

Simple Games

- Games: a mathematical formalism for rational interaction

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- Games: a mathematical formalism for rational interaction
- What is the best solution concept? (Nash Equilibrium)

Types of Uncertainty

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Alleatory

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Markov Decision Process

Types of Uncertainty

Alleatory

Epistemic (Static)

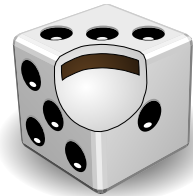


Markov Decision Process

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Markov Decision Process

Reinforcement Learning

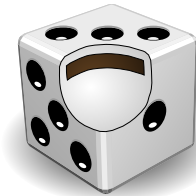
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POMDP

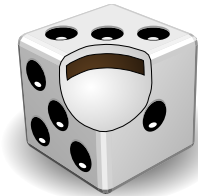
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POMDP

Interaction

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Game

Normal Form Games

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Called a **Normal Form, Simple**, or **Bimatrix** Game

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Question for today: What **solution concept** should we use for games?

Dominant Strategies

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Is the dominant strategy equilibrium always the best outcome for the players?

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Player 2

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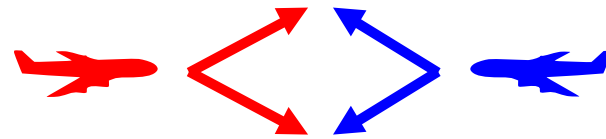
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Do all games have a dominant strategy equilibrium?

Collision Avoidance Game

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Example: Airborne Collision Avoidance



