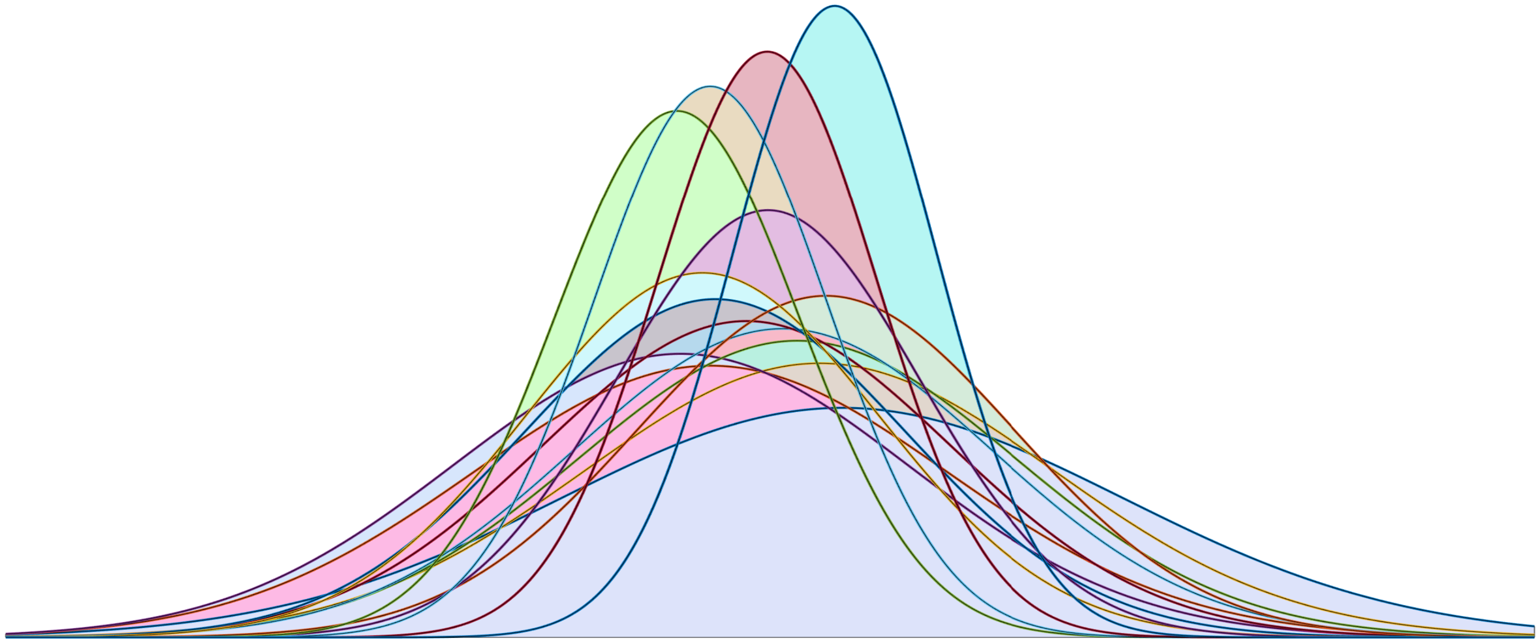
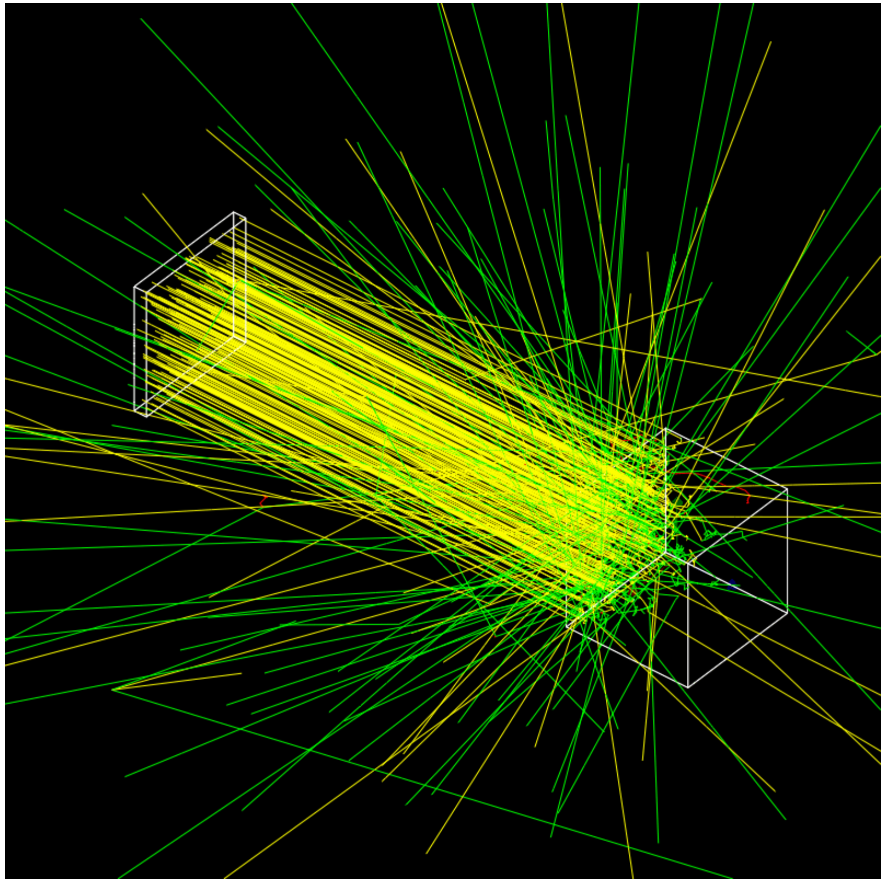
Towards an interval particle transport monte carlo method



Ander Gray
Institute for Risk and Uncertainty
University of Liverpool

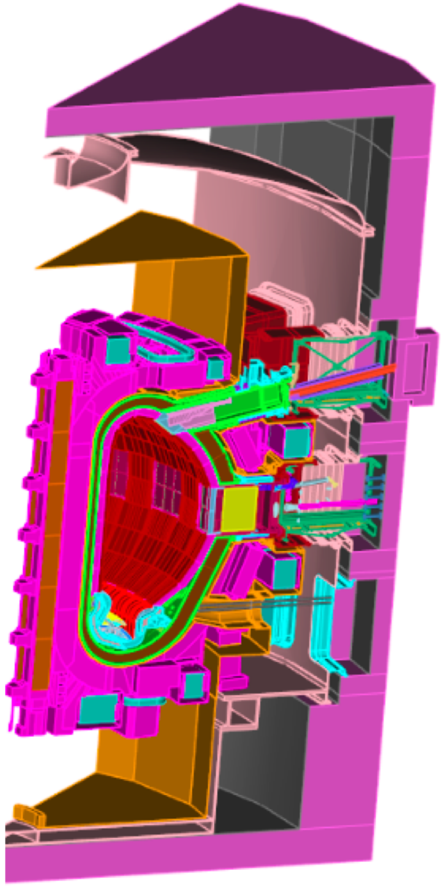
Neutron transport monte carlo

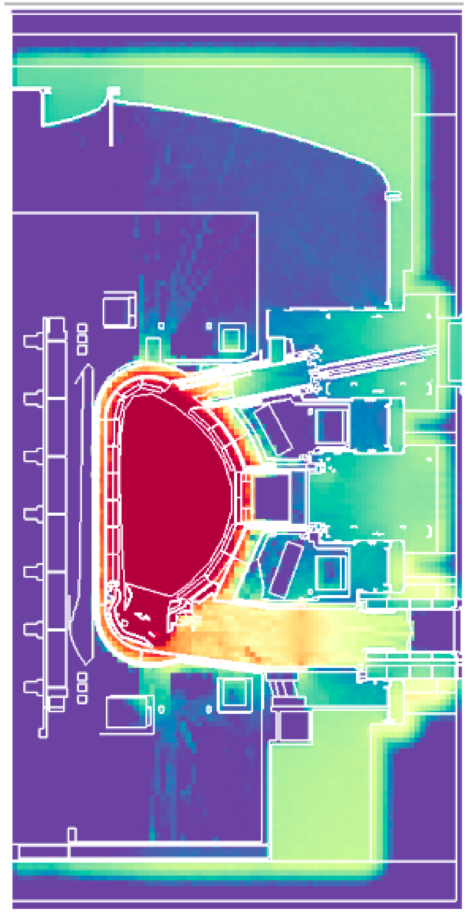


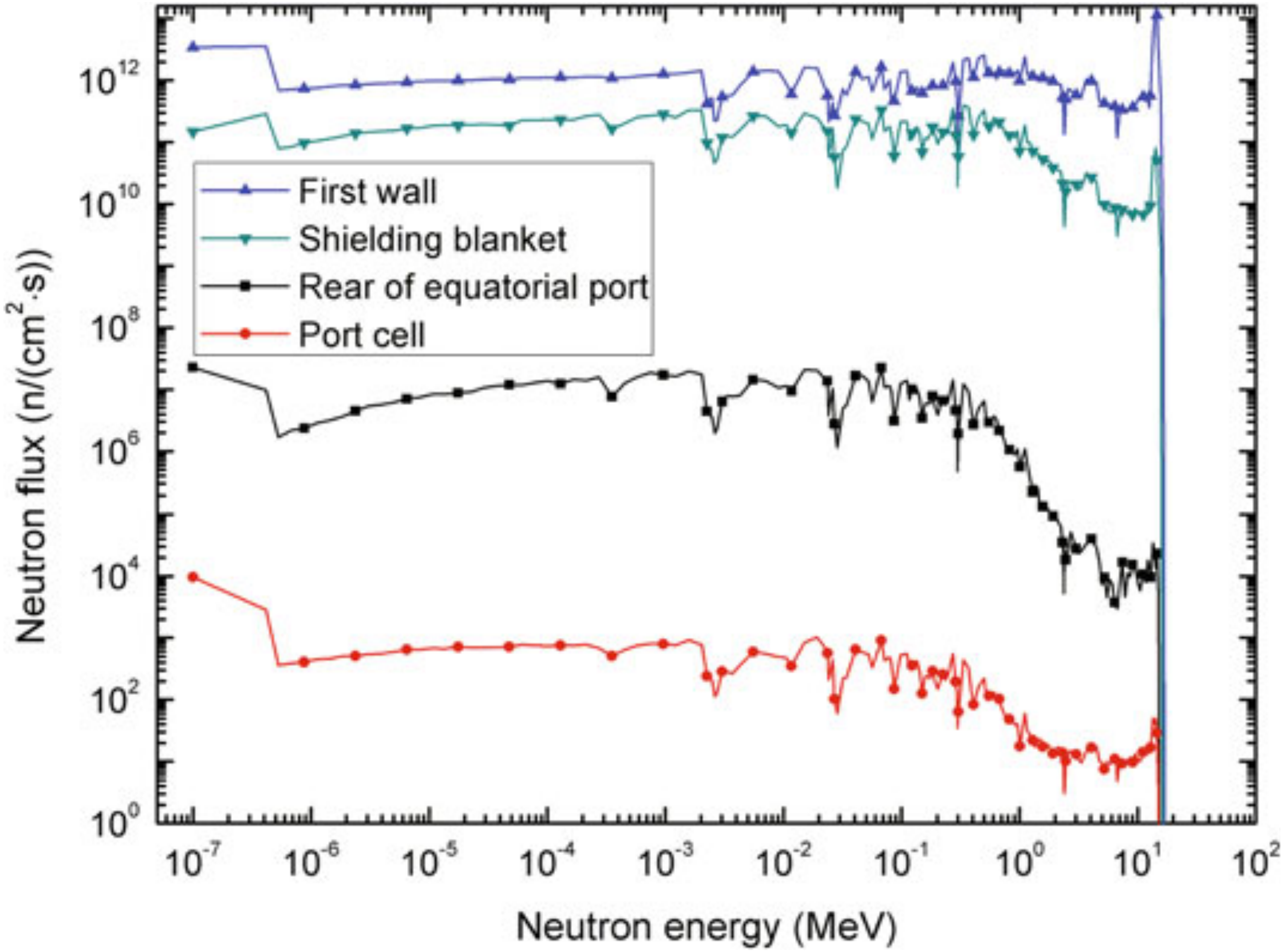
Inputs:

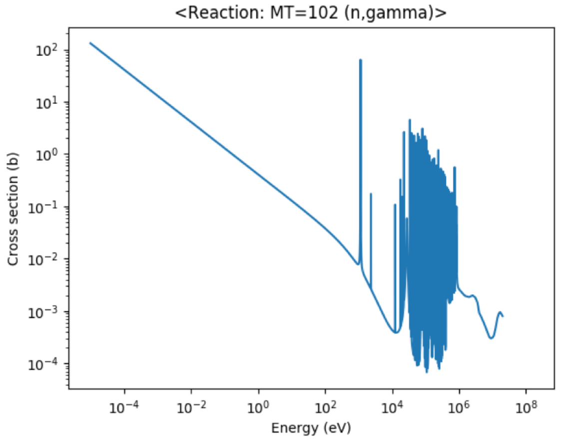
- Particle source
- Geometry
- A definition of particle/matter interaction rules (Nuclear Data)

