CS404 Agent-based Systems Coursework: Auction Games

Jiyu Yan
Student ID:1851015

I. Introduction

In this project, four different strategies need to be made against four auctions. There are four types of paintings in the auction, which correspond to four different values.

There are some common principles: each game is a finite extensive and one-shot game against other participants, each agent play depends on the games played before such as who has won which items and how much money everyone left. The knowledge of agents have is the auction type, the items sold to whom for how much, the budget left to everyone, the amount of items available and the order sometimes.

How important one item is for a certain player at a certain round could be different. For example, in five-of-a-kind game, the type of painting each player aims for could be variant. The bids players make are not independent of the budget they left and others have.The key principle is to propose a strategy using what one player knows, estimate what one does not know to maximise the expected utility, which variance in different situations.

It is nearly impossible to solve the game by brute force. However, some backward induction solutions and other considerations could be used. The attitude to risk also needs to take into account other participants. Intuitively, it is better to be aggressive in five-of-a-kind game since no matter which one get enough which type, the game ends. Waiting will help in some situations, for instance when someone else is going to win, one could know it clearly and could choose to overbid if possible.

II. Five of a kind

A. Order known

When order of auction is known, everyone could know which artist's painting have the first number to reach 5. In theory, in the best case, players should try their best to get this artist's first five works to end the game in the first time and win.

1	2	3	4	5	6
P	V	P	V	P	V
7	8	9	10	11	12
V	P	P	V	V	V
13	14				
V	V				

Fig. 1. Auction order1

The money left in the end doesn't count so the basic rule of strategy is trying to spend all money out in the auction. Assuming a winning, at least five paintings of the same type are required. In order to be competitive enough on every bid for the type of target, the money should not be spent on other types of painting in most cases. In conclusion, the strategy should based on following rules, to try to get 5 same type as soon as possible:

1) Minimum the money left at the end
2) Maximum each bidding
3) Spend minimum money on paintings other than the 5 same type

1) strategy1-first 5, long-term consideration: Based on these rules, theoretically, if the level of importance on each bidding is same and the money waste on paintings other than the 5 same type is 0, then $Bidding = budget/needtowin$. As the budget for each player is 1000 and 'need to win' is 5, the bidding is 200 according to this formula.

The rule behind this strategy is that if you get a kind of painting, you only keep bidding for this type of painting for 200, and bid 0 for the rest. Then the strategy for finding the type of painting to bid at first is extremely important.

A simple consideration is to find the first 5 streak paintings, bid on the first one, if not get it, find another winning sequence. However, there are some situations in which the first sequence may not be a good choice.

As shown in Figure1, the type of first same 5 items is P and it ends at position 9. Supposing there are many players.If you bid on P at position 1 and win it, according to this strategy, you have to win P at position 3,5,8,9 to win the game, which is difficult, since you don't have the flexibility to lose only one auction of them. If you bid on V at position 2 and win it, since there are a lot paintings of V latter, you have bigger opportunity to win the game. So a good strategy need to consider the total auction list, although the earlier the more important, which could make player tend to be aggressive, as our analyzed, just consider the first 5-same-type is not enough. This is the first problem.

Apart from the auction list, another consideration is your opponents potential option. The same list as shown in Figure1, supposing now there are 20 players already have 2 type P paintings in their hand and they all will bid only on P, and no one wants V now. Since you are still empty in your hands, it is a much better choice to wait for type V to bid, as there are many players who want P so the ratio of success is a lot smaller. This is the second problem.

Here we simply and safely assume they will bid on each type of painting if they don't have any before. So here are my assumptions and considerations:

1) Player will bid on the type they already have
2) Player will bid on each type if they have nothing
3) For auction list, not just the first 5 same type matters
4) The earlier, the more important

The strategy is proposed based on these assumptions. At first, the potential opponents number(PO) for each artist type are calculated according to what players already have in their hands. If one player has P and V in hands, it is assumed that both P and V would be bidden by this player. Assuming others with nothing will bid for each type. The probability of getting each type of painting can be estimated using $R = 1/(PO_{type} + 1)$. Location is another crucial factor, in order to using 'the earlier, the more important' and consider all auction list rather than just the first 5, the following formula is used to compute the score of each type once it has enough 5 same types, the initial score is 0 and i is the serial number of each work relative to the current round:

$$Score_{artist} = Score_{artist} + R * (1/i^5) \qquad (1)$$

Using the power of 5, rather than $R * log(1/i)$ or R^i to control the influence of position on the score is an empirical choice. Using this strategy, in Figure3, supposing the first problem, the R for type P and V is the same, score for P at position 9 is $R * (1/9^5) = R * 1.69 * 10^{-5}$, score for V at position 11 is $R * (1/10^5) + R * (1/11^5) = R * 1.62 * 10^{-5}$, $1.62 < 1.69$, at position 12 is $R * (1/10^5) + R * (1/11^5) + R * (1/12^5) = R * 2.02 * 10^{-5}$,$2.02 > 1.69$, so for this auction list, when agent find the V at position 12, the score for V is higher than P. So the player will not bid for P at position 1, as the best choice is wait for V at position 2, to make the first bid. For the second problem, similarly, the score would also be more reasonable according to the number of potential opponents for each type. Since too many players want P at position 1, this agent will wait for bidding V at position 2 as the score computed.

