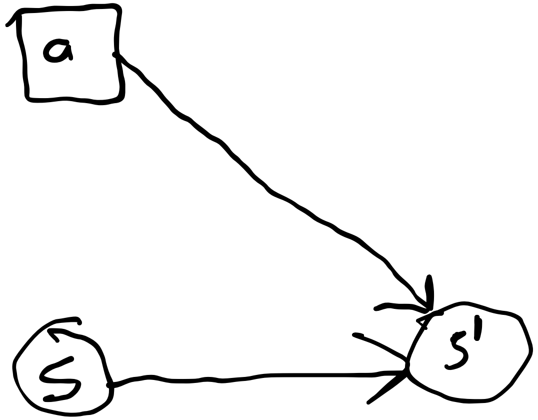

Decision Networks and MDPs

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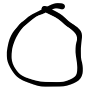
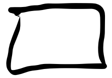
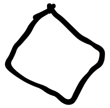
 Chance node

 Decision node

 Utility node

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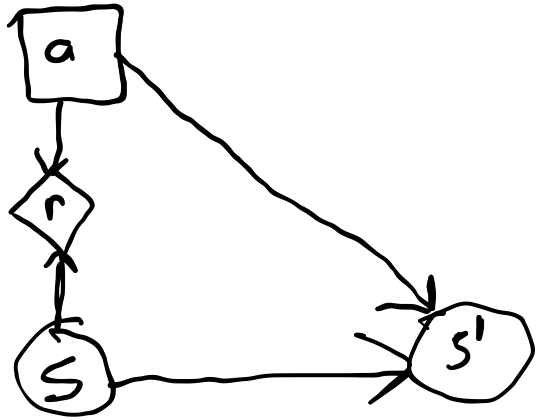

Decision Networks and MDPs

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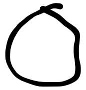
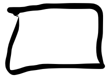
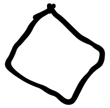
 Chance node

 Decision node

 Utility node

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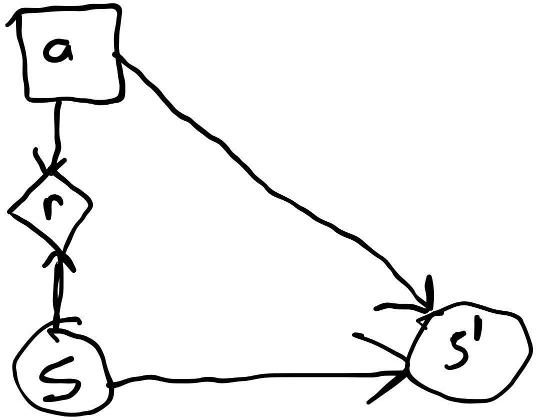
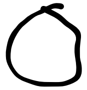
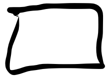


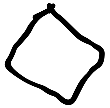
MDP Optimization problem

Decision Networks and MDPs

Decision Network

 Chance node

 Decision node

 Utility node

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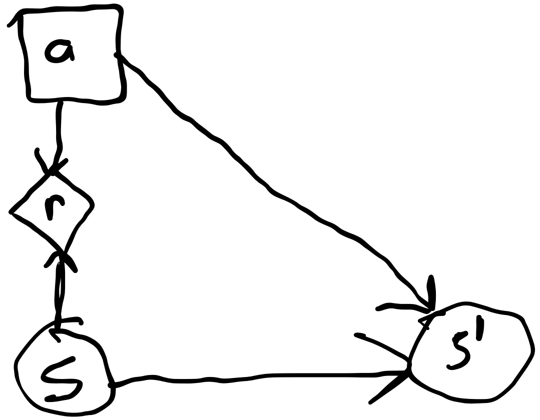
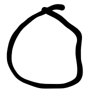


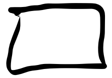
MDP Optimization problem

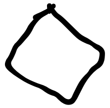
$$\text{maximize} \quad \mathrm{E}\left[\sum_{t=1}^{\infty} r_t\right]$$