Player 1

Player 2

Up

Down

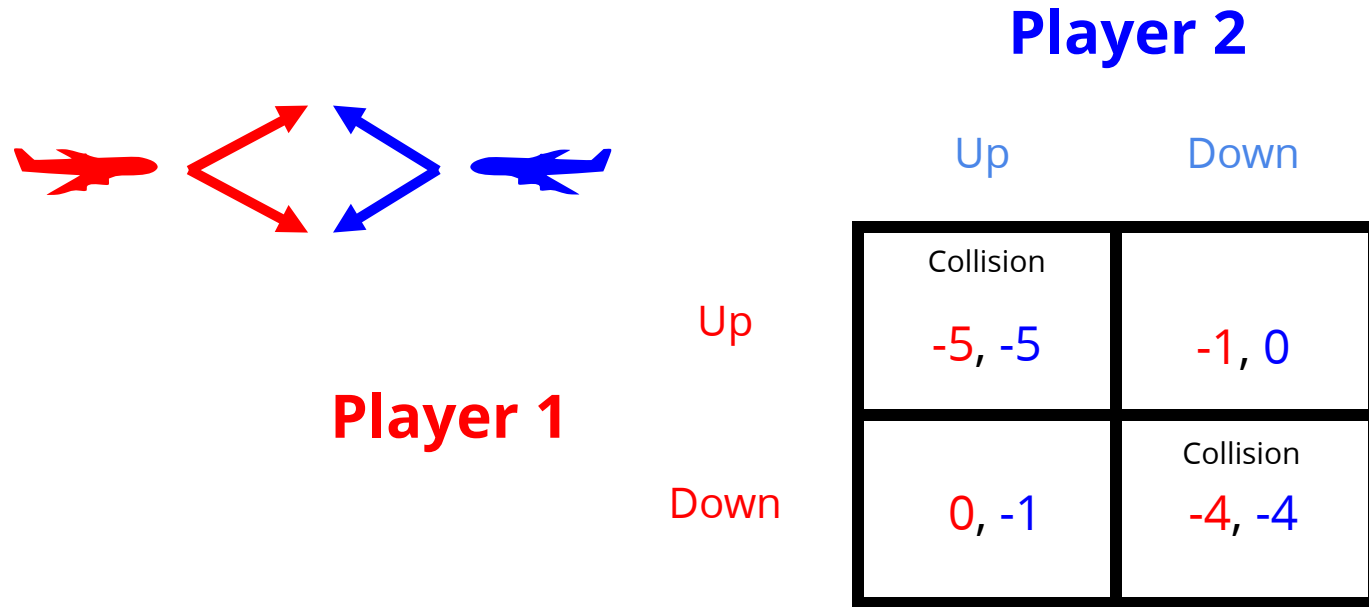
Up

Down

Up	Down
Collision -5, -5	-1, 0
0, -1	Collision -4, -4

Collision Avoidance Game

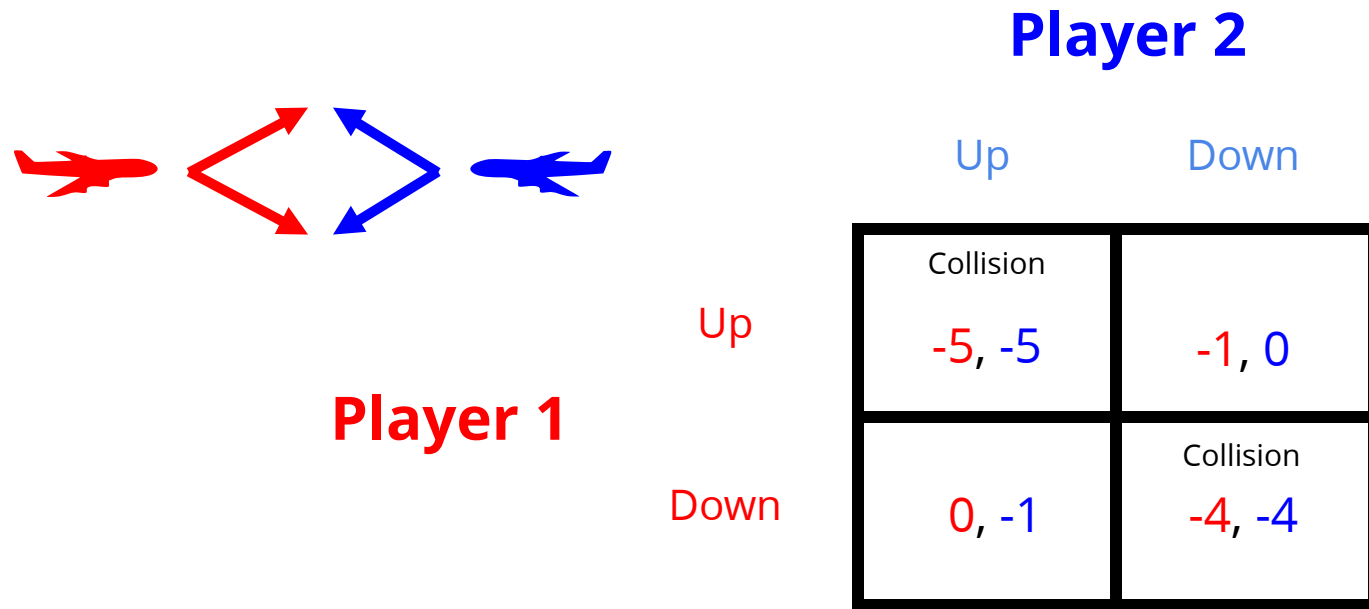
Example: Airborne Collision Avoidance



Pure Nash Equilibrium: All players play a deterministic best response.

Collision Avoidance Game

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Do all simple games have a pure Nash equilibrium?

