What is a R.V.?

How to infer info about A given B

How to (efficiently) encode relationships between R.V.s

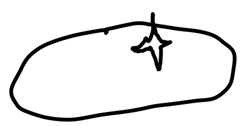
What is a R.V.

- Quantity we don't know precisely but do know distribution
- Mapping from $\Omega \to E$ $\leftarrow$

Variable take several values
- Discrete
- Prob.

C $P(C = h) = 0.5$

Chipotle



Variable
- Continuous
- Discrete
- Related to other R.V.s

Conditional Dist

Bayes Rule

Prob. density

$\uparrow$

Elway's

$X : \Omega \to E$

(Ω, F, P)

Today Only!   R.V.s - capital

Distributions

Discrete

$P(A)$ table

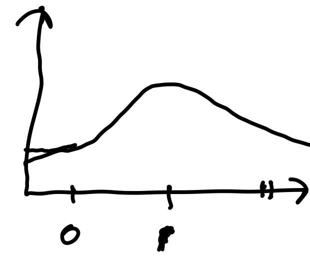
$P(A=a)$ $p(a)$ $p_A(a)$ scalars

a	$P(a) = P(A=a)$
0	0.5
1	0.5

Continuous R.V.

$p(a)$

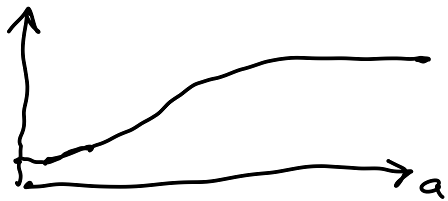
density function pdf



$P(A=a) = 0$

Measure Theory

$c(a)$



$c(a) = P(A \le a)$

$p(a) = \frac{dc}{da}$

"Deterministic" R.V.s $\underline{A=1}$

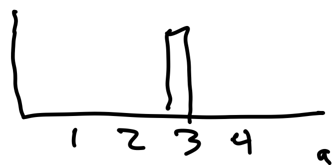
δ-functions

$$P(A=a) = \delta_1(a)$$

Discrete: "Kroenecker δ"

$$\delta_x(y) = \begin{cases} 1 & \text{if } x=y \\ 0 & \text{o.w.} \end{cases}$$

$P(A=a)$



Continuous: "Dirac δ"

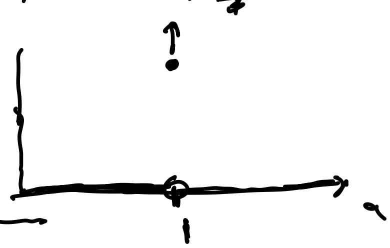
$$\delta_x(y) = \begin{cases} \infty & \text{if } x=y \\ 0 & \text{o.w.} \end{cases}$$

(not actually a real-valued fn.)

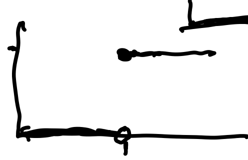
$$\int_{-\infty}^{\infty} \delta_x(y)\, dy = 1$$

$A=1$

$p(a)$



$c(a)$



Multiple Related R.V.s

3 Rules ~~Chipotle~~ Pete's

Elways Axiomatizations

$$P(A \mid B)$$

1) a) $0 \le P(A \mid B) \le 1$

   b) $\sum_a P(a \mid B) = 1$

2) "Law of total Probability"

$$P(A) = \sum_{b \in B} P(A, b)$$

3) "Def of conditional Probability"

$$P(A \mid B) = \frac{P(A,B)}{P(B)}$$

A, B

$P(A,B)$ Joint Distribution

a	b	P(a,b)
0	0	0.1
1	0	0.2
0	1	0.3
1	1	0.4

3 indep. parameters

All information

support = set

supp(A) = set of vals A can take

Conditional

$P(A \mid B)$

"Dist.-valued function"

$b = 0$

a	P(a)
0	1/3
1	2/3

$b = 1$

a	P(a)
0	3/7
1	4/7

"Marginalization"

