Last Time

Last Time

- Does value iteration always converge?
- Is the value function unique?

Guiding Questions

Guiding Questions

- What are the differences between online and offline solutions?
- Are there solution techniques that are *independent* of the state space size?

Offline vs Online Solutions

Offline

Online

Offline vs Online Solutions

Offline

- Before Execution: find V^*/Q^*

Online

Offline vs Online Solutions

Offline

- Before Execution: find V^*/Q^*
- During Execution: $\pi^*(s) = \operatorname{argmax} Q^*(s, a)$

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Offline vs Online Solutions

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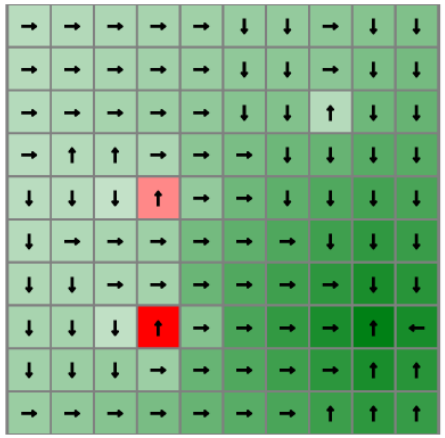

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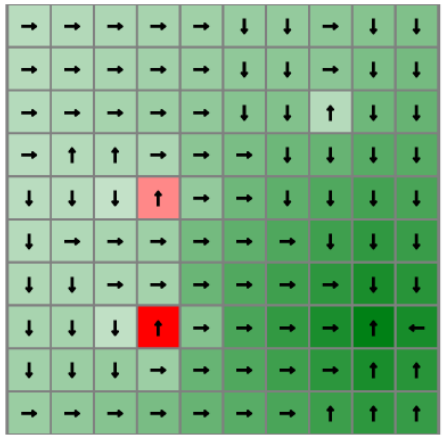

<u>Online</u>

- Before Execution: <nothing>

Offline vs Online Solutions

<u>Offline</u>

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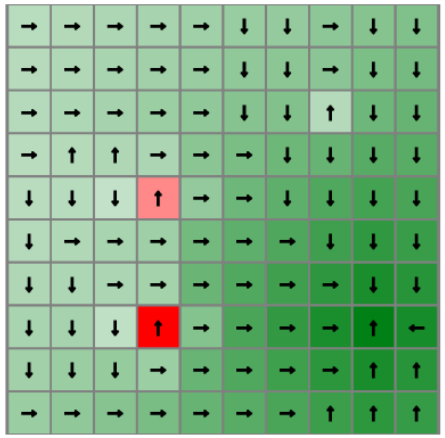

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Offline vs Online Solutions

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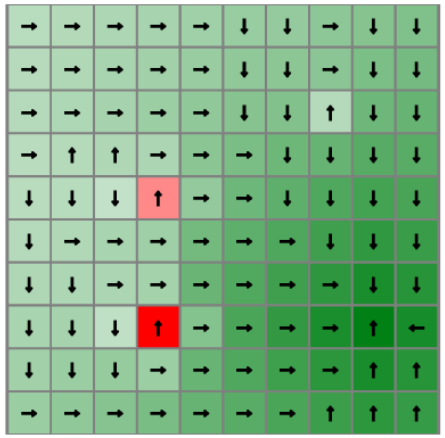

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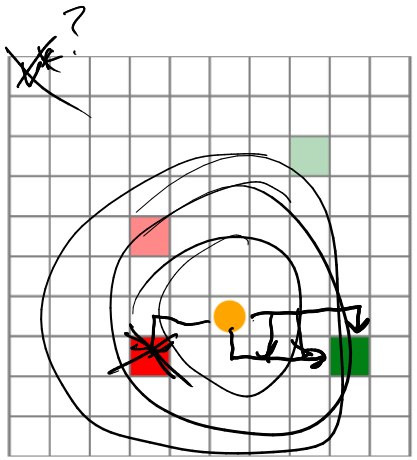

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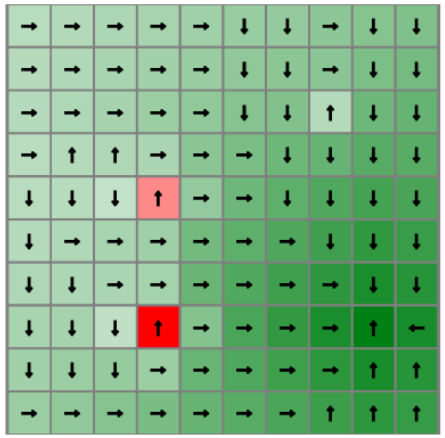

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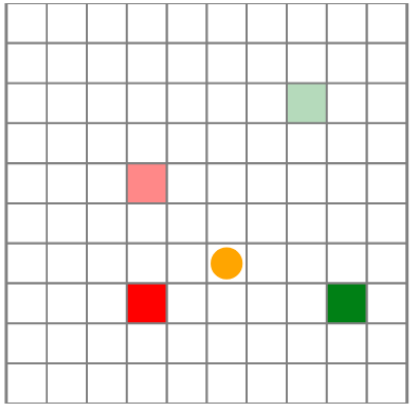

- Why?