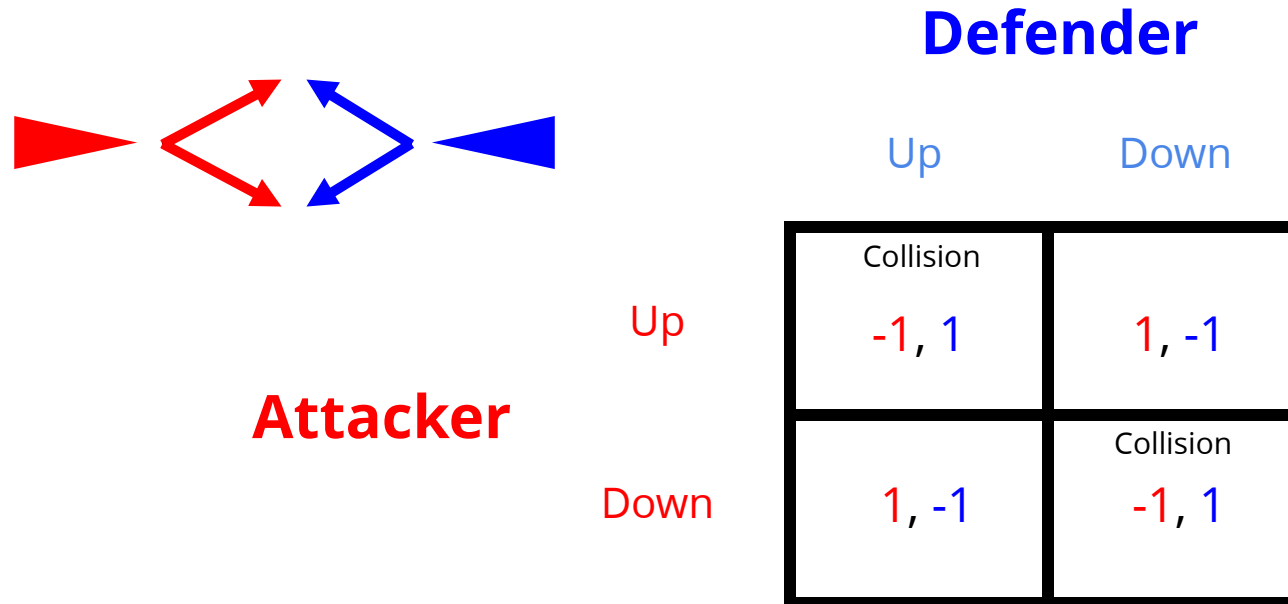
Practice: Find Pure Nash Equilibria

		Player 2		
		a	b	c
Player 1	a	4,4	2,5	0,0
	b	5,2	3,3	0,0
	c	0,0	0,0	10,10

Missile Defense Game

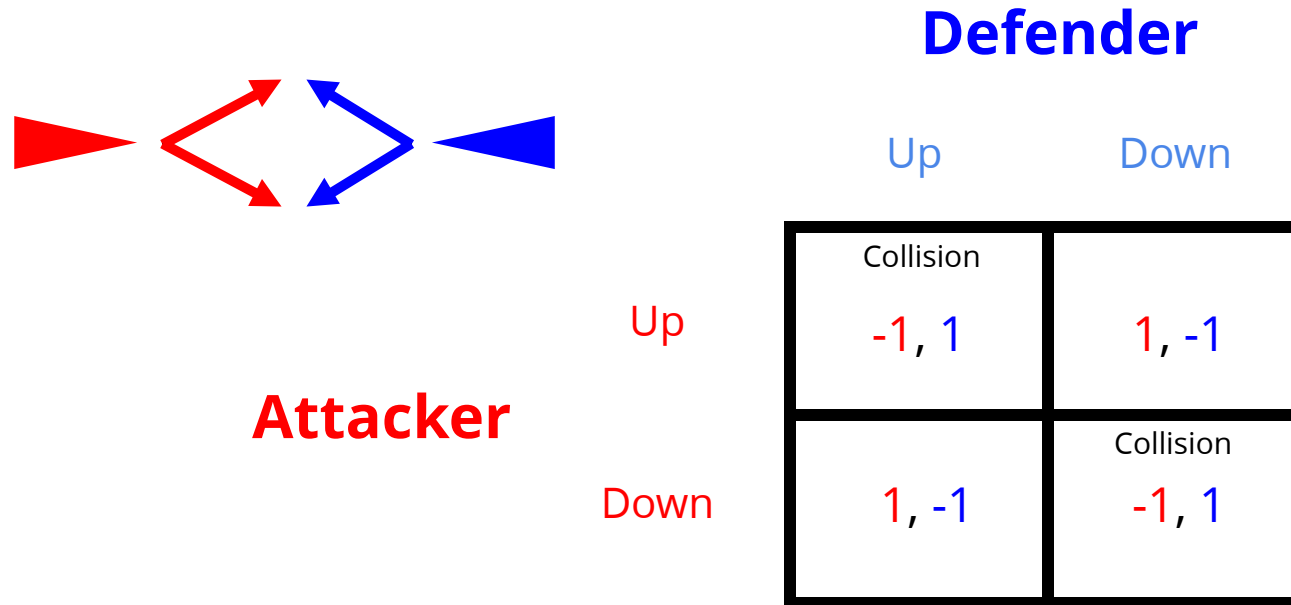
Missile Defense Game

Missile Defense (simplified)



