

## Task RT2.3.1: Derive iterative formula

1. Directly expanding the base case, we quickly find a patterns that evaluates to the wanted expression:

$$\begin{aligned} \mathbf{c}_b &= (1 - \alpha_0)\mathbf{c}_0 + \alpha_0\mathbf{c}^1 \\ &= (1 - \alpha_0)\mathbf{c}_0 + \alpha_0((1 - \alpha_1)\mathbf{c}_1 + \alpha_1\mathbf{c}^2) \\ &= (1 - \alpha_0)\mathbf{c}_0 + (1 - \alpha_1)\alpha_0\mathbf{c}_1 + \alpha_1\alpha_0\mathbf{c}^2 \\ &= (1 - \alpha_0)\mathbf{c}_0 + (1 - \alpha_1)\alpha_0\mathbf{c}_1 + (1 - \alpha_2)\alpha_1\alpha_0\mathbf{c}_1 + \alpha_2\alpha_1\alpha_0\mathbf{c}^3 \\ &= \sum_{i=0}^{\infty} (1 - \alpha_i)(\alpha_0 \cdot \dots \cdot \alpha_{i-1})c_i \\ &= \sum_{i=0}^{\infty} (1 - \alpha_i) \left( \prod_{k=0}^{i-1} \alpha_k \right) c_i \end{aligned}$$

2. Starting from the case where we have  $N$  reflections, we get

$$\sum_{i=0}^N (1 - \alpha_i) \left( \prod_{k=0}^{i-1} \alpha_k \right) c_i$$

3. Following the formula, the code is built up by an outer loop over the total number of reflections we wish to adress. within this loop, each ray of light is checked for an intersection with an object in the scene, and the corresponding light intensity is calculated based on the mode of shading. These light intensities are multiplied to the term  $(1 - \text{mirror})$  as is describing the surface mirror constant. Summing over these equations for the amount of reflections, provide us with the desired result.