Offline vs Online Solutions

Offline

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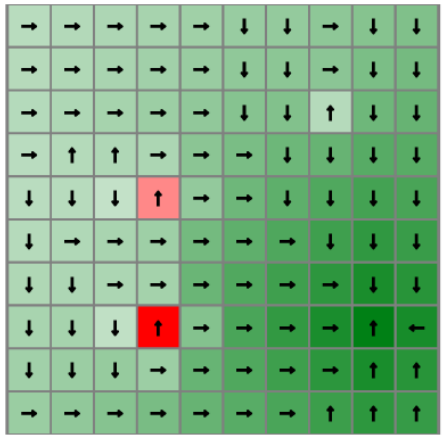

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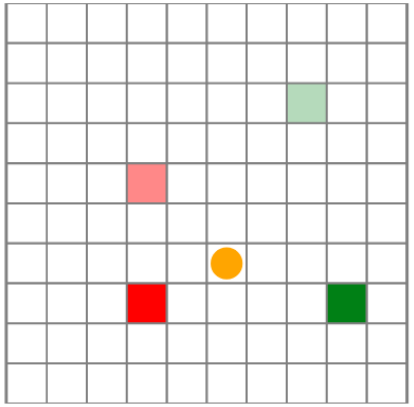
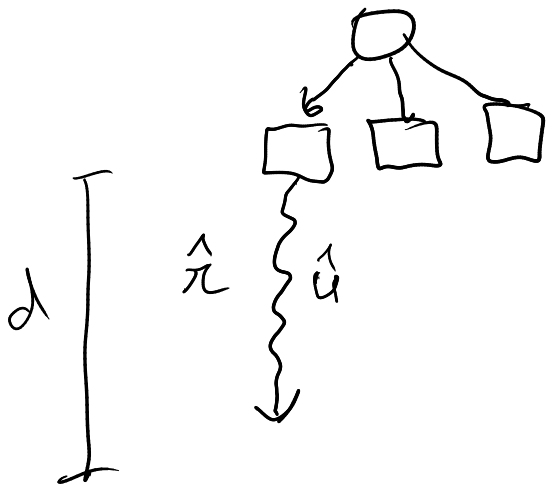


- Why?
- Online methods are insensitive to the size of S !

Lookahead

Lookahead



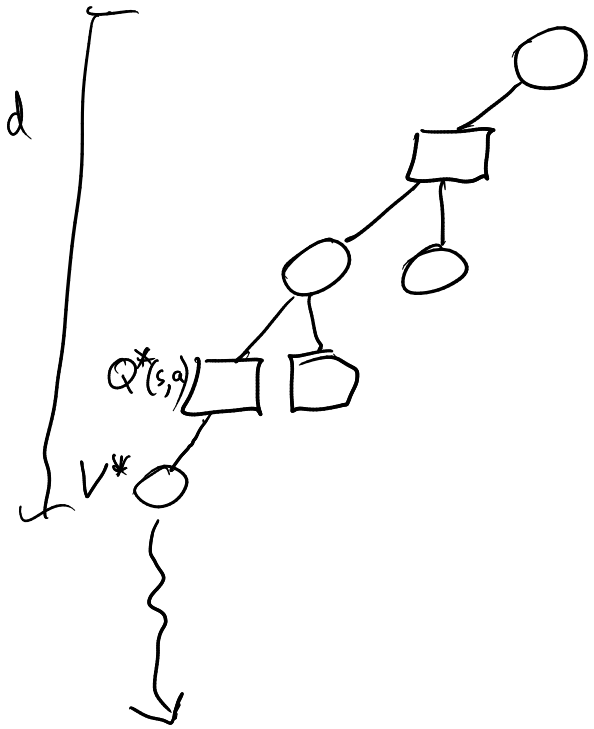

```
randstep(𝒫::MDP, s, a) = 𝒫.TR(s, a)

function rollout(𝒫, s, π, d)
    ret = 0.0
    for t in 1:d
        a = π(s)
        s, r = randstep(𝒫, s, a)
        ret += 𝒫.γ^(t-1) * r
    end
    return ret
end

function (π::RolloutLookahead)(s)
    U(s) = rollout(π.𝒫, s, π.π, π.d)
    return greedy(π.𝒫, U, s).a
end
```