Missile Defense Game

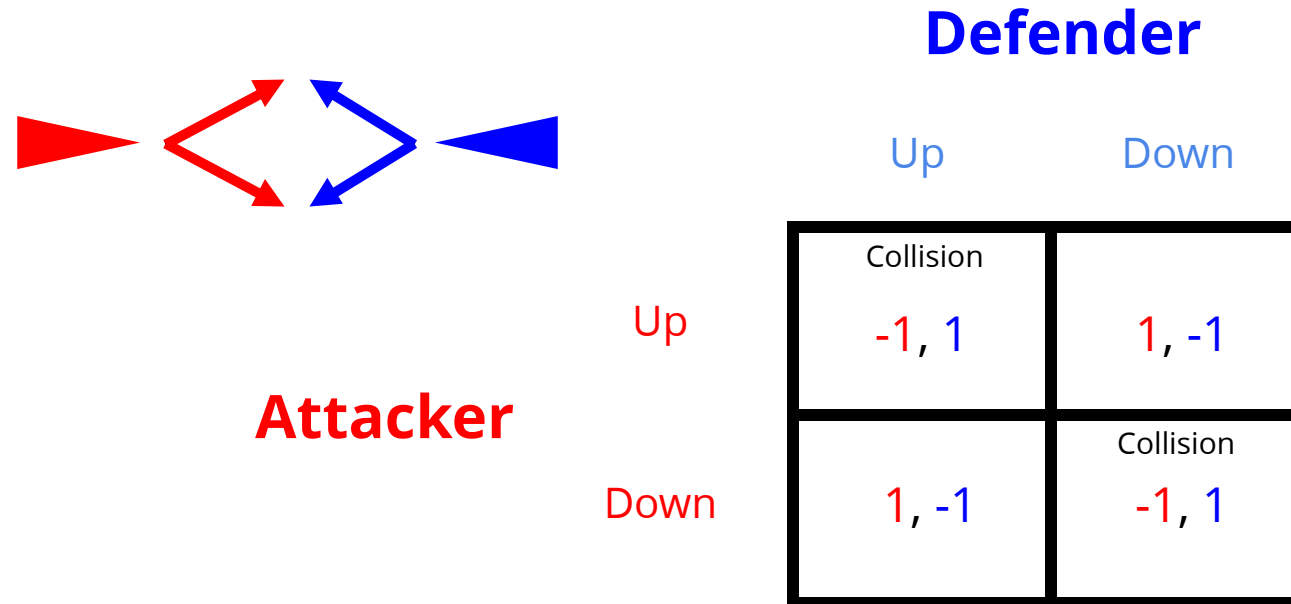
Missile Defense (simplified)



No Pure Nash Equilibrium!

Missile Defense Game

Missile Defense (simplified)



No Pure Nash Equilibrium!

Need a broader solution concept: Mixed Nash equilibrium.

Vocabulary and Notation for Mixed Strategies

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Single Player

Joint

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	Single Player	Joint
• Action	$a^i \in A^i$	$a \in A$

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• Reward	$R^i(a)$	$R(a)$
• Utility	$U^i(\pi) = \sum_a R^i(a)\pi(a)$	$U(\pi) = \sum_a R(a)\pi(a)$

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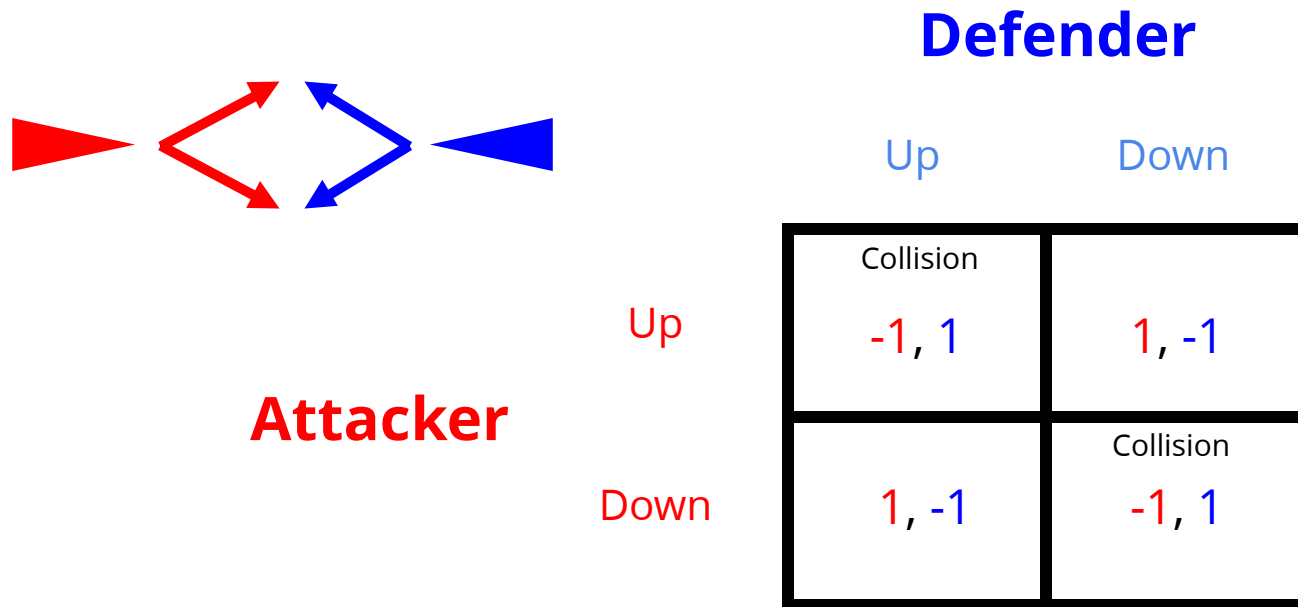
Best Response: Given a joint policy of all other agents, π^{-i} , a best response is a policy π^i that satisfies

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for all other $\pi^{i'}$.