Decision Networks and MDPs

Decision Network

 Chance node

 Decision node

 Utility node

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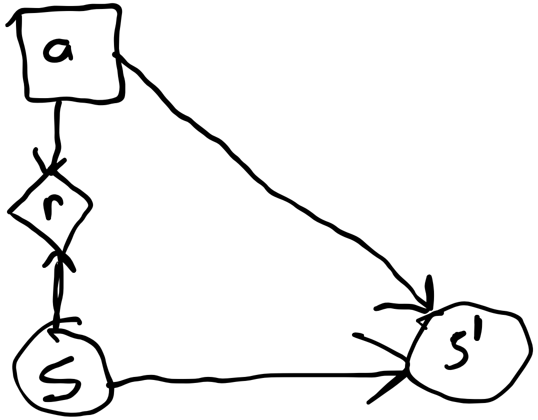


MDP Optimization problem

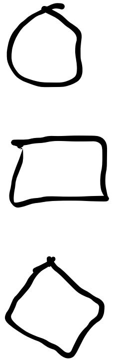
$$\text{maximize} \quad \mathrm{E}\left[\sum_{t=1}^{\infty} r_t\right]$$

Not well formulated!

$r_t = 1$

Decision Networks and MDPs

Decision Network

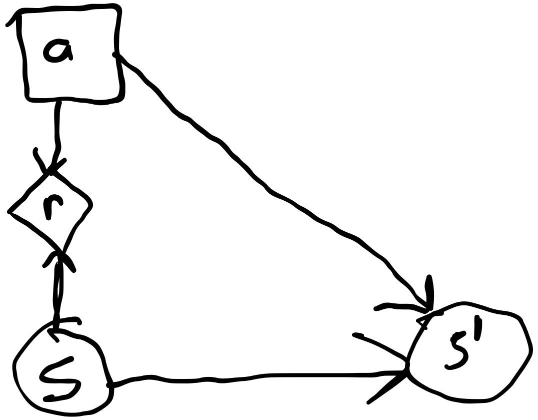


Chance node

Decision node

Utility node

MDP Dynamic Decision Network



MDP Optimization problem

$$\text{maximize} \quad \mathrm{E}\left[\sum_{t=1}^{\infty} r_t\right]$$

Not well formulated!
Infinite

Finite MDP Objectives

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1. Finite time

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$$\mathbf{E}\left[\sum_{t=0}^{T} r_t\right]$$

Finite MDP Objectives

1. Finite time $$\mathbf{E}\left[\sum_{t=0}^{T} r_t\right]$$

2. Average reward

Finite MDP Objectives

1. Finite time

$$\mathrm{E}\left[\sum_{t=0}^{T} r_t\right]$$

2. Average reward

$$\lim_{n\to\infty} \mathrm{E}\left[\frac{1}{n}\sum_{t=0}^{n} r_t\right]$$

Finite MDP Objectives

1. Finite time

$$\mathbf{E}\left[\sum_{t=0}^{T} r_t\right]$$

2. Average reward

$$\lim_{n\to\infty} \mathbf{E}\left[\sum_{t=0}^{n} r_t\right]$$

3. Discounting

Finite MDP Objectives

1. Finite time

$$\mathbf{E}\left[\sum_{t=0}^{T} r_t\right]$$

2. Average reward

$$\lim_{n\to\infty} \mathbf{E}\left[\sum_{t=0}^{n} r_t\right]$$

3. Discounting

$$\mathbf{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t\right]$$

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3. Discounting

$$\mathbf{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t\right]$$

discount $\gamma \in [0, 1)$

$\gamma = 0.9$

$\gamma^0 = 1$

Finite MDP Objectives

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2. Average reward

$$\lim_{n\to\infty} \mathbf{E}\left[\sum_{t=0}^{n} r_t\right]$$

3. Discounting

$$\mathbf{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t\right]$$

discount $\gamma \in [0, 1)$

typically 0.9, 0.95, 0.99

$\gamma^0 = 1$ $\gamma^1 = 0.99$

$\gamma^2 = 0.99^2$

Finite MDP Objectives

1. Finite time

$$\mathbf{E}\left[\sum_{t=0}^{T} r_t\right]$$

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$$\lim_{n\to\infty} \mathbf{E}\left[\sum_{t=0}^{n} r_t\right]$$

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$$\mathbf{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t\right]$$

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if $\underline{r} \le r_t \le \bar{r}$

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discount $\gamma \in [0,1)$

typically 0.9, 0.95, 0.99

if $\underline{r} \leq r_t \leq \bar{r}$

then

$$\frac{\underline{r}}{1-\gamma} \leq \sum_{t=0}^{\infty} \gamma^t r_t \leq \frac{\bar{r}}{1-\gamma}$$

Finite MDP Objectives

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4. Terminal States

Finite MDP Objectives

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4. Terminal States

Infinite time, but a terminal state (no reward, no leaving) is always reached with probability 1.

