

The errorfunction

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1 Definition

The error function (Gauss error function, erf) is defined as

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt, \quad (1)$$

and can be seen on figure 1, along with some of the tabulated values.

This function is generally used in probability and statistics to describe diffusion.

2 Usage

In statistics, the errorfunction is used as a probability for a single measurement. When some measurements can be described as a normal distribution with a standard deviation σ and expected value 0, $\operatorname{erf}\left(\frac{a}{\sigma\sqrt{2}}\right)$ is the probability that a single measurement lies between -a and +a. This is very useful in determining bit error rate of a digital communication system.

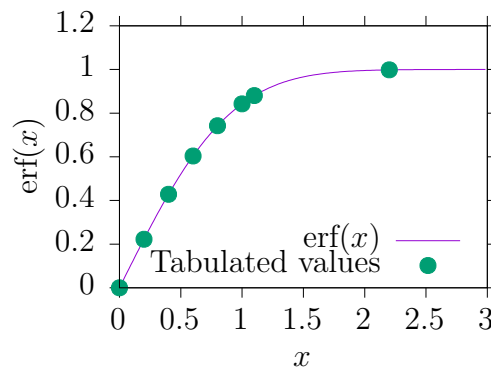


Figure 1: Plot of the error function with tabulated values

3 Properties

The errorfunction is an odd function, meaning that $\operatorname{erf}(-z) = -\operatorname{erf}(z)$.
The derivative of the errorfunction is

$$\frac{d}{dz}\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}}e^{-z^2}. \quad (2)$$