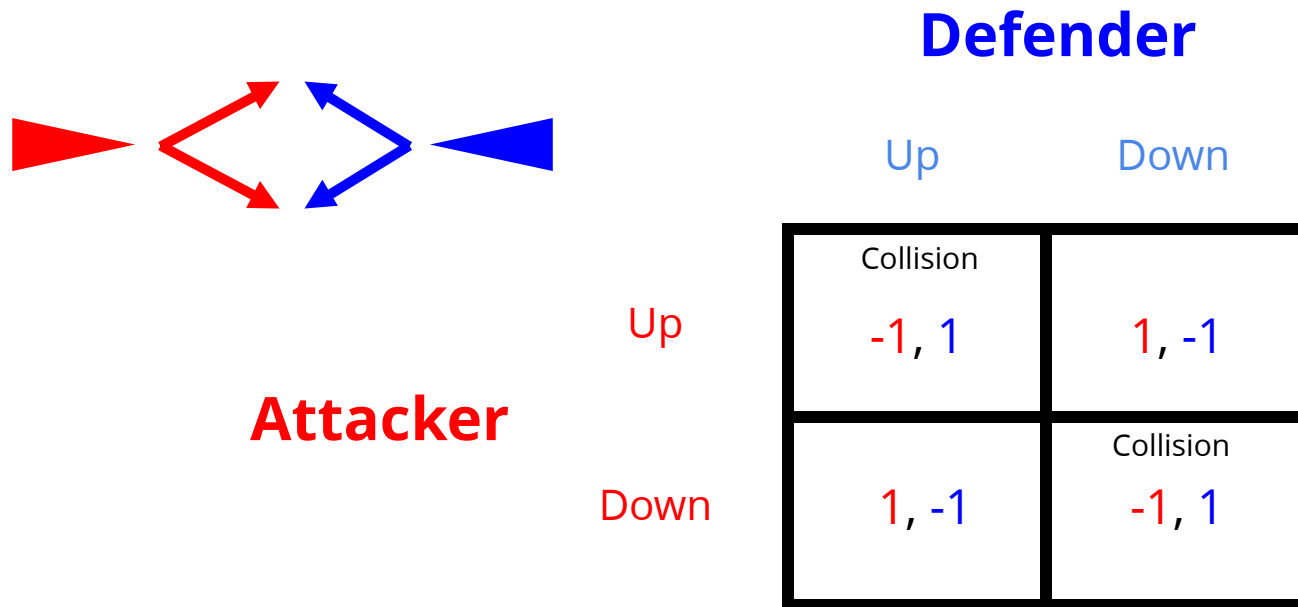
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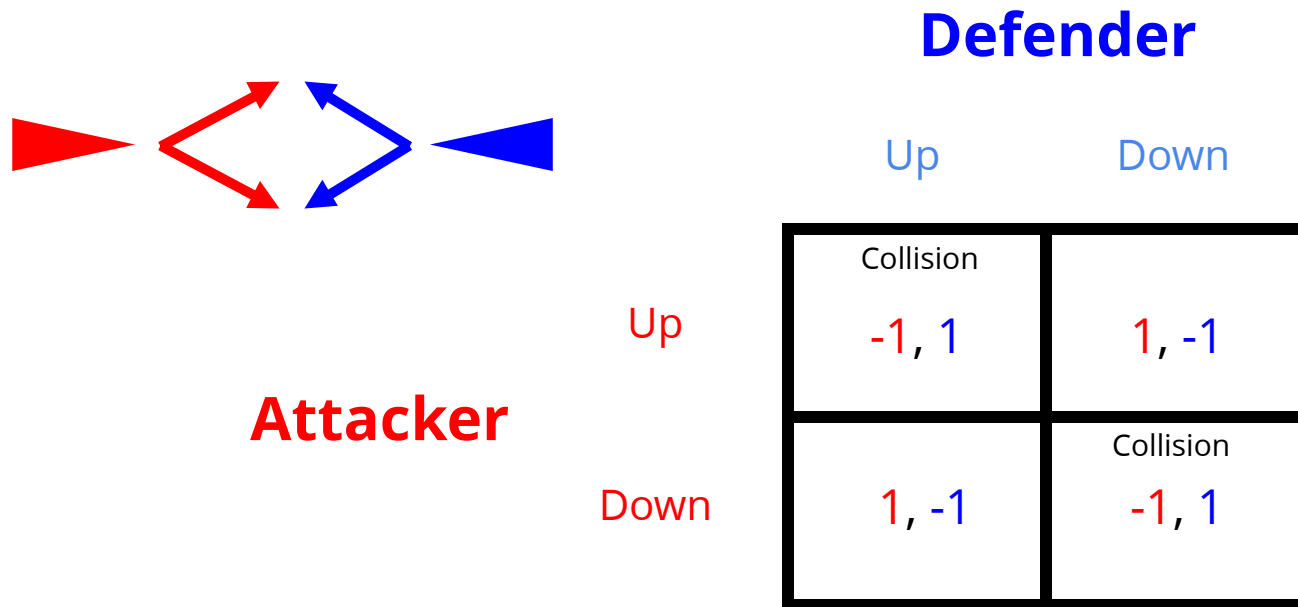
Missile Defense (simplified)