Finite MDP Objectives

1. Finite time $\mathbf{E}\left[\sum_{t=0}^{T} r_t\right]$

maximize$_{a_t}$ $E\left[\sum_{t=0}^{\infty} \gamma^t r_t\right]$

2. Average reward $\lim_{n\to\infty} \mathbf{E}\left[\sum_{t=0}^{n} r_t\right]$

3. Discounting $\boxed{\mathbf{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t\right]}$

discount $\gamma \in [0, 1)$

typically 0.9, 0.95, 0.99

if $\underline{r} \leq r_t \leq \bar{r}$

then

$$\frac{\underline{r}}{1-\gamma} \leq \sum_{t=0}^{\infty} \gamma^t r_t \leq \frac{\bar{r}}{1-\gamma}$$

4. Terminal States

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MDP "Tuple Definition"

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$$(S, A, T, R, \gamma)$$

MDP "Tuple Definition"

(S, A, T, R, γ) (and b in some contexts)

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(S, A, T, R, γ) (and b in some contexts)

- S (state space) - set of all possible states

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(S, A, T, R, γ) (and b in some contexts)

$\{1, 2, 3\}$

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$\{\text{healthy}, \text{pre-cancer}, \text{cancer}\}$

MDP "Tuple Definition"

(S, A, T, R, γ) (and b in some contexts)

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$\{1, 2, 3\}$ $\mathbb{R}^2$

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$\{1, 2, 3\}$ $\mathbb{R}^2$ $\{0, 1\} \times \mathbb{R}^4$

$\{\text{healthy}, \text{pre-cancer}, \text{cancer}\}$

MDP "Tuple Definition"

(S, A, T, R, γ) (and b in some contexts)

state space

$\{1, 2, 3\}$ $(x, y) \in \mathbb{R}^2$ $\{0, 1\} \times \mathbb{R}^4$

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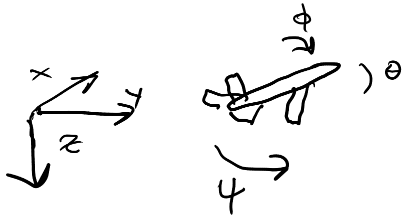
MDP "Tuple Definition"

$(x, y, z, u, v, w, \phi, \theta, \psi, p, q, r) \in \mathbb{R}^{12}$



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$\{\text{test}, \text{wait}, \text{treat}\}$

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- T (transition distribution) - explicit or implicit ("generative") model of how the state changes

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$T(s' \mid s, a)$

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$T(s' \mid s, a)$

$s', r = G(s, a)$

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- T (transition distribution) - explicit or implicit ("generative") model of how the state changes

  $T(s' \mid s, a)$

  $s', r = G(s, a)$

- R (reward function) - maps each state and action to a reward

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  $\{1, 2, 3\}$ $\quad (x, y) \in \mathbb{R}^2$ $\quad \{0, 1\} \times \mathbb{R}^4$

  $\{\text{healthy}, \text{pre-cancer}, \text{cancer}\}$ $\quad (s, i, r) \in \mathbb{N}^3$

- A (action space) - set of all possible actions

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  $\{\text{test}, \text{wait}, \text{treat}\}$

- T (transition distribution) - explicit or implicit ("generative") model of how the state changes