Marginal

2 indep param

$P(A)$

a	P(A)
0	0.4
1	0.6

$P(B)$

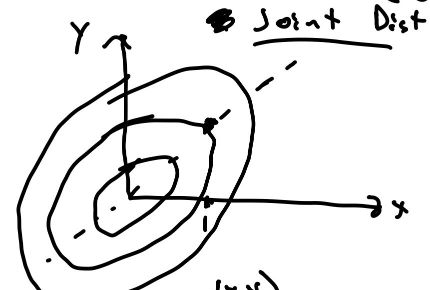
b	P(B)
0	0.3
1	0.7

$$P(A) = \sum_b P(A \mid b) P(b)$$

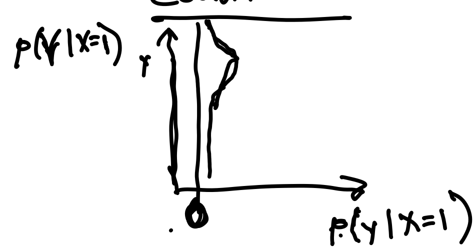
2 indep param

Related Continuous R.V.s

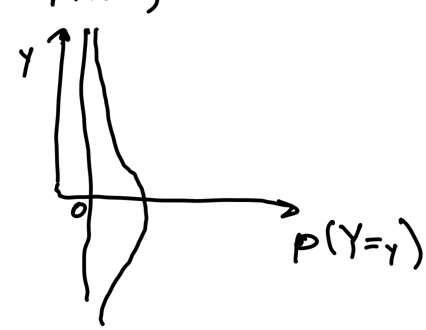
Joint Dist

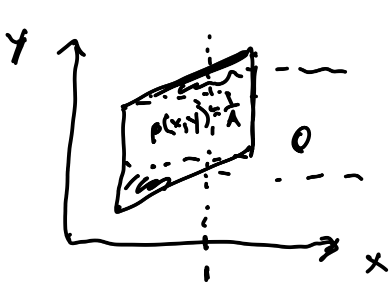


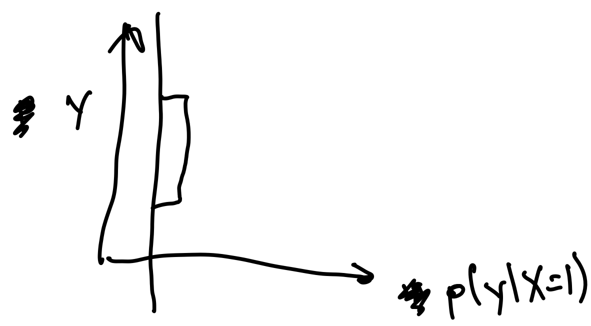
Conditional

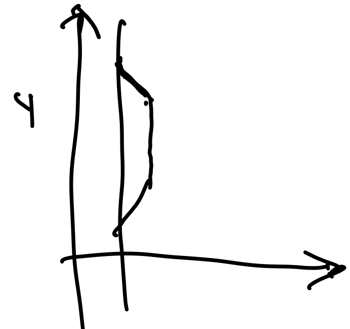


Marginal









A: hidden
B: measured

know $P(B|A)$

C: Cancer
T: test

$$P(C=1) = 0.01$$
$$P(T=1|C=1) = 0.8$$
$$P(T=1|C=0) = 0.096$$

$$P(C=1|T=1)$$

Bayes Rule:

$$P(A|B) = \frac{P(A,B)}{P(A)} \qquad P(B|A) = \frac{P(B,A)}{P(B)}$$

$$P(A|B)P(A) = P(A,B) = P(B,A) = P(B|A)P(B)$$

$$P(A|B)P(A) = P(B|A)P(B)$$

$$\boxed{P(A|B) = \frac{P(B|A)P(A)}{P(B)}}$$

$$P(T) = \sum_{C} P(T|C)P(C)$$

$$P(T=1) = 0.01 \times 0.8 + 0.99 \cdot 0.096$$
$$= 0.103$$

$$P(C|T) = \frac{P(T|C)P(C)}{P(T)}$$

$$P(C=1|T=1) = \frac{0.8 \cdot 0.01}{0.103}$$
$$= 0.078$$

Independence

Def: Indep. iff $\quad P(A,B) = P(A)\,P(B)$

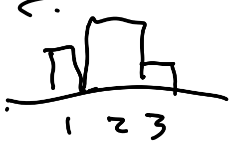
$$P(A,B) = P(A|B)\,P(B) \quad = \quad P(A|B) = P(A)$$

$$P(A|B) = P(A)$$

$A \perp B$

Def: Cond. Indep. iff $\quad P(A,B|C) = P(A|C)\,P(B|C)$

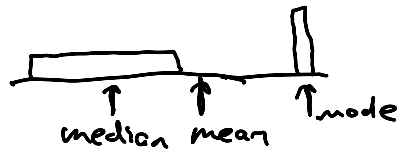
$$\Leftrightarrow P(A|B,C) = P(A|C)$$



mean

median

mode $\longrightarrow$



Expectation

$$E[A] = \sum_a a\,P(a) \qquad E[A] = \int_{-\infty}^{\infty} a\,p(a)\,da$$

$$E[A|b] = \sum_a a\,P(a|b) \qquad E[A|b] = \int_{-\infty}^{\infty} a\,p(a|b)\,da$$

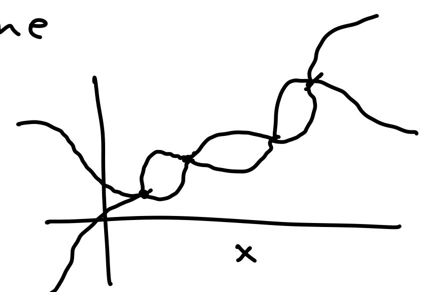
function $E[A|B] : \mathrm{supp}(B) \to \mathbb{R}$

Stochastic Process

Warning: Lowercase may be R.V.s

Collection of R.V.'s indexed by e.g. time

$$\{x_t\}_{t=1}^{\infty} = \{x_1, x_2, x_3, \dots\}$$



G.P.

Example

$x_1 = 0 \qquad x_{t+1} = x_t + v_t \qquad x' = x + v$

v_t are i.i.d. R.V.s $\qquad v_t \sim U(\{0,1\})$

$P(v_t)$

v	
0	0.5
1	0.5

$$P(x_{t+1} | x_t) = P(v_t = x_{t+1} - x_t)$$

$x_t = 4$

x_{t+1}	$x_{t+1} - x_t$	$P(x_{t+1}\|x_t)$
5	1	0.5
4	0	0.5