Forward Search

Forward Search



$$O\left((|S| \times |A|)^d\right)$$

```
function forward_search(𝒫, s, d, U)
    if d ≤ 0
        return (a=nothing, u=U(s))
    end
    best = (a=nothing, u=-Inf)
    U′(s) = forward_search(𝒫, s, d-1, U).u
    for a in 𝒫.𝒜
        u = lookahead(𝒫, U′, s, a)
        if u > best.u
            best = (a=a, u=u)
        end
    end
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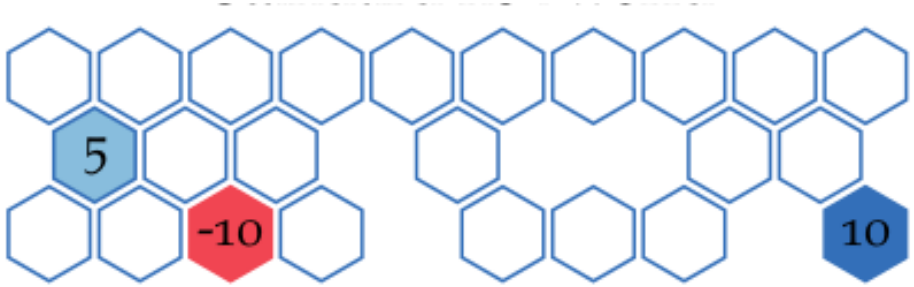

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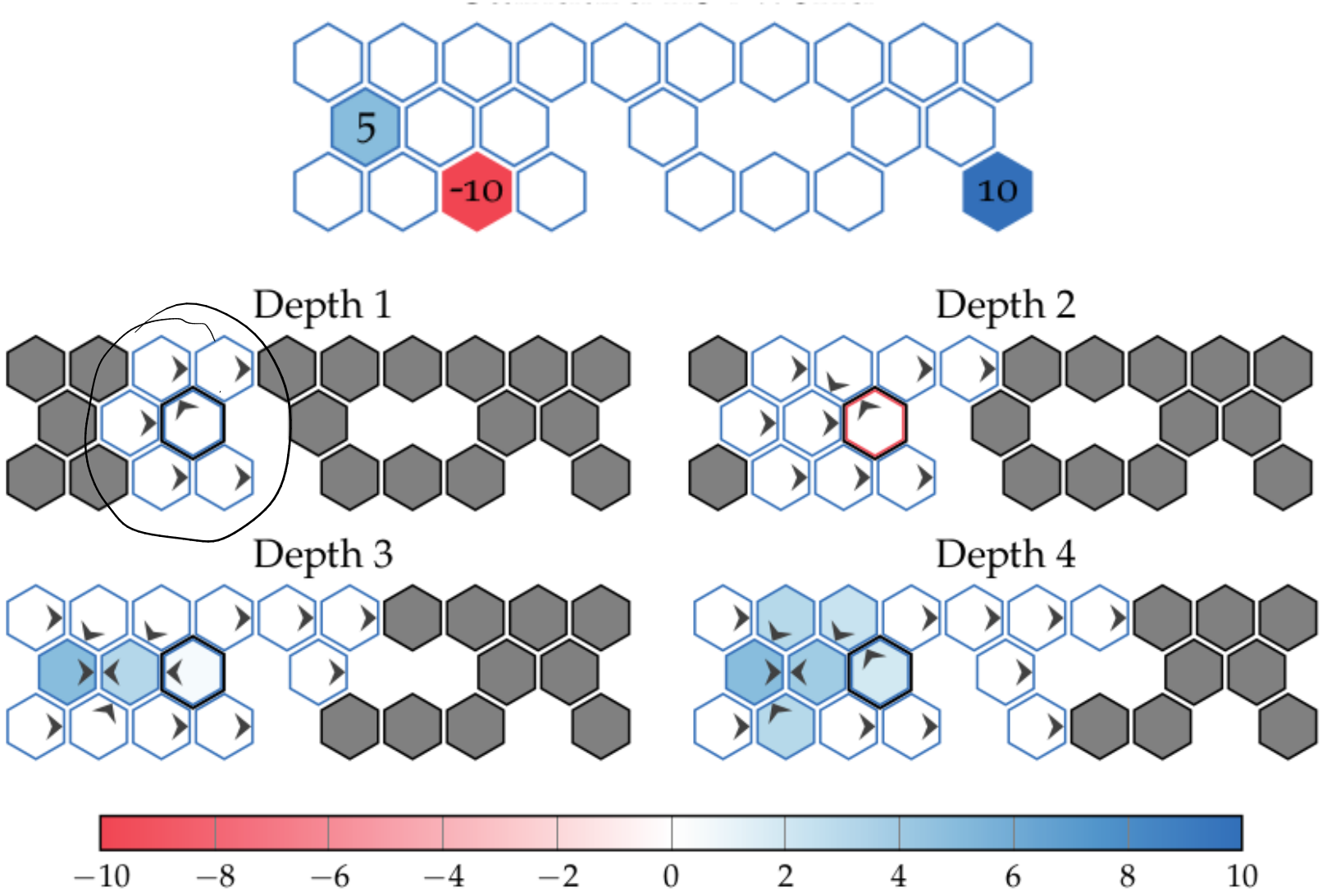

Branch and Bound

Branch and Bound

Assume you have $\underline{V}(s)$ and $\bar{Q}(s, a)$

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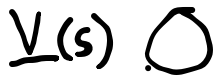


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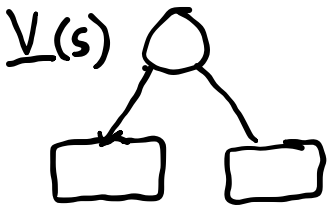