  $T(s' \mid s, a)$

  $s', r = G(s, a)$

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$S = \{1,2,3\}$ $A = \{1,2\}$

s'	$T(s' \mid s=1, a=1)$
1	0.5
2	0.2
3	0.3

- b: initial state distribution

$s_0 \sim b$

MDP Example

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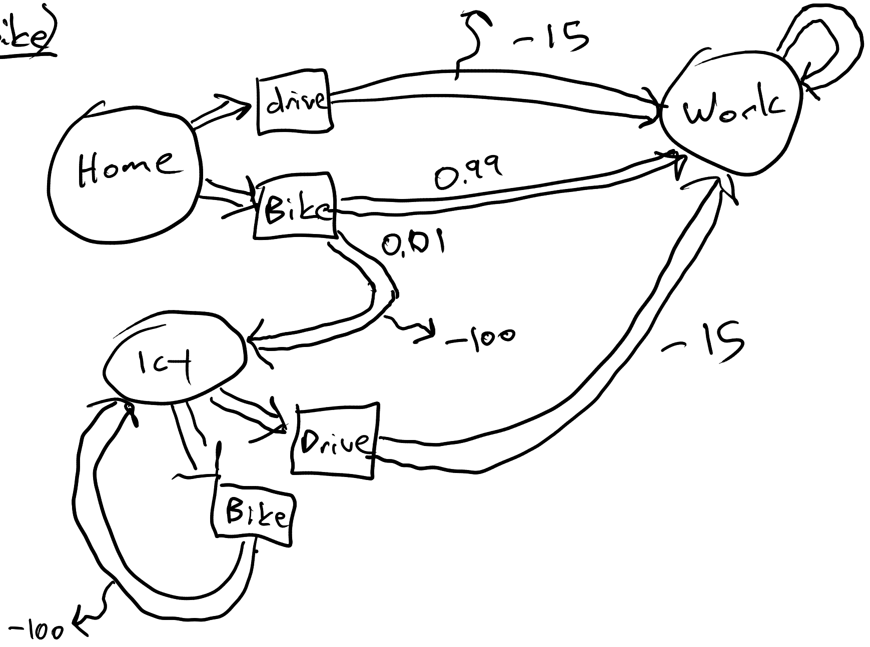
- If you drive, you will have to pay \$15 for parking; biking is free.
- On 1% of cold days, the ground is covered in ice and you will crash if you bike, but you can't discover this until you start riding. After your crash, you limp home with pain equivalent to losing \$100.

$$\begin{cases} S = \{\text{home}, \text{icy}, \text{work}\} \\ A = \{\text{drive}, \text{bike}\} \\ R(s,a,s') = \begin{cases} -15 & \text{if } a=\text{drive} \\ -100 & \text{if } a=\text{bike},\ s'=\text{icy} \\ 0 & \text{o.w.} \end{cases} \\ \gamma = 0.99 \\ T(s' \mid s,a) = \begin{cases} 1 & \text{if } s=\text{home},\ a=\text{drive},\ s'=\text{work} \\ 0.99 & \text{if } s=\text{home},\ a=\text{bike},\ s'=\text{work} \\ \vdots & \\ 0 & \text{if } a=\text{drive},\ s'=\text{icy} \end{cases} \end{cases}$$

s'	$T(s' \mid s=\text{home}, a=\text{bike})$
work	0.99
icy	0.01
home	0



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- An optimal policy, π^*, maximizes the sum of expected rewards:

$a_t = \pi(s_t)$

$$\pi^* = \underset{\pi}{\operatorname{argmax}} \, \mathrm{E}\left[\sum_{t=0}^{\infty} \gamma^t \, R\left(s_t, \pi(s_t)\right)\right]$$

Breakout Rooms

- Name, "I think Colorado winters are ______"
- Suggest a policy that you think is optimal for the icy day problem

$$\pi(s) = \begin{cases} \text{bike} & \text{if } s = \text{home} \\ \text{drive} & \text{if } s = \text{icy} \end{cases}$$

MDP Simulation

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<u>Algorithm: Rollout Simulation</u>

Given: MDP (S, A, R, T, γ, b)

$s \leftarrow \text{sample}(b)$

$\hat{u} \leftarrow 0$

for t in $0 \ldots T-1$

  $s', r \leftarrow G(s, a)$

  $\hat{u} \leftarrow \hat{u} + \gamma^t r$

  $s \leftarrow s'$

return $\hat{u}$

$\gamma^T < \varepsilon$

$T(s'|s,a)$

$R(s,a)$

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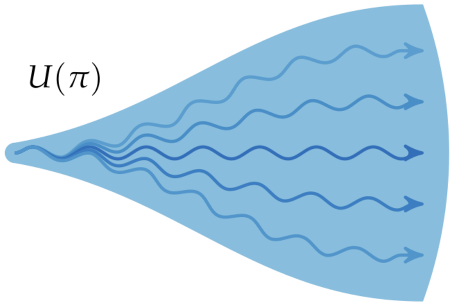

