MFx – Macroeconomic Forecasting

IMFx

Properties of Time Series

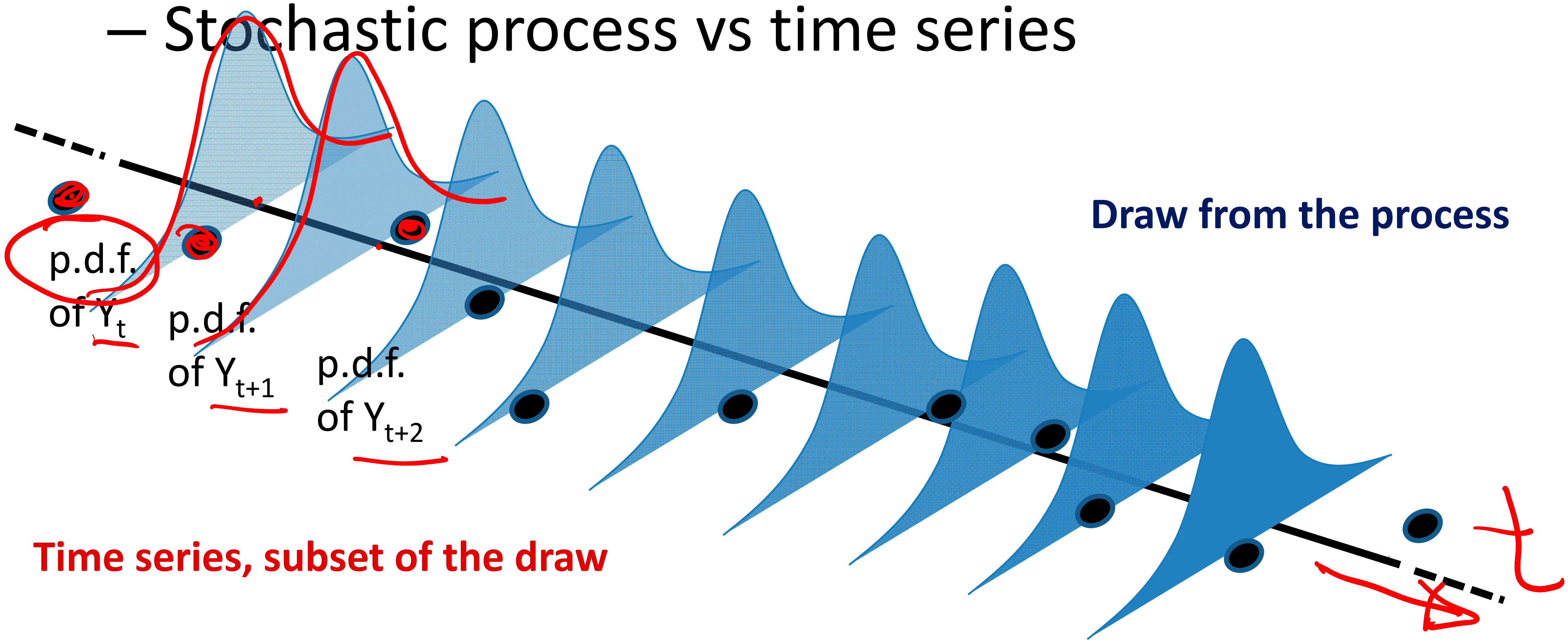
L-1: Introduction

Introduction

- General comments
  - Univariate analysis
  - Two general “classes” of processes
  - Both science and art (judgement):
    - Understanding behavior and forecasting
    - Assessing/testing

Introduction

- Univariate analysis
  - Stochastic process vs time series



Introduction

- Two general "classes" of processes
  - Stationary vs nonstationary
  - Unchanged distribution (pdf) over time?
  - Covariance stationary:
    - Unconditional mean and variance constant

$$E(Y_t) = E(Y_{t+j}) = \mu \qquad Var(Y_t) = Var(Y_{t+j}) = \sigma_Y^2$$

    - Covariance depends on time j that has elapsed between observations, not on reference period:

$$Cov(Y_t, Y_{t+j}) = Cov(Y_s, Y_{s+j}) = \gamma_j$$

y_{12}

y_8

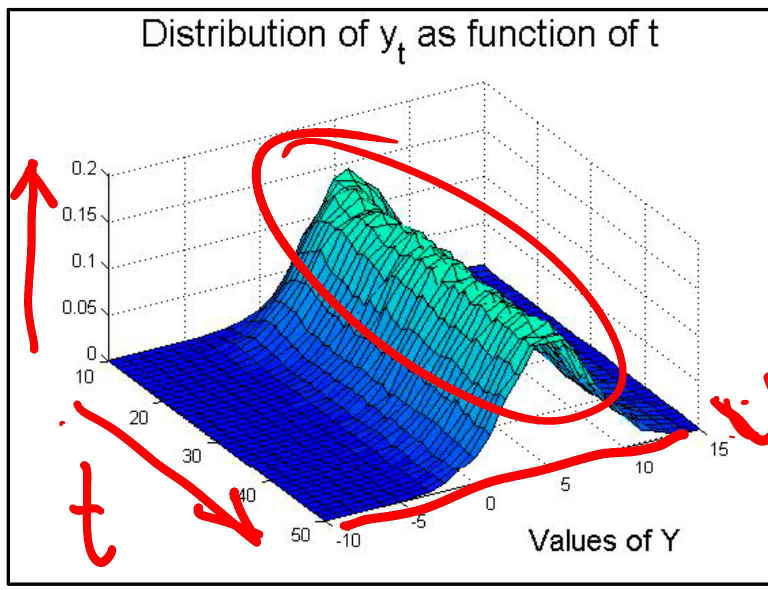
$\boxed{j = 4}$

Introduction

- Stationary?