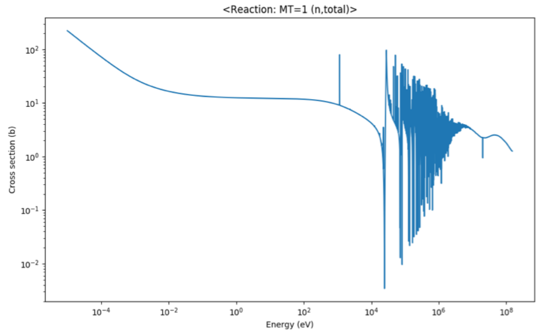
Neutron transport monte carlo

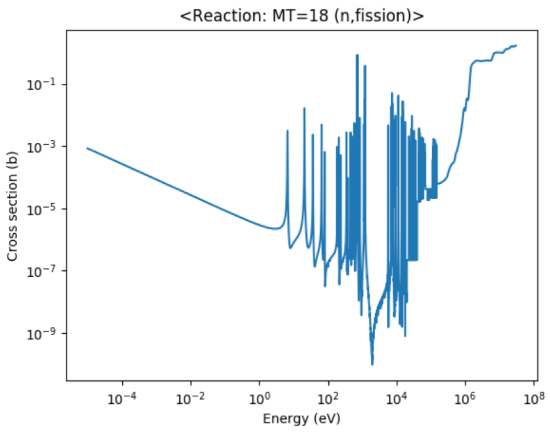




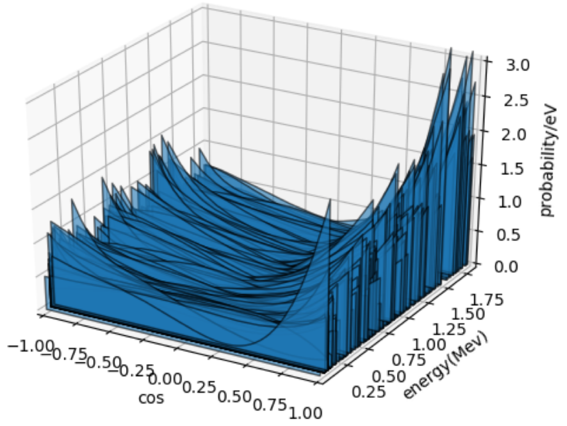


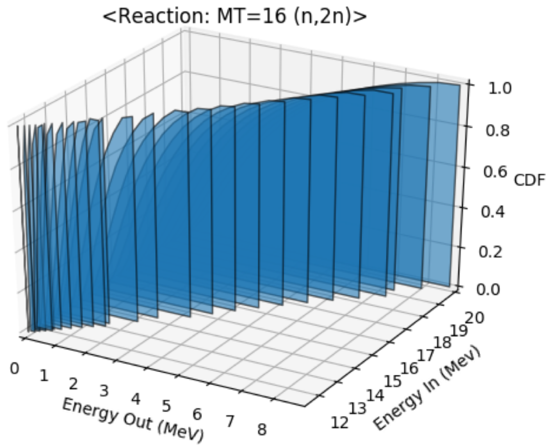






Nuclear Data





- For Tokamaks you would need to consider 100s of different nuclides
- Each nuclide has ~ 30 different reaction channels (events)
- All of which are uncertain

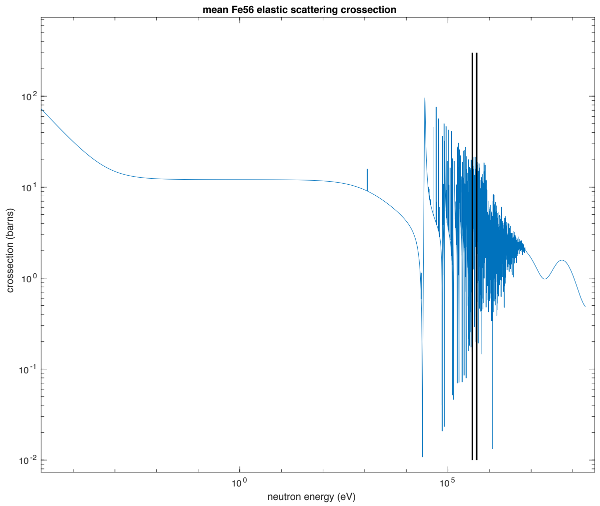
~ 10^{12} histories needed for reasonable monte carlo error

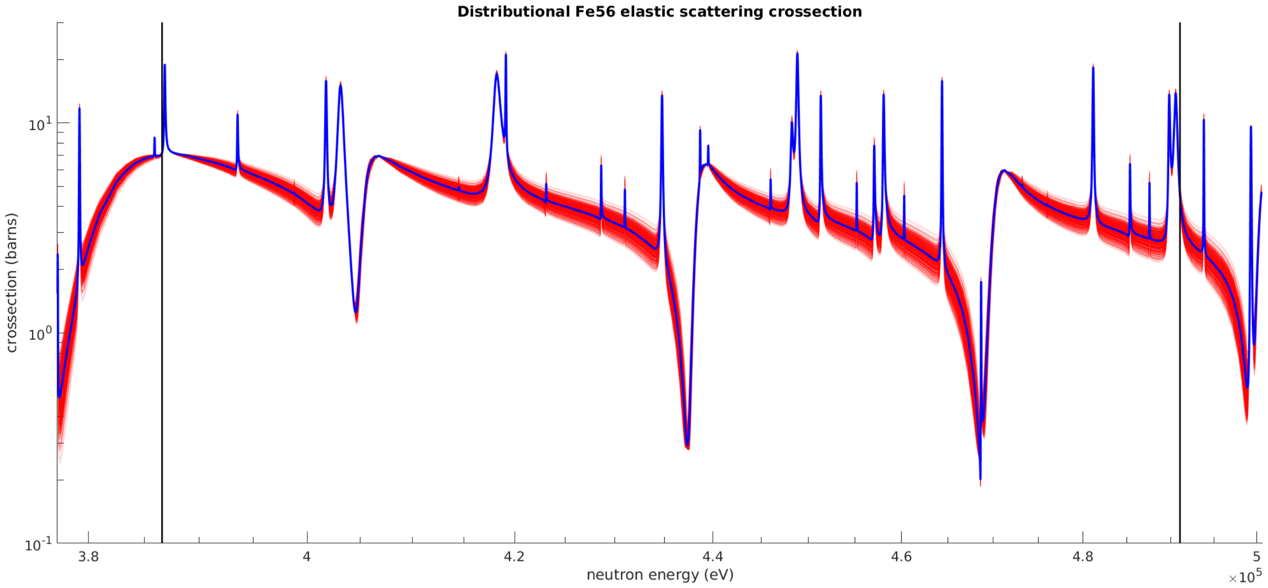
We can simulate ~1000 histories per s

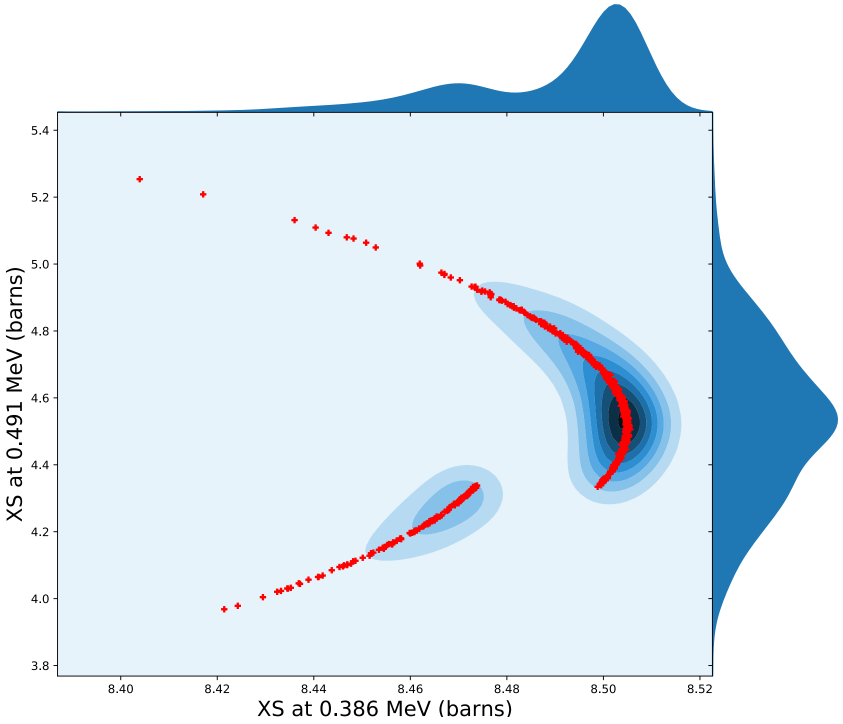
On 1000 cpus

~1.5 weeks for one simulation!

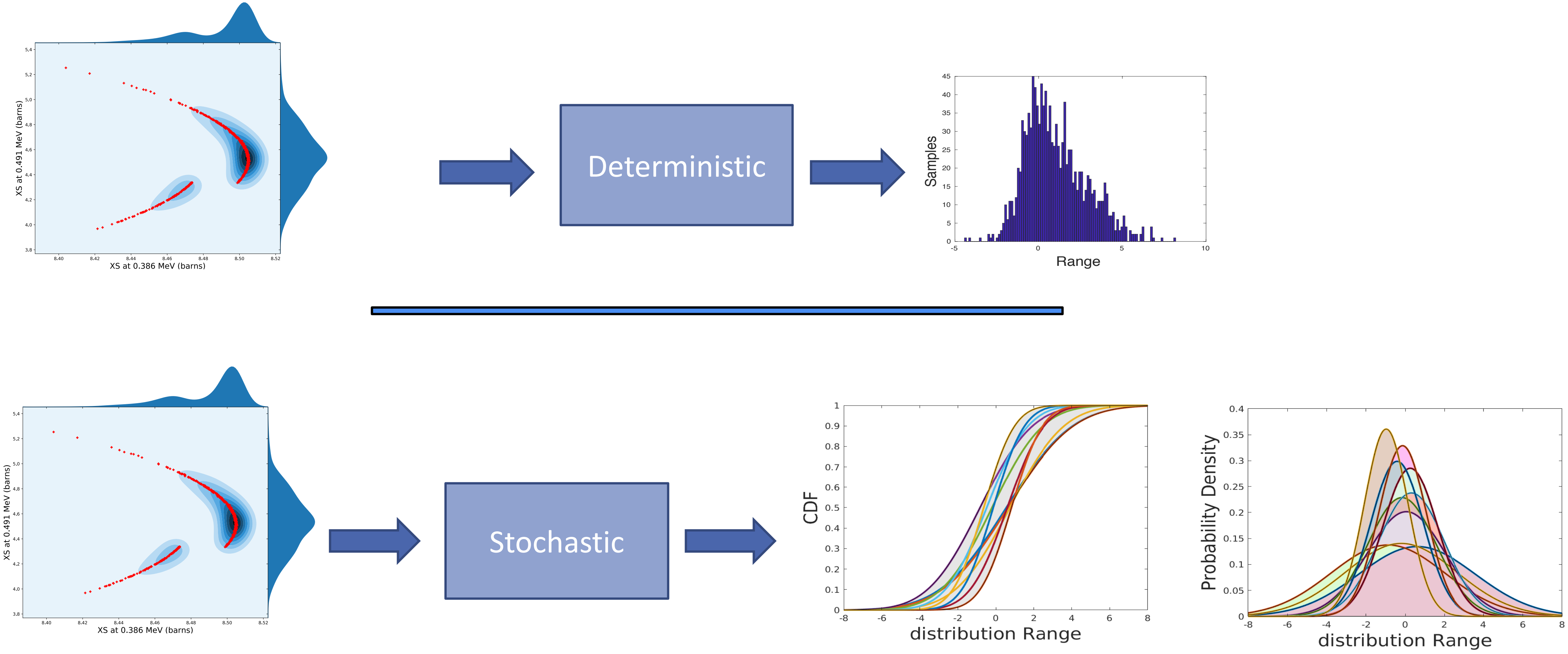
Uncertainty in cross sections







Monte Carlo uncertainty propagation

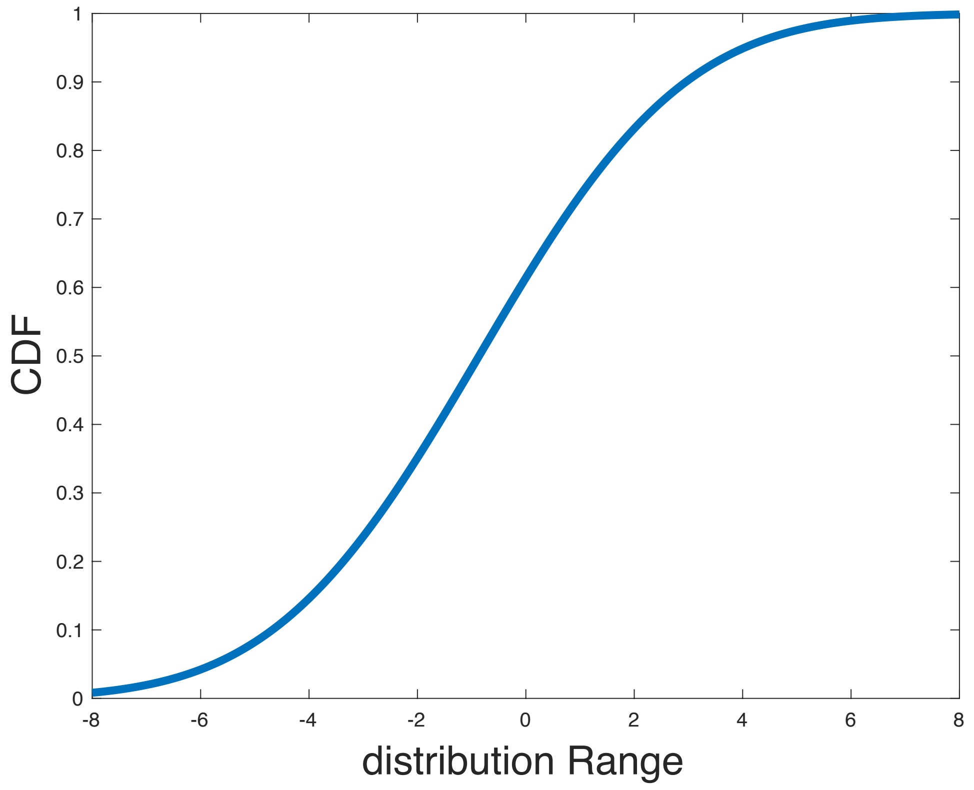


2^{nd} order distributions

Consider:

