

Point Location in Voronoi Diagrams

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Abstract. Voronoi diagrams are used in a variety of contexts in mathematics, biology and computing. We are interested in analysing the composition of the voronoi diagram to say something about the underlying data that the diagram represents. In this paper we describe a solution that generates a voronoi diagram from an arbitrary set of data points and returns a data structure for performing point location in $O(\log n)$ time in the generated diagram. The solution makes use of an existing open-source implementation of Fortunes algorithm for generating voronoi diagrams, which is merged with our own implementation of a trapezoidal map for performing point location. Querying the trapezoidal map data structure will return the site of the voronoi cell in which the query point lies. As a concrete example of the usefulness of this, we discuss how the algorithm could be used for answering point location queries on a data set of fast-food restaurants. The idea is that one could seek to answer questions like which restaurants would cover or attract the most customers, or, in general, which restaurant is closest to a given point of interest. Secondly, we provide an analysis of the expected number of customers that will end in the biggest voronoi cell when distributed randomly across the fast-food restaurant diagram. We find that such information querying on the target data set, could be of potential interest when dealing with location planning of new restaurants, or analysing customer behavior.

Keywords: algorithms, computational geometry, point location, software development

1 Introduction

Voronoi diagrams have been known for... In YYYY Fortune invented the algorithm for ... They are interesting because this and that. Get examples from http://voronoi.com/wiki/index.php?title=Voronoi_Applications We use restaurants, defined as sites.

Various interesting questions could be asked about the diagram, that would also pose of interest in algorithm design. In which cell does a given point reside? Which cell is the largest? The questions are not just interest for the algorithmic challenge, but also due to how we can interpret the answer to such questions in relation to the sites that compose the diagram. E.g. we could assume that if a given

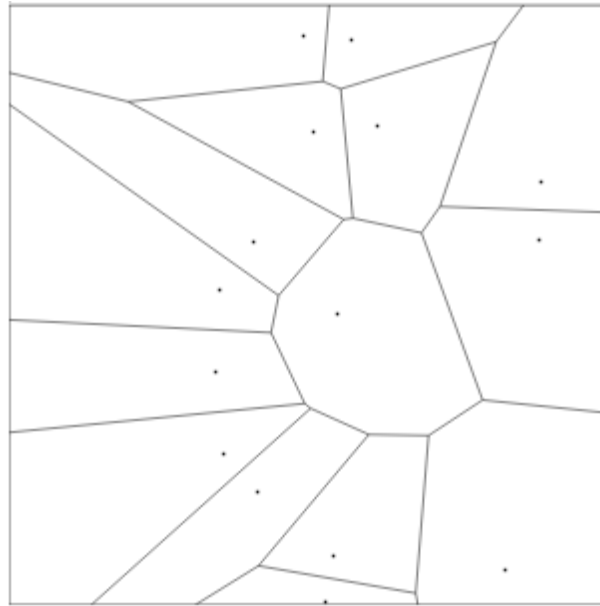


Fig. 1. A Voronoi Diagram with 15 sites

2 Background

3 Problem Analysis and Requirements

3.1 Problem Analysis

4 Solution

The solution is two fold and described in the following.

4.1 Point location in Voronoi

While there are existing algorithms for the two problems with good and optimal running times, an analysis of the two algorithms makes it clear that, for the scope of this project, it would not make much sense to look for an improvement or more elegant way of accomplishing what the two algorithms do. By realizing that the trapezoidal map simply requires a random edge at the time until all edges have been traversed, makes it somewhat trivial to see that the running output of the Voronoi algorithm can be used as input for the trapezoidal map algorithm. Seen in another way, while merging the two algorithms might be a learningful experience, it would in the general case make sense to just keep the two algorithms separated and run them in sequence, as the complexity and result is exactly the same. On the down side one could argue that merging them

is just a minus as it simply makes the implementation harder to understand. In fact, by merging them we do save some $n \log n$ computations, but unless we come up with some extreme case, we see this as a premature optimization. However, we find the task of combining the two of sufficient academic interest to have carried it through and provided an implementation. describe voronoi and how one needs to modify to the creation of segments upon circle events, maybe discuss why this is actually a final segment, refer to DCEL data structure and half edges not knowing their other ends. Describe the open source solution that we use, problems with it, and bridge it to our trapezoidal map implementation. Round off with nice features of our trapezoidal map implementation. And show the simplicity in getting the tree DS from it for querying.

4.2 Expected Customers

5 Evaluation

Test with a few screen shots showing that algo actually found the right cell.

6 Threats to Validity

Buggy software til generering af Voronoi.

7 Plan For Future Work

7.1 Future Work

Create a clean and generic voronoi implementation in .NET 4.x Find best point for planar location. Not trivial, would probably be Hawaii, but that would be winning a lot of oceanic area.

7.2 Something else?

8 Conclusion

We have shown the design of two algorithms in the field of computational geometry. We found that altering the the Fortune algorithm to return a fast search data structure of a trapezoidal map for point location was indeed feasible, and showed that by querying random points at the datastructure we could reason about which cell is the biggest. The algorithms presented are implemented in a generic fashion and it should be easy to see the solution used on top of another any other data set.

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References