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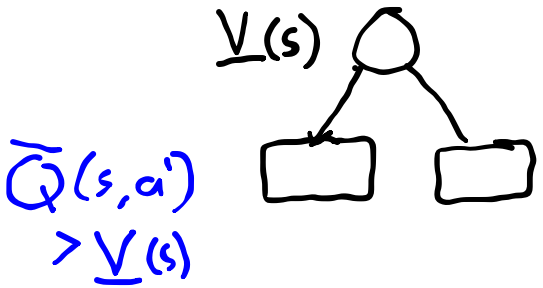


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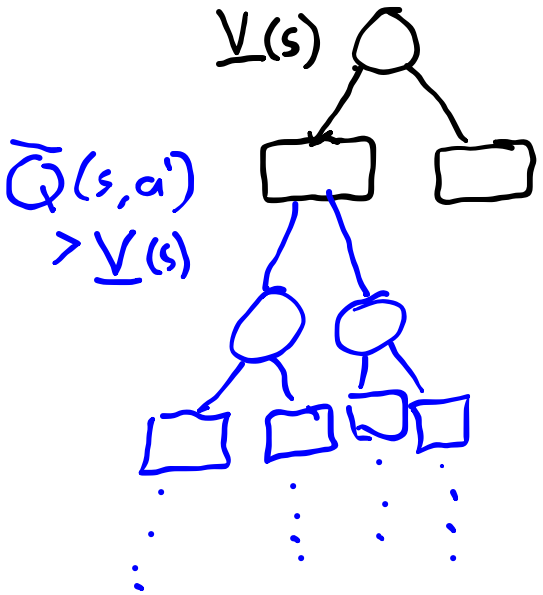


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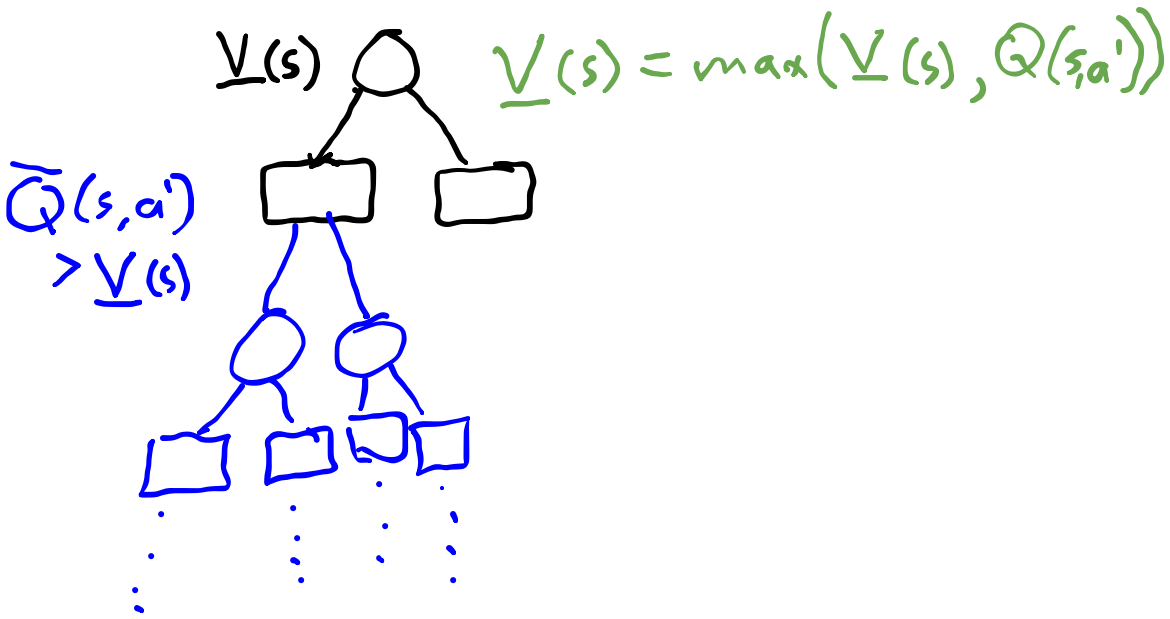


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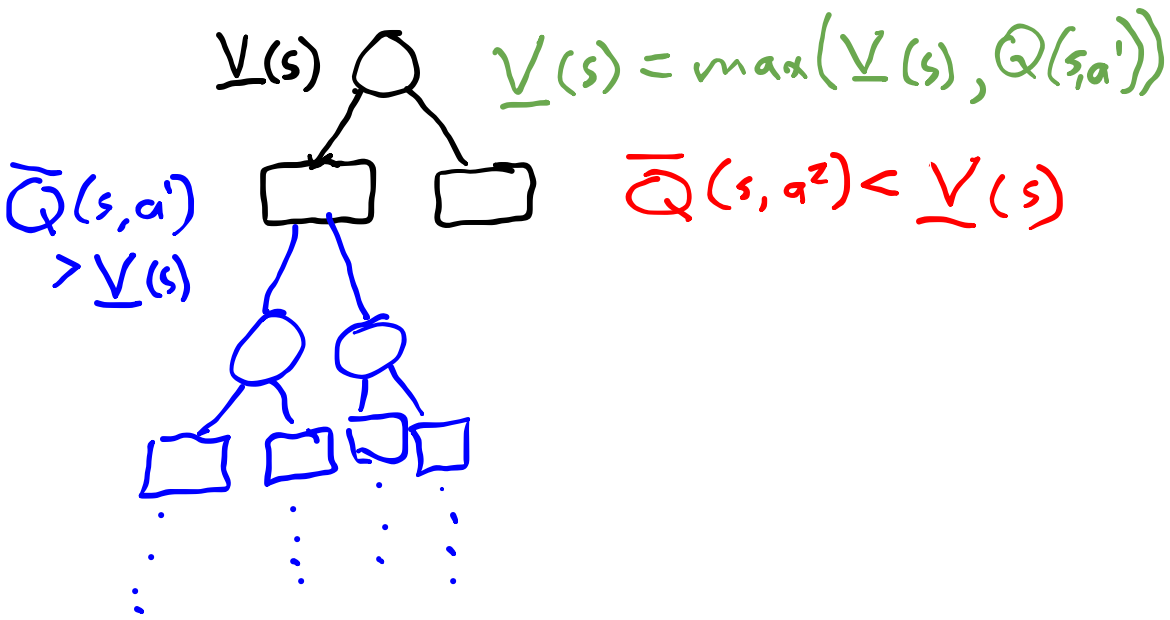


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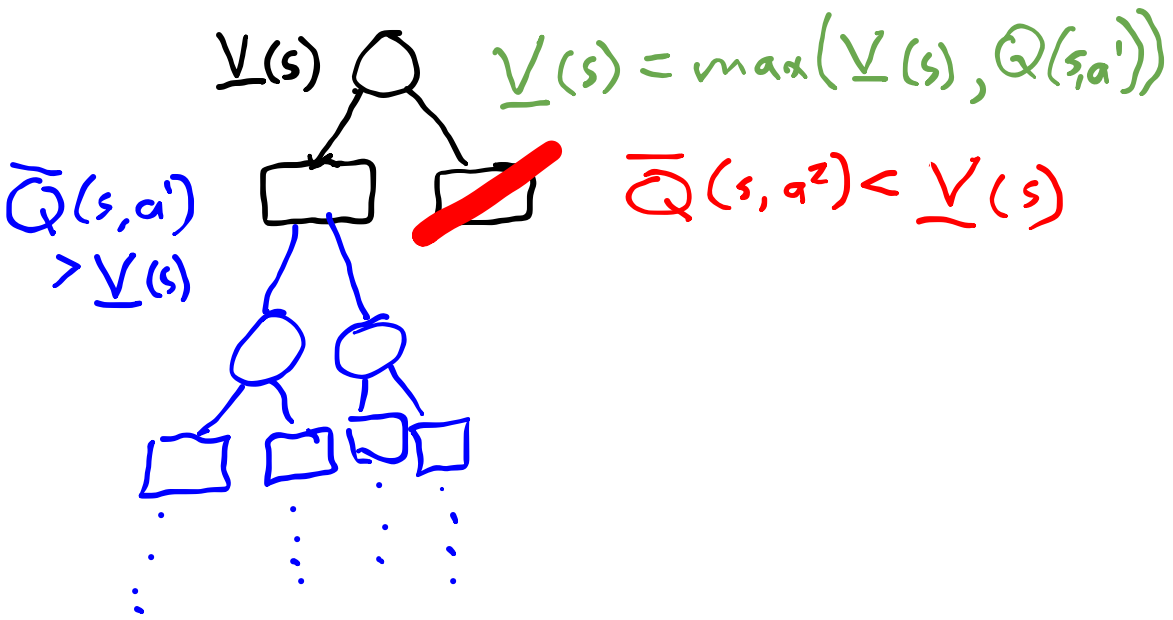


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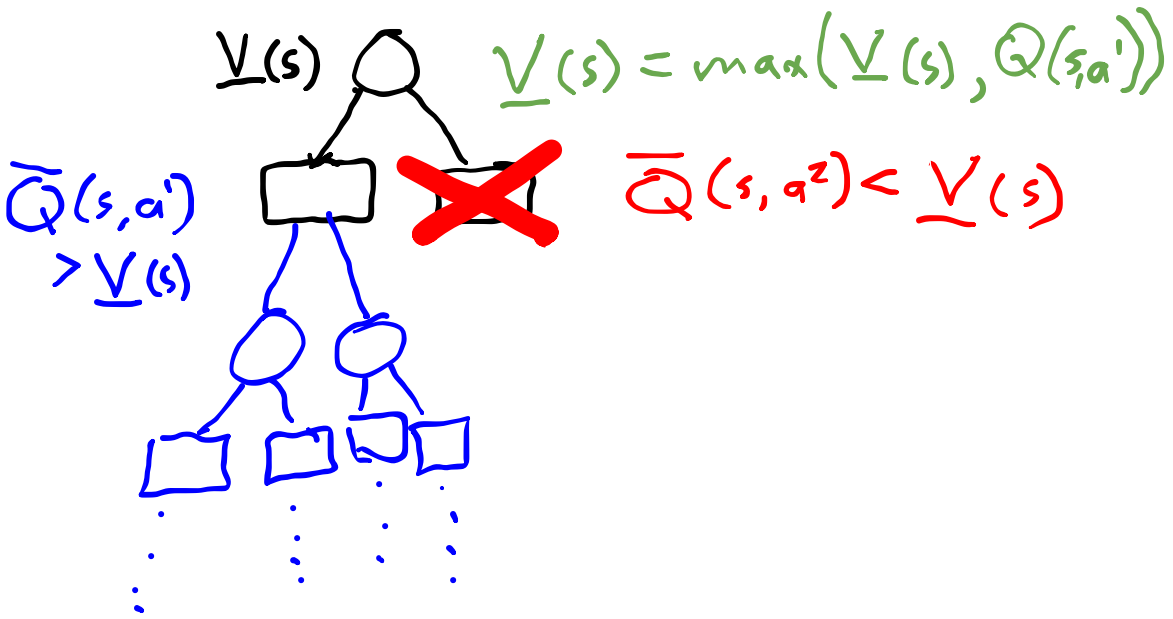


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Sparse Sampling

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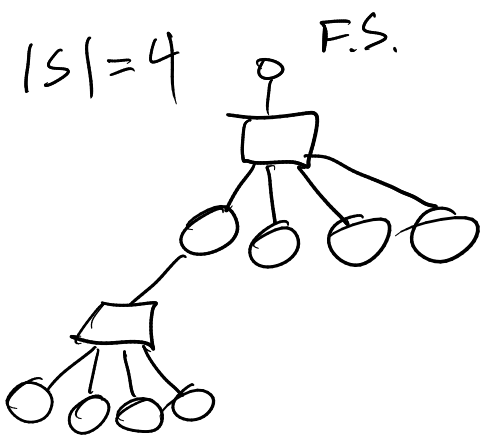

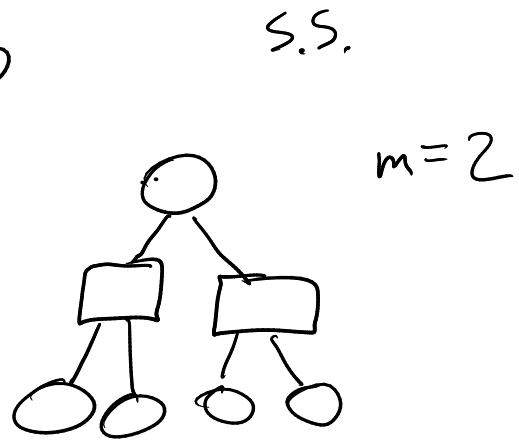