2) strategy2 for small number of bidders(< 5): The first strategy is not so aggressive as the main idea is to avoid the unreasonable bidding for the first auction. However, when the number of players is rather small, one has to be more aggressive for the first 5 streak painting. In an extreme case, there are only two competitors. If you lose the first 5 auctions that can be completed, you must consider stopping the opponent from succeeding, instead of focusing on the next 5 consecutive auctions, which will decline success rate. So the strategy must consider the number of bidders.

The first consideration is trying to make a draw if somebody is going to win. That's not a good choice if other players are willing to do that. Since the number of player is small, oneself need to do it. In the case of many players, it is hoped that other people may do this for everyone. Of course, it is also possible that everyone wants others to do it, and no one does it, resulting in direct loss. But a good strategy always needs to take certain risks. So I only choose to use the strategy of making a draw when the number of players is rather small($numbidders < 5$).

When the number is small, since not every painting is easy to receive the bidding of 200, there may be more uncertain situations. Taking a bidding strategy that differs from 200 is possible to have an advantage in the face of all 200 strategies when there are few people. Since the first bidding would decide the type and the last bidding relates to whether winning, these biddings must be bigger than 200. Since at least one bidding need to be less than 200, a good thinking is to make just the second bidding smaller than 200, and all others are bigger than 200. However, after several tests, these strategies are not very stable. So for ($numbidders < 5$), always focus on first 5 winning paintings with bidding 200 is chosen.

3) strategy3 - Only two players: It is interesting to propose a strategy for the situation with only two players, since each step is much more predictable. There are some considerations:

1) Player will bid on the type they already have in most cases
2) Player will try their best to bid on the first 5 winning paintings if they have nothing

3) Consider your opponents using always 200 strategy

There are only two players, once you lose the first winning list, if you focus on finding the next winning list, and the artist type of the second winning list is different from the first one, then you will lose. So if the type of first and second winning list is different, it is very important to snatch the first one. Here the second winning list means, if you remove the first painting you won, the type of the remaining 5 consecutive paintings.

My strategy is, find out the first and second winning list, if the type is different, use (330,10,220,220,220) as my bidding. Since if that 330 helps me winning the first one, if your opponent find the next winning list and doesn't consider there are only two players, he won't bid on my type the next, so the 10 is safe here.

1	2	3	4	5
P	P	V	V	P
6	7	8	9	10
V	P	P	V	V
11				
P				

Fig. 2. Auction order2

As shown in Figure2, the first winning list is P at 8 and the second is V at 10(consider the list without the first P at 1 now). P is different from V, suppose you win the P at 1 with 330, if your opponent now find out at position 2, that the following 5 P will end at position 11, the V is the best one which will end at position 10, he will bid 0 on P at 2 and decide to bid on V at 3, and plan to finish at position 10, then you are going to win at position 9.

Suppose the first and second winning list has the same type of artist, then use always 200 strategy as it is a stable choice. If you still use 330,10 bidding, your 10 is likely to lose, because your opponent is likely to compete with you for the same type of painter.

Suppose you win the first one with 330, and lose the second one with 10. This explains that your opponent does not choose the next winning list, but chooses to target you or is free to place a bet. This is the biggest weakness of this adventurous strategy. At this time, you can only see what the next winning list is, and distribute the remaining money evenly, but the hope of winning is not big.

Suppose you lose your first bidding with 330, it suggests that your opponent is even more adventurous than you. Then you must not consider the next 5 winning list, but use the always 200 strategy at your opponent's type(which he cost more than 330 for the first one). If your opponent continues to focus on this type, it is very likely that he will lose.

1	2	3	4	5	6
P	V	R	D	P	R
7	8	9	10	11	12
R	P	R	P	P	P
13	14				
R	R				

Fig. 3. Auction order3

4) strategy4-another assumption:second 5: A defect of the assumption in strategy1 is that every one will bid 200 on every item if they have nothing, which is basically impossible. For those players empty in their hands, we can assume that they will bid only on the type of first 5 streak painting, which is more reasonable.

As shown in Figure3, at position 1,2,3,4 and 5, type P is the first list for winning, and R is the second option. If a lot of people are competing for P, then it is a good choice to wait for 3,R from position 1. At this time, not many people notice it, not too many competitors, looking at the second list that can win maybe is a good choice. It depends on how many opponents you estimate will focus on the first winning list.

