Math assignment

2018年11月30日 星期五　　上午11:42

Name: Jiyu Yan
ID: 1851015
CS917 Maths and Stats Assignment

1.(a) $(a \to c \wedge d) \vee (\neg a \to b \vee \neg d)$

(b)

a	b	c	d	$c\wedge d$	$\neg a$	$\neg d$	$b\vee\neg d$	$a\to c\wedge d$	$\neg a\to b\vee\neg d$	S
0	0	0	0	0	1	1	1	1	1	1
0	0	0	1	0	1	0	0	1	0	1
0	0	1	0	0	1	1	1	1	1	1
0	0	1	1	1	1	0	0	1	0	1
0	1	0	0	0	1	1	1	1	1	1
0	1	0	1	0	1	0	1	1	1	1
0	1	1	0	0	1	1	1	1	1	1
0	1	1	1	1	1	0	1	1	1	1
1	0	0	0	0	0	1	1	0	1	1
1	0	0	1	0	0	0	0	0	1	1
1	0	1	0	0	0	1	1	0	1	1
1	0	1	1	1	0	0	0	1	1	1
1	1	0	0	0	0	1	1	0	1	1
1	1	0	1	0	0	0	1	0	1	1
1	1	1	0	0	0	1	1	0	1	1
1	1	1	1	1	0	0	1	1	1	1

(c) $(a \to c\wedge d) \vee (\neg a \to b \vee \neg d) \Leftrightarrow [\neg a \vee (c \wedge d)] \vee [a \vee (b \vee \neg d)] \Leftrightarrow$

$a \vee (\neg a) \vee (c\wedge d) \vee (b \vee \neg d) \Leftrightarrow T \vee [(c\wedge d) \vee (b \vee \neg d)]$

$\Leftrightarrow T$

2. (a) a: $\{x>2 \mid P(x)\} \Rightarrow \forall x: (\{x \mid x = 2k+1, k \in N\} \wedge \{x \mid \frac{x}{1} \in N\})$

b: $\{x>2 \mid P(x)\} \Rightarrow \forall x: (\{x \mid x=2k+1, k\in N\} \wedge \{x \mid \exists y \in N, \frac{x}{y} \in N\})$

c: $\{x \mid P(x)\} \Rightarrow$ $\{x = \sum_{i=1}^{n} k_i \mid k \in S, \exists S \subseteq P(x)\}$

n is the number of elements in S.

(b) a: T. If x is not odd, then it has a divisor of 2 which is contradict to prime numbers. All prime numbers can't be divided by any whole number except itself and the number 1. So this statement is true.

b: T. Suppose $y=1$ or $y=x$ here, from above it is true.

C: F. 11 is prime number. can't find any subset of P here, by adding all elements could get 11. So it's false.

3. (a) Negative. $\emptyset$ is not a set. So it's not a power set of any set.

(b) $\{a\}$

(c) Negative.

(d) Negative.

(e) Negative.

4. Transitive means $R_p: A \leftrightarrow A$, if $\forall a,b,c \in A: (apb \wedge bpc) \Rightarrow apc$.

So for $R_\#: X \leftrightarrow X$, it is transitive only if $x\#y$, $y\#z$ and $x\#z$.

Through the existence of $x\#y$ and $y\#x$ we can't deduce it's transitive.

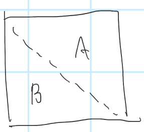
z must be different from x.

5. (a) partial order. For example, $X=\{a,b\}$, $Y=\{a,c\}$. Either $X \subseteq Y$ or $Y \subseteq X$ is not right so it doesn't have totality. So it is partial order.

(b) $\{\emptyset\}$ is least (sole minimal), no greatest or maximal.

6. (a) R on A is reflexive $\Rightarrow$ $(x,x) \in R$, $\forall x \in A$. This means we must include all n diagonal pairs in all relation. Other pairs are optional.

So we have 2^{n^2-n} reflexive relations.

(b) 

each pair $(a,b) \in A$ must be the same state with $(b,a) \in B$.

Apart from diagonal elements, we have $\frac{n^2-n}{2}$ free elements.

So the total free elements are $\frac{n^2-n}{2} + n = \frac{n^2+n}{2}$.

So there are $2^{\frac{n^2+n}{2}}$ symmetric elements.

(c) For each pair (a,b), $a \neq b$, there are 3 possibilities, only (a,b), only (b,a), or none of them. Apart from diagonal there are $\frac{n^2-n}{2}$ free elements.

All diagonal n elements have 2 possibilities none of them. Apart from diagonal there are $\frac{n^2-n}{2}$ free elements.

All diagonal n elements have 2 possibilities.

So there are $3^{\frac{n^2-n}{2}} \cdot 2^n$ antisymmetric relations.

7. $(gf)^{-1}(x) = y \Rightarrow x = (gf)y = g(f(y)) \Rightarrow g^{-1}(x) = f(y)$

$\Rightarrow f^{-1}g^{-1}(x) = y$ So $(gf)^{-1}(x) = f^{-1}g^{-1}(x)$

8. $\{\forall x \in N: \text{isPrime}(x) = T\} \subseteq N$. The set of prime number is a subset of N. N is countably infinite and a subset of a countably infinite set is either countably infinite or finite. Since the set of prime number is infinite, it is countably infinite.