```
function sparse_sampling(𝒫, s, d, m, U)
    if d ≤ 0
        return (a=nothing, u=U(s))
    end
    best = (a=nothing, u=-Inf)
    for a in 𝒫.𝒜
        u = 0.0
        for i in 1:m
            s′, r = randstep(𝒫, s, a)
            a′, u′ = sparse_sampling(𝒫, s′, d-1, m, U)
            u += (r + 𝒫.γ*u′) / m
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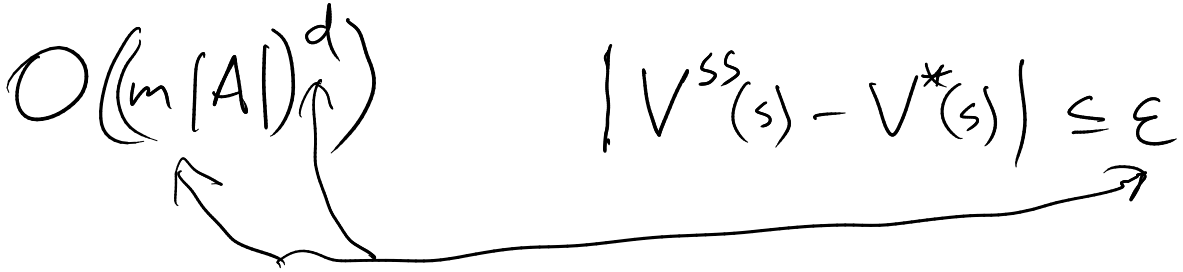

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$$O\left((m|A|)^d\right)$$

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https://www.cis.upenn.edu/~mkearns/papers/sparsesampling-journal.pdf

Monte Carlo Tree Search (MCTS/UCT)

Monte Carlo Tree Search (MCTS/UCT)

Search

Monte Carlo Tree Search (MCTS/UCT)

Search

Expansion

Monte Carlo Tree Search (MCTS/UCT)

Search Expansion Rollout

Monte Carlo Tree Search (MCTS/UCT)

Search Expansion Rollout Backup

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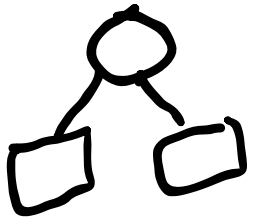
Search