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Missile Defense Game

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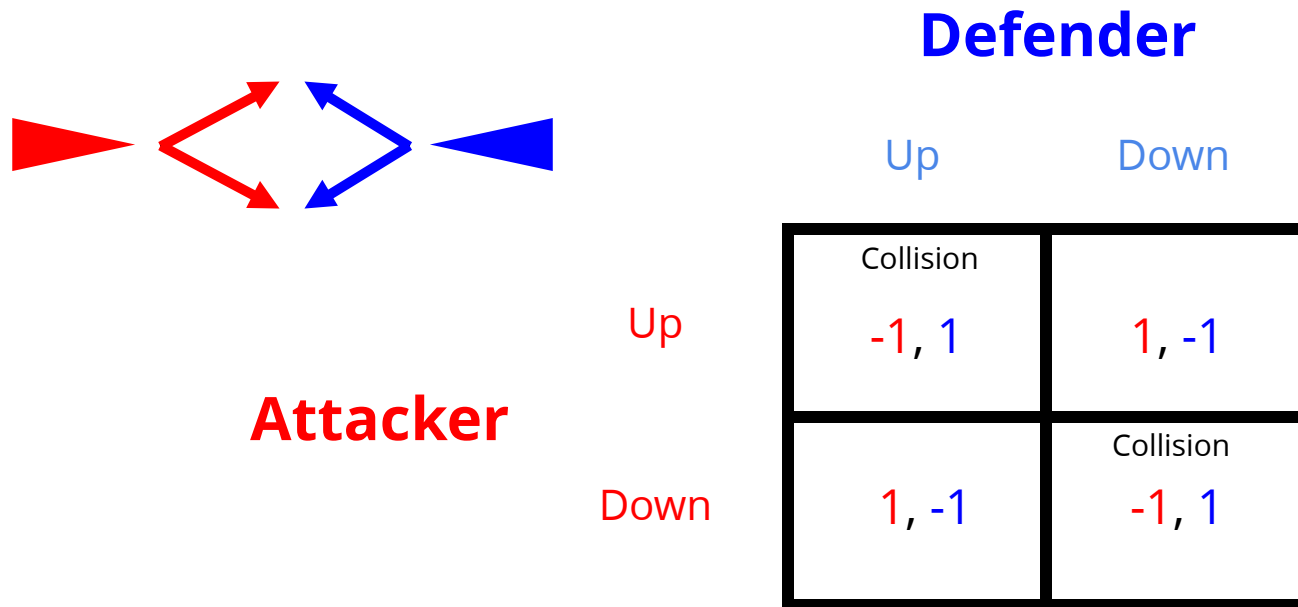


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Do all simple games have at least one Nash equilibrium?

Missile Defense Game

Missile Defense (simplified)



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Do all simple games have at least one Nash equilibrium?

Yes!! (might be mixed) 10.3

Every finite game has a Nash Equilibrium

Every finite game has a Nash Equilibrium

EQUILIBRIUM POINTS IN N -PERSON GAMES

BY JOHN F. NASH, JR.*

PRINCETON UNIVERSITY

Communicated by S. Lefschetz, November 16, 1949

One may define a concept of an n -person game in which each player has a finite set of pure strategies and in which a definite set of payments to the n players corresponds to each n -tuple of pure strategies, one strategy being taken for each player. For mixed strategies, which are probability distributions over the pure strategies, the pay-off functions are the expectations of the players, thus becoming polylinear forms in the probabilities with which the various players play their various pure strategies.

Any n -tuple of strategies, one for each player, may be regarded as a point in the product space obtained by multiplying the n strategy spaces of the players. One such n -tuple counters another if the strategy of each player in the countering n -tuple yields the highest obtainable expectation for its player against the $n - 1$ strategies of the other players in the countered n -tuple. A self-countering n -tuple is called an equilibrium point.