$$X \sim N(\mu, \sigma)$$

μ, σ are scalar

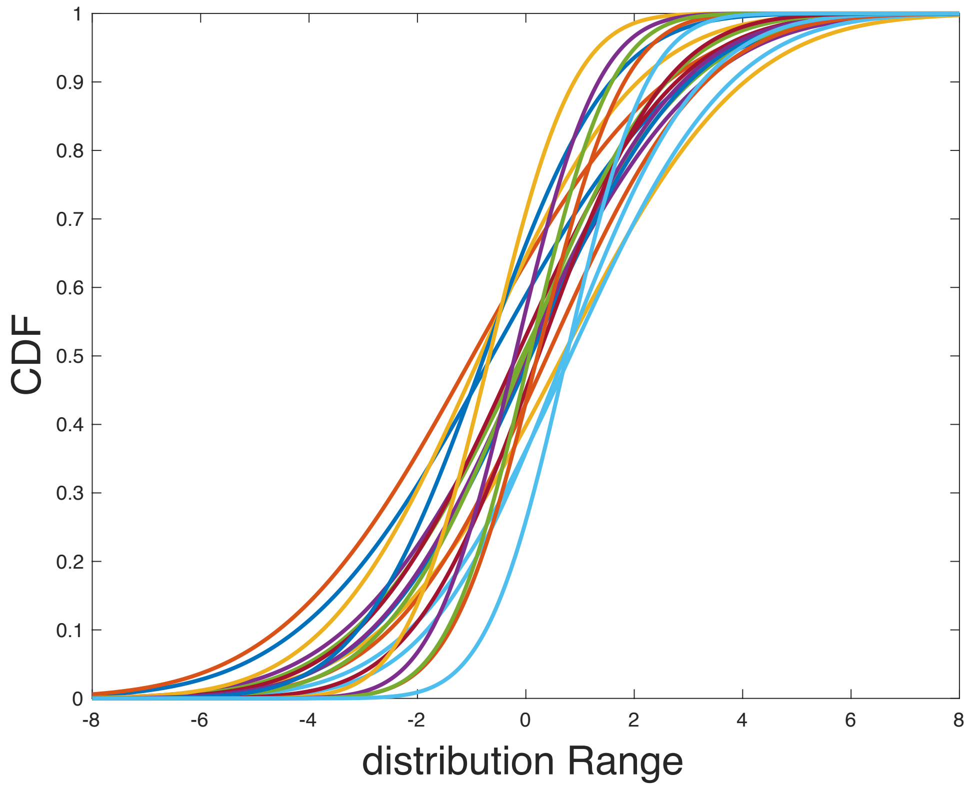


2^{nd} order distributions

Consider:

$$X \sim N(\mu, \sigma)$$

μ, σ have a joint distribution

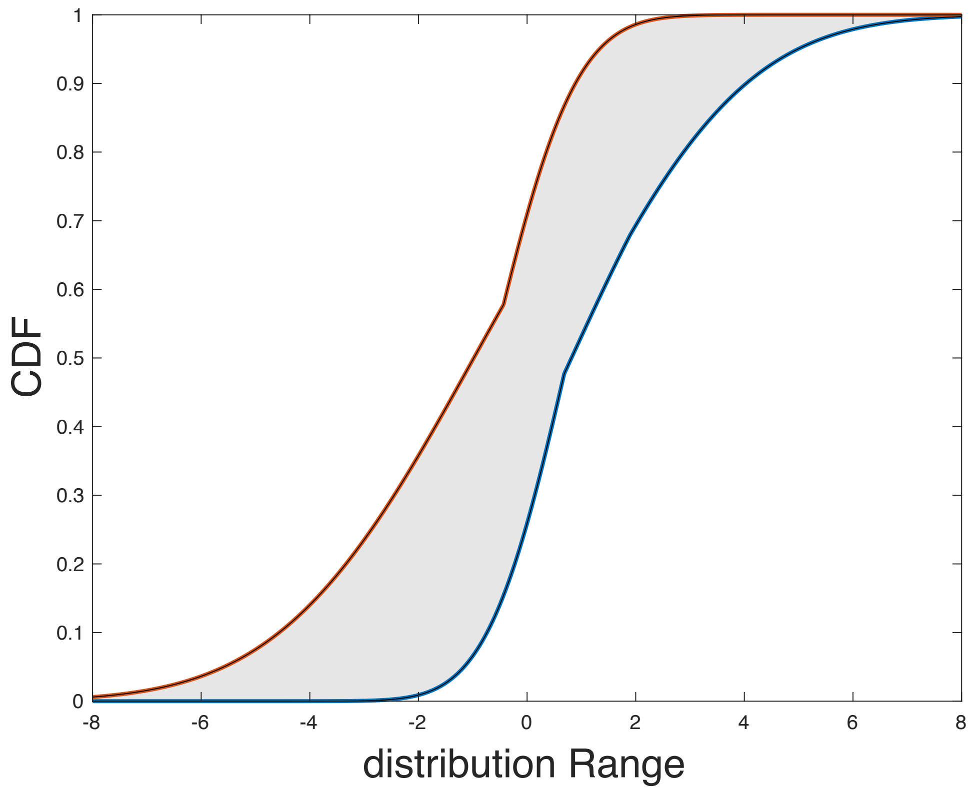


2^{nd} order distributions

Consider:

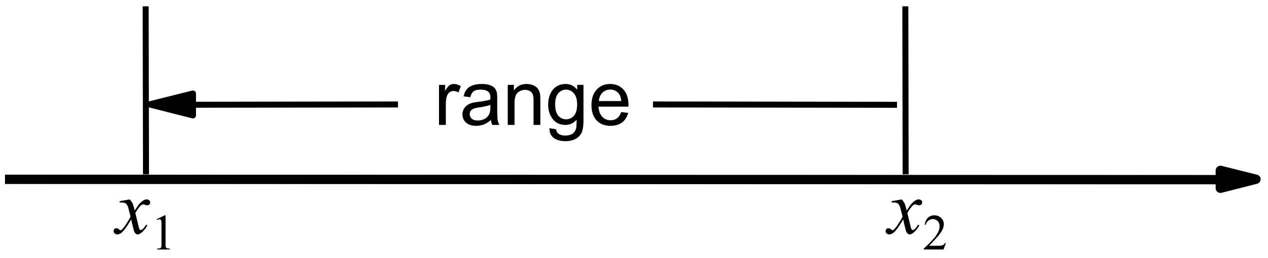
$$X \sim N(\mu, \sigma)$$

μ, σ are intervals



The interval

$$x = [x_1, x_2] = [\underline{x}, \bar{x}]$$



- Used when upper and lower bounds are known reliably
- Sometimes absolutely sure, sometimes just a judgment

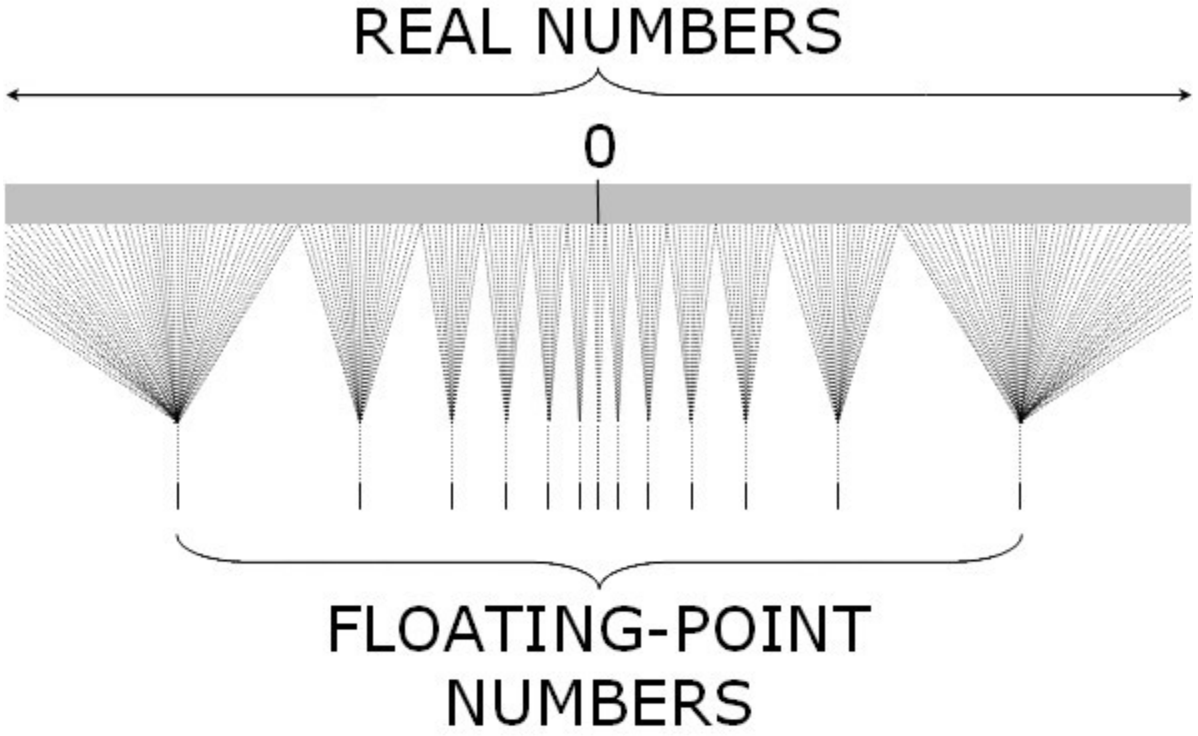
Why intervals are a good idea

- Intervals are easy to understand
- Calculations with them are easy
- They work with even the crappiest data
- Often sufficient for a decision
- Quicker than Monte Carlo

Intervals are rigorous

- The computations are guaranteed to enclose the true results (so long as the inputs do)
- You can still be wrong, but the *method* won't be the reason if you are
- Outward directed rounding

Intervals are rigorous



- Outward directed rounding

Interval arithmetic

$x + y = [x_1 + y_1,\ x_2 + y_2]$

$x - y = [x_1 - y_2,\ x_2 - y_1]$

$x \times y = [\min(x_1y_1, x_1y_2, x_2y_1, x_2y_2), \max(x_1y_1, x_1y_2, x_2y_1, x_2y_2)]$

$x \div y = x \times [1/y_2, 1/y_1]$, if y doesn't include zero

Interval arithmetic

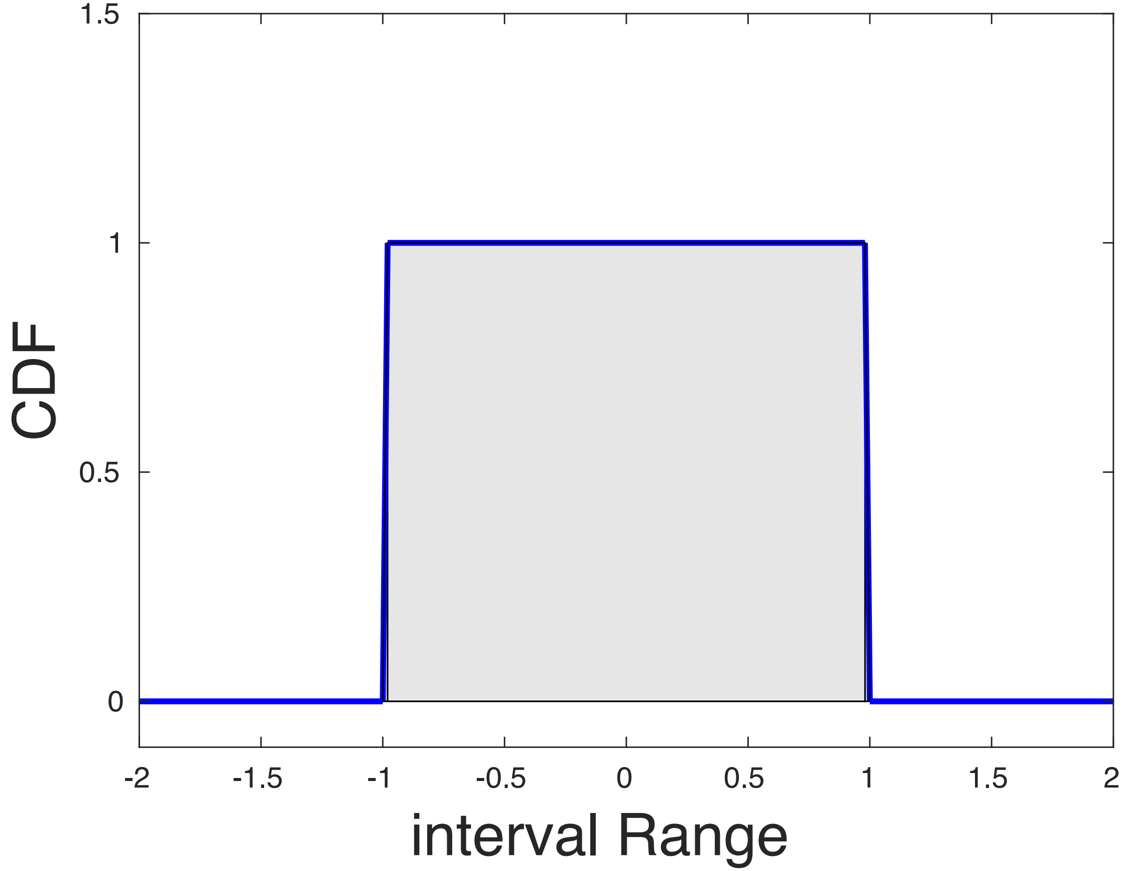
$\min(x, y) = [\min(x_1, y_1), \min(x_2, y_2)]$