Expansion

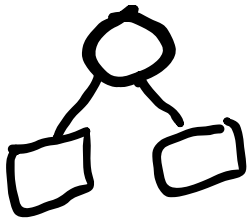


Rollout

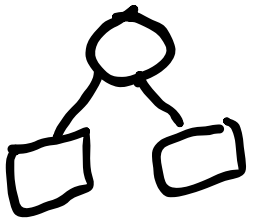
Backup

Monte Carlo Tree Search (MCTS/UCT)

Search



Expansion

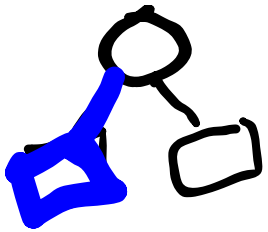


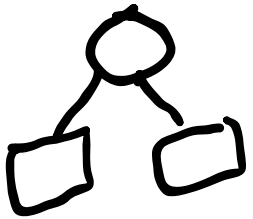
Rollout

Backup

Monte Carlo Tree Search (MCTS/UCT)

Search | Expansion | Rollout | Backup





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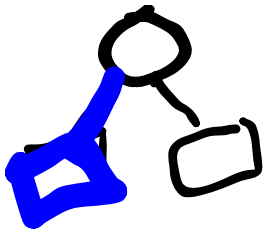
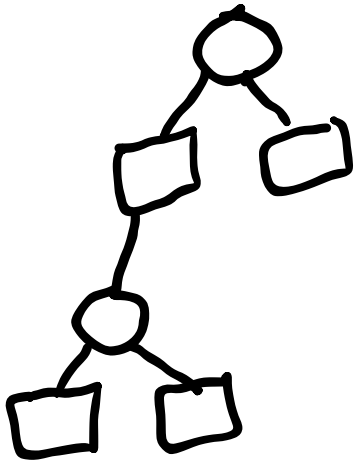
low $N(s,a)/N(s)$ = high bonus

start with $c = 2(\bar{V} - \underline{V})$

Monte Carlo Tree Search (MCTS/UCT)

Search | Expansion | Rollout | Backup





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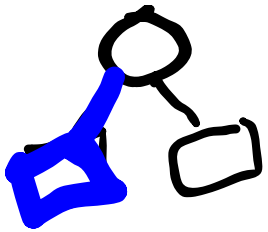

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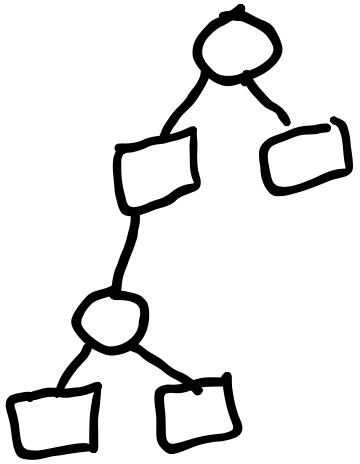

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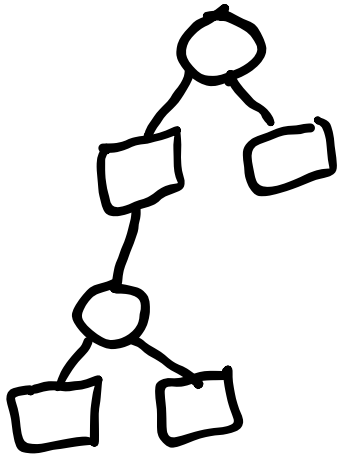

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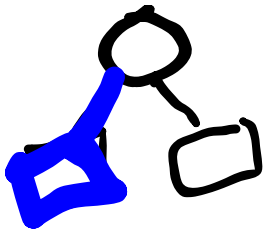

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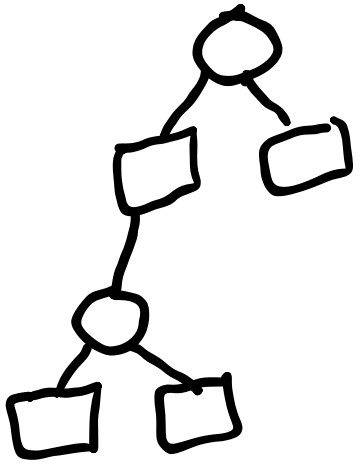

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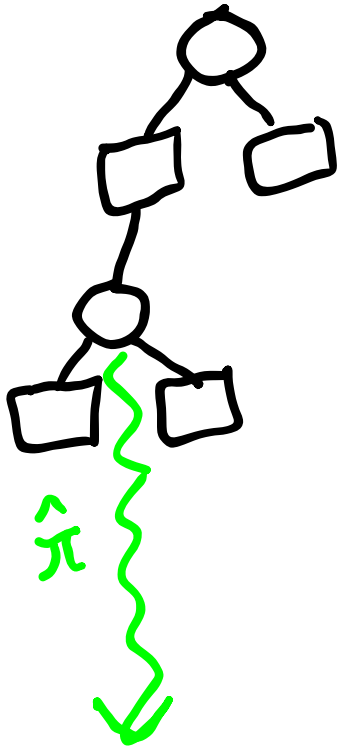

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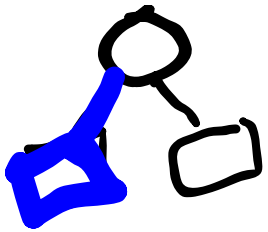

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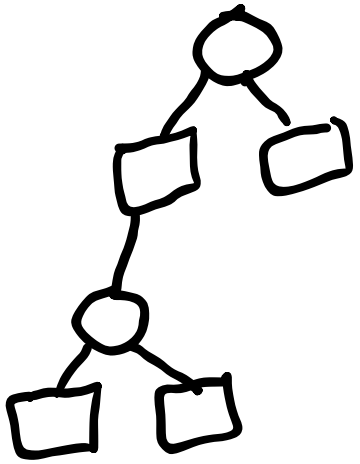

