Task RT1.2.1: Derive the expression for a Ray-Cylinder intersection

The equation for a straight cylinder of infinite height is

$$(x - c_x)^2 + (y - c_y)^2 = r^2$$

where (x, y, z) are the coordinates of a point on the cylinder, (c_x, c_y, c_z) are the coordinates of the center of the cylinder, and r is the radius of the cylinder. We know that the ray equation is defined as $r(t) = o + td$
As this equation only holds for straight cylinders, we want to perform a rotation on our cylinder so that the a vector is directed toward the z axis. Hence, we compute the required rotation with the difference from a to z and treat the cylinder as a straight cylinder. We also rotate the direction of the ray accordingly. In the following computations, d is the trace rotated by the same angle as for a to become the z axis. To solve for the intersection, we substitute the equation of the ray into the equation of the cylinder and simplify:

$$\begin{cases} (x - c_x)^2 + (y - c_y)^2 = r^2 \\ ((o_x + td_x) - c_x)^2 + ((o_y + td_y) - c_y)^2 = r^2 \end{cases}$$

Expanding the squares and simplifying, we get:

$$(at^2 + bt + c) = 0$$

where:

$$\begin{cases} a = d_x^2 + d_y^2 \\ b = 2(o_x - c_x)d_x + 2(o_y - c_y)d_y \\ c = (o_x - c_x)^2 + (o_y - c_y)^2 - r^2 \end{cases}$$

Also taking into account the rotation, the coefficients become (**a** is for the axis, and a for the quadratic coefficient):

$$\begin{cases} a = d \cdot d \\ b = 2d \cdot (o - (d \cdot \mathbf{a})\mathbf{a}) \\ c = (o - (o \cdot \mathbf{a})\mathbf{a}) \cdot (o - (o \cdot \mathbf{a})\mathbf{a}) - r^2 \end{cases}$$

This is a quadratic equation in t, which can be solved using the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If we want to limit the intersection of the ray with the cylinder to a finite height h, we can simply add an additional constraint to our equation. We can add the condition that the z-coordinate of the intersection point must be between the z-coordinate of the starting point and the z-coordinate of the starting point plus h. That is,

$$\begin{cases} o_z + td_z \leq c_z + \frac{h}{2} \\ o_z + td_z \geq c_z - \frac{h}{2} \end{cases}$$

where (o_x, o_y, o_z) is the starting point of the ray, (d_x, d_y, d_z) is the direction of the ray, and (c_x, c_y, c_z) is the center of the cylinder.
We can rearrange these equations to solve for t:

$$\begin{cases} t \leq \frac{(c_z + \frac{h}{2} - o_z)}{d_z} \\ t \geq \frac{(c_z - \frac{h}{2} + o_z)}{d_z} \end{cases}$$

The intersection of the ray with the cylinder will occur only if both of these conditions are satisfied. We can take the smaller of the two possible values of $tthatsatisfytheseconstraints.$