Var changes

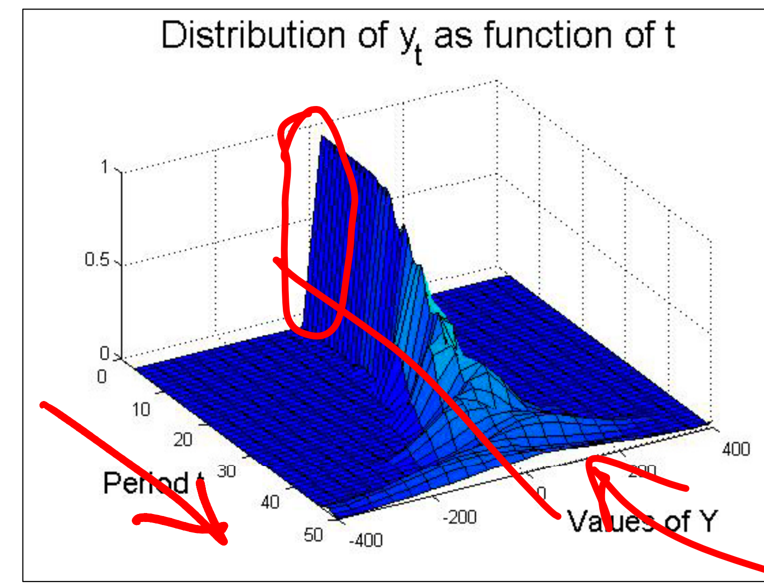
pdf



t

y

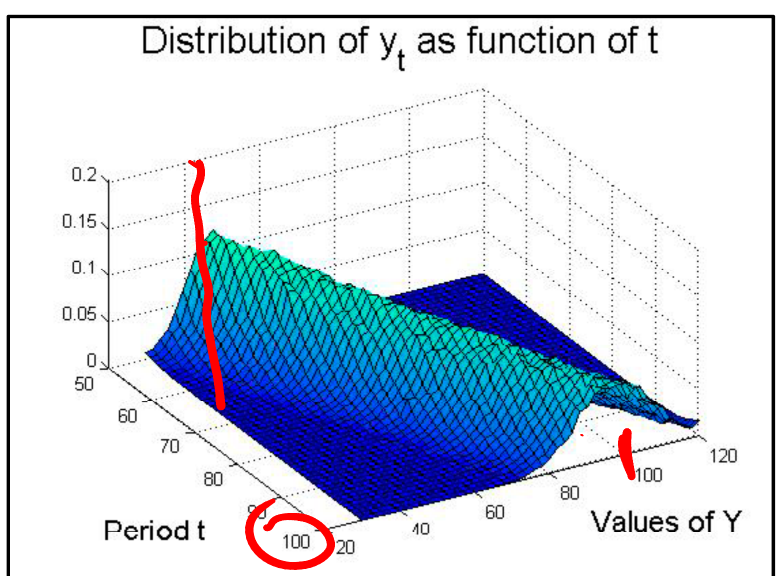
√



X

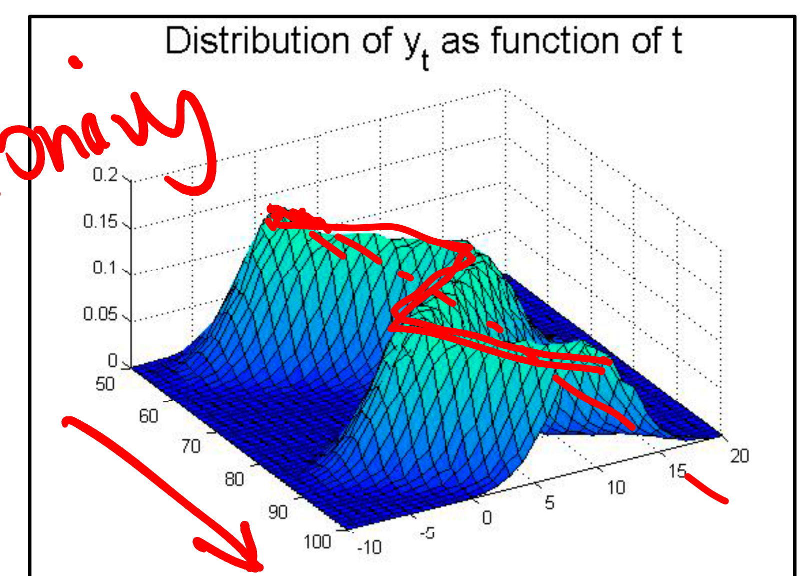
↑t ↑σ^2

Mean changes



X

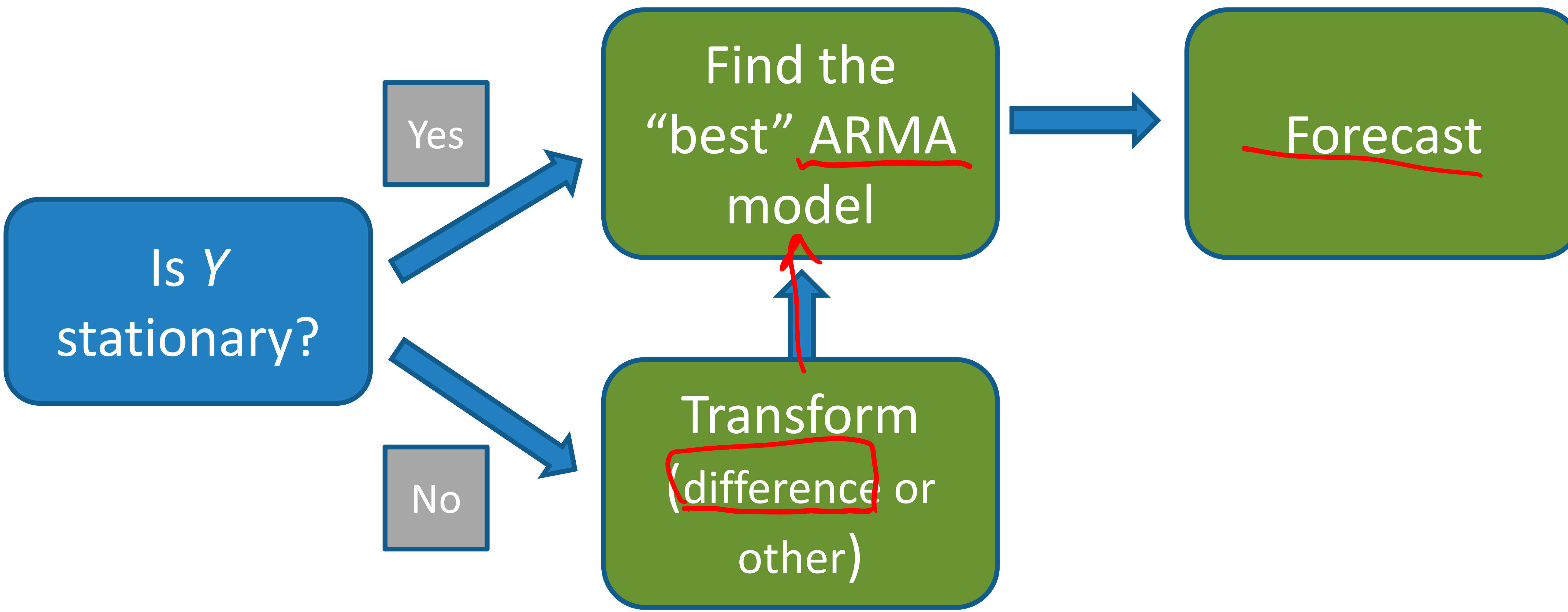
Non-Stationary



X

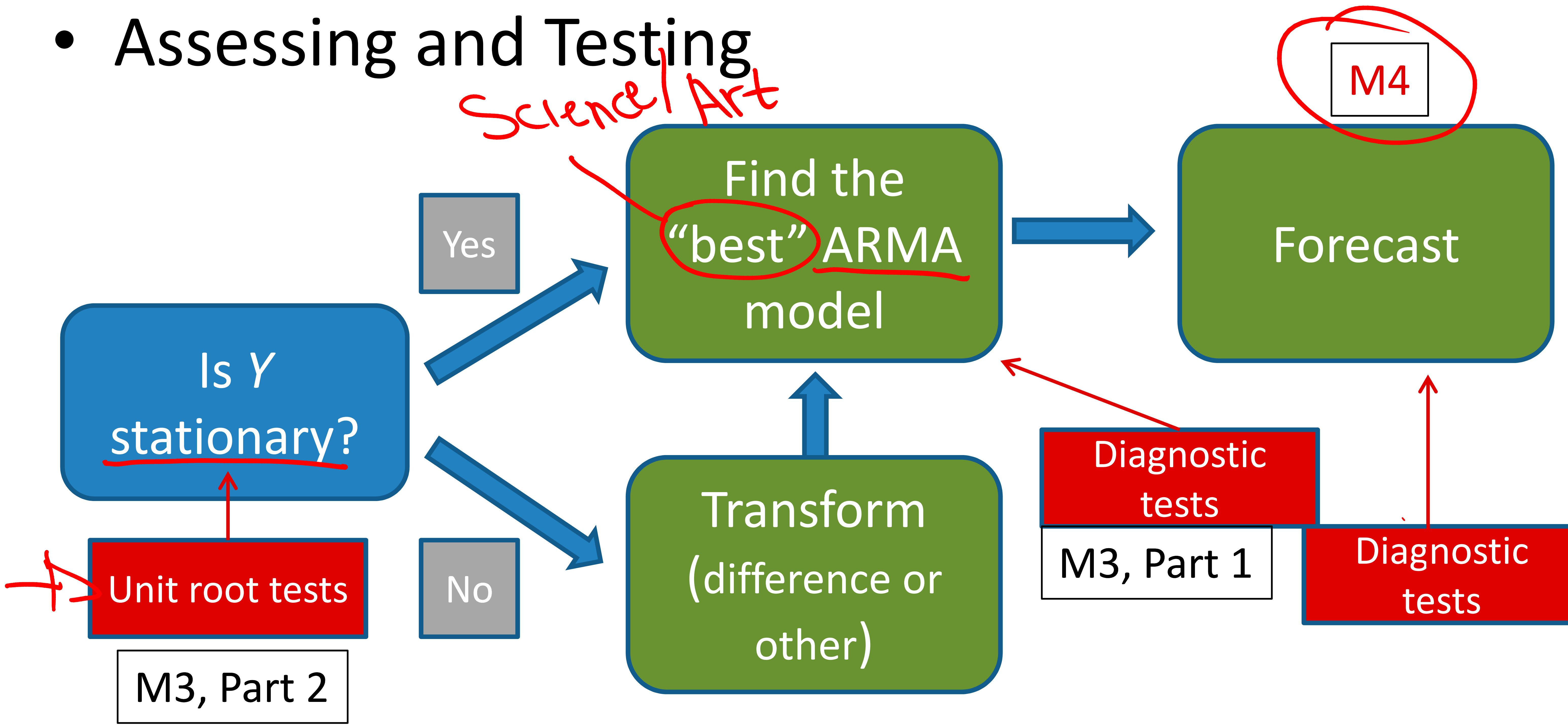
Introduction

- Understanding behavior and forecasting



Introduction

- Assessing and Testing



Outline

- *Part 1: Stationary processes*
  - *Identification*
  - *Estimation & Model Selection*
  - *Putting it all together* (simulated, real)
- *Part 2: Nonstationary processes*
  - *Characterization*
  - *Testing* unit roots

Properties of Time Series

Part 1: Stationary Time Series

L-2: Identification

Part 1: Stationary Time Series

Just to remind you….

- Identification
- Estimation & Model Selection
- Putting it all together

Identification

The first step is visual inspection: graph and observe your data.

"You can observe a lot just by watching"

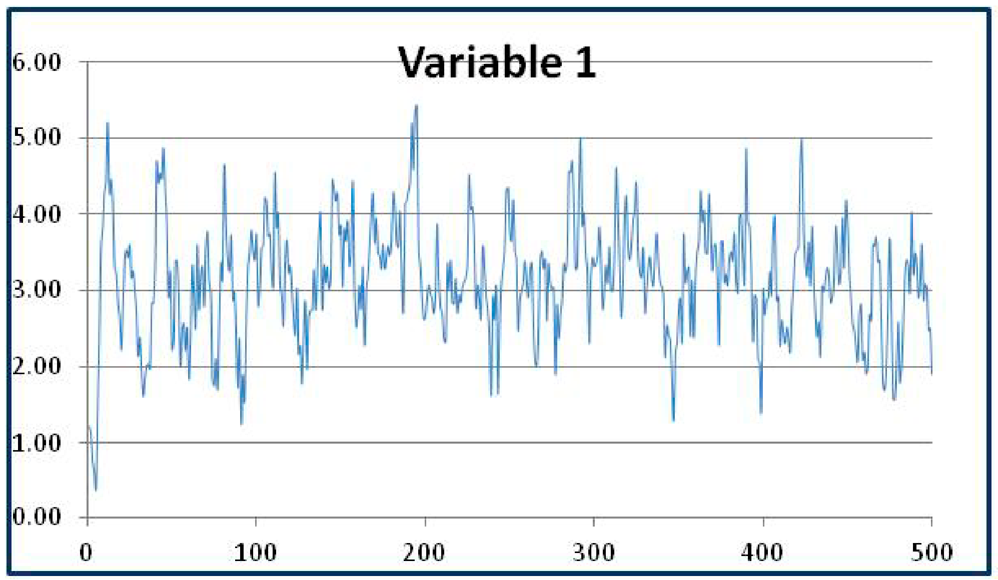
Yogi Berra

Identification

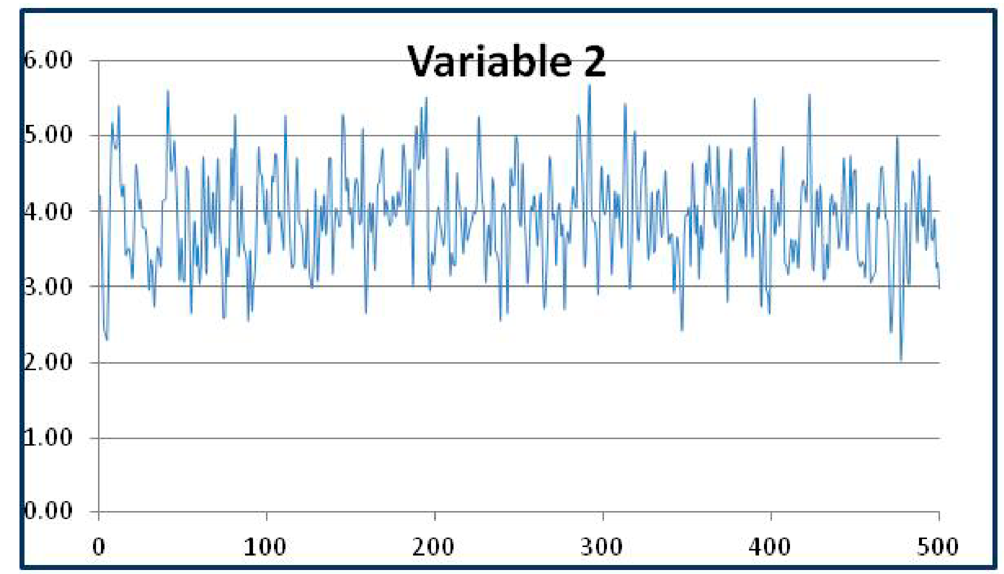
Does the series look stationary?