The correspondence of each n -tuple with its set of countering n -tuples gives a one-to-many mapping of the product space into itself. From the definition of countering we see that the set of countering points of a point is convex. By using the continuity of the pay-off functions we see that the graph of the mapping is closed. The closedness is equivalent to saying: if P_1, P_2, \dots and $Q_1, Q_2, \dots, Q_n, \dots$ are sequences of points in the product space where $Q_n \rightarrow Q$, $P_n \rightarrow P$ and Q_n counters P_n then Q counters P .

Since the graph is closed and since the image of each point under the mapping is convex, we infer from Kakutani's theorem¹ that the mapping has a fixed point (i.e., point contained in its image). Hence there is an equilibrium point.

In the two-person zero-sum case the "main theorem"² and the existence of an equilibrium point are equivalent. In this case any two equilibrium points lead to the same expectations for the players, but this need not occur in general.

* The author is indebted to Dr. David Gale for suggesting the use of Kakutani's theorem to simplify the proof and to the A. E. C. for financial support.

¹ Kakutani, S., *Duke Math. J.*, 8, 457-459 (1941).

² Von Neumann, J., and Morgenstern, O., *The Theory of Games and Economic Behaviour*, Chap. 3, Princeton University Press, Princeton, 1947.

Every finite game has a Nash Equilibrium

Kakutani's fixed-point theorem

A correspondence $f: X \rightarrow X$ has a fixed point (i.e., $\mathbf{x} \in f(\mathbf{x})$ for some $\mathbf{x} \in X$) if all of the following conditions hold.

- (1) X is a non-empty, closed, bounded, and convex set.
- (2) $f(\mathbf{x})$ is non-empty for any \mathbf{x} .
- (3) $f(\mathbf{x})$ is convex for any \mathbf{x} .
- (4) The set $\{ (\mathbf{x}, \mathbf{y}) \mid \mathbf{y} \in f(\mathbf{x}) \}$ is closed.

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- A fixed point of BR is a Nash Equilibrium
- The BR operator and policy space for finite games meet the conditions above
- BR has a fixed point for every finite game, i.e. every finite game has a Nash Equilibrium

General approach to find Nash Equilibria

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$$\begin{aligned} &\underset{\pi, U}{\text{minimize}} && \sum_i \left(U^i - U^i(\pi) \right) \\ &\text{subject to} && U^i \geq U^i(a^i, \pi^{-i}) \text{ for all } i, a^i \\ & && \sum_{a^i} \pi^i(a^i) = 1 \text{ for all } i \\ & && \pi^i(a^i) \geq 0 \text{ for all } i, a^i \end{aligned}$$