$\max(x, y) = [\max(x_1, y_1), \max(x_2, y_2)]$

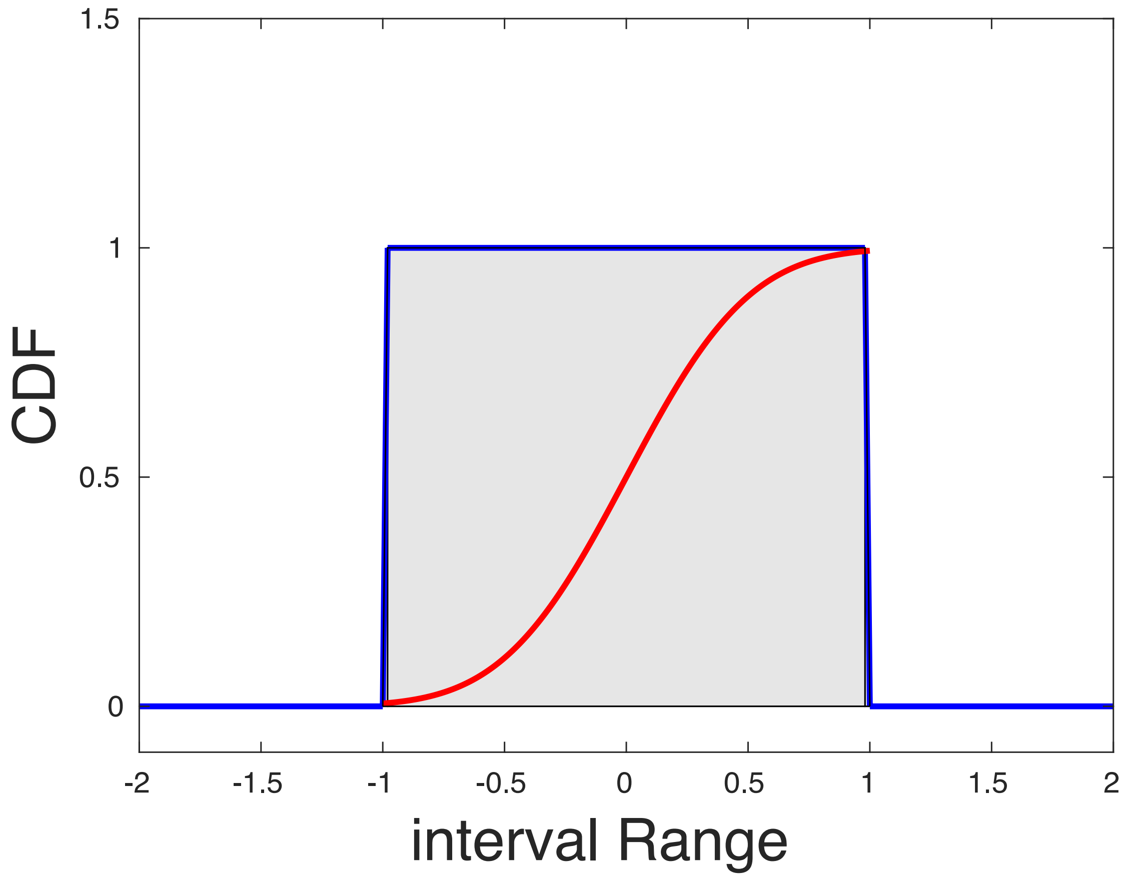
$\mathrm{env}(x, y) = [\min(x_1, y_1), \max(x_2, y_2)]$

$\mathrm{intersect}(x, y) = [\max(x_1, y_1), \min(x_2, y_2)]$, if x and y overlap