Autoregressive

AR(1)



Moving Average

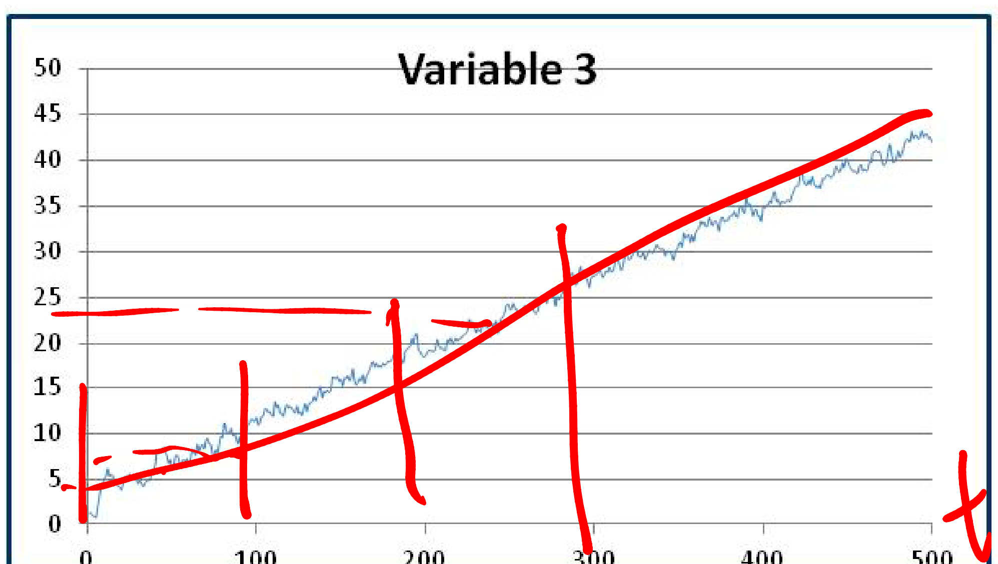


MA(1)

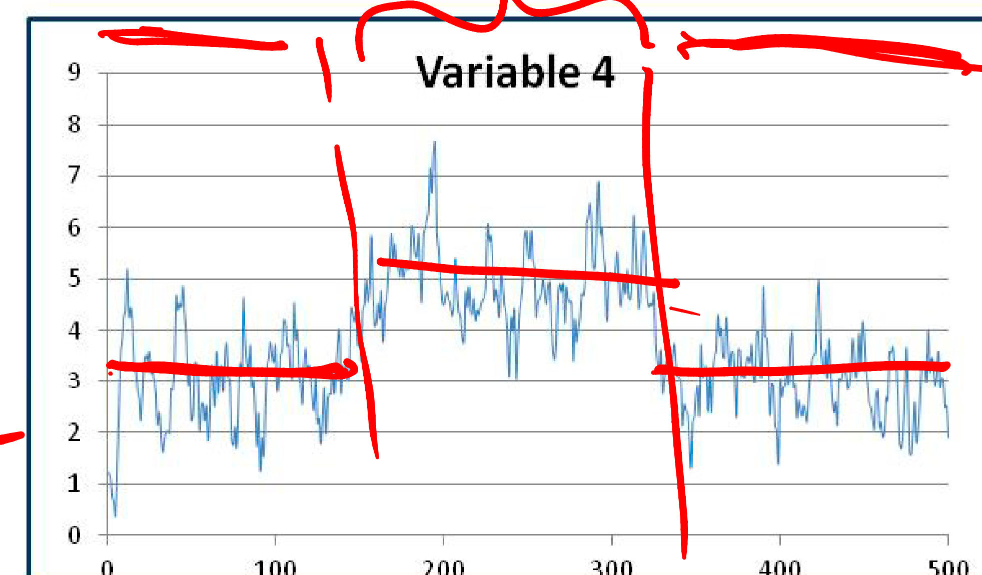
Nonstat

Mean changes

AR(1) with trend



Struct break



AR(1) with break

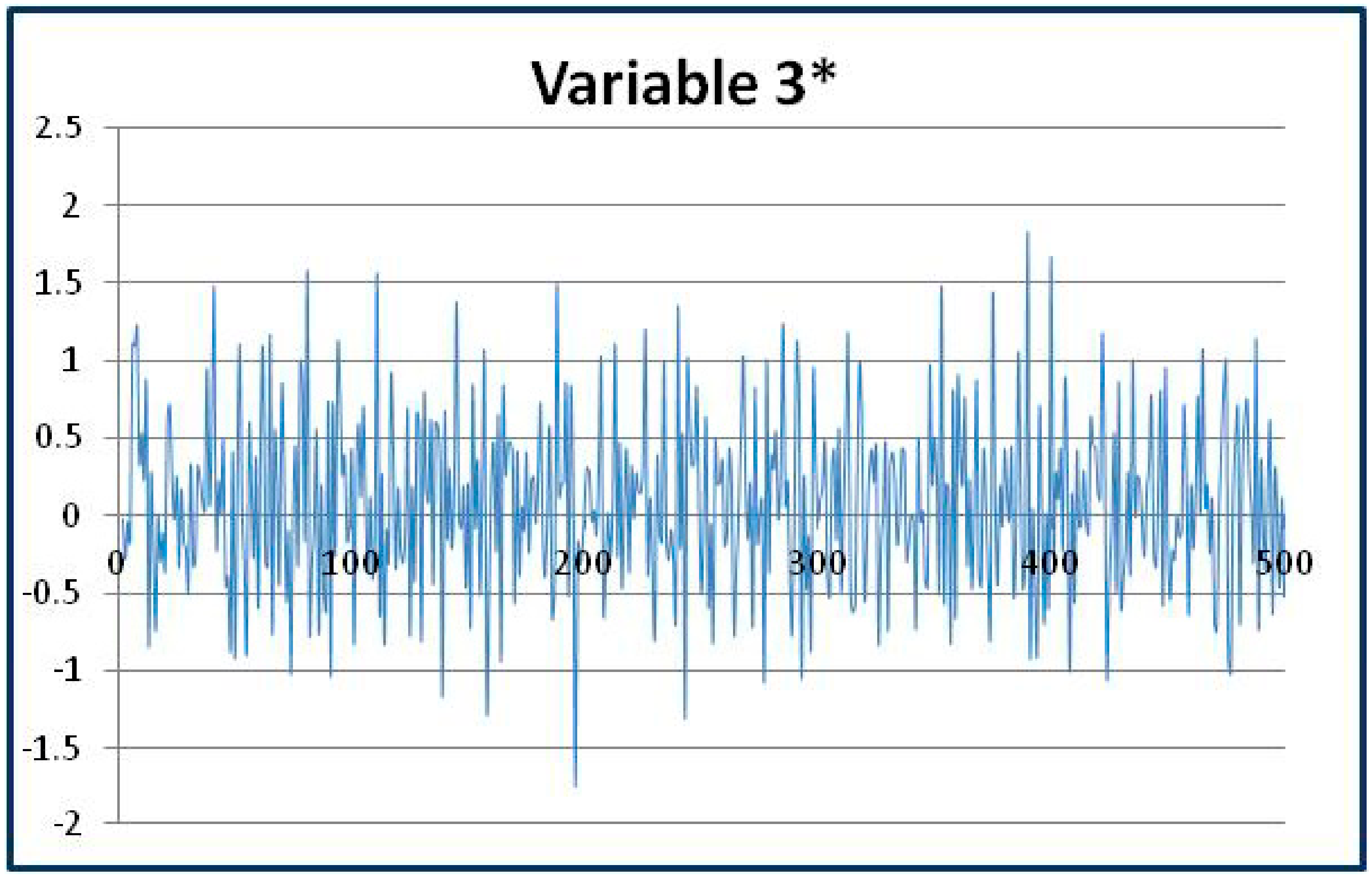
Identification

Note: *differencing* can remove the trend:

$$y_t^* = y_t - y_{t-1}$$

$y_{12} - y_{11}$

$y_7 - y_8$



Stationary

↓

Methods

Identification

Assuming that the process is stationary, there are three basic types that interest us:

random disturbance

- Autoregressive (AR) $$y_t = a + b_1 y_{t-1} + b_2 y_{t-2} + ... + b_p y_{t-p} + \varepsilon_t$$

max

- Moving Average (MA) $$y_t = \mu + u_t + \phi_1 u_{t-1} + \phi_2 u_{t-2} + ... + \phi_q u_{t-q}$$
- Combined (ARMA) $$y_t = a + b_1 y_{t-1} + b_2 y_{t-2} + ... + b_p y_{t-p} + u_t + \phi_1 u_{t-1} + \phi_2 u_{t-2} + ... + \phi_q u_{t-q}$$

Identification

Some notation:

- AR(p), MA(q), ARMA(p,q), where p,q refer to the *order* (maximum lag) of the process

ut

independently iid

- ε_t is a "white noise" disturbance:

$$\boxed{E[\varepsilon_t]=0, Var[\varepsilon_t]=E\left[\varepsilon_t^2\right]=\sigma^2, Cov[\varepsilon_t,\varepsilon_s]=0, \text{ if } t\neq s}$$

zero constant mean, constant variance → identically

Properties of Time Series

Part 1: Stationary Time Series

L-3: Some tools for identification

Part 1: Stationary Time Series

Where are we? Where are we going?

- Stationary process (visual inspection) *y*
- Learned about possible processes for *y*
- Need to identify which one in order to understand, then eventually forecast *y*

→ *tools to help identify*

Part 1: Stationary Time Series

Autocovariance and autocorrelation

- Relations between observations at different lags:
  - Autocovariance: $\gamma_j = E\left[\left(y_t - \mu\right)\left(y_{t-j} - \mu\right)\right]$

    expected value of y
  - Autocorrelation: $\rho_j = \dfrac{\gamma_j}{\gamma_0}$

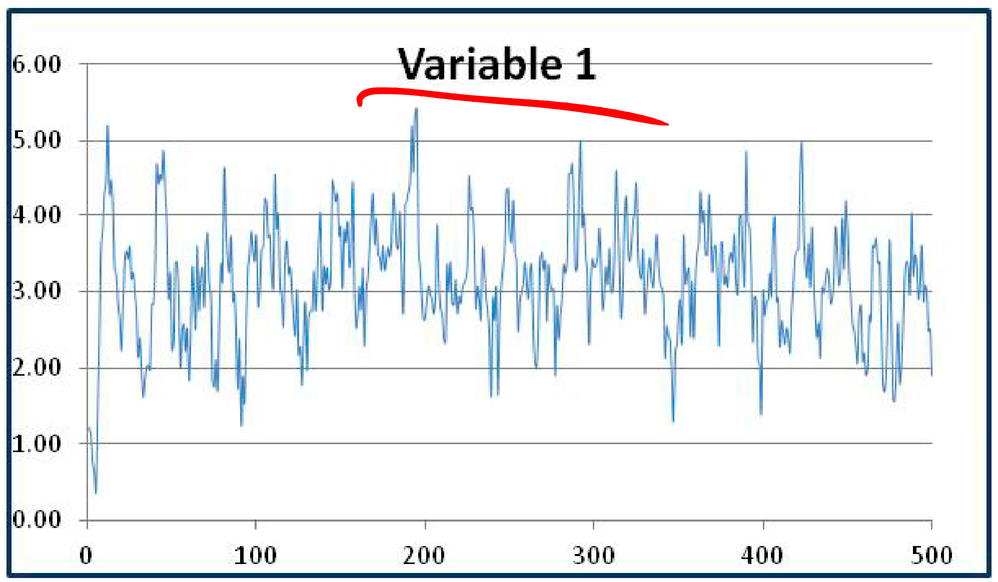
    variance of y
  - *ACF or "Correlogram"*: graph of autocorrelations at each lag

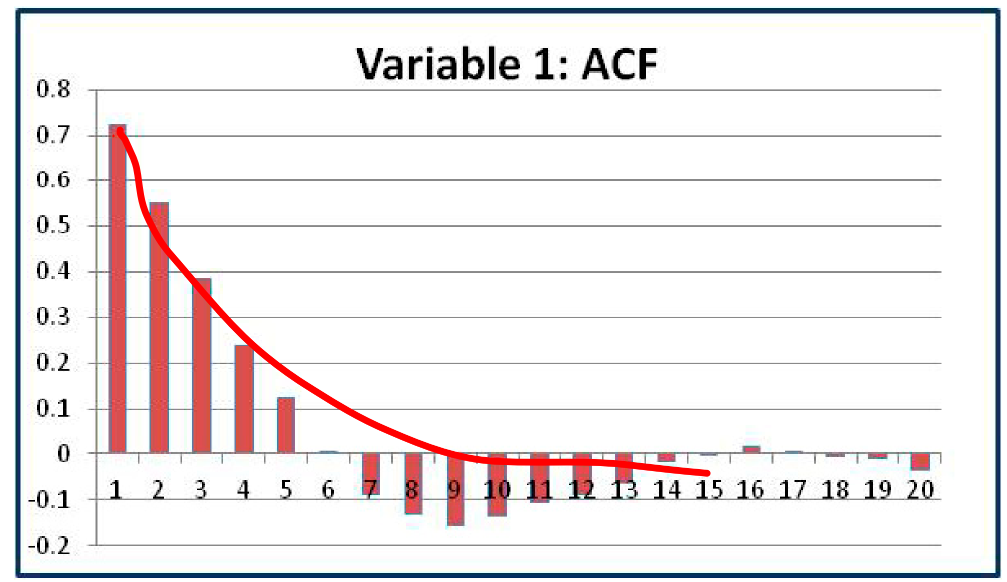
Part 1: Stationary Time Series

Back to our previous examples…

Different patterns:

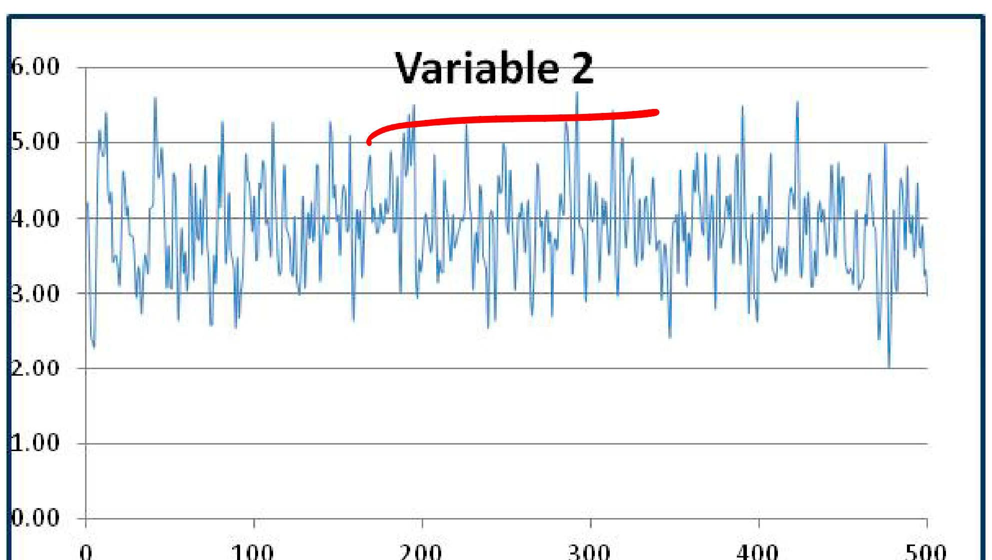
AR(1), $b_1 = 0.7$

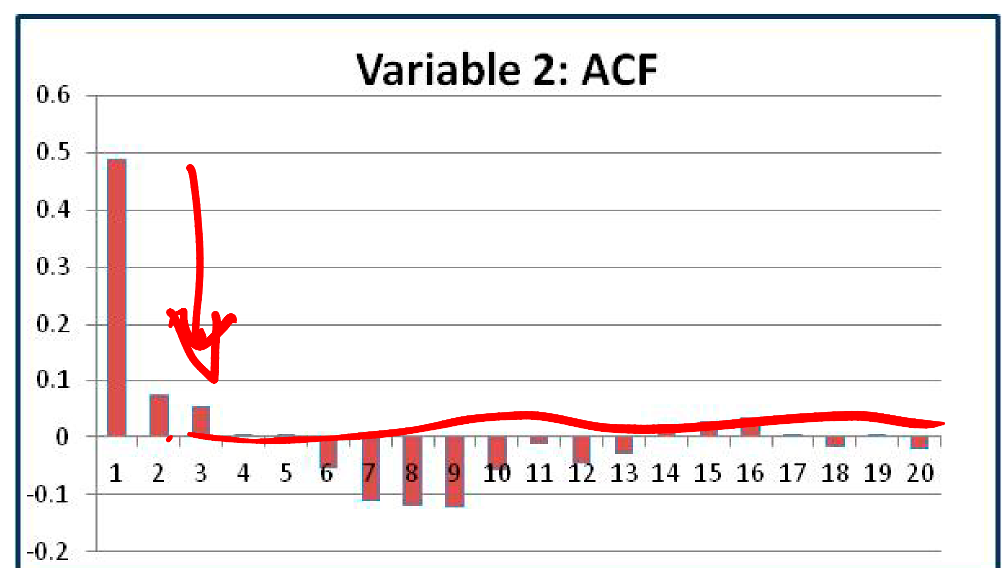




Geometric decay

MA(1), $\phi_1 = 0.7$





Cutoff

Part 1: Stationary Time Series

Partial autocorrelation

- The p^{th} partial autcorrelation is the p^{th} coefficient of a linear regression of y_t on its lags up to p:

$$y_t = \hat{a} + \boxed{\hat{b}_1 y_{t-1} + \hat{b}_2 y_{t-2}} + \ldots + \hat{b}_p y_{t-p} + e_t$$

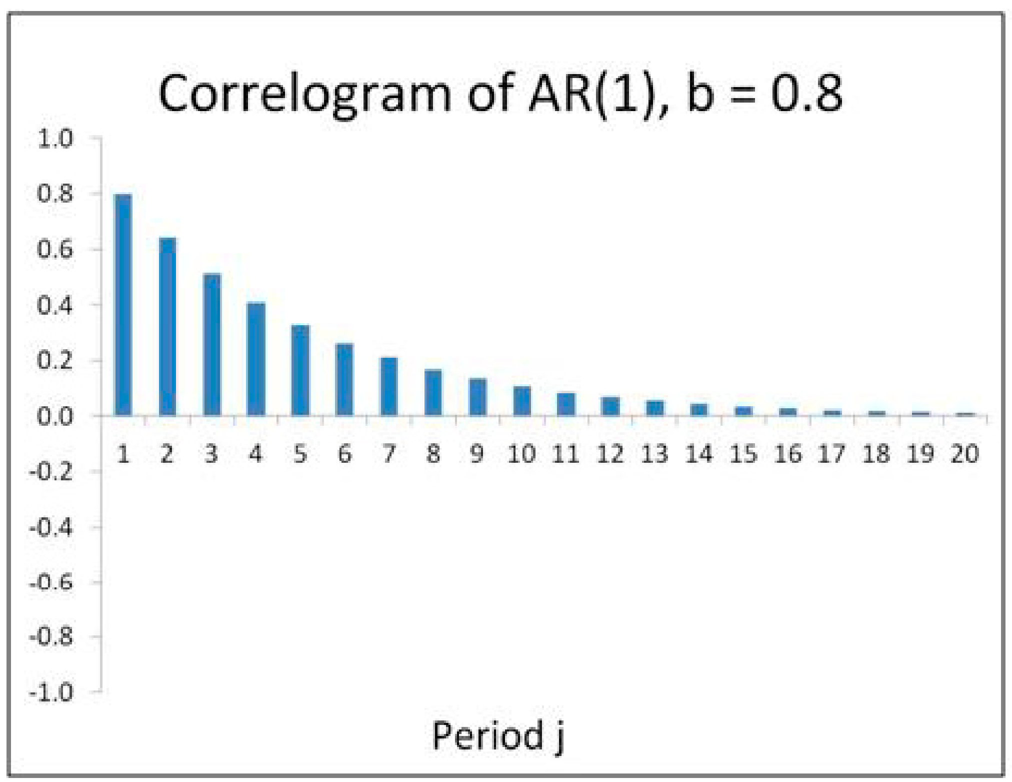
lags < p

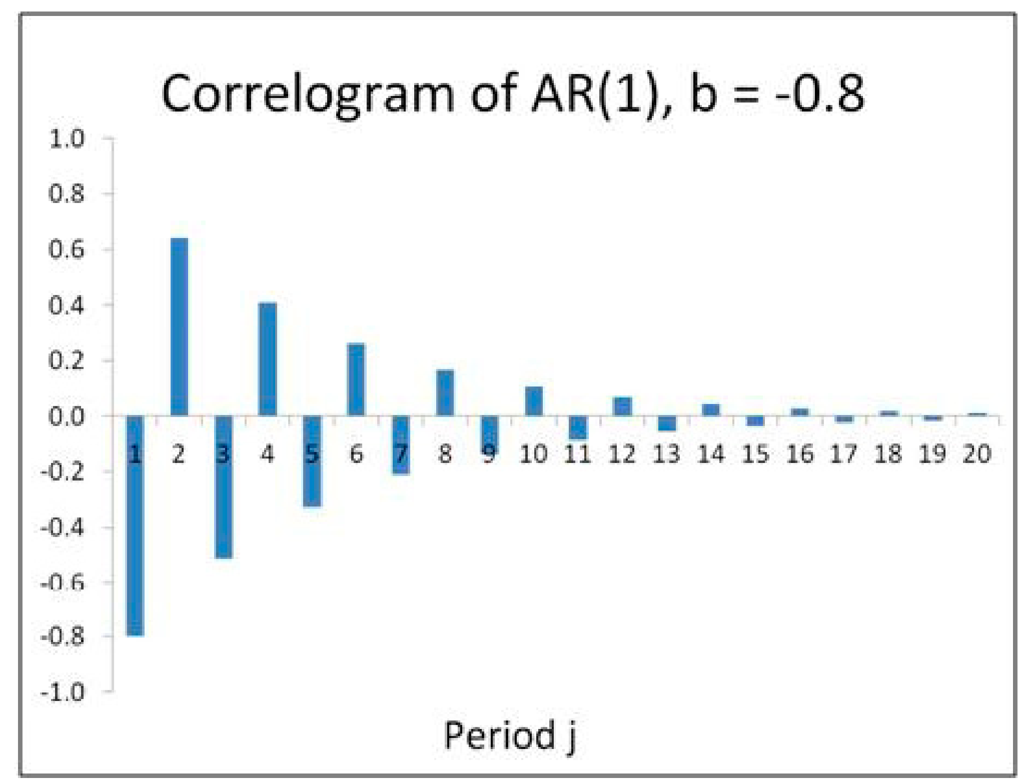
- Thus, $PAC_p = \hat{b}_p$
- Relationship between y_t and y_{t-p}, controlling for effects of other lags up to p
- For example, for $p = 3$, regress y_t on y_{t-1}, y_{t-2}, y_{t-3}

Part 1: Stationary Time Series

Some identifiable patterns for ACF, PACF

- Geometric decay of ACF in AR(1), oscillating if b<0.