9. Flush: $52-5=47$ remaining $13-3=10$ ♡ $\frac{\binom{10}{2}}{\binom{47}{2}} = \frac{10 \cdot 9}{47 \cdot 46} \approx 0.0416$

Straight: $(9, 10)$ or $(10, A)$ $\frac{4}{47} \times \frac{4}{46} \times 2 \times 2 \approx 0.0296$

10. (a) $P(den) = 0.0001$ $P(head \mid den) = 0.64$ $P(head \mid \overline{den}) = 0.6$

$P(test \mid den) = 0.99$ $P(test \mid \overline{den}) = 0.04$.

now test for Barry was positive, also has headaches.

$P(test) = P(den) \cdot P(test \mid den) + P(\overline{den}) \cdot P(test \mid \overline{den}) = 0.0001 \times 0.99 + 0.9999 \times 0.04 = 0.040095$

$P(head) = P(den) \cdot P(head \mid den) + P(\overline{den}) \cdot P(head \mid \overline{den}) = 0.0001 \times 0.64 + 0.9999 \times 0.6 = 0.60004$

$$P(den \mid test \cdot head) = \frac{P(test \cdot head \mid den) \cdot P(den)}{P(test \cdot head)} = \frac{0.99 \times 0.64 \times 0.0001}{P(test) \cdot P(head)} = \frac{0.00006336}{0.040095 \times 0.60004} \approx 0.0026337$$

Based on 0.0026337, it's not enough evidence for doctor to conclude that Barry has dengue.

(b) $P(test2 \mid den) = 0.99 \times 0.99$ $P(test2 \mid \overline{den}) = 0.04 \times 0.04$

$$p(test2) = p(den)\cdot p(test2|den) + p(\overline{den})\cdot p(test2|\overline{den}) = 0.0001 \times 0.99 \times 0.99 + 0.9999 \times 0.04 \times 0.04$$

$$p(den|test2) = \frac{p(test2|den)\cdot p(den)}{p(test2)} = \frac{0.0001 \times 0.99 \times 0.99}{0.0001\times 0.99\times 0.99 + 0.9999 \times 0.04 \times 0.04} \approx 0.05773$$

$0.05773 > 0.02415$, but it's still very small. Couldn't affect the belief of diagnosis now.

(c) $$p(den|test\ n) \geq 0.5 = \frac{p(test\ n|den)\cdot p(den)}{p(test\ n)} = \frac{p(test\ n|den)\cdot p(den)}{p(den)\cdot p(test\ n|den) + p(\overline{den})\cdot p(test\ n|\overline{den})}$$

$$= \frac{0.0001 \times (0.99)^n}{0.0001 \times (0.99)^n + 0.9999 \times (0.04)^n}$$

when $n=3$, $p(den|test\ 3) = 0.603 > 0.5$

The minimum number is 3.

11. This is my output after one running:
estimate pai is 3.156
95 % confidence interval in 5 estimates:[3.057, 3.183]
95 % confidence interval in 10 estimates:[3.096, 3.154]
95 % confidence interval in 100 estimates:[3.110, 3.179]
95 % confidence interval in 1000 estimates:[3.123, 3.160]
t=3.808 in 5 estimates
t=5.873 in 10 estimates
t=5.799 in 100 estimates
t=10.496 in 1000 estimates

From the output we could conclude that averagely the more number of estimates are, the more accuracy of the 95% confidence interval is.
Although in 5 and 10 estimates, sometimes t is not big enough to conclude that there is a significant difference at the p = 99% level. When the number of estimates is 100, 1000 or more, we could always conclude that there is a significant difference at the p = 99% level.
My main methods are shown as below:

```
# Estimate pai by throwing 1000 darts uniformly
def pred_uniform():
    num_less = 0
    for i in range(1000):
        dart = throwUniformDart() # change here to:dart=throwNormalDart() is the function pred_normal()
        if distanceToOrigin(dart) < 1:
            num_less += 1
    pred = 4 * num_less / 1000
    # print(pred)
    return pred

# Compute the confident interval. n is the number of estimate.
def confident(n):
    ans = mean_sig(n)
    mean = ans[0]
    sig = ans[1]
    left = mean - 1.96 * sig/(n ** 0.5)
    right = mean + 1.96 * sig/(n ** 0.5)
    print("95 % confidence interval in {} estimates:[{:.3f}, {:.3f}]".format(n, left, right))

# Compute the mean and standard deviation of uniformly thrown from n estimates.
```

```python
22 def mean_sig(n):
23     all = []
24     sum = 0
25     for i in range(n):
26         pred = pred_uniform() #normally thrown just change here to:pred = pred_normal()
27         all.append(pred)
28         sum += pred
29     mean = sum / n
30     # print(mean)
31     temp_sum = 0
32     for i in all:
33         temp_sum += (i - mean) ** 2
34     sig = (temp_sum / (n ** 0.5)) ** 0.5
35     return mean, sig
36 
37 # Compute t-test from n estimates
38 def diff(n):
39     ans1 = mean_sig(n)
40     ans2 = mean_sig2(n) #change pred=pred_normal() in line26, then mean_sig becomes mean_sig2()
41     nom = abs(ans1[0] - ans2[0])
42     dnom = (ans1[1] **2 /n + ans2[1]**2/n) ** 0.5
43     t = nom/dnom
44     print("t={:.3f} in {} estimates".format(t, n))
45     return t
46 if __name__ == "__main__":
47     pred = pred_uniform()
48     print('estimate pai is {:.3f}'.format(pred))
49     confident(5)
50     confident(10)
51     confident(100)
52     confident(1000)
53     diff(5)
54     diff(10)
55     diff(100)
56     diff(1000)
```