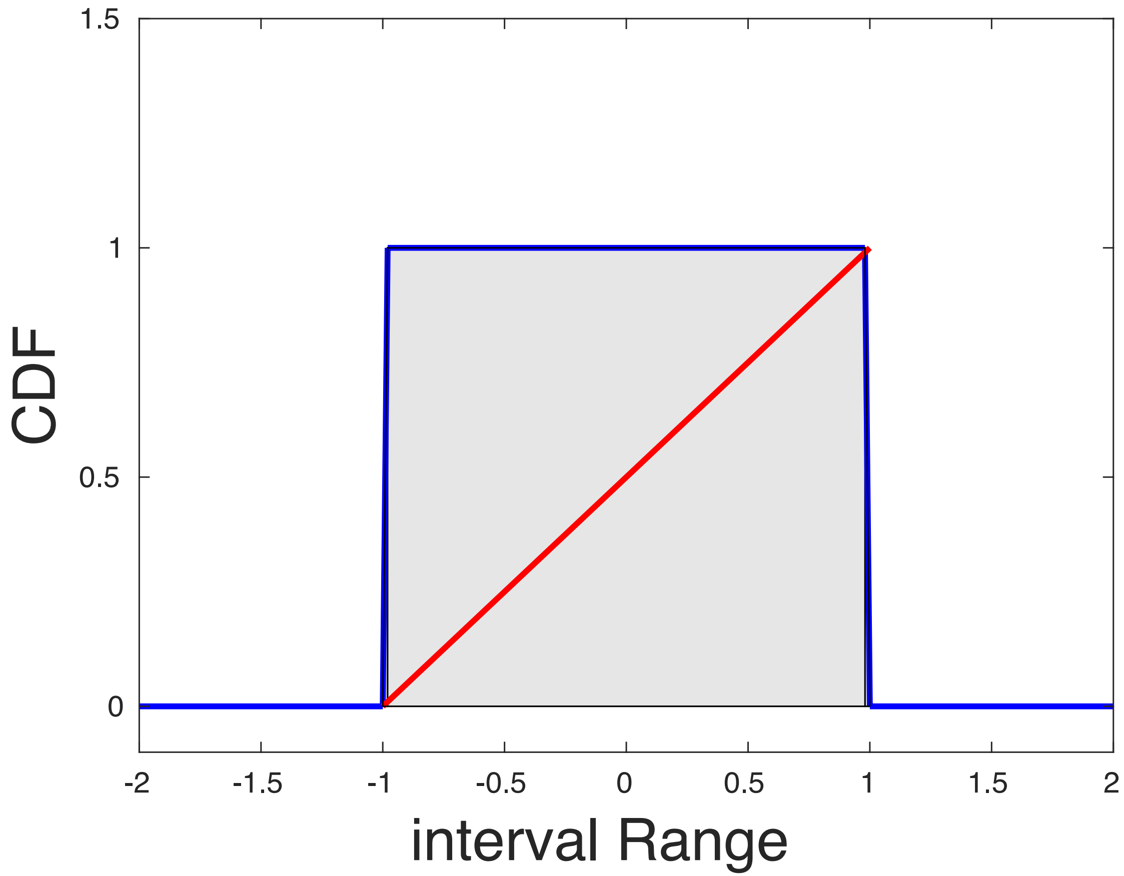
Intervals as a model for uncertainty



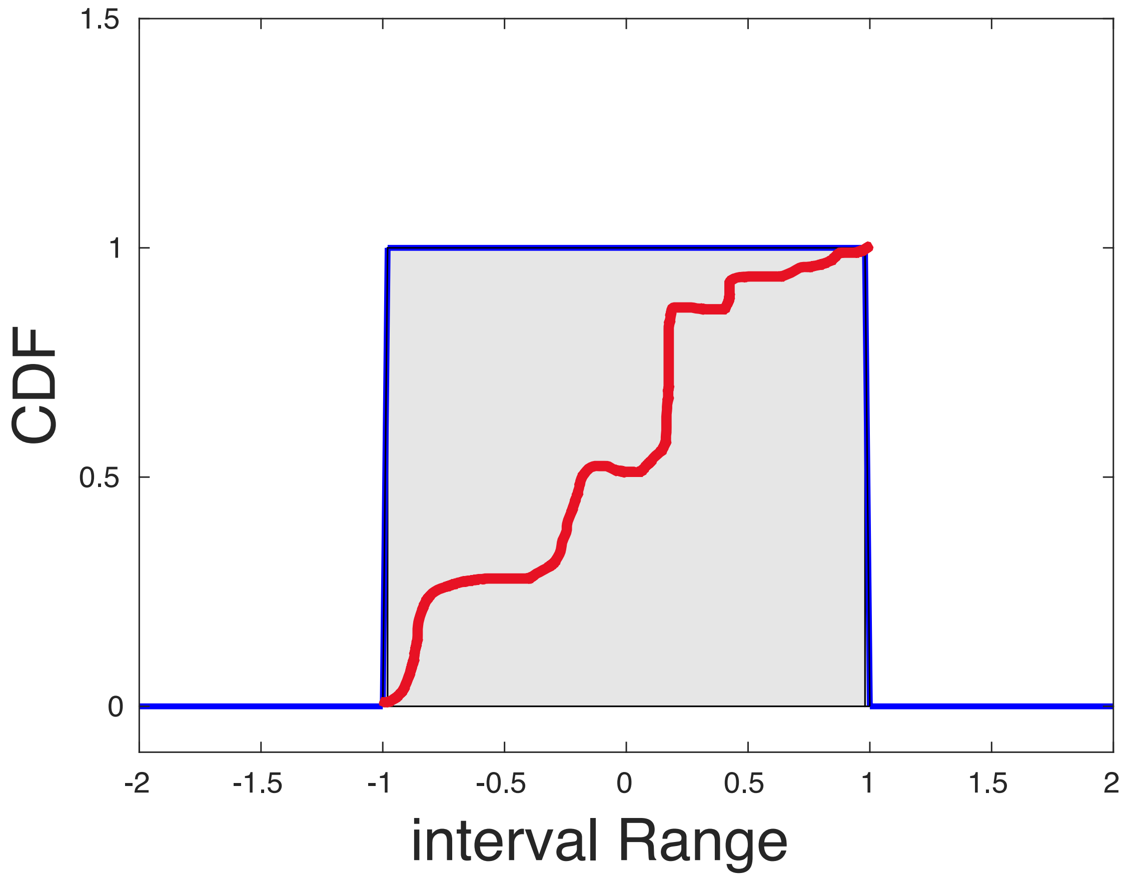
Intervals as a model for uncertainty



Intervals as a model for uncertainty

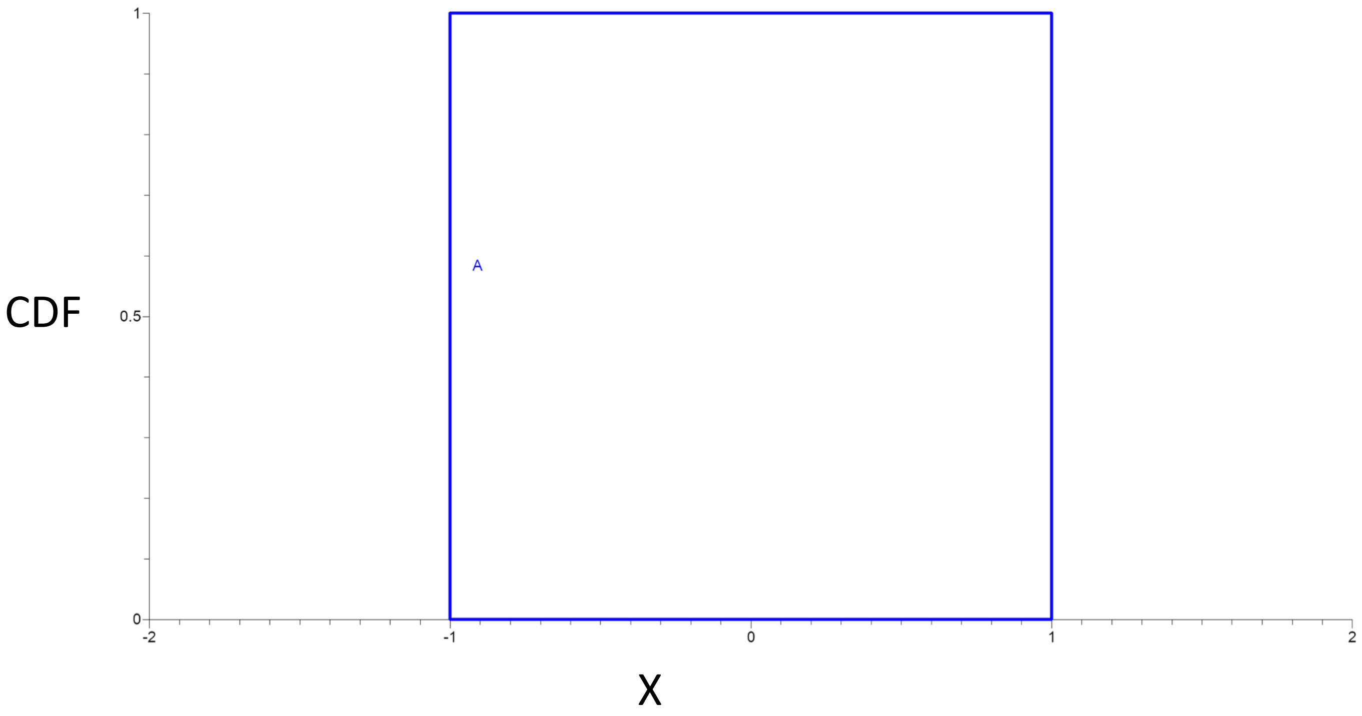


Intervals as a model for uncertainty



Intervals and probability distributions

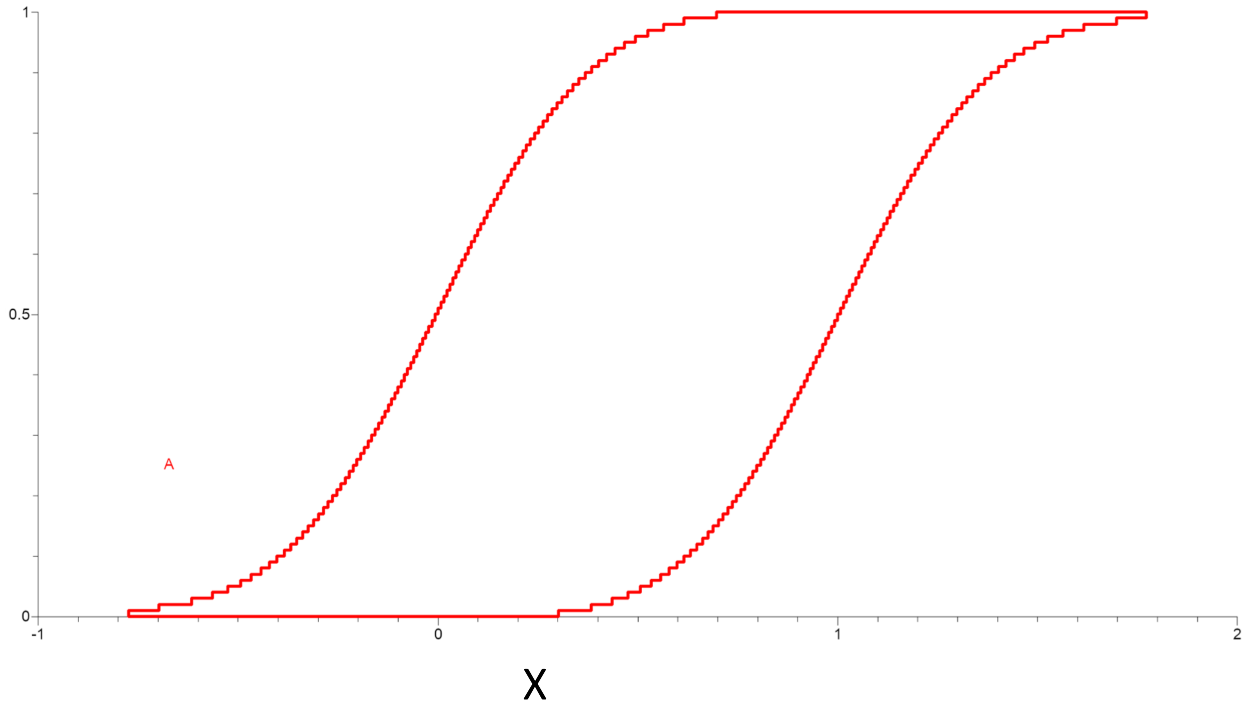
$$\underline{x} < x < \overline{x}$$



Intervals and probability distributions

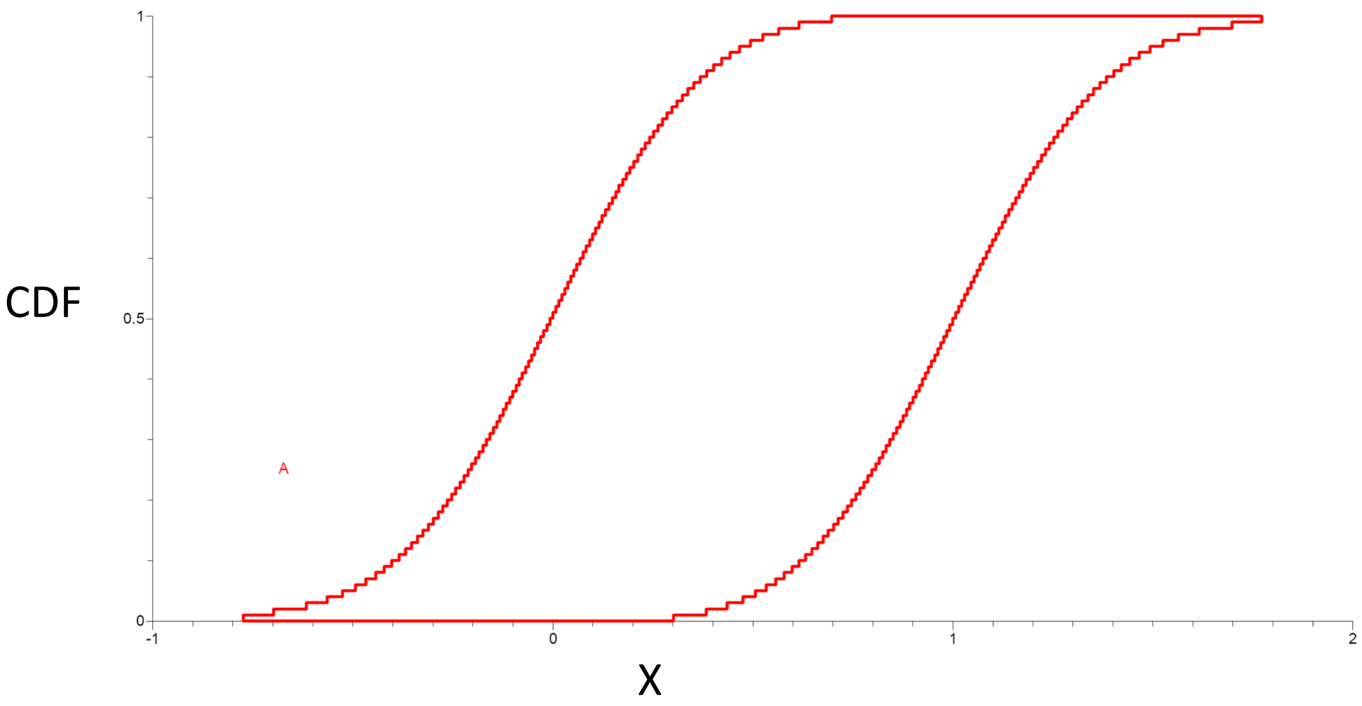
$$\underline{f(x)} < f(x) < \overline{f(x)}$$