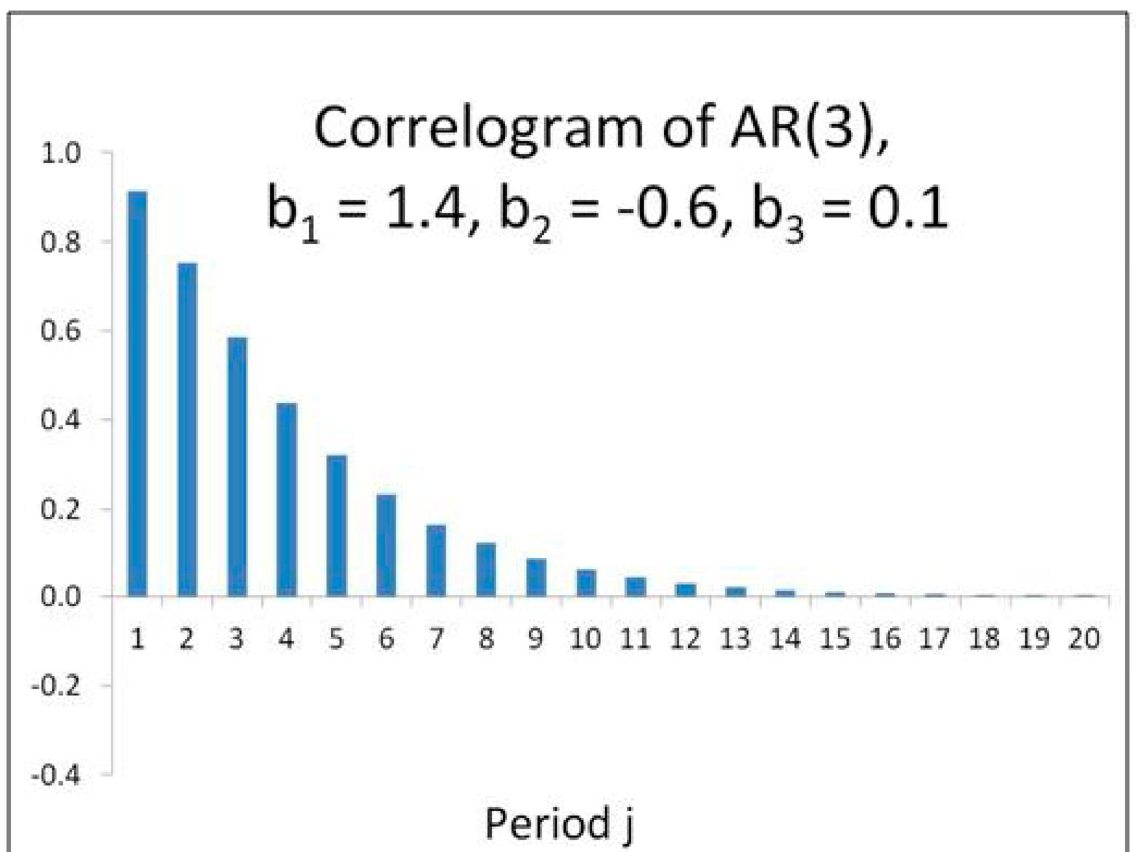


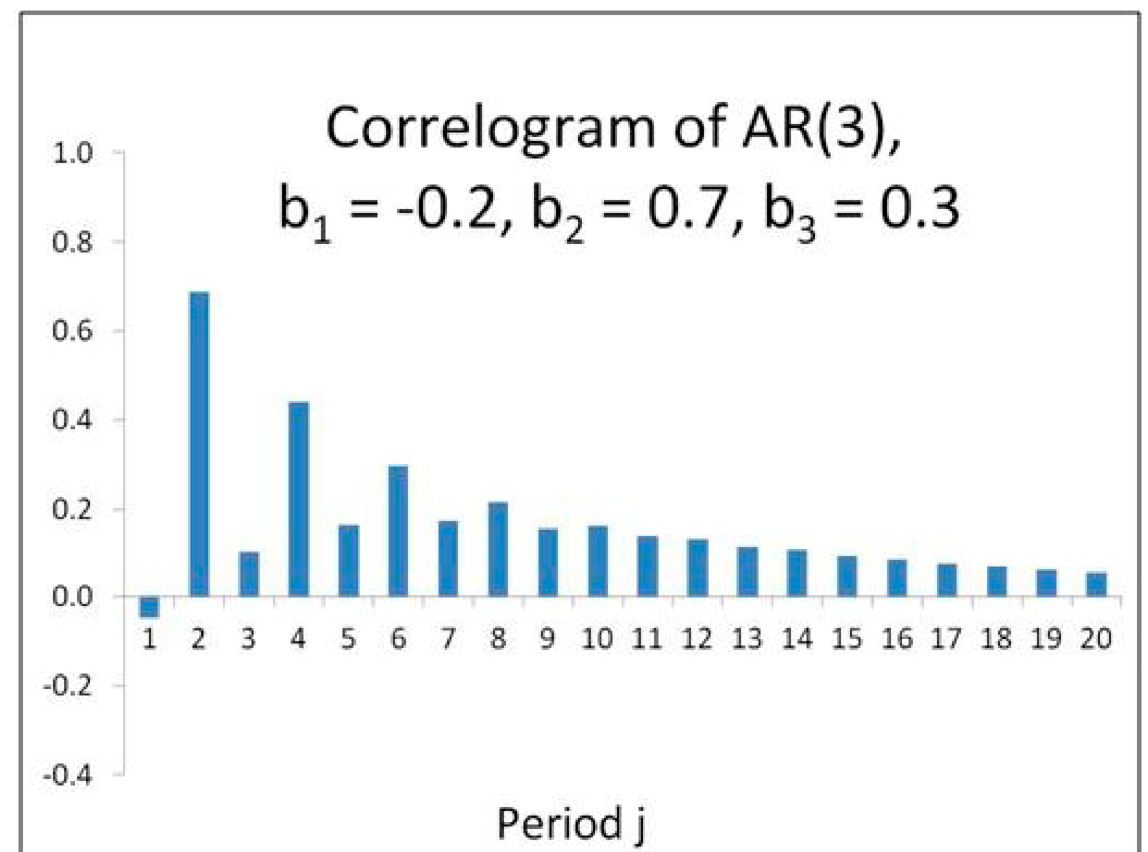


Part 1: Stationary Time Series

Some identifiable patterns for ACF, PACF

- Gradual decay of ACF in AR(p), tending toward zero relatively quickly.

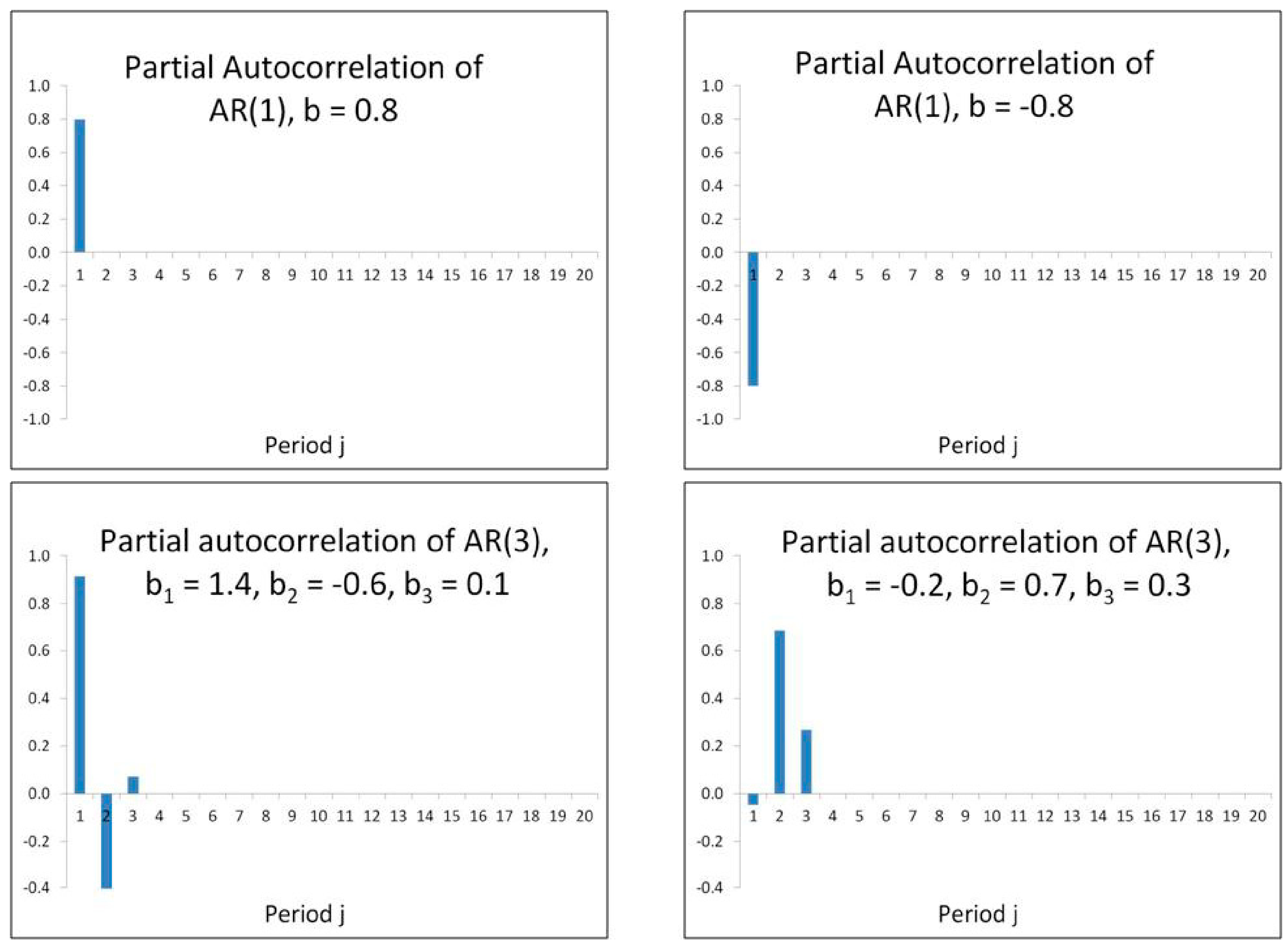




Part 1: Stationary Time Series

Some identifiable patterns for ACF, PACF

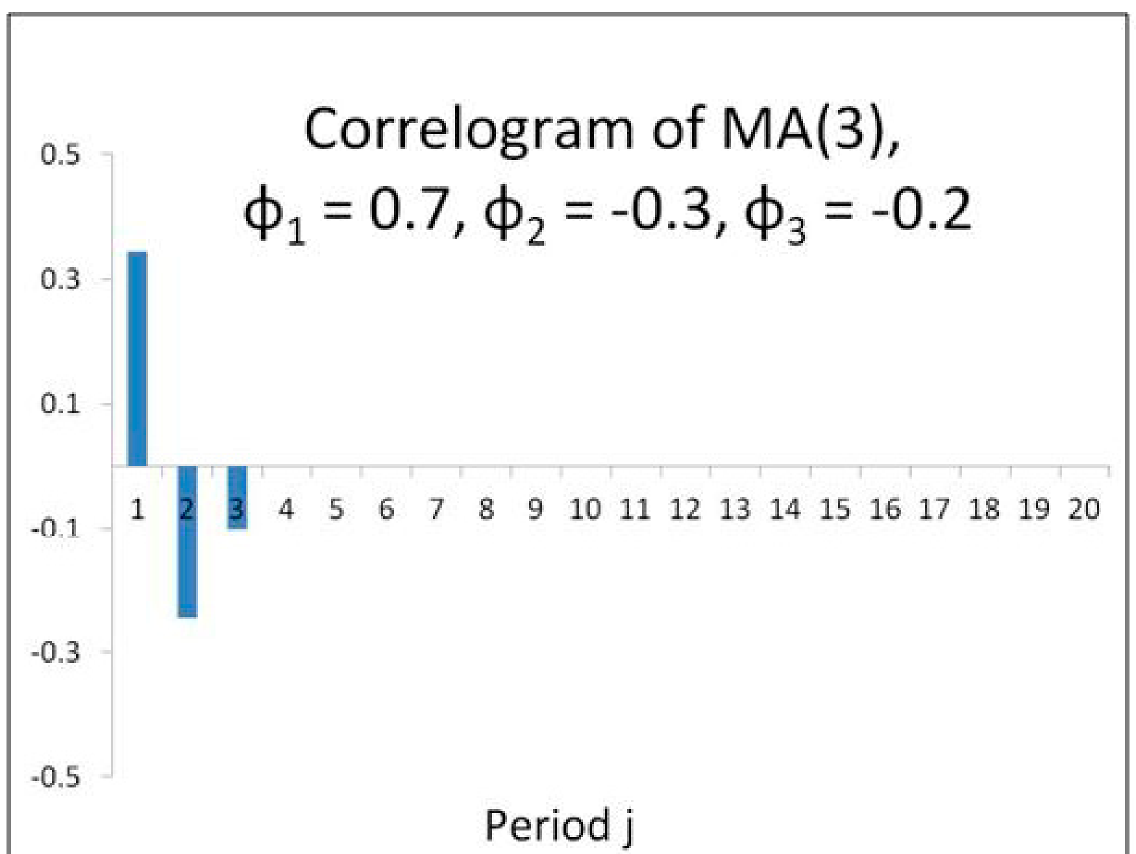
- Abrupt dropoff in PACF in AR(p) after lag p.

