

ASEN 5519-003 Decision Making under Uncertainty

Homework 1: Probabilistic Models

January 16, 2023

1 Questions

Question 1. (20 pts) Consider the following joint distribution of three binary-valued random variables, A , B , and C :

A	B	C	$P(A, B, C)$
0	0	1	0.15
0	1	0	0.05
0	1	1	0.01
1	0	0	0.14
1	0	1	0.18
1	1	0	0.29
1	1	1	0.07

a) What is the probability of the outcome $A = 0$, $B = 0$, $C = 0$?

b) What is the marginal distribution of A ?

c) What is the conditional distribution of A given $B = 0$ and $C = 1$?

Question 2. (20 pts) 2% of women at age forty who participate in routine screening have breast cancer. 86% of those with breast cancer will get positive mammograms. 8% of those without breast cancer will also get positive mammograms. A woman in this age group had a positive mammogram in a routine screening. What is the probability that she actually has breast cancer?

Question 3. (40 pts) Suppose that a stationary stochastic process $\{x_t\}$ is defined by the following equation: $x_{t+1} = 1.5x_t - x_{t-1} + v_t$ where v_t are independent, identically distributed random variables with $v_t \sim \mathcal{N}(0.0, 0.2^2)$.

- (a) Simulate and plot 10 20-step trajectories sampled from this process with $x_0 = x_{-1} = 1$ (as always, submit your code for this).
- (b) Is this process a Markov process if the state is defined as x_t ? Why or why not?
- (c) If you only had access to the trajectories you plotted what evidence could you use to convince someone that this process is or is not Markov?
- (d) What would need to be included in the state at time t to make this a Markov process?

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2 Auto-graded Programming

Question 4. (20 pts) In this exercise, you will write and test a Julia function to ensure that you can get Julia and the course-specific code running and help you learn how to do a task that sometimes trips students up in homework 2. Your function should take two arguments:

- **a**: A matrix, and
- **bs**: A non-empty vector of vectors.

The function should multiply all of the vectors in **bs** by **a** and then return the *elementwise* maximum of the resulting vectors.

In order to get full-credit, the function must be completely “type-stable” (see the “Performance Tips” section of the Julia manual). Your function should always return a vector with the same element type as **a**. You can assume the vectors in **bs** will have the same element type as **a**, but you should be able to handle **a** with any numeric element type.

Evaluate this function with `DMUStudent.HW1.evaluate` and submit the resulting json file *along with a listing of the code*. A score of 1 will receive full credit.