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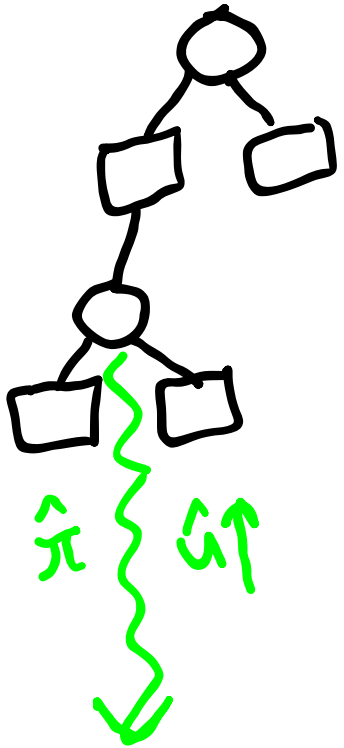

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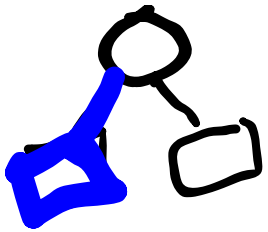

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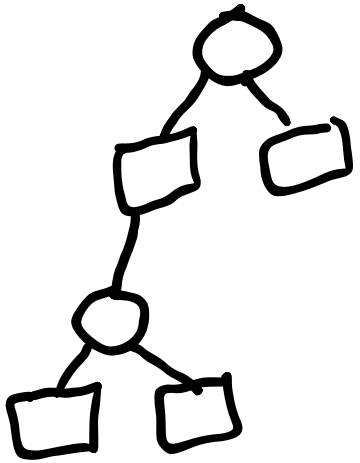
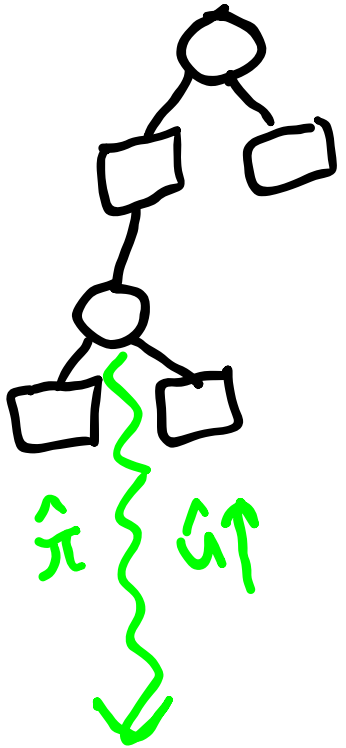


Rollout



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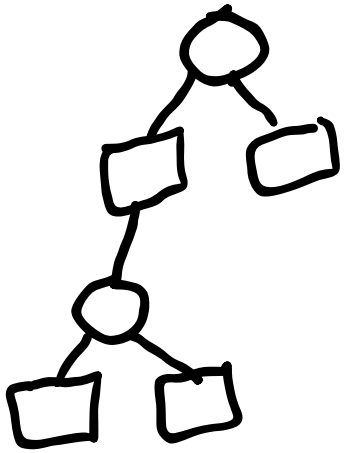

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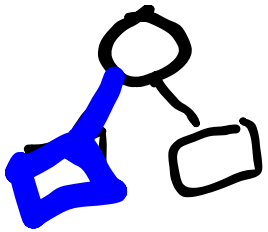

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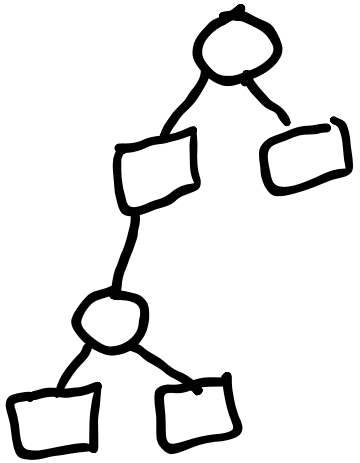

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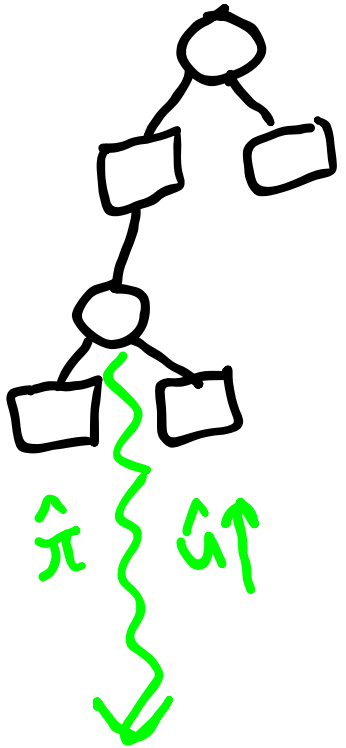

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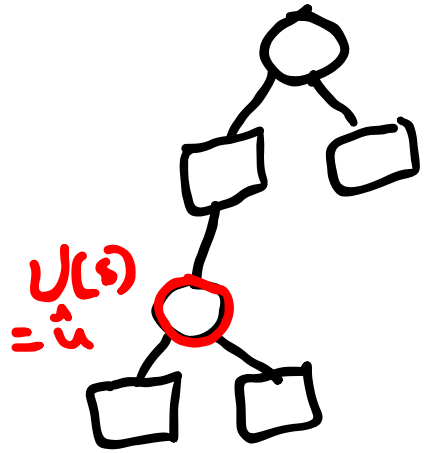

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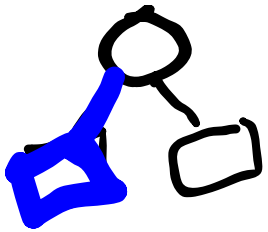