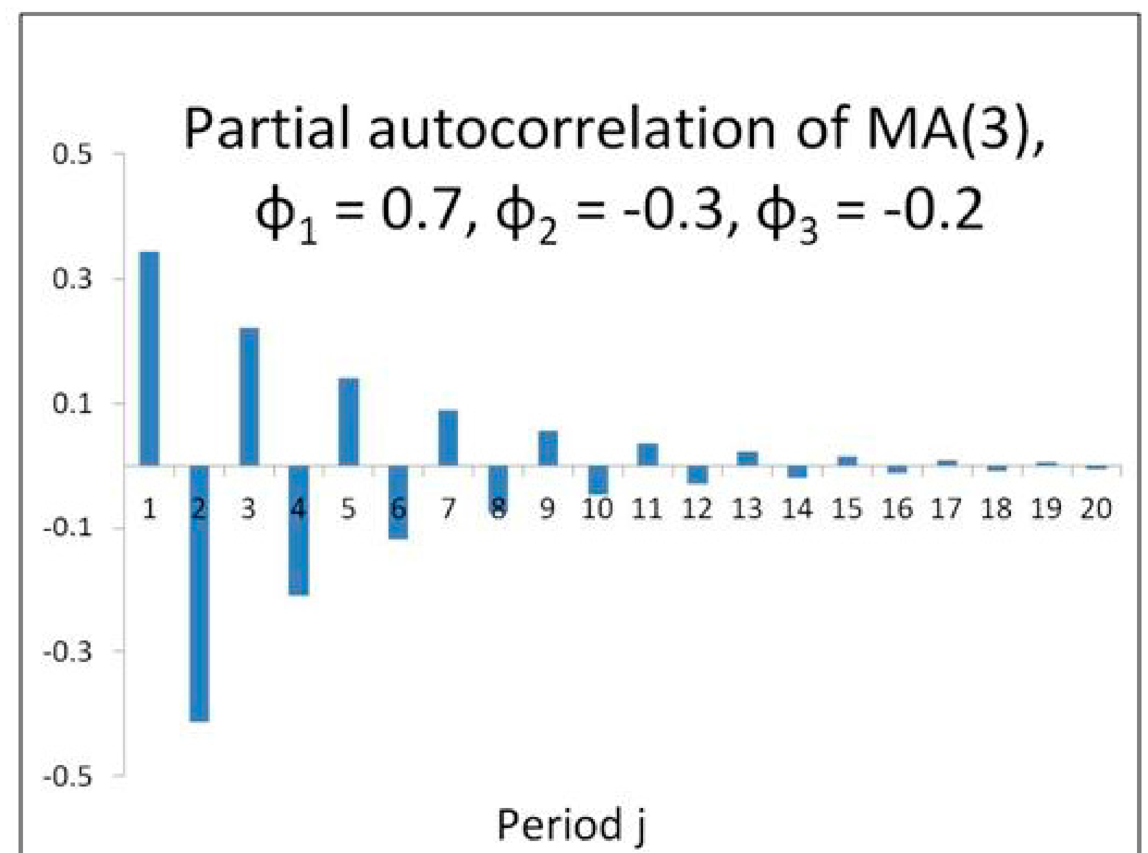


Part 1: Stationary Time Series

Some identifiable patterns for ACF, PACF

- Opposite patterns for MA(q): abrupt dropoff in ACF after *q*, gradual decay toward zero in PACF.





Properties of Time Series

Part 1: Stationary Times Series

L-4: Looking closer at identification

Part 1: Stationary Time Series

We now have a tool (ACF, PACF) to help us identify the stochastic process underlying a time series we are observing.

Now we will:

- Summarize the basic patterns to look for
- Observe an actual data series and make an initial guess
- Next step: estimate (several alternatives) based on this guess

Part 1: Stationary Time Series

Summary of the patterns to look for:

	ACF	PACF
White noise	All ρ's = 0	All b's = 0
AR(1)	**Geometric decay (oscillating if b<0)**	**Cutoff after lag 1; $\rho_1 = b_1$**
AR(p)	Decays toward zero, may oscillate	**Cutoff after lag p.**
MA(1)	**Cutoff after lag 1.**	**Geometric decay (oscillating if ϕ <0)**
MA(q)	**Cutoff after lag q.**	Decay (oscillating if ϕ <0)
ARMA(1,1)	Geometric decay after lag 1 (oscillating if b <0)	Geometric decay after lag 1 (oscillating if b<0)
ARMA(p,q)	Decay (direct or oscillatory) after lag q	Decay (direct or oscillatory) after lag p

Part 1: Stationary Time Series

Some tips

- ACF's that do not go to zero could be sign of nonstationarity
- ACF of both AR, ARMA decay gradually, drops to 0 for MA
- PACF decays gradually for ARMA, MA, drops to 0 for AR
- *Possible approach*: begin with parsimonious low order AR, check residuals to decide on possible MA terms.

Part 1: Stationary Time Series

When looking at ACF, PACF

- Box-Jenkins provide sampling variance of the observed ACF and PACFs (r_s and b_s)
- Permits one to construct confidence intervals around each ⇒ assess whether significantly ≠ 0
- Computer packages (EViews) provide this automatically!

Properties of Time Series

Part 1: Stationary Times Series

L-5: Estimation

Part 1: Stationary Time Series

Estimation & Model Selection:

- Decide on plausible alternative specifications (ARMA)
- Estimate each specification
- Choose "best" model, based on:
  - Significance of coefficients
  - Fit vs parsimony (criteria)
  - White noise residuals
  - Ability to forecast
  - Account for possible structural breaks

Part 1: Stationary Time Series

Fit vs parsimony (information criteria):

- Additional parameters (lags) automatically improve fit but reduce forecast quality.
- Tradeoff between fit and parsimony; widely used *criteria*:
  - Akaike Information Criterion (AIC)

    $AIC = T\, ln(SSR) + 2(p + q + 1)$
  - Schwartz Bayesian Criterion (SBC)

    $SBC = T\, ln(SSR) + (p + q + 1)\ln(T)$
- SBC will tend to prefer more parsiminious models than AIC.

Part 1: Stationary Time Series

White noise errors:

Q

- Aim to eliminate autocorrelation in the residuals

  (could indicate that model does not reflect the lag structure well)

- Plot "standardized residuals" (ε_{it}/σ)   Structural breaks

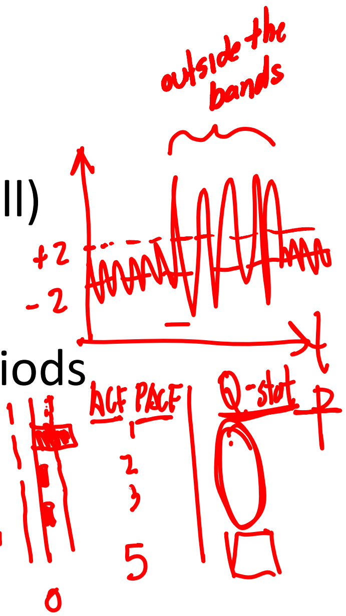
  No more than 5% of them should lie outside [-2,+2] over all periods

sample autocorrel

- Look at r_s, b_s (and significance) at different lags

estimation

- Box-Pierce Statistic: joint significance test up to lag s:



$$Q = T\sum_{k=1}^{s} r_k^2 \sim \chi_s^2$$

H$_0$: all $r_k = 0$, $k = 1,2,3,\ldots s$

H$_1$: at least one $r_k \neq 0$

Part 1: Stationary Time Series

(Anticipate M4)

Forecast ability:

- Can assess how well the model forecasts " out of sample":
  - Estimate the model for a sub-sample (for example, the first 250 out of 300 observations).
  - Use estimated parameters to forecast for the rest of the sample (last 50)
  - Compute the " forecast errors" and assess:
    - Mean Squared Prediction Error
    - Granger-Newbold Test
    - Diebold-Mariano Test

Part 1: Stationary Time Series

(Anticipate M4)

Account for possible structural breaks:

- Does the same model apply equally well to the entire sample, or do parameters change (significantly) within the sample?

How to approach:

- Own priors/suspicion: Chow test for parameter change
- If priors not strong, recursive estimation, tests for parameter stability over the sample, for example, CUSUM.

Properties of Time Series

Part 1: Stationary Times Series

L-6: Putting it all together—Simulated Data

Part 1: Stationary Time Series

Let's first work with simulated data

- Look at how MA(1), AR(1) series are simulated
  - in Excel
  - in EViews

Part 1: Stationary Time Series

Let's first work with simulated data

- View graph
- View ACF, PACF ➡ do they behave as expected?
- Decide on alternative specifications (one *correct*, one or more incorrect)
- Estimate and compare the results
- Use the EViews "Automatic ARIMA Modeling" feature

➡ Does the correct specification "win"?

Properties of Time Series

Part 1: Stationary Times Series

L-7: Putting it all together—Real world data

Part 1: Stationary Time Series

Now let's work with real world data

Price - Earnings
• mean - reverting
• stationary

File "M3_Series.wf1"

- View Sheet "PE_ratios" and choose a series:
  - Look at the graph and correlogram for a specific time series
  - Does it appear to be stationary?
- Again, choose two (or more) possible specifications
  - Estimate, compare results (coefficients, AIC/SBC, ACF of residuals)
  - Use "Automatic ARIMA Modeling"

white noise

➡ Which specification "wins"?

Properties of Time Series

Part 2: Nonstationary Times Series

L-8: Introduction

Part 2: Nonstationary Time Series

Introduction:

Key Questions:

- What is nonstationarity?
- Why is it important?
- How do we determine whether a time series is nonstationary?

Part 2: Nonstationary Time Series

What is nonstationarity?

Recall from Part 1:

- Covariance stationarity of *y* implies that, over time, *y* has:
  - Constant mean
  - Constant variance
  - Co-variance between different observations that do not depend on time (*t*), only on the "distance" or "lag" between them (*j*):

$$Cov(Y_t, Y_{t+j}) = Cov(Y_s, Y_{s+j}) = \gamma_j$$

Part 2: Nonstationary Time Series

What is nonstationarity?