CDF



Examples of uncertain distributions

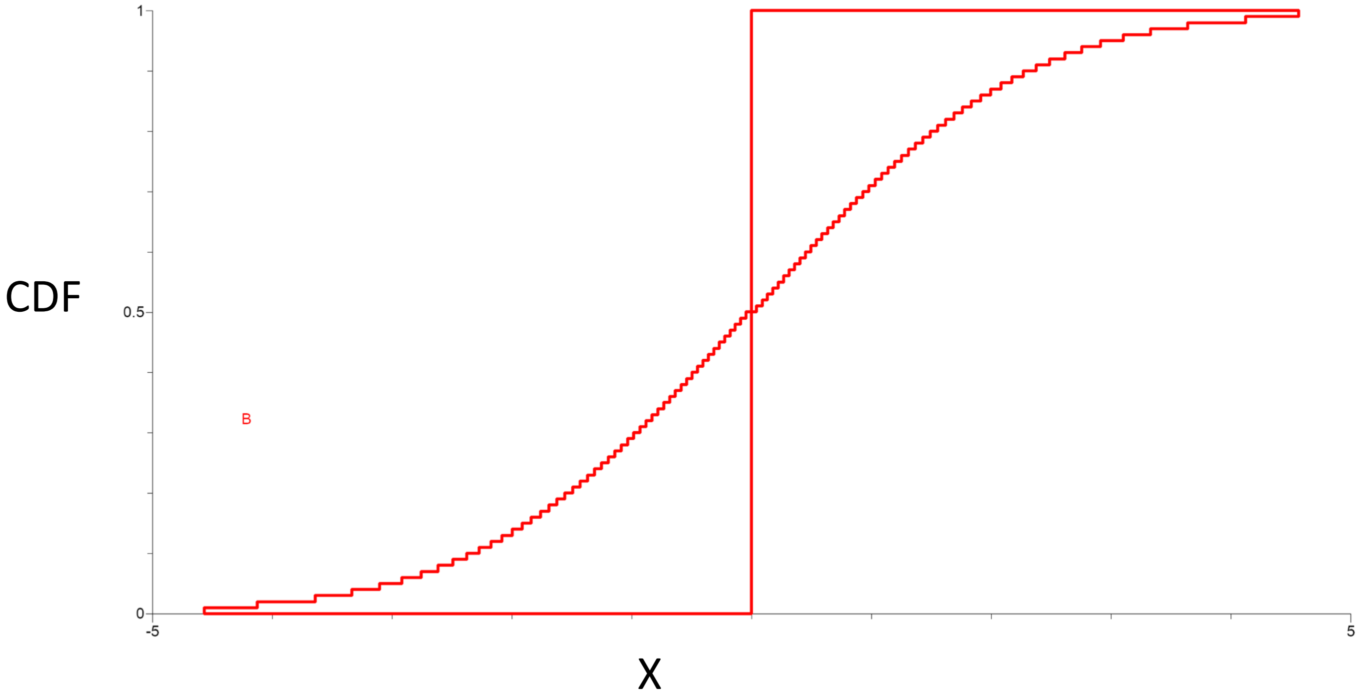
$$X = N([-1,1], 0.3)$$



Examples of uncertain distributions

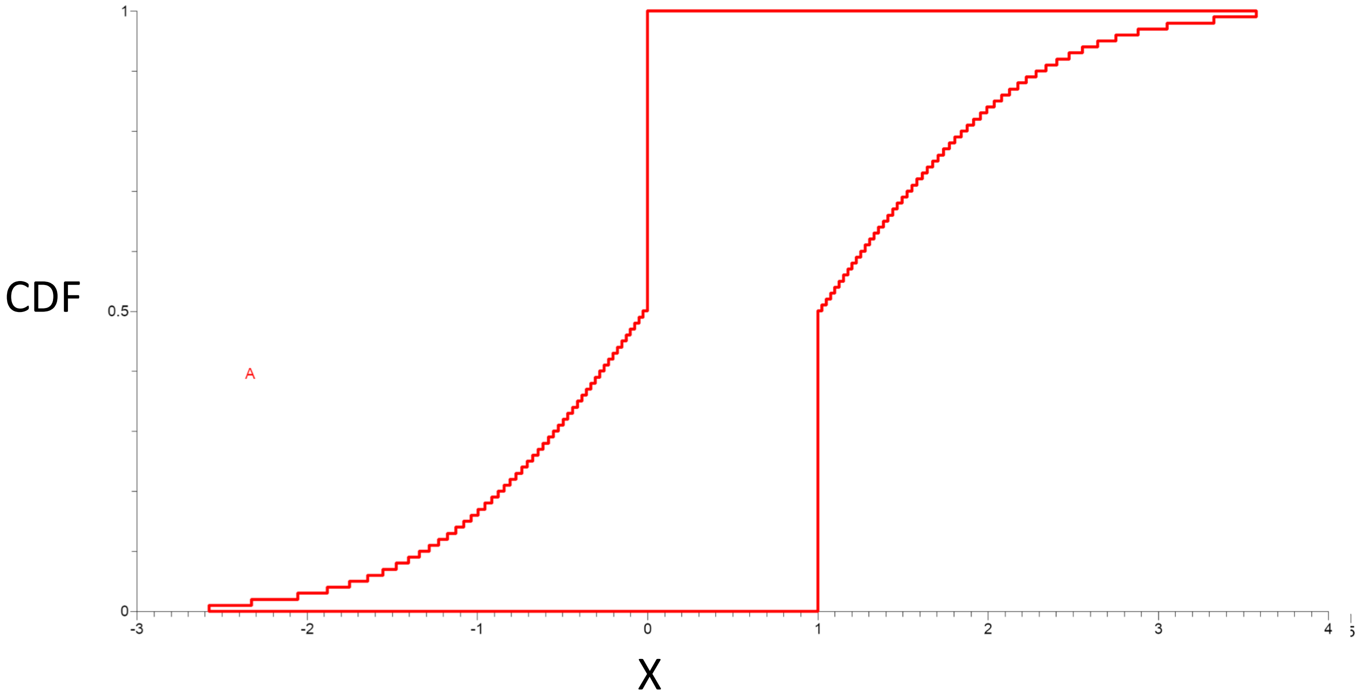
$$\text{A} = N([-1,1], 0.3)$$

$$\text{X} = N(0, A)$$



Examples of uncertain distributions

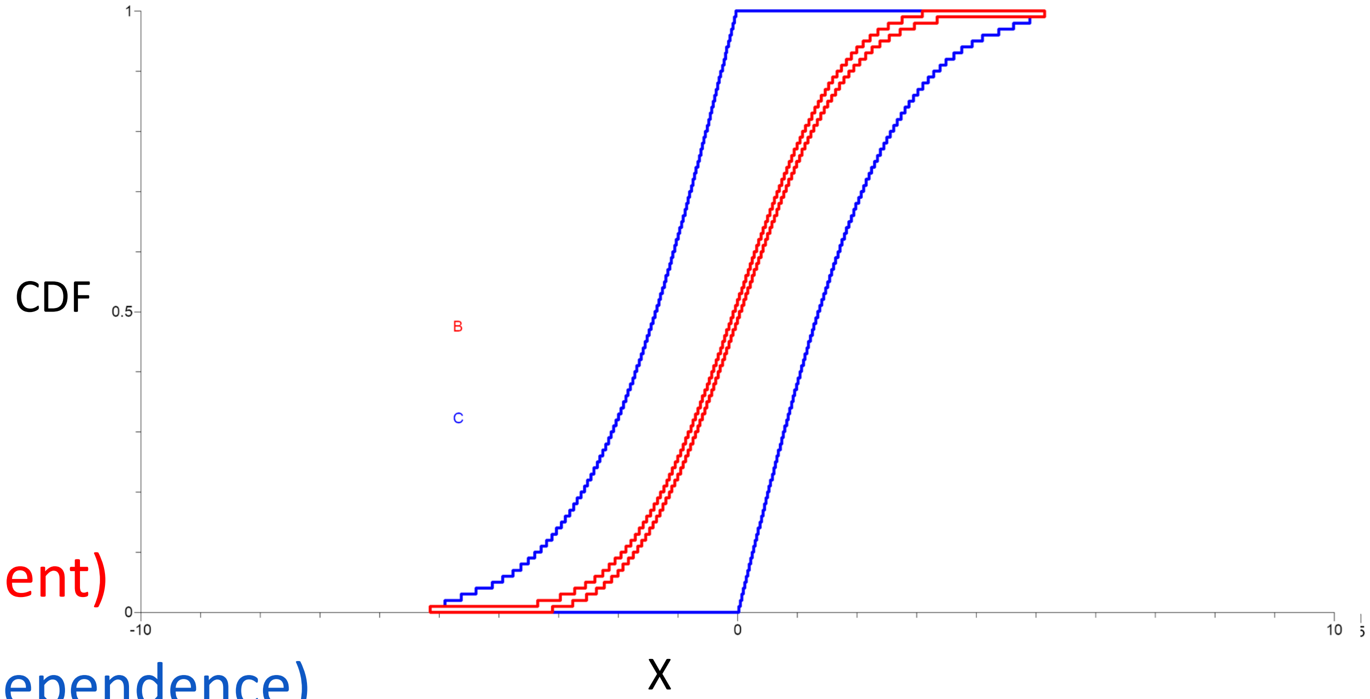
$$X = N([0,1],[0,1])$$



Examples of uncertain distributions

$A = N(0,1)$
$B = N(0,1)$

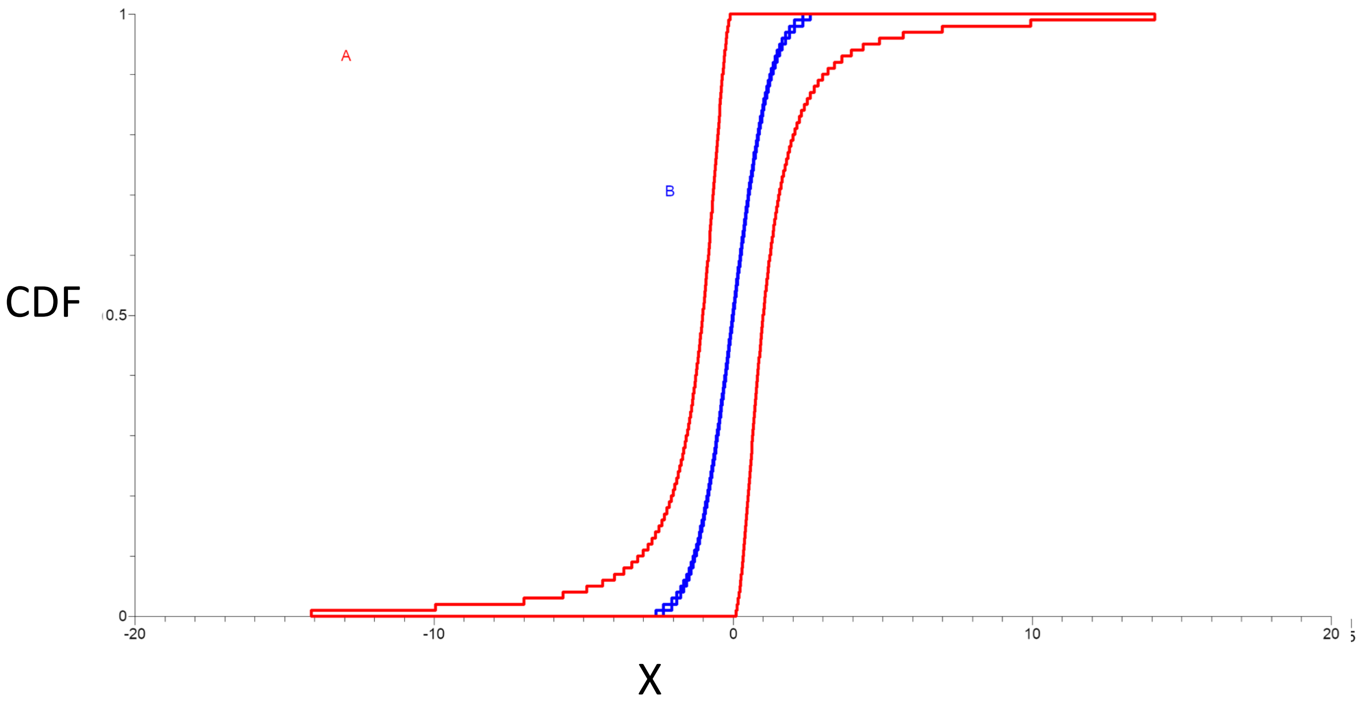
$X = A \;|+|\; B$ (Independent)
$X = A + B$ (Unknown dependence)



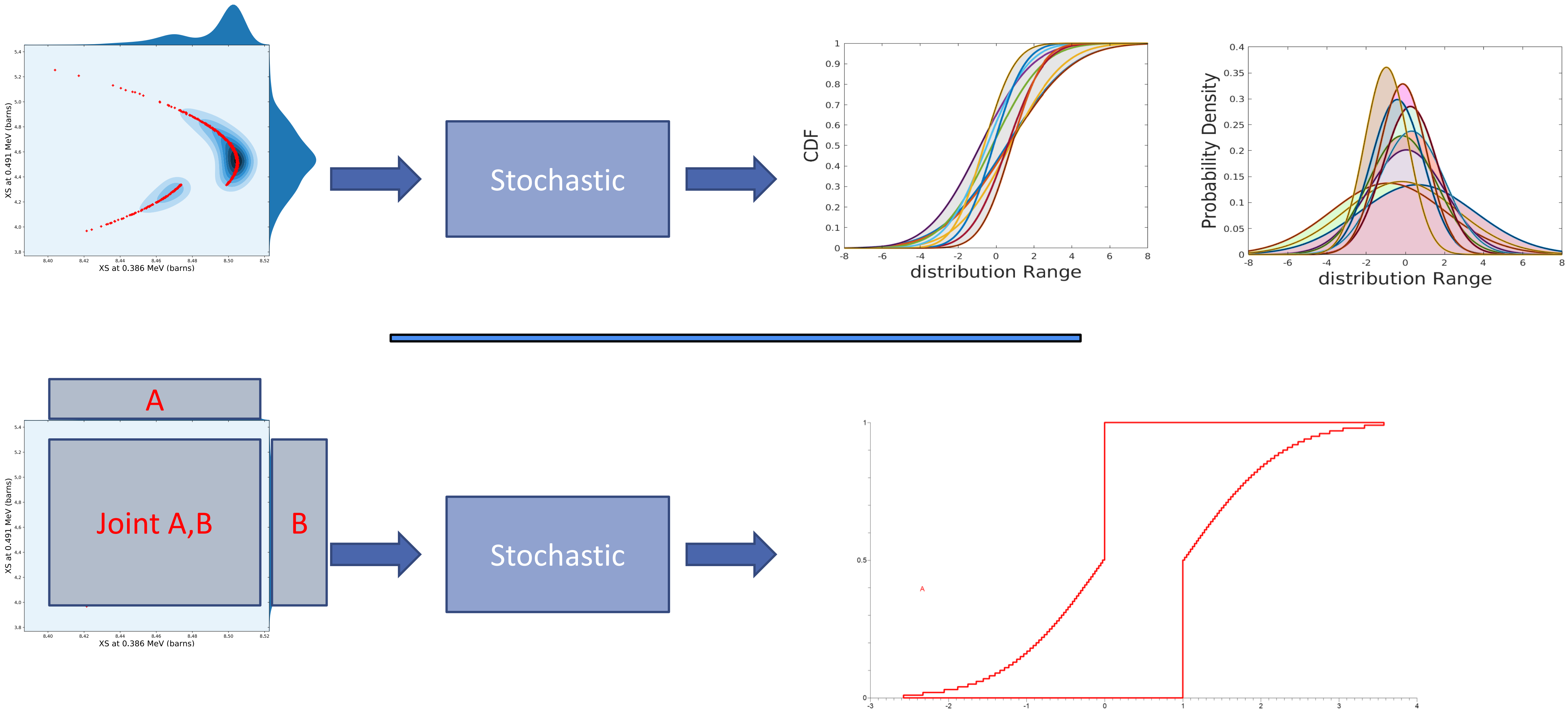
Examples of uncertain distributions

$\mathrm{X} = N(0,1)$

$\mathrm{X} = Chebyshev(0,1)$



=> Bounds on all distributions (parametric and non-parametric) with a known mean and variance


XS at 0.491 MeV (barns)
XS at 0.386 MeV (barns)
Stochastic
CDF
distribution Range
Probability Density
distribution Range
A
Joint A,B
B
XS at 0.491 MeV (barns)
XS at 0.386 MeV (barns)
Stochastic
A

Interval particle transport

1) Specify particle attributes as intervals
2) Characterize samplers as imprecise distributions
3) Propagate imprecise events concurrently

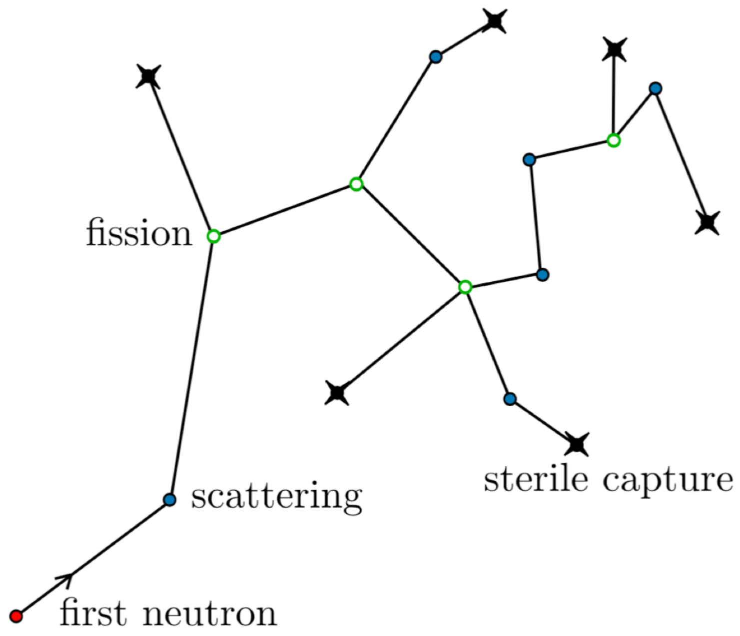
Particle attributes

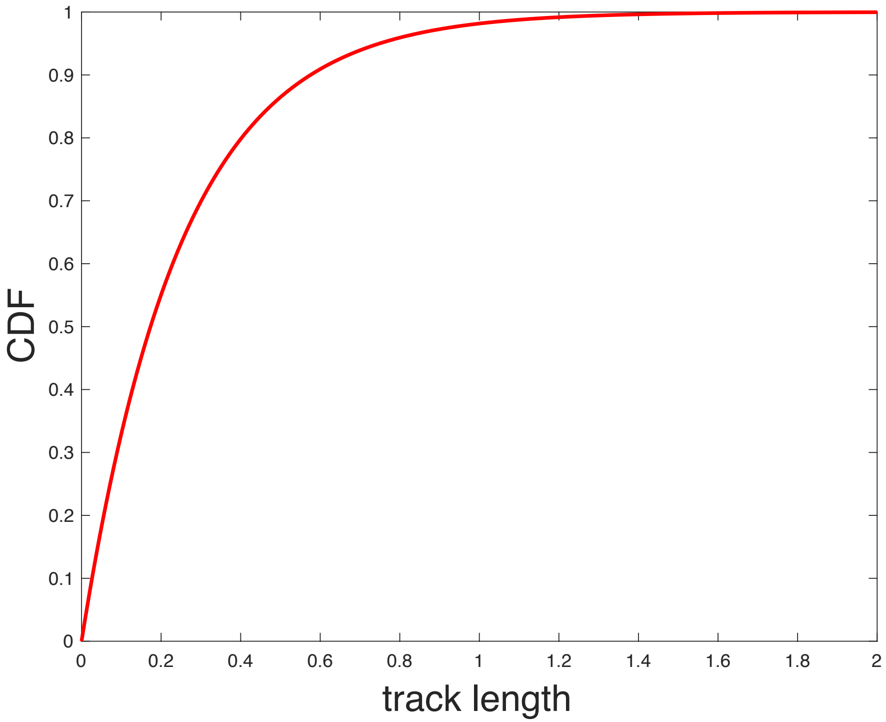
- Energy (scalar)
- Position (3 vector)
- Direction (3 vector)
- Alive state (Boolean)

- Statistical weight (scalar)

→ Random initial interval states can created by specifying the particle source as an imprecise distribution