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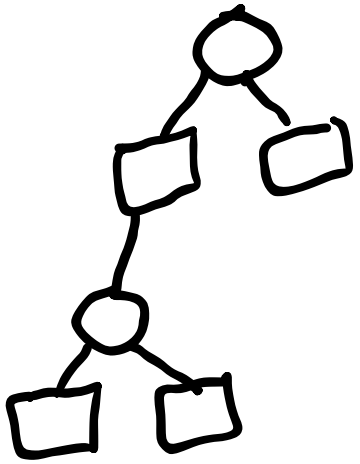

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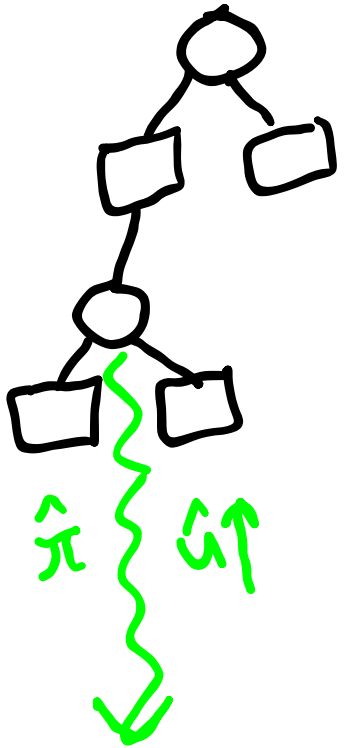

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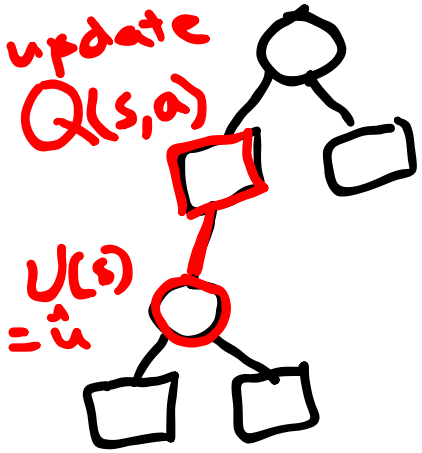

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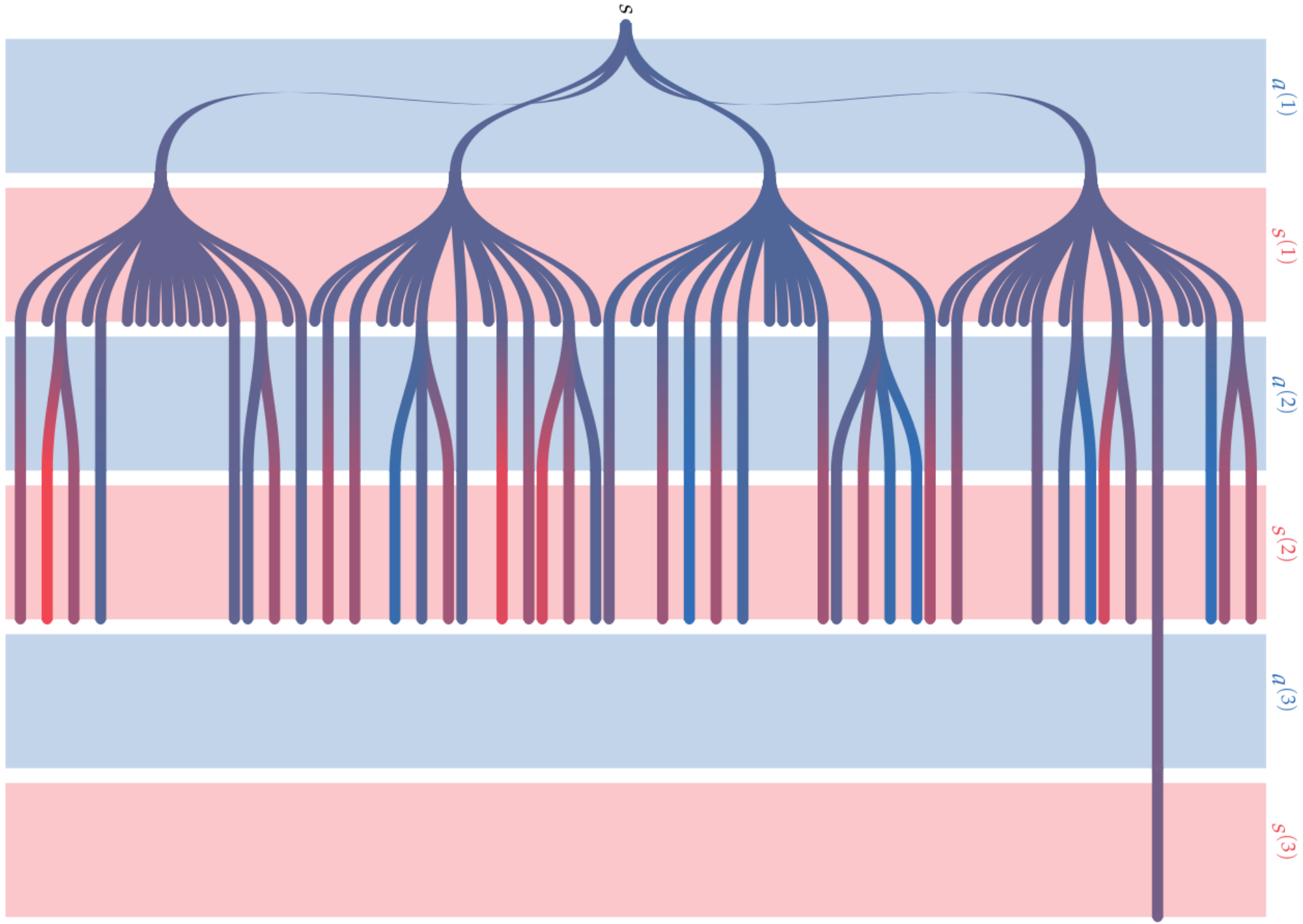
s
$a^{(1)}$
$s^{(1)}$
$a^{(2)}$
$s^{(2)}$
$a^{(3)}$
$s^{(3)}$

Guiding Questions

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- What are the differences between online and offline solutions?
- Are there solution techniques that are *independent* of the state space size?