- Thus, if any of these conditions does not hold, we say that *y* is nonstationary:
  - There is no long-run mean to which the series returns (economic concept of long-term *equilibrium*)
  - The variance is time-dependent. For example, could go to infinity as the number of observations goes to infinity
  - Theoretical autocorrelations do not decay, sample autocorrelations do so very slowly. If we ACF that decay slowly → non-stat

Part 2: Nonstationary Time Series

Nonstationary series can have a trend:

- Deterministic: nonrandom function of time:

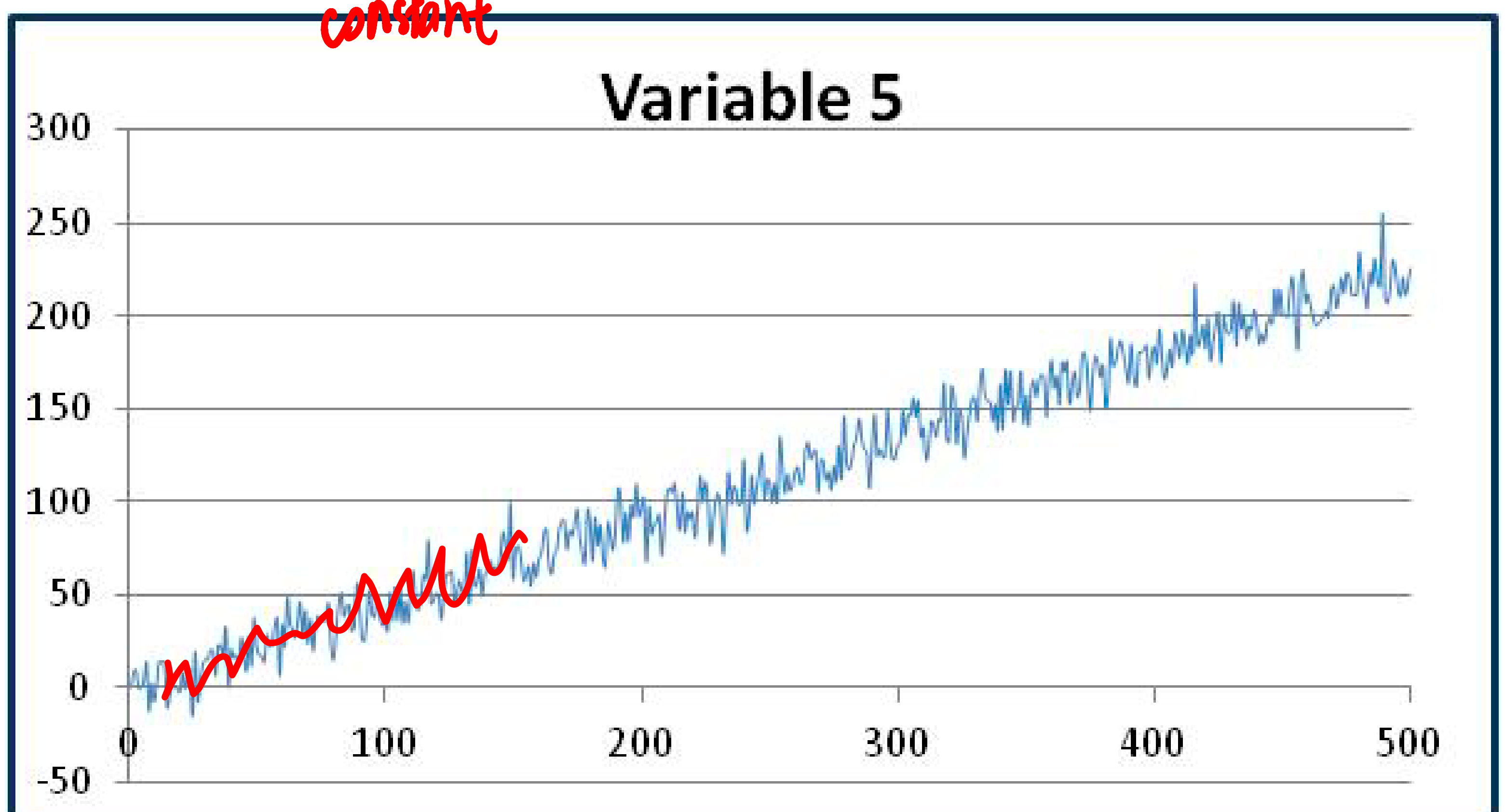
time

$$y_t = \mu + \beta t + u_t$$, where u_t is "iid"

constant

- Example:

  $\beta = 0.45$



Part 2: Nonstationary Time Series

Non-stationary series can have a trend:

- Stochastic: random trend, varies over time
  - Random Walk: $y_t = b y_{t-1} + u_t$
  - Random Walk with Drift: $y_t = \mu + b y_{t-1} + u_t$

  (as before, u_t is iid)

AR autoreg

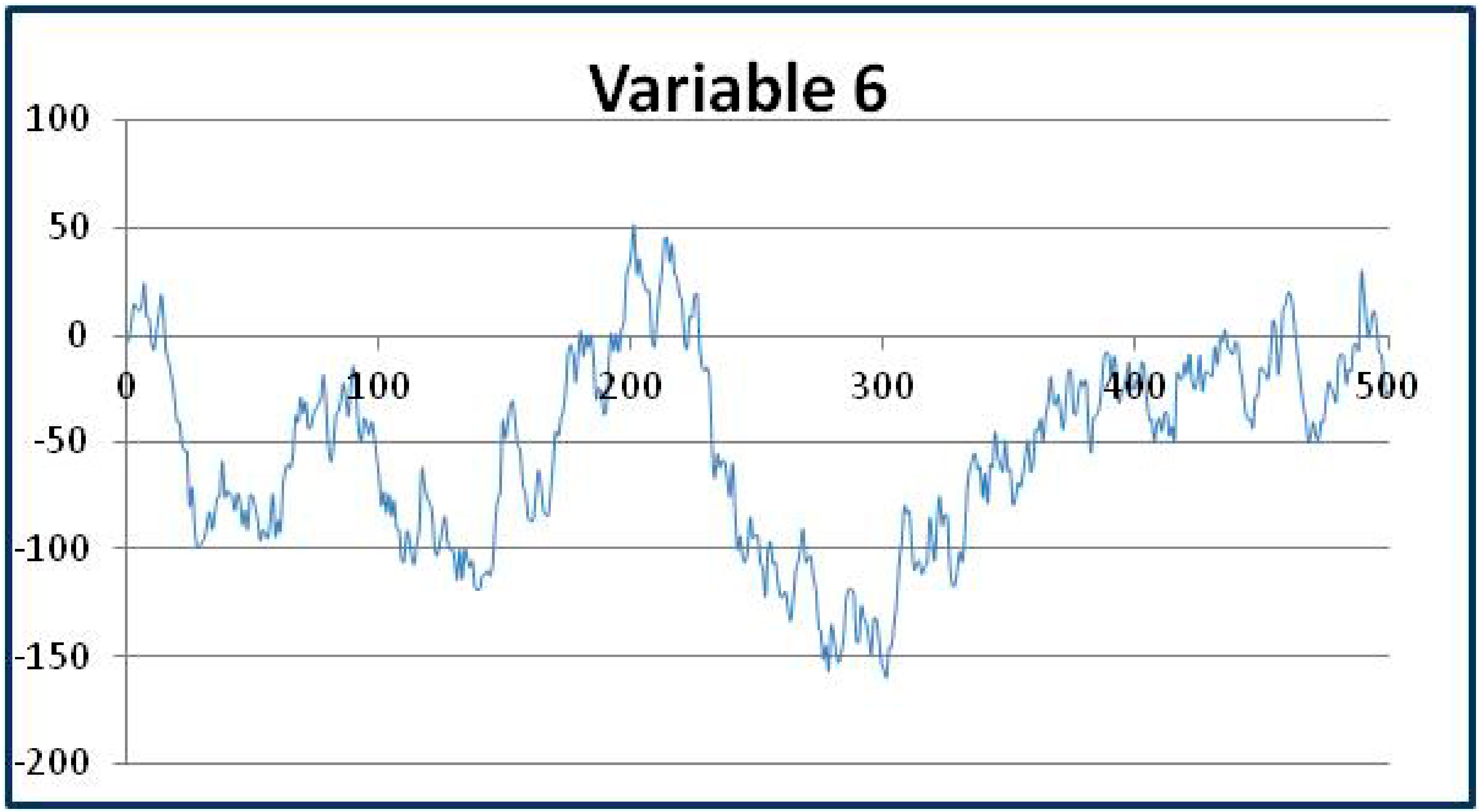
$b = 1$

- μ is the "Drift"; if μ>o, then y_t will be increasing

Question: RW is a special case of what process?

Part 2: Nonstationary Time Series

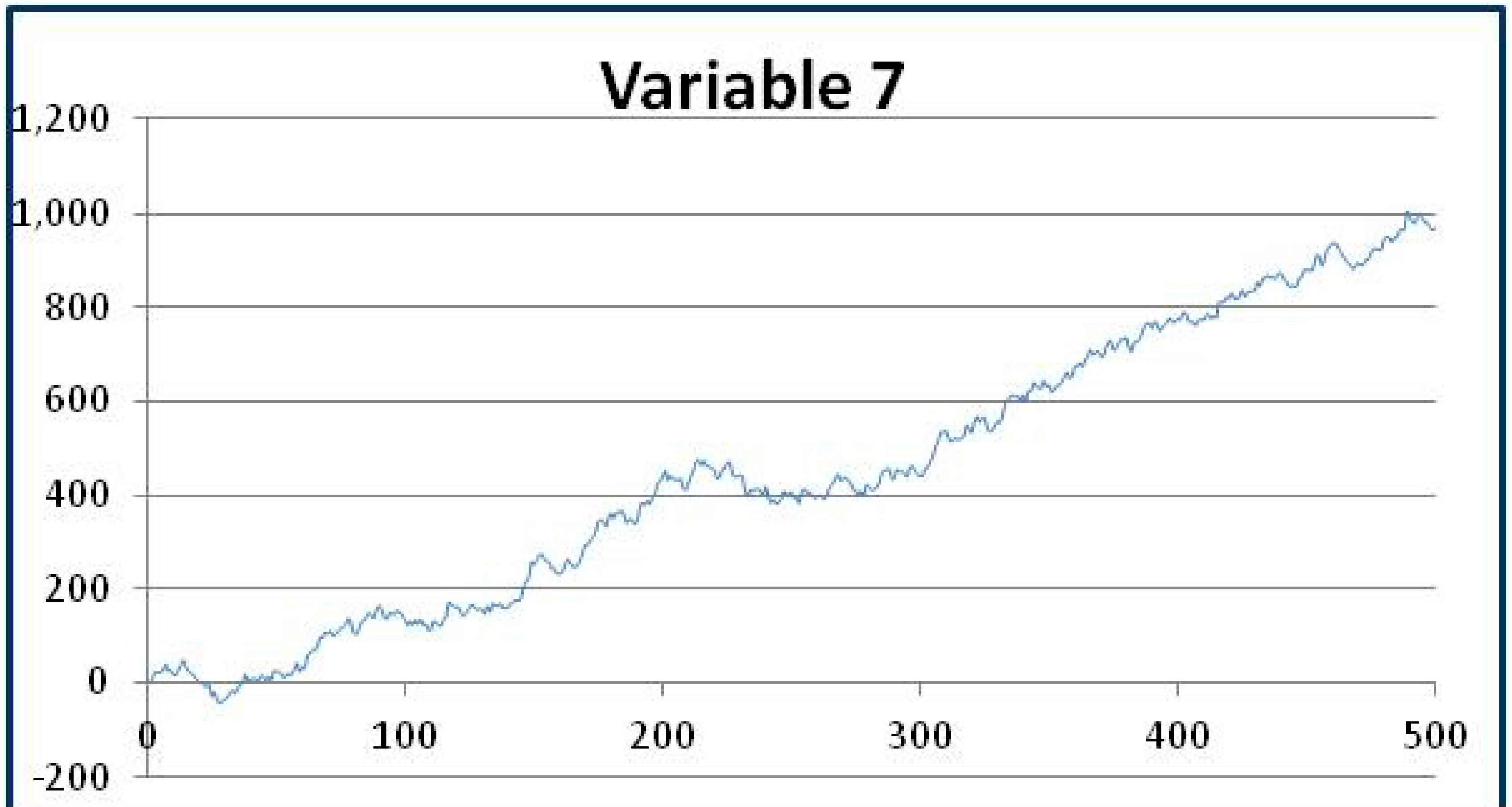
Example of a random walk:



Part 2: Nonstationary Time Series

Example of a random walk with drift:

$\mu = 2.0$



Note: simulated with the same disturbances as in the Random Walk in previous slide.

