Derivadas e Integrales Fundamentales

I. Funciones Trigonométricas

Derivadas:

$$\begin{aligned} (\sin x)' &= \cos x & (\cos x)' &= -\sin x \\ (\tan x)' &= \sec^2 x & (\sec x)' &= \sec x \tan x \\ (\csc x)' &= -\csc x \cot x & (\cot x)' &= -\csc^2 x \end{aligned}$$

$$\begin{aligned} (\sin(u))' &= u' \cos(u) & (\cos(u))' &= -u' \sin(u) \\ (\tan(u))' &= u' \sec^2(u) & (\sec(u))' &= u' \sec(u) \tan(u) \\ (\csc(u))' &= -u' \csc(u) \cot(u) & (\cot(u))' &= -u' \csc^2(u) \end{aligned}$$

Integrales:

$$\begin{aligned} \int \sin x \, dx &= -\cos x + C & \int \cos x \, dx &= \sin x + C \\ \int \tan x \, dx &= -\ln|\cos x| + C & \int \sec x \, dx &= \ln|\sec x + \tan x| + C \\ \int \csc x \, dx &= -\ln|\csc x + \cot x| + C & \int \cot x \, dx &= \ln|\sin x| + C \end{aligned}$$

$$\begin{aligned} \int \sin(ax) \, dx &= -\tfrac{1}{a} \cos(ax) + C & \int \cos(ax) \, dx &= \tfrac{1}{a} \sin(ax) + C \\ \int \tan(ax) \, dx &= -\tfrac{1}{a} \ln|\cos(ax)| + C & \int \sec(ax) \, dx &= \tfrac{1}{a} \ln|\sec(ax) + \tan(ax)| + C \\ \int \csc(ax) \, dx &= -\tfrac{1}{a} \ln|\csc(ax) + \cot(ax)| + C & \int \cot(ax) \, dx &= \tfrac{1}{a} \ln|\sin(ax)| + C \end{aligned}$$

II. Funciones Exponenciales

Derivadas:

$$\begin{aligned} (e^x)' &= e^x & (a^x)' &= a^x \ln a \\ (e^{ax})' &= a e^{ax} & (a^{ax})' &= a^{ax} \, a \ln a \end{aligned}$$

Integrales:

$$\begin{aligned} \int e^x \, dx &= e^x + C & \int a^x \, dx &= \tfrac{a^x}{\ln a} + C \\ \int e^{ax} \, dx &= \tfrac{1}{a} e^{ax} + C & \int a^{ax} \, dx &= \tfrac{a^{ax}}{a \ln a} + C \end{aligned}$$

III. Funciones Logarítmicas

Derivadas:

$$\begin{aligned} (\ln x)' &= \tfrac{1}{x} & (\log x)' &= \tfrac{1}{x \ln 10} \\ (\ln(ax))' &= \tfrac{1}{x} & (\log(ax))' &= \tfrac{1}{x \ln 10} \end{aligned}$$

Integrales:

$$\begin{aligned} \int \ln x \, dx &= x(\ln x - 1) + C & \int \log x \, dx &= \tfrac{x(\ln x - 1)}{\ln 10} + C \\ \int \ln(ax) \, dx &= x(\ln(ax) - 1) + C & \int \log(ax) \, dx &= \tfrac{x(\ln(ax) - 1)}{\ln 10} + C \end{aligned}$$

IV. Funciones Inversas Trigonométricas

Derivadas:

$$(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}} \qquad (\cos^{-1} x)' = -\frac{1}{\sqrt{1-x^2}}$$
$$(\tan^{-1} x)' = \frac{1}{1+x^2} \qquad (\cot^{-1} x)' = -\frac{1}{1+x^2}$$
$$(\sec^{-1} x)' = \frac{1}{|x|\sqrt{x^2-1}} \qquad (\csc^{-1} x)' = -\frac{1}{|x|\sqrt{x^2-1}}$$

Derivadas con constante a :

$$(\sin^{-1}(ax))' = \frac{a}{\sqrt{1-a^2x^2}} \qquad (\cos^{-1}(ax))' = -\frac{a}{\sqrt{1-a^2x^2}}$$
$$(\tan^{-1}(ax))' = \frac{a}{1+a^2x^2} \qquad (\cot^{-1}(ax))' = -\frac{a}{1+a^2x^2}$$
$$(\sec^{-1}(ax))' = \frac{a}{|ax|\sqrt{a^2x^2-1}} \qquad (\csc^{-1}(ax))' = -\frac{a}{|ax|\sqrt{a^2x^2-1}}$$

Integrales:

$$\int \tfrac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C \qquad \int \tfrac{1}{\sqrt{1-a^2x^2}}\,dx = \tfrac{1}{a}\sin^{-1}(ax) + C$$
$$\int \tfrac{1}{1+x^2}\,dx = \tan^{-1} x + C \qquad \int \tfrac{1}{1+a^2x^2}\,dx = \tfrac{1}{a}\tan^{-1}(ax) + C$$
$$\int \tfrac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1}|x| + C \qquad \int \tfrac{1}{|ax|\sqrt{a^2x^2-1}}\,dx = \tfrac{1}{a}\sec^{-1}|ax| + C$$

V. Propiedades Extra

Identidades Pitagóricas:

$$\sin^2 x + \cos^2 x = 1 \quad 1 + \tan^2 x = \sec^2 x \quad 1 + \cot^2 x = \csc^2 x$$

Ángulo Medio:

$$\sin^2 x = \tfrac{1-\cos(2x)}{2} \qquad \cos^2 x = \tfrac{1+\cos(2x)}{2} \qquad \tan^2 x = \tfrac{1-\cos(2x)}{1+\cos(2x)} \qquad \sin x\cos x = \tfrac{\sin(2x)}{2}$$

Propiedades Recíprocas:

$$\sin x = \tfrac{1}{\csc x} \quad \cos x = \tfrac{1}{\sec x} \quad \tan x = \tfrac{1}{\cot x}$$
$$\csc x = \tfrac{1}{\sin x} \quad \sec x = \tfrac{1}{\cos x} \quad \cot x = \tfrac{1}{\tan x}$$

$$\sin\theta = \tfrac{\text{opuesto}}{\text{hipotenusa}} \quad \cos\theta = \tfrac{\text{adyacente}}{\text{hipotenusa}} \quad \tan\theta = \tfrac{\text{opuesto}}{\text{adyacente}}$$

$$\csc\theta = \tfrac{\text{hipotenusa}}{\text{opuesto}} \quad \sec\theta = \tfrac{\text{hipotenusa}}{\text{adyacente}} \quad \cot\theta = \tfrac{\text{adyacente}}{\text{opuesto}}$$

No.	Expresión	Identidad	Triángulo
1	$\sqrt{a^2-u^2}$	$\sin\theta = \frac{\text{opuesto}}{\text{hipotenusa}}$	Opuesto: u Hipotenusa: a Adyacente: $\sqrt{a^2-u^2}$
2	$\sqrt{a^2+u^2}$	$\tan\theta = \frac{\text{opuesto}}{\text{adyacente}}$	Opuesto: u Hipotenusa: $\sqrt{a^2+u^2}$ Adyacente: a
3	$\sqrt{u^2-a^2}$	$\sec\theta = \frac{\text{hipotenusa}}{\text{adyacente}}$	Opuesto: $\sqrt{u^2-a^2}$ Hipotenusa: u Adyacente: a