NP-Complete graph problems - KombSøg 2017

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The graph problems and reductions below were introduced by Thomas Dueholm Hansen, at the lecture on NP-Complete graph problems may 3 2017.

1 Graph problems

1.1 INDEPENDENT SET

Given: Graph G = (V,E), target K.
Question: Does there exist $\subseteq$

1.2 CLIQUE

Given:
Question:

1.3 VERTEX COVER

Given:
Question:

1.4 MAX CUT

Given:
Question:

1.5 BISECTION

Given:
Question:

1.6 HAMILTONIAN PATH

Given:
Question:

1.7 TSP

Given:
Question:

2 Reductions

As always, in a reduction $L_1 \leq L_2$, we describe a polynomial time computable function r, such that $\forall x : x \in L_1 \iff r(x) \in L_2$. We then argue that it is indeed polynomial, and that both directions of the bi-implication holds.

2.1 3SAT $\leq$ INDEPENDENT SET

We prove **Theorem 9.4: INDEPENDENT SET is NP-Complete**, by reducing from 3SAT, which we know is NP-Complete.

For this reduction we need a gadget, the triangle. The logic behind this is that, if a graph contains a triangle, then at most one of the nodes can be in the independent set. We restrict the class of graphs we consider, to graphs whose nodes can be partitioned in m disjoint triangles.

Read more in papadimitriou page 188-190.

2.2 INDEPENDENT SET $\leq$ CLIQUE

2.3 INDEPENDENT SET $\leq$ VERTEX COVER

2.4 NAESAT $\leq$ MAX CUT

2.5 3SAT $\leq$ HAMILTONIAN PATH

2.6 HAMILTONIAN PATH $\leq$ TSP