Properties of Time Series

Part 2: Nonstationary Times Series

L-9: Consequences

Part 2: Nonstationary Time Series

Key Questions:

- What is nonstationarity?
- Why is it important?
- How do we determine whether a time series is nonstationary?

Part 2: Nonstationary Time Series

Consequences of non-stationarity

- Shocks do not "die out"
- Statistical consequences
  - Non-normal distribution of test statistics
  - Bias in AR coefficients; poor forecast ability

Part 2: Nonstationary Time Series

Shocks do not die out

- Consider a general AR(1):

$$y_t = by_{t-1} + \varepsilon_t$$

- Can be expressed as an MA(q):

$$y_t = b^t y_0 + \varepsilon_t + b\varepsilon_{t-1} + b^2\varepsilon_{t-2} + \ldots b^{t-2}\varepsilon_2 + b^{t-1}\varepsilon_1$$

The impact of shocks (disturbances) will depend on value of *b*.

Part 2: Nonstationary Time Series

$$y_t = b^t y_0 + \varepsilon_t + b\varepsilon_{t-1} + b^2\varepsilon_{t-2} + \ldots b^{t-2}\varepsilon_2 + b^{t-1}\varepsilon_1$$

Three cases:

happen in R data

1. $b < 1$, $b^t \to 0$ as $t \to \infty$, so the effect of a shock will diminish as time elapses

shocks have equal weight

2. $b = 1$, $b^t = 1$ for all t; effect persists, $y_t = y_0 + \sum_{i=0}^{t-1} \varepsilon_{t-i}$ variance grows indefinitely with time
3. $b > 1$, shocks become *more influential* over time

Part 2: Nonstationary Time Series

Statistical consequences of nonstationarity

Non-normal distribution of test statistics

nonstationary

- Bias in autorregressive coefficients (*b's);* we might mistakenly estimate an AR(1), deficient forecast
- Usual confidence intervals for coefficients not valid

inference breaks down

Part 2: Nonstationary Time Series

Statistical consequences of non-stationarity for multivariate regressions (anticipating M6)

- For example, two *unrelated* nonstationary series *y* and *z* might appear to be related through a standard OLS regression
  - High R^2
  - t-statistics that appear to be significant
  - The true test: are the *regression residuals* stationary? (i.e., long-run equilibrium relationship between *y* and *z*?)

cointegration

Part 2: Nonstationary Time Series

Spurious regression practical exercise:

- Simulate two random walk series: *y* and *z*
  (each with its own disturbances, and either can have drift or not)
- Note that *by construction*, they are unrelated
- Run OLS regression of *y* on *z*, evaluate coefficients, R^2, and plot residuals.

Possibility of a "spurious" relationship

M6 (cointegration)

Properties of Time Series

Part 2: Nonstationary Times Series

L-10: Unit root tests

Part 2: Nonstationary Time Series

Key Questions:

- What is nonstationarity?
- Why is it important?
- How do we determine whether a time series is non-stationary?

Part 2: Nonstationary Time Series

Testing for non-stationarity

- Recall AR(1) model: $y_t = by_{t-1} + \varepsilon_t$
- Special case: RW, when *b* = 1
- Stationarity requires $|b| < 1$
- Generalizing to AR(p):
  - Roots of the polynomial below must all be >1 in abs value
    $1 - b_1 z - b_2 z^2 - b_3 z^3 - \ldots - b_3 z^p$
    nonstationary
  - If one of the roots = 1, then *y* is said to have a *unit root*.

Part 2: Nonstationary Time Series

Testing for non-stationarity

- AR(1) model: $y_t = b y_{t-1} + \varepsilon_t$
- Can test for whether *y* is a driftless random walk:
  - H_0: $b = 1$

  Or, equivalently: $\overset{y_t - y_{t-1}}{\Delta y_t} = \underset{\text{t-stat}}{\psi}\, y_{t-1} + \varepsilon_t \;,\; \overset{=0}{\psi} = b - 1$

  $H_0 \; \psi = 0$
- This is the “Dickey-Fuller” (DF) test:
  - Regress Δy on its lag, test for significance of coefficient.

Part 2: Nonstationary Time Series

Testing for non-stationarity

Can extend simple DF test in previous slide:

- Intercept: $$\Delta y_t = \mu + b y_{t-1} + \varepsilon_t$$
- Intercept and time trend: $$\Delta y_t = \mu + b y_{t-1} + \alpha t + \varepsilon_t$$
- In all three cases, H_0: $b = 0$; y has a unit root

rejection

Rejecting the unit root test = finding that y is stationary

Note: critical values for the t-statistics of *b* will vary depending on whether intercept, trend are included.

Part 2: Nonstationary Time Series

Some terminology

- *Order of integration*: number of times a series *y* must be differenced to become stationary
- Thus, if *y* is "integrated of order zero", I(0), then it is stationary (no differencing needed).
  - That is, it is stationary in *levels* (no differencing needed)
- If *y* is I(1), then its *first* difference (Δy) is stationary

…and so on… I(2) difference y twice $\Delta(\Delta y)$ stationary

Part 2: Nonstationary Time Series

Moving beyond white noise disturbances

DF test assumes that ε_t is white noise.

- However, if ε_t is autocorrelated, need different version of the test, allowing for higher-order lags:
- Augmented Dickey-Fuller (ADF) test:

unit root   autocorres coefficients

$$\Delta y_t = \mu + \alpha t + \overset{\approx 0}{\gamma} y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta y_{t-i+1} + \varepsilon_t , \quad \gamma = -\left(1 - \sum_{i=1}^{p} \underset{\to 1}{b_i}\right) \text{ and } \beta_i = -\sum_{j=1}^{p} b_j$$

Part 2: Nonstationary Time Series

ADF test

- As with DF, ADF tests whether coefficient on y_{t-1} (γ)$\neq 0$
- Must make choices
  - Intercept, trend, both, none?
  - p: how many lags? (test statistics are very sensitive to p)
    - AIC ✓
    - SBC ✓
    - General-to-specific (start out with large p, then re-estimate with successively smaller p)

EViews ADF → choose the "optimum" lag p → $\boxed{\text{AIC}}$

SBC more parsimonious

Properties of Time Series

Part 2: Nonstationary Times Series
L-11: Testing for nonstationarity, alternative tests

Part 2: Nonstationary Time Series

DF, ADF have been found to have low power in certain circumstances:

power to reject

- Stationary processes with near-unit roots
  - For example, difficulty distinguishing between $b = 1$ and $b = 0.95$, especially with small samples.

DF ADF fail to reject y $I(1)$ when it is truly $I(0)$

- Trend stationary processes

So alternative tests have been designed.

Part 2: Nonstationary Time Series

Phillips–Perron (PP) Test:

unit root test

more general

- Formulation: $\Delta y_t = \mu^* + \delta^* t + \psi y_{t-1} + u_t$, where u_t is I(0) and may be *heteroskedastic and autocorrelated*, that is, following an ARMA(p,q).

changing variance

- H_0: $\psi = 0$
- PP corrects for any serial correlation and heteroskedasticity in the errors u_t by directly modifying the test statistics.
- One advantage of PP: no need to specify lag length.

Part 2: Nonstationary Time Series

Kwiatkowski–Phillips–Schmidt–Shin (KPSS)Test:

- Null hypothesis: y is *trend stationary*
- Formulation:

int + time

white noise

var = 0 ?

$$y_t = \beta_0 D_t + \mu_t + u_t$$

$$\mu_t = \underline{\mu_{t-1}} + \varepsilon_t$$

mean zero

0

  - Where D_t contains deterministic components (constant or constant plus time trend), μ_t is a random walk
  - H_0: $\sigma_\varepsilon^2 = 0$ therefore μ is a constant, y is trend stationary.
  - H_1: $\sigma_\varepsilon^2 > 0$
  - KPSS critical values are obtained by simulation methods.

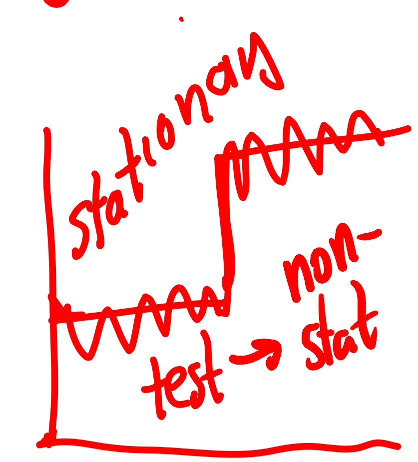
Part 2: Nonstationary Time Series

A few notes:

- DF, ADF, and PP are called “unit root tests”; the null hypothesis is that y_t has a unit root; is I(1) or higher.
- KPSS, on the other hand, is a “stationarity test”, null hypothesis is that y_t is I(0).

significane

- Correct specification is key: intercept and trend should be included when appropriate.
- Structural breaks can complicate matters further.



Part 2: Nonstationary Time Series

A unified way of looking at the unit root tests

Slightly different representation:

$$y_t = \mu + \alpha t + u_t$$
$$u_t = \rho u_{t-1} + \varepsilon_t$$

H$_0$: $\rho = 1$ *y* has a unit root

H$_1$: $|\rho| < 1$ *y* is stationary

- If ε_t is white noise, then DF can be used
- If ε_t is ARMA(p,q) then use ADF or PP.

In practice, this is what EViews does (test for ρ).

Properties of Time Series

Part 2: Nonstationary Times Series

L-12: Some exercises with simulated data

Part 2: Nonstationary Time Series

Simulate three processes in EViews

- Stationary process with near-unit roots
- Trend stationary process
- An I(1) process
- Graph them and observe their behavior
- Conduct Unit Root/Stationarity Tests on all three.

Part 2: Nonstationary Time Series

In "Simulated Times Series Examples.xlsx"

Simulate an I(0) process with a structural break

Import into EViews

- Graph and observe
- Conduct Unit Root/Stationarity Tests

Properties of Time Series

Part 2: Nonstationary Times Series

L-13: Some exercises with real-world data

Part 2: Nonstationary Time Series

Now let's work with real world data

- Choose a series:
  - Look at graph and correlogram for a specific time series
  - Does it appear to be non-stationary?
  - Does it appear to have a trend, or a structural break?
- Undertake Unit Root/Stationarity Tests
  - Do the different tests agree?
  - If you suspect a structural break, re-test for two sub-samples