5) conclusion for the first game: How to decide which strategy to use based on the number of players is very difficult. After evaluation, strategy3 for $numbdidder = 2$, strategy2: always 200 with first winning list for $2 < numbdidder < 5$, strategy 4 for $5 < numbdidder < 30$, strategy 1 for $30 < numbdidder$ as it is the most stable one with longer consideration.

B. Order is not known

When order is not known, the main difference is one can't find the first or second 5 same type paintings anymore. The other assumptions and considerations basically remain the same.

The valuable knowledge now is the total and sold number of each type paintings. For each type, the more the total number, the fewer others already have, the less potential competitors you have, the more likely you are to win.

So here is the strategy. When there is a certain painting in hand, the bidding is 200 for this category and 0 otherwise. When empty in hands, calculating the score for each type by using the total number of pieces minus the number of people who want to get it. The way to calculate the potential opponent number is equal to PO_{type} in the last section, by supposing player will bid on the type they have and all types if they have nothing.

After getting this score, when the item being auctioned is listed in the lowest score, it will not bid, otherwise the bid will be 200. Use a strategy to fight for a draw when the number of people is less than 3.

III. Highest total value

A. The highest bidder pays

Supposing there are n players, every player has the same best strategy and nobody could do better further, then the value each person get in the end is the average of total value. Although every player wants to get the highest total value in the end, suppose they all use the best strategy(if there is) and everyone will not feel regret, the only result each player could get is the average of total value.

Suppose players past bids follow this rule. If every one has spent half of the money, everyone has already got about half of the total value of the work, and the ratio of (money-spent, value-got) keeps. Then the total remaining score will also be divided by the number of bidders in the second half of auction. In conclusion, suppose there are n players, during the auction, the value each player would get is

$$value_{each\ player\ get} = value_{total\ remaining}/n \qquad (2)$$

The $value_{type}$ denotes the value of each type, in this auction the value is 4,6,8,12 accordingly. Then the bidding of each painting could be computed by

$$my\ money\ left * value_{type}/value_{each\ player\ get} \qquad (3)$$

This is the final bidding for each item. The thinking behind this evaluation, which seems not ambitious, is very simple. No matter how much I have spent and how much I have earned, I still use the remaining money to get the average one that belongs to me. Although it is only a tactical pursuit of the average, but because other players are more or less, and are likely to be too restrained, relatively low bids, this strategy is always possible to obtain higher value. And still based on the remaining value and money calculation ratio, you can get more value as much as possible. If there are more failures in the previous auction, this strategy can also adjust the bid based on the remaining money and value to cope with the change.

B. The second highest bidder pays

It is a second-price sealed-bid auction at each round. Previous research has reached a conclusion, in the second-price auction, if a bidder knows the value of the object to himself, then his dominant strategy is to submit a sealed bid equal to that value[1].

Then the main question now is to figure out the true value for each painting at every round. However, there are some rules which make this situation more difficult. This is a finite game and money left doesn't count.

It is possible that if some people adopt a more aggressive strategy and spend too much money in the early stage, the average price in the later period may be lower, because the maximum bid for each person can only be equal to the money left in their hands. Even if the bid price is very high, only the second highest bid price is paid at this time, and the second highest bid price is not likely to be high at this time, so it is easier to earn a higher score than the average.

Another thinking is the target value. If there are only 2 players, average value plus 1 could assure winning and a higher value is meaningless. It is difficult to predict the final score of winner if there are more players, After several tests, 1.1 is chosen to be the coefficient of the final target value, as it is also slightly bigger than the original value.

At the beginning of the auction, bids are made according to the target value. This means that in a strategy that wants to get more points, the bid is lower than the average price, and it is likely to leave some more money after the early period. When judging that the remaining value is not enough to reach the target, the strategy changes the target to the original average rather than average*1.1, to try to spend all money left to win as much as possible in the second stage, which is the stage where the average price of the item is predicted to be lower.

IV. Conclusion

In the first game, although the alwasys-200 strategy is advantageous in the long run, for two people, a more aggressive and clear strategy can be used, and while many others focus on robbing the first 5 consecutive paintings, consider taking precedence over the second five consecutive paintings is a good choice. The second game doesn't tell the order, the total number of each type of paintings and the paintings that others already have could optimize the bid. The basic point of the latter two games is to set a target value and average remaining money on these values. The bidding strategy can be changed in the game according to certain conditions. For example, it is easier to make a profit in the latter part of auction as the lower price of the item is assumed.

References

[1] P. R. Milgrom and R. J. Weber, "A theory of auctions and competitive bidding," *Econometrica: Journal of the Econometric Society*, pp. 1089–1122, 1982.