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4	1	6	5	4	3	2	1	6	5	4	3	2	1	
2	3	4	2	3	4	2	3	4	2	3	4	2	3	2 3
1	4	1	4	1	4	1	4	1	4	1	4	1	4	1 4
Heg. Stability	Samaritan _{su}	Samaritan _{su}	Clock _{su}	Clock _{su}	Endless	Called Bluff	Bully	Unfair	Skewed BoS	Asym BoS	Chicken			—
3	3	4	2	3	4	2	3	4	2	3	4	2	3	3 2
1	4	1	4	1	4	1	4	1	4	1	4	1	4	1 4
Samson	Asym Sd _{su}	Asym Sd _{su}	Cycle _{su}	Cycle _{su}	Inspector	Self-serving _{su}	Protector _{su}	Protector _{su}	Favorites _{su}	Battle of Sexes	Asym BoS			
4	3	4	2	3	4	2	3	4	2	3	4	2	3	3 1
1	4	1	4	1	4	1	4	1	4	1	4	1	4	2 4
Delliah	Asym Sd _{su}	Asym Sd _{su}	Pursuit	Pareto	Missile Crisis	Self-serving _{su}	Protector _{su}	Protector _{su}	Hero	Favorites _{su}	Skewed BoS			X
5	3	4	2	3	4	2	3	4	2	3	4	2	3	2 1
1	4	1	4	1	4	1	4	1	4	1	4	1	4	3 4
Hostage	Benevolence _{su}	Benevolence _{su}	2nd Best _{su}	2nd Best _{su}	Big Bully	Tragedy	Delight _{su}	Pure Delight	Protector _{su}	Protector _{su}	Unfair			
6	3	4	2	3	4	2	3	4	2	3	4	2	3	1 2
1	4	1	4	1	4	1	4	1	4	1	4	1	4	3 4
Blackmailer	Benevolence _{su}	Benevolence _{su}	2nd Best _{su}	2nd Best _{su}	Hamlet	Total Conflict	Mixed Delight	Delight _{su}	Protector _{su}	Protector _{su}	Bully			I
1	3	4	2	3	4	2	3	4	2	3	4	2	3	1 3
1	4	1	4	1	4	1	4	1	4	1	4	1	4	2 4
Id. Hegemony	Samaritan _{su}	Samaritan _{su}	Revelation	Alibi	Asym Pd	Prisoners D.	Total Conflict	Tragedy	Self-serving _{su}	Self-serving _{su}	Called Bluff			—
2	3	4	2	3	4	2	3	4	2	3	4	2	3	1 4
1	4	1	4	1	4	1	4	1	4	1	4	1	4	2 3
Win-win	C. Aligned _{su}	C. Aligned _{su}	C. Assurance _{su}	C. Assurance _{su}	Stag Hunt	Asym Pd	Hamlet	Big Bully	Missile Crisis	Inspector	Endless			—
3	3	4	2	3	4	2	3	4	2	3	4	2	3	1 4
1	4	1	4	1	4	1	4	1	4	1	4	1	4	3 2
R Assurance	Commons _{su}	Commons _{su}	Coordination _{su}	Coordination _{su}	R Assurance	Alibi	2nd Best _{su}	2nd Best _{su}	Pareto	Cycle _{su}	Clock _{su}			
4	3	4	2	3	4	2	3	4	2	3	4	2	3	2 4
1	4	1	4	1	4	1	4	1	4	1	4	1	4	3 1
R Assurance	Commons _{su}	Commons _{su}	Coordination _{su}	Coordination _{su}	R Assurance	Revelation	2nd Best _{su}	2nd Best _{su}	Pursuit	Cycle _{su}	Clock _{su}			X
5	3	4	2	3	4	2	3	4	2	3	4	2	3	3 4
1	4	1	4	1	4	1	4	1	4	1	4	1	4	2 1
Row Aligned	Harmony _{su}	Harmony-mix	Commons _{su}	Commons _{su}	Row Aligned _{su}	Samaritan _{su}	Benevolent _{su}	Benevolent _{su}	Asym Sd _{su}	Asym Sd _{su}	Samaritan _{su}			
6	3	4	2	3	4	2	3	4	2	3	4	2	3	3 4
1	4	1	4	1	4	1	4	1	4	1	4	1	4	1 2
Row Aligned	Harmony-pure	Harmony	Commons _{su}	Commons _{su}	Row Aligned _{su}	Samaritan _{su}	Benevolent _{su}	Benevolent _{su}	Asym Sd _{su}	Asym Sd _{su}	ActiveSam _{su}			I
2	3	4	2	3	4	2	3	4	2	3	4	2	3	2 4
1	4	1	4	1	4	1	4	1	4	1	4	1	4	1 3
No Conflict	C. Aligned _{su}	C. Aligned _{su}	C. Assurance _{su}	C. Assurance _{su}	Win-win	Id. Hegemony	Blackmailer	Hostage	Delliah	Samson	Heg. Stability			
3	3	4	2	3	4	2	3	4	2	3	4	2	3	2
1	4	1	4	1	4	1	4	1	4	1	4	1	4	1

Recap

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- Games may not have a single "optimal" solution; instead there are equilibria
- If every player is playing a best response, that joint policy is a Nash Equilibrium
- Every finite game has at least one Nash Equilibrium (pure or mixed)

Battle of the Sexes

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- Gabby and Max are going on a date

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A hand-drawn blue payoff matrix for the Battle of the Sexes game. The matrix is a square divided into four quadrants by a horizontal and vertical line. The labels 'G' and 'M' are written in blue ink around the matrix. 'G' is written to the left of the matrix, and 'M' is written above the matrix. The quadrants are labeled with 'G' and 'M' as follows: the top-left quadrant is labeled 'G' above it and 'G' to its left; the top-right quadrant is labeled 'M' above it and 'M' to its right; the bottom-left quadrant is labeled 'G' above it and 'M' to its left; the bottom-right quadrant is labeled 'M' above it and 'M' to its right.

	G	M
G		
M		

Battle of the Sexes

- Gabby and Max are going on a date
- Gabby wants to go to a football game
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		M	
		G	M
G	G	2, 1	0, 0
	M	0, 2	1, 2

Battle of the Sexes

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- Gabby wants to go to a football game
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Correlated Equilibrium

- A *correlated joint policy* is a single distribution over the joint actions of all agents.
- A *correlated equilibrium* is a correlated joint policy where no agent i can increase their expected utility by deviating from their current action to another.

A hand-drawn payoff matrix for the Battle of the Sexes game. The matrix is a 2x2 grid with 'G' and 'M' as row and column headers. The payoffs are written in the cells: (G, G) is 2,1; (G, M) is 0,0; (M, G) is 0,2; and (M, M) is 1,2.

	G	M
G	2,1	0,0
M	0,2	1,2