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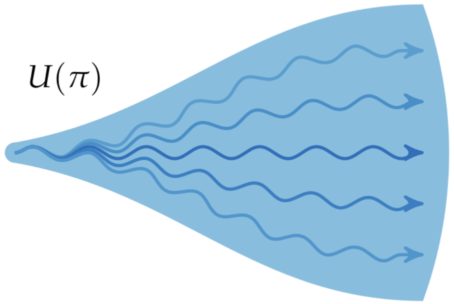

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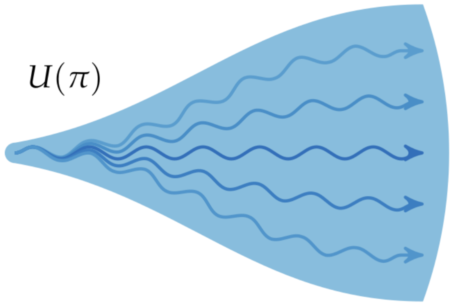

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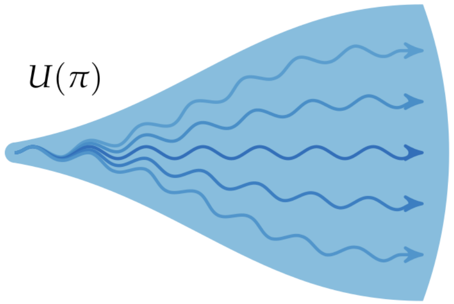

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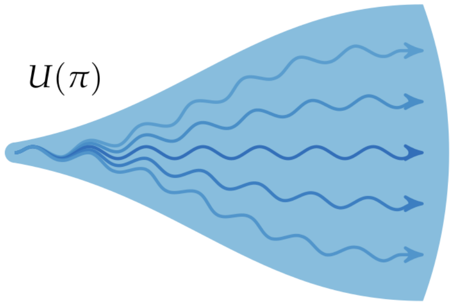

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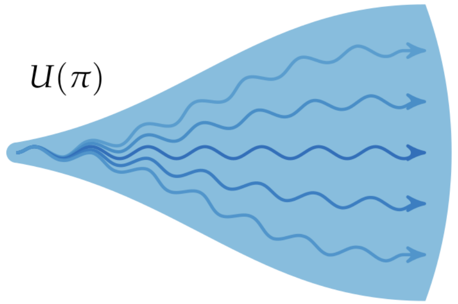

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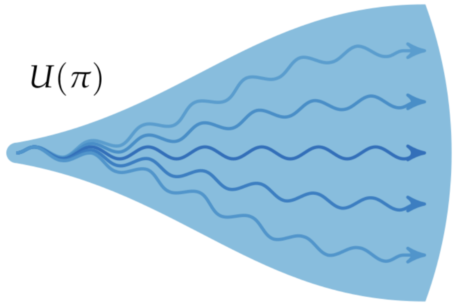

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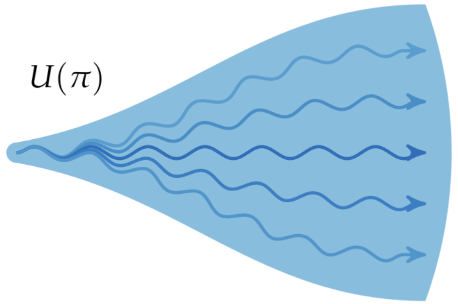

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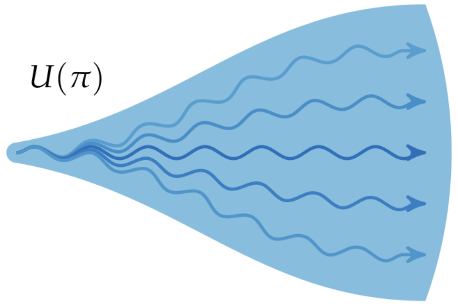

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$$\text{s.e.m.} = \frac{\text{std}(\hat{u})}{\sqrt{m}}$$

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