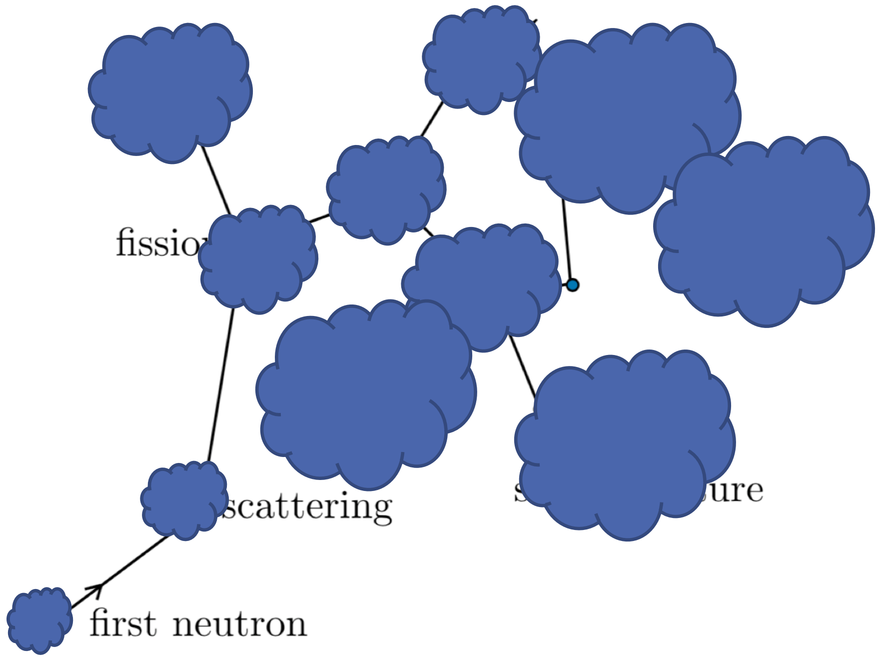
Imprecise exponential random walks

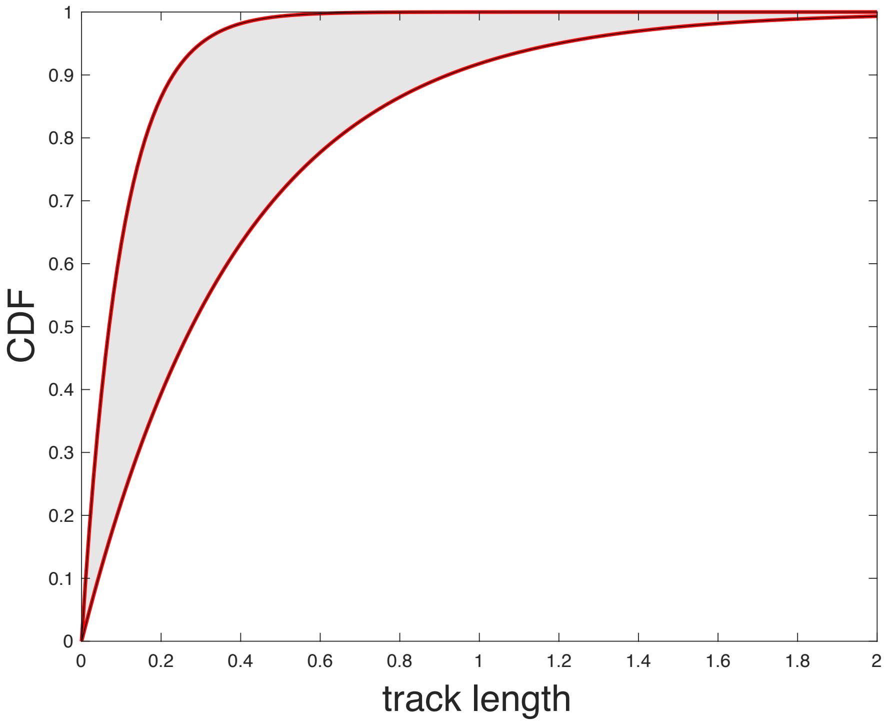




$$d = -\frac{ln(\varepsilon)}{\Sigma_T}$$

Imprecise exponential random walks





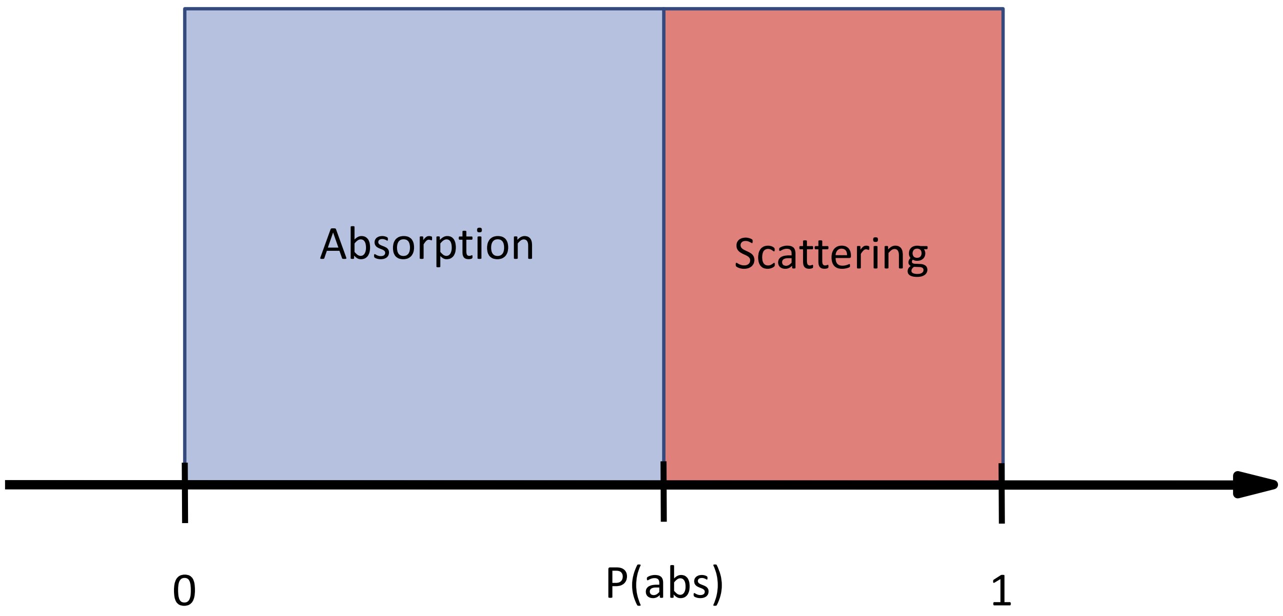
$$[\underline{d}, \overline{d}] = [-\frac{\ln(\varepsilon)}{\overline{\Sigma_T}}, -\frac{\ln(\varepsilon)}{\underline{\Sigma_T}}]$$

Imprecise events

Consider 2 events: Absorption and scattering

$$P(abs) = \frac{\sigma_{abs}}{\sigma_{scat} + \sigma_{abs}}$$

$$P(scat) = 1 - P(abs)$$

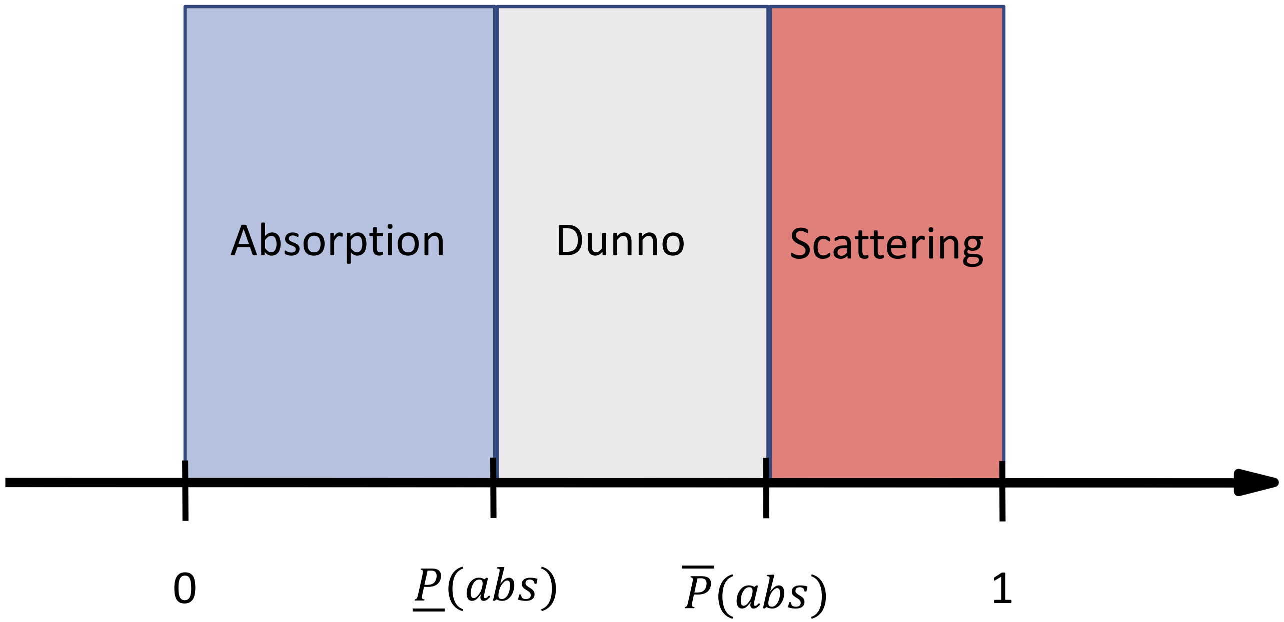


Imprecise events

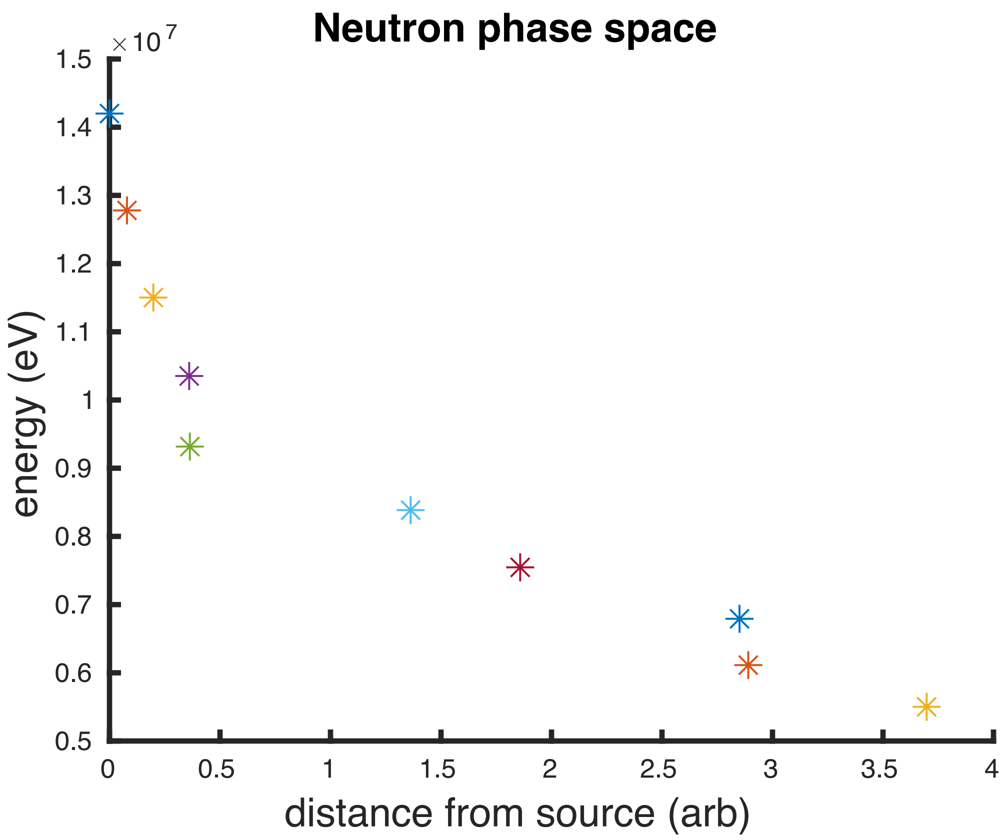
Consider 2 events: Absorption and scattering

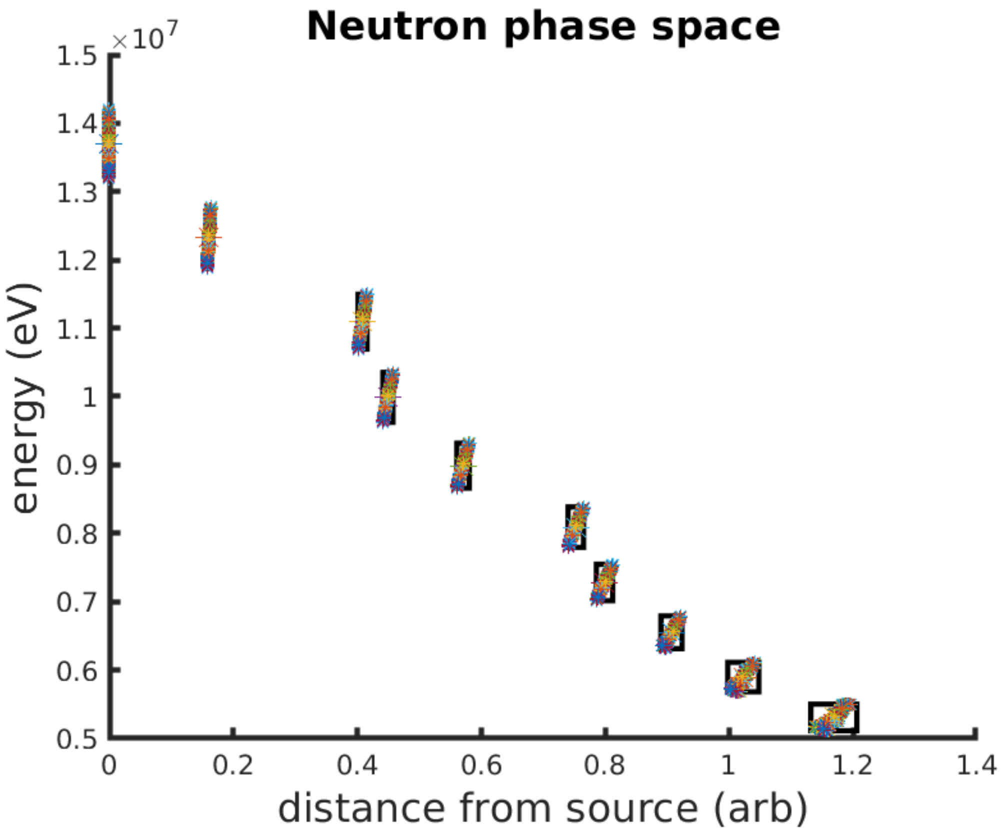
$$[\underline{P}(abs), \overline{P}(abs)] = [\frac{1}{1+\frac{\overline{\sigma_{scat}}}{\underline{\sigma_{abs}}}}, \frac{1}{1+\frac{\underline{\sigma_{scat}}}{\overline{\sigma_{abs}}}}]$$

$$P(scat) = 1 - P(abs)$$

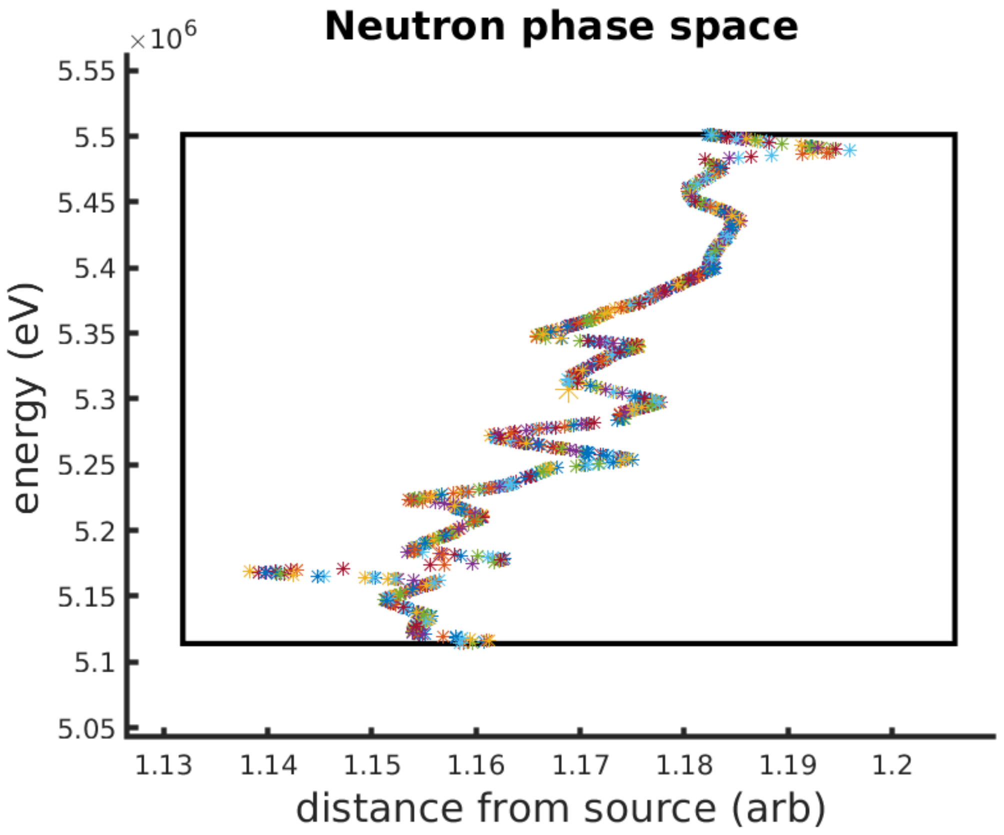


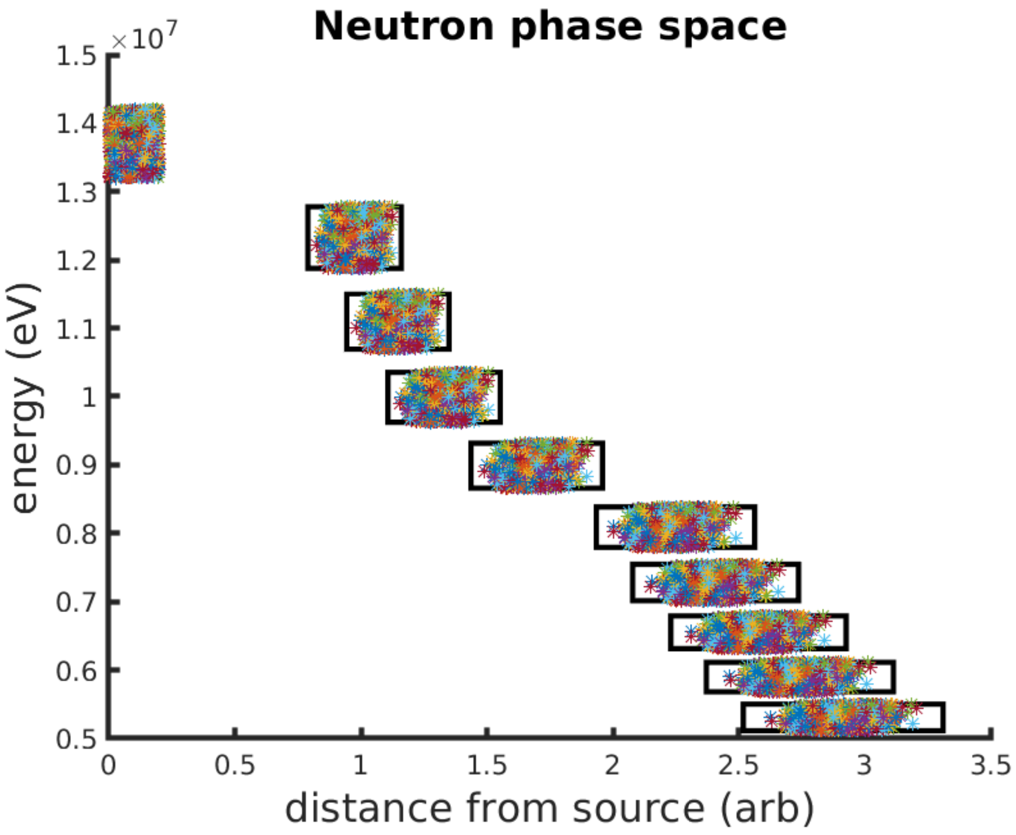
Imprecise histories





Imprecise histories





Tallies

The monte carlo estimator for the particle flux is:

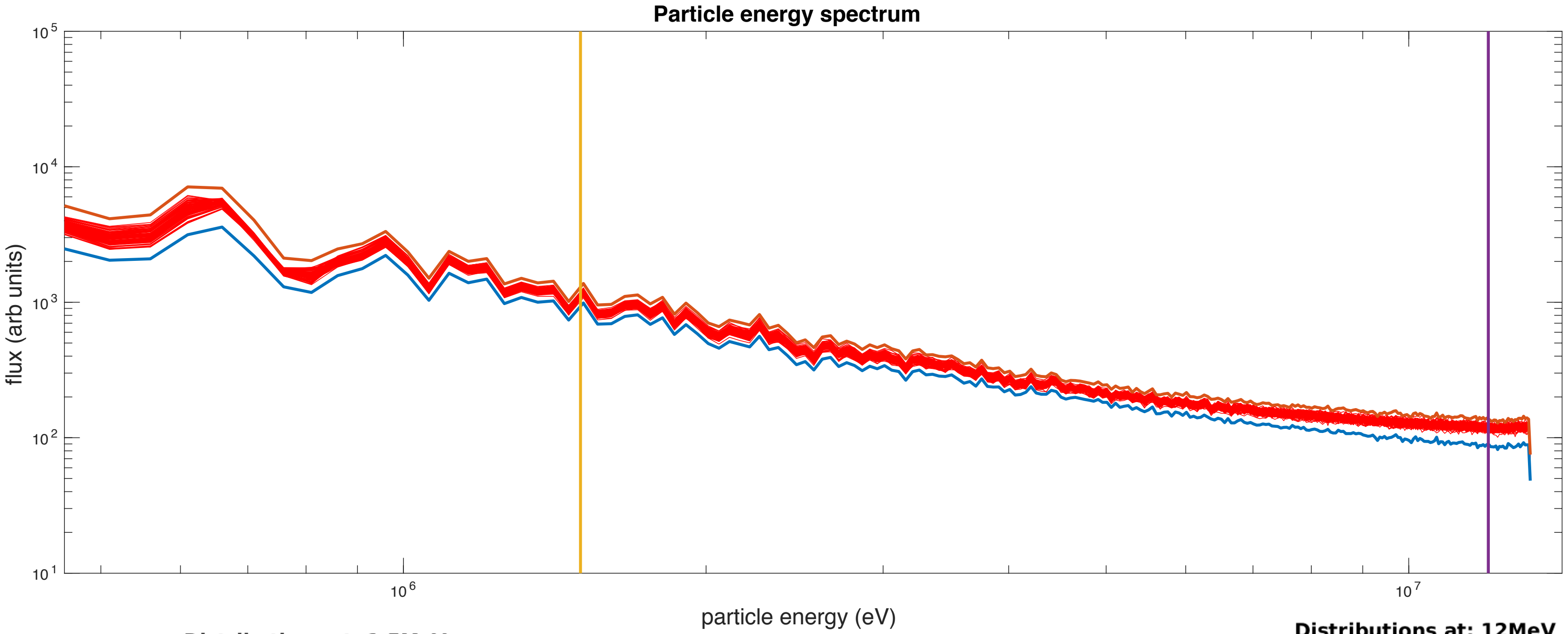
$$\phi(E) = \frac{1}{N}\sum_{n} d_i$$

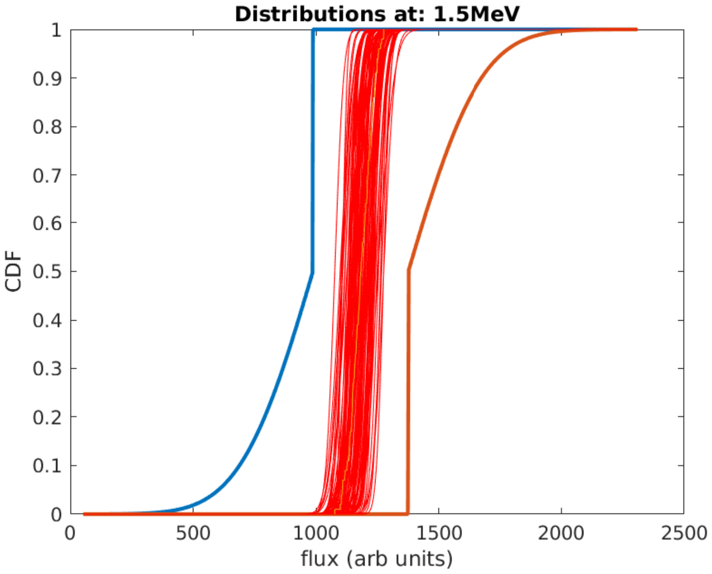
Tallies

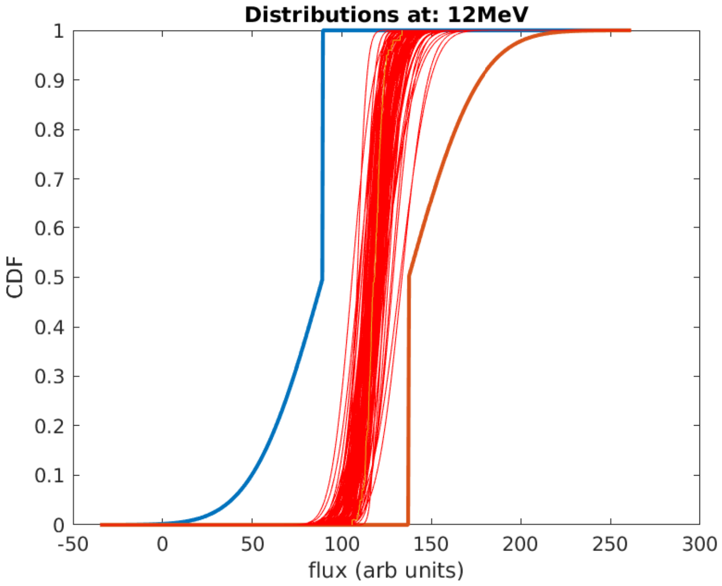
The monte carlo estimator for the particle flux is:

$$\overline{\phi}(E) = \frac{1}{N}\sum_{n} \overline{d_i}$$

$$\underline{\phi}(E) = \frac{1}{N}\sum_{n} \underline{d_i}$$


Particle energy spectrum
flux (arb units)
particle energy (eV)
10^5
10^4
10^3
10^2
10^1
10^6
10^7


Distributions at: 1.5MeV
CDF
flux (arb units)
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
0 500 1000 1500 2000 2500


Distributions at: 12MeV
CDF
flux (arb units)
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
-50 0 50 100 150 200 250 300

Conclusions

- We are working towards the (near) automatic propagation of uncertainty in particle transport monte carlo
- We have shown that this works under very simple simulation conditions
- For the method to be generally applicable:
    - The action to be taken when the particle intersects a geometric barrier
    - The simulation of a material with multiple nuclides
    - The concurrent propagation of interval